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Abstract

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Reference


DOI : 10.1016/0370-2693(92)91189-G
LIGHT VECTOR RESONANCES IN THE EFFECTIVE CHIRAL LAGRANGIAN FOR HEAVY MESONS*

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UGVA-DPT 1992/07-780
BARI-TH/92-116
July 1992

* Partially supported by the Swiss National Foundation
ABSTRACT

We modify a chiral effective lagrangian recently suggested to describe interactions of the light pseudoscalars with mesons containing a heavy quark, so as to incorporate light vector resonances, such as \( \rho \), etc. The modification uses the hidden gauge symmetry approach. As a preliminary example we present an application to the semileptonic \( D \to K^* \) decay.
1 Introduction

Much work has been recently devoted to the formulation of an effective heavy quark theory and to the study of its predictions [1], [2], [3], [4], [5]. Recently Wise [6] has proposed an effective lagrangian to describe at low momentum the interactions of a meson containing a heavy quark with π, K, η (see also [7], [8]). The effective lagrangian possesses the heavy-quark SU(2N) spin-flavor symmetry (for N heavy quarks) and a non-linearly realized SU(3)L ⊗ SU(3)R chiral symmetry in the light sector, corresponding to spontaneous symmetry breaking of the chiral group to the diagonal SU(3)\text{V}.

The aim of the present paper is to present an extension of such a description to accommodate inside the scheme also the "light" vector resonances as ρ, K*, etc. The effective lagrangian incorporating the vector resonances is given below (see eq. (4.3)). We will then apply the formalism, in a preliminary way, to the decay D → K*ℓν.

To introduce vector resonances we will follow the standard hidden gauge symmetry technique already used in previous cases [3], [10], [11].

A further extension of the method allowing for the introduction of axial-vector resonances has been described in [10] and in [12]. For the applications described in this paper we will consider only the case of the octet of vector resonances, and we will not discuss the also known enlargement to U(3)L ⊗ U(3)R, that would be necessary to study the nonet, besides the extension to the axial-vector case.

2 Non linearly realized chiral symmetry

Let us recall the basic elements leading to the effective lagrangian of Wise [6].

The description of a light pseudoscalar meson can be encoded in the 3 × 3 unitary matrix

\[
\Sigma = \exp \frac{2iM}{f}
\]

where \( f \approx 132 \text{ MeV} \) is the pseudoscalar pion decay constant and \( M \) is the matrix

\[
M = \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & \pi^+ & K^+\\
\pi^- & -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^0 \\
K^- & K^0 & -\sqrt{\frac{1}{2}} \eta
\end{pmatrix}
\]

At the lowest order in the derivatives, and in the massless limit, the chiral lagrangian is

\[
\mathcal{L}_{\text{chiral}} = \frac{f^2}{8} Tr \left[ \partial^\mu \Sigma \partial_\mu \Sigma^\dagger \right]
\]

showing the invariance under the transformations of \( SU(3)_L \otimes SU(3)_R \):

\[
\Sigma \rightarrow g_L \Sigma g_R^\dagger
\]

To describe interactions with other fields it turns out convenient to go to the CCWZ realization [13]:

\[
\Sigma = \xi^2
\]
with $\xi$ transforming in the following way

$$
\xi \rightarrow g_L U^\dagger \xi = U \xi g_R^\dagger
$$

(2.6)

where $U \in SU(3)$ is a non-linear function of $g_L, g_R$ and $\xi$.

The effective description of the ground state of the system $Q\bar{q}^a$ ($Q$ is $c$ or $b$ and $q^1 = u, q^2 = d, q^3 = s$) uses the $4 \times 4$ matrix $H_a$ [14]. [6] ($a = 1, 2, 3$)

$$
H_a = \frac{(1 + \not{v})}{2} [P_{a\mu}^* \gamma^\mu - P_a \gamma_5]
$$

(2.7)

which transforms under $SU(3)_L \otimes SU(3)_R$ as

$$
H_a \rightarrow H_b U^\dagger_{ba}(x)
$$

(2.8)

We recall that in (2.7) $P_{a\mu}^*$ and $P_a$ annihilate respectively a spin-1 and spin zero meson $Q\bar{q}_a$ of velocity $v_\mu$ ($v^\mu P_{a\mu}^* = 0$). The heavy quark spin symmetry $SU(2)_v$ acts as

$$
H_a \rightarrow SH_a
$$

(2.9)

for $S \in SU(2)_v$, satisfying $[\not{v}, S] = 0$, and under Lorentz transformations

$$
H_a \rightarrow D(\Lambda) H_a D(\Lambda)^{-1}
$$

(2.10)

Let us also recall that one defines

$$
\bar{H}_a = \gamma_0 H_a^\dagger \gamma_0
$$

(2.11)

### 3 Description through a linear gauge symmetry

Eq. (2.6) implies in particular, that the transformation $U$ depends on the space-time. It is then possible to get a simpler, linear realization, by using the hidden gauge symmetry approach. This consists in using two new $SU(3)$-matrix valued fields $L$ and $R$ to build up $\Sigma$

$$
\Sigma = LR^\dagger
$$

(3.1)

The chiral lagrangian in eq. (2.3) is then invariant under the group $SU(3)_L \otimes SU(3)_R \otimes SU(3)_H$

$$
L \rightarrow g_L h^\dagger L(x), \quad R \rightarrow g_R h^\dagger R(x)
$$

(3.2)

where $h \in SU(3)_H$ is a local gauge transformation. The local symmetry associated to the group $SU(3)_H$ is called the hidden gauge symmetry because the field $\Sigma$ belongs to the singlet representation. It should be noticed that this description is equivalent to the previous one by the gauge choice $L = R^\dagger$, which is always possible to make (at least locally). From the fields $L$ and $R$ we can construct two currents

$$
\mathcal{V}_\mu = \frac{1}{2} \left( L^\dagger \partial_\mu L + R^\dagger \partial_\mu R \right)
$$

(3.3)

$$
\mathcal{A}_\mu = \frac{1}{2} \left( L^\dagger \partial_\mu L - R^\dagger \partial_\mu R \right)
$$

(3.4)
which are singlets under $SU(3)_L \otimes SU(3)_R$ and transform as

$$\mathcal{V}_\mu \rightarrow h\mathcal{V}_\mu h^\dagger + h\partial_\mu h^\dagger \quad (3.5)$$

$$\mathcal{A}_\mu \rightarrow h\mathcal{A}_\mu h^\dagger \quad (3.6)$$

under the local group $SU(3)_H$.

In this notation, the transformation for $H_a$ reads

$$H_a \rightarrow H_b h_{ba}^\dagger(x) \quad (3.7)$$

and the covariant derivative is defined as

$$D_\mu \bar{H} = (\partial_\mu + \mathcal{V}_\mu) \bar{H} \quad (3.8)$$

4 Inclusion of vector mesons

The octet of vector resonances ($\rho$, etc.) are introduced as the gauge particles associated to the group $SU(3)_H$. We put

$$\rho_\mu = i \frac{g_V}{\sqrt{2}} \bar{\hat{\rho}}_\mu \quad (4.1)$$

where $\bar{\hat{\rho}}$ is a hermitian $3 \times 3$ matrices analogous to the one defined in equation (2.2). This field transforms under the full symmetry group as $\mathcal{V}_\mu$

$$\rho_\mu \rightarrow h\rho_\mu h^\dagger + h\partial_\mu h^\dagger \quad (4.2)$$

The vector particles acquire a common mass through the breaking of $SU(3)_L \otimes SU(3)_R \otimes SU(3)_H$ to $SU(3)_V$. In fact, 8 out of the 16 Goldstone bosons coming from the breaking are the light pseudoscalar mesons, whereas the other 8 are eaten up by the $\rho$ field.

From all the preceding transformation laws one is lead to the following simple lagrangian which incorporates the Wise lagrangian and the vector-octet interactions with the heavy mesons and the pseudoscalars

$$\mathcal{L} = \mathcal{L}^{\text{light}} + iTr[H_a v_\mu \partial^\mu \bar{H}_a] + iTr[H_b v^\mu (\mathcal{V}_\mu)_{ba} \bar{H}_a] + igTr[H_b \gamma_\mu \gamma_5 (A_\mu)_{ba} \bar{H}_a] + i\beta Tr[H_b v^\mu (\mathcal{V}_\mu - \rho_\mu)_{ba} \bar{H}_a] + \frac{\beta^2}{2f^2a} Tr[H_b H_a H_a H_b] + \lambda_0 Tr[m_q \Sigma + m_q \Sigma^\dagger] + \lambda_1 Tr[H_a (R^\dagger m_q L + L^\dagger m_q R)_{ab} \bar{H}_b] + \lambda_1' Tr[H_a \bar{H}_a (m_q \Sigma + \Sigma^\dagger m_q)_{ab} + \cdots] \quad (4.3)$$

where the ellipsis denotes terms with additional derivatives as well as higher order mass corrections ($1/m_Q$, etc.), and $\mathcal{L}^{\text{light}}$ is

$$\mathcal{L}^{\text{light}} = -\frac{f^2}{2} \left\{ tr[A_\mu A^\mu] + atr[(\mathcal{V}_\mu - \rho_\mu)^2] \right\} + \frac{1}{2g_V^2} tr[F_{\mu\nu}(\rho)F^{\mu\nu}(\rho)] \quad (4.4)$$
In eqs. (4.3) and (4.4) \( g \) (the same one occurring in the Wise lagrangian), \( \beta, a \) are constants (\( f \) and \( g_V \) had been defined above),

\[
F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu]
\]

(4.5)

the trace operations are \( Tr \) on spinor indices and \( tr \) on group indices. In (4.3) a sum on the velocities \( v_\mu \) is understood.

The quartic term in the heavy meson fields in (4.3) follows from the requirement that the lagrangian goes back to the Wise lagrangian in the formal limit \( g_V \to \infty \) (\( m_\rho \to \infty \)) in which the \( \rho \) field decouples.

The lagrangian \( \mathcal{L}^{\text{light}} \), eq. (4.4), describes light pseudoscalars and vector resonances. It reproduces in this sector all the good results of vector dominance [11]. Of the three terms in eq. (4.4), the first one reproduces the chiral lagrangian of eq. (2.3), as it can be seen by using the identity

\[
Tr[\mathcal{A}_\mu \mathcal{A}^\mu] = -\frac{1}{4} Tr[\partial_\mu \Sigma \partial^\mu \Sigma^\dag]
\]

(4.6)

By coupling to the electromagnetic field one gets that the first KSRF relation [13] is automatically satisfied, i.e. \( g_\rho = g_{\rho\pi\pi} f^2 \), where \( g_\rho \) is the \( \rho - \gamma \) mixing. Furthermore by the choice \( a = 2 \) it is also possible to satisfy the second KSRF relation, \( m_\rho^2 = g_{\rho\pi\pi}^2 f^2 \).

Using (4.3) we get

\[
m_\rho^2 = \frac{1}{2} ag_V^2 f^2
\]

(4.7)

and for \( a = 2 \)

\[
g_V = \frac{m_\rho}{f} \approx 5.8
\]

(4.8)

Finally, the light-quark mass-terms in eq. (4.3) are the same as defined by Wise.

Eq. (4.3) is the lagrangian we propose as an attempt to improve on the chiral Wise lagrangian by taking into account also light vector mesons effects. The extension to include axial-vector resonances can easily be made following references [10] and [12]. Also, one can easily enlarge to \( U(3)_L \otimes U(3)_R \) [10], necessary to study the full resonance nonets. As an example, we will present in the next section a preliminary, and as we will see probably incomplete, discussion of the semileptonic decay \( D \to K^* \ell \nu \), which does not require at this stage the extensions we have mentioned.

5 Application to the semileptonic decay \( D \to K^* \)

We will apply the previous formalism to the calculation of semileptonic decays of \( B \) and \( D \) mesons to “light” vector states as \( \rho \) and \( K^* \). We will consider the explicit calculation of \( D^+ \to \bar{K}^{0}\alpha \ell \nu \).

First of all we need to construct an effective current between heavy mesons and light vectors transforming as \((3_L, 1_R)\) under \( SU(3)_L \otimes SU(3)_R \) (remember that the quark current is \( J^\mu_a = \bar{q}_a \gamma^\mu (1 - \gamma_5) Q \)). The lowest dimension relevant operator is

\[
L^\mu_a = i \frac{\alpha}{2} Tr[\gamma^\mu (1 - \gamma_5) H_b \xi^\dag_{ba}] + \alpha_3 Tr[\gamma_5 H_6 (\rho^\mu - \mathcal{V}^\mu)_{bc} \xi^\dag_{ca}] + \cdots
\]

(5.1)
where the ellipsis denotes terms vanishing in the limit $m_q \to 0$, $m_Q \to \infty$ or terms with derivatives. The constant $\alpha$ can be obtained by considering the matrix element of $L^\mu$ between the meson state and the vacuum, with the result

$$
\langle 0|\bar{q}\gamma^\mu\gamma_5 Q|P\rangle = if_pm_p\nu^\mu
$$

(5.2)

In the limit $m_Q \to \infty$

$$
\alpha = f_P\sqrt{m_p}
$$

(5.3)

and $\alpha$ has a calculable logarithmic dependence on the heavy quark masses.

For the following we will need also the parametrization of the matrix element of the quark current in terms of form factors

$$
\langle \bar{K}_0^*(p',\epsilon)|J_\mu|D^+(p)\rangle = \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^{*\rho}p'^{*\sigma} \frac{2V(q^2)}{(MD + MK^*)} + i\epsilon^{*}_\mu (MD + MK^*)A_1(q^2)
$$

$$
- i(\epsilon^* \cdot q) \left[(p + p')_\mu \frac{A_2(q^2)}{(MD + MK^*)} + 2\frac{MK^*}{q^2}q_\mu A(q^2)\right]
$$

(5.4)

where $q = p - p'$ and $A(0) = 0$. Using the effective lagrangian given in eq. (4.3) we find two contributions to the calculation of the matrix element of the current. The first one (see Fig. 1) is just the matrix element of $L^\mu$ between meson states, and it gives

$$
- 2i\frac{gV}{\sqrt{2}}\alpha_1\sqrt{MD}\epsilon^*_\mu
$$

(5.5)

Then we have the contribution from the exchange of $D^+_s$ (see Fig. 2). The trilinear coupling between $D_s$, $D$ and $K^*$ can be easily derived from the last term in the lagrangian (4.3). We find

$$
- i\beta\frac{gV}{\sqrt{2}}\frac{f_D(\epsilon^* \cdot q)}{MD(v \cdot p' + \Delta')}q_\mu
$$

(5.6)

where $\Delta' = MD_s - MD$, $v_\mu$ is the four-velocity of $D_s$, and $f_D = (\alpha/\sqrt{MD})$ is the decay coupling constant of the meson $D$ (see [3]). The mass splittings among the heavy mesons are treated by adding convenient terms to the lagrangian (4.3) (a color magnetic moment operator). This is discussed in ref. [3] together with the modifications induced on the heavy quark propagators.

We see that the lagrangian (4.3) gives contributions only to the form factors $A_1(q^2)$ and $A(q^2)$. However there are other possible pole terms that could contribute. For instance the ones coming from the p-wave meson excitations. The necessary formalism to deal with these states has been discussed by Falk [16], but, at least in this paper, we will not consider these contributions, our main motivation being to illustrate the formalism, rather than to perform a complete calculation. However, there is another possible contribution that we can take into account, that is the pole contribution from a $D^*_s$ meson as illustrated in Fig. 3. The lagrangian (4.3) does not contribute to the trilinear coupling among $D$, $D^*_s$ and $K^*$, so we need to consider a higher dimensional invariant operator to add to it

$$
i\lambda Tr[H_a\sigma_{\mu\nu}F_{\mu\nu}^a(p)\bar{H}_b]
$$

(5.7)

The contribution from the diagram of Fig. 3 is then

$$
- \frac{gV}{\sqrt{2}v \cdot p' + \Delta} \epsilon_{\mu\nu\rho\sigma}^{*\nu}p^{*\rho}p'^{*\sigma}
$$

(5.8)
with \( \Delta = M_{D^*} - M_D \).

Summarizing, we get the following expressions for the form factors defined in eq. (5.4) at \( q^2_{\text{max}} \)

\[
V(q^2_{\text{max}}) = -\frac{g_V \lambda f_D (M_D + M_{K^*})}{\sqrt{2} \left(M_{K^*} + \Delta\right)}
\]

\[
A_1(q^2_{\text{max}}) = -\frac{g_V \beta f_D}{\sqrt{2} (M_D + M_{K^*})} (M_D - M_{K^*})^2
\]

\[
A_2(q^2_{\text{max}}) = 0
\]

\[
A(q^2_{\text{max}}) = \frac{g_V \beta f_D}{\sqrt{2} M_{K^*} (\Delta') \left(2M_D + M_{K^*} + \Delta\right)}
\]

(5.9)

As we have discussed previously, the reason why we find \( A_1 \) independent of \( q^2 \) and \( A_2 = 0 \), is that we are neglecting the p-wave contributions. However these states do not contribute to the form factor \( V(q^2) \).

We expect our calculation to be reliable only around the region of maximum recoil \( q^2 = (M_D - M_{K^*})^2 \), so in order to compare with experimental data and with other models we need to extrapolate our form factor down to \( q^2 = 0 \). For this, we assume the pole dominance of \( V \) from the \( D^*_s \) exchange

\[
V(q^2) = V(0) \frac{M_{D^*_s}^2}{M_{D^*_s}^2 - q^2}
\]

(5.10)

The expression we get for \( V(0) \) is

\[
V(0) = -\frac{g_V \lambda f_D (M_D + M_{K^*}) (2M_D - M_{K^*} + \Delta)}{\sqrt{2} M_{D^*_s}^2}
\]

(5.11)

An evaluation for \( \lambda \) can be obtained using the recent data from the E653 Collaboration [17]

\[
V(0) = 0.99 \pm 0.27
\]

(5.12)

We get (using \( f_D \approx 200 \text{ MeV} \) as obtained from QCD sum rules [18])

\[
\lambda = -(0.63 \pm 0.17) \text{ GeV}^{-1}
\]

(5.13)

This number turns out to be of the right order of magnitude, as we expect that the momentum expansion is regulated by the chiral symmetry breaking scale, which is believed to be around 1 GeV.

Unfortunately, from this process, it is impossible to draw any conclusion about the \( \beta \) parameter, because the form factor \( A(q^2) \) is practically not accessible to the experiments. We may have a further check on our approach by studying the form factor \( V \) for the companion semileptonic processes \( D \to \rho \) and \( B \to \rho \). We can easily evaluate these processes following the same steps as for \( D \to K^* \), with obvious changes (for example \( D^*_s \to D^* \) and \( D^*_s \to B^* \) respectively, \( M_{K^*} \to M_{\rho} \), and \( \Delta = 46 \text{ MeV} \) for \( B \to \rho \)).

However, due to the lack of experimental data about these reactions we can only compare our results with the ones obtained in other calculations. A comparison is shown in the following table, for \( V(q^2_{\text{max}}) \equiv V^m \) in the various processes assuming as normalization \( D \to K^* \), and using the scaling relation \( f_B/f_D = \sqrt{(m_D/m_B)} \).
Note however that the above scaling law from the meson decay constants is most probably afflicted by order $\mathcal{O}(1/m_Q)$ corrections, presumably still big at the charm mass. This appears from studies of QCD sum rules [18], and lattice QCD calculations [22].

In fact, besides the non inclusion of the p-waves excitations in the calculations, that we have already mentioned, the lack of a discussion of the non-leading contributions in the derivative expansion and of neglected $\mathcal{O}(1/m_Q)$ corrections is one of the weakest points of the present calculation, which we have presented mostly as a concrete example of application of the effective lagrangian (4.3). The fact that the $K^*$ mass is not negligibly small as compared to the $D$ mass leads additional work as inevitable before coming to final conclusions.

6 Conclusions

The lagrangian in eq. (4.3) is proposed as a convenient way to incorporate light vector resonances, such as $\rho$, $K^*$, etc., into the effective lagrangian for mesons containing a heavy quark. We have applied such a lagrangian to the $D \rightarrow K^*$ semileptonic decay, to see how the formalism may work in a particular case but leaving apart the discussion of non-dominant effects.

ACKNOWLEDGMENT: We would like to thank S. De Curtis and D. Dominici for many fruitful and enlightening discussions during the early stage of this work.
References


[14] see H.Georgi in ref. [1]


Figure Captions

Fig. 1 Feynman diagram giving rise to the contribution (5.5). The shaded square denotes the current of eq. (5.1).

Fig. 2 Feynman diagram giving rise to the contribution (5.6). The shaded square denotes the current of eq. (5.1).

Fig. 3 Feynman diagram giving rise to the contribution (5.8). The shaded square denotes the current of eq. (5.1).