The pseudo-Goldstone mass spectrum

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Abstract

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Reference


DOI : 10.1016/0370-2693(92)91307-U
The pseudo-Goldstone mass spectrum

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Received 15 April 1992

In schemes of strong electroweak symmetry breaking, independently of the details of the underlying fundamental theory, we develop a framework for the quantitative evaluation of the pseudo-Goldstone masses, including, beside the gauge contributions, also the contributions from those interactions which are responsible for the masses of the ordinary fermions. The effective low-energy Yukawa interactions among pseudo-Goldstone bosons and ordinary fermions appears to play an essential role. All those states which, neglecting such Yukawa terms, would remain massless, tend to acquire mass and we find that these masses, barring special cancellations, are close to those of the heaviest fermions of the theory, the top and the bottom quarks.

1. Introduction

Theories of the electroweak symmetry breaking (SB) which avoid the introduction of fundamental scalars are usually characterized by a large global invariance under a chiral symmetry group G. The dynamical mechanism which is responsible for the electroweak SB triggers, at the same time, a spontaneous breaking of G into a subgroup H. To each generator of G/H then corresponds a Goldstone boson, which is exactly massless as long as one neglects additional interactions. Such a situation is, however, not realistic and, typically, gauge interactions explicitly breaking the symmetry group G are present from the beginning. For instance (but it cannot be the whole story) the standard model gauge interactions, related to the local group $G_W = SU(3) \times SU(2) \times U(1)_Y$, will in general break the chiral symmetry associated to G. This fact will force an interaction which will break the degeneracy among the previously equivalent vacua $|\Omega\rangle$. In the absence of such an induced orientation, all the states $|\Omega\rangle$ would have the same energy and the small oscillations along the directions connecting the different vacua would correspond to strictly massless Goldstone bosons. Due to the selection of a particular orientation some of the previous oscillations will now correspond to massive modes [1], or, as it is commonly referred to, to pseudo-Goldstone bosons.

Because of the possible lightness of some of these states, their presence is at the same time desirable and embarrassing. Desirable, for the possibility they offer to be discovered soon at the available or planned facilities, embarrassing for the potential conflict they could cause with the amount of data already accumulated. It is therefore of primary importance to try to give a description as accurate as possible of the properties of these particles.

From the previous discussion it follows that basic to the prediction of the mass spectrum of the pseudo-Goldstones is the precise knowledge of the possible interactions which explicitly break the chiral symmetry G. Among those, the gauge interactions associated to $G_W$ were first studied and their effects on the pseudo-Goldstone boson mass spectrum of simple technicolor models were quantitatively assessed [2,3].

It is however clear that other interactions explicitly breaking G must also be present and taken into account. We are here referring in particular to the...
mechanism which is responsible for the generation of the masses of the ordinary fermions.

If, as an illustration of these ideas, we think of an extended technicolor scheme, the gauge interactions associated to the generators connecting ordinary fermions to technifermions will in general break the chiral symmetry $G$, which in this model is related to the technifermion sector. Since the interactions considered are those responsible for the generation of the fermion masses, it is natural to expect that the induced pseudo-Goldstone masses are somehow related to the fermionic mass spectrum.

Aim of this note is to provide a framework for the explicit and quantitative evaluation of the contribution to the pseudo-Goldstone mass spectrum, coming from the interactions which provide masses to the ordinary fermions. In doing this we will avoid the detailed introduction of a specific fundamental theory. Instead, we will work with the low energy effective theory, essentially characterized by the groups $G$, $H$ and $G_w$.

Our strategy will consist in writing the allowed effective Yukawa couplings between the ordinary fermions and the pseudo-Goldstones. In the low-energy theory, such effective Yukawa couplings are supposed to carry the relevant information independently of the fundamental mechanism which gives origin to the observed fermion spectrum.

Allowed couplings are those which are invariant under the gauge symmetry $G_w$, surviving in the low-energy sector. According to the general prescription of ref. [4], such couplings are specific functions of the fermion fields and of the pseudo-Goldstone bosons, which transform nonlinearly both under $G$ and $G_w$. The expansion of these couplings contains, at lowest order, the mass terms for the ordinary fermions. This allows us to obtain a set of relations among the Yukawa couplings and the fermion masses.

In general, the number of independent Yukawa couplings exceeds the number of masses, so that some free parameters are still present in the analysis. Nevertheless, as explicitly illustrated in the following, this fact does not prevent the possibility of making certain quantitative statements about the pseudo-Goldstone masses.

Once the Yukawa couplings are given, the natural tool to analyse the problem of vacuum orientation is the effective potential. We will evaluate the one-loop effective potential including the effects of both ordinary gauge interactions and Yukawa couplings. Finally, from the one-loop effective potential we will derive the pseudo-Goldstones mass spectrum.

For illustrative purposes here we will discuss two commonly used choices for $G$ and $H$: case A with $G=SU(4)\times SU(4)$, $H=SU(4)$, and case B with $G=SU(8)\times SU(8)$, $H=SU(8)$. These special possibilities correspond to the cases discussed in ref. [2].

When fermion masses are taken into account, the chiral symmetry $G$ is so completely broken that, in general, no massless Goldstone boson survives in the theory. Pseudo-Goldstone bosons remaining massless when only the ordinary gauge interactions $G_w$ are considered, receive in general masses. Those masses have a natural range situated around the heaviest among the ordinary fermions, namely $m_{\text{loop}}$ and $m_{\text{button}}$. Special choices of the parameters characterizing the fermion masses are nevertheless possible so as to make some of these pseudo-Goldstones light, or even worse, to destabilize the theory.

In the next section we illustrate our formalism in the example A. We then consider the case B. Finally we present our conclusions.

2. Case A: $G=SU(4)\times SU(4)$, $H=SU(4)$

In this section we consider a toy model, just to illustrate in a simple case how the mechanism of generation of pseudo-Goldstone masses works. This model has a chiral symmetry $SU(4)\times SU(4)$ spontaneously broken to the diagonal $SU(4)$ subgroup.

Although irrelevant to our effective low-energy formalism, it is suggestive to think of technicolor interactions, associated to some technicolor group $G_{\text{TC}}$ with technifermions given by two doublets of techniquarks: $(U, D)$, $(C, S)$. These doublets have the same quantum numbers as the corresponding ordinary quarks, with respect to $SU(2)_L\times U(1)_Y$. This realizes the case $d=0$ of ref. [2]. Such techniquarks are colorless with respect to the ordinary color interactions, so that, in the limit of massless techniquarks, the chiral symmetry is $SU(4)\times SU(4)$ (the singlet axial current is expected to be afflicted by anomaly).

The spontaneous breaking of $G=SU(4)\times SU(4)$ down to the diagonal subgroup $H=SU(4)$ produces 15 Goldstone bosons, associated to the generators of
G/H. For reference a list of these generators is given in table 1. The Goldstone bosons \(\pi^a (a = 1, 2, 3)\), associated to the generators \(T^a\), are adsorbed by the electroweak gauge bosons \(W^\pm\) and \(Z\). The remaining 12 bosons \(\pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha, \pi^\alpha\) (\(\mu = 0, a\)) are physical particles in the spectrum and may acquire a mass.

The low-energy effective lagrangian will contain interaction terms among the Goldstones and the ordinary gauge bosons, collected in \(\mathcal{L}_G\):

\[
\mathcal{L}_G = \frac{1}{4} v^2 \text{Tr}(\partial_\mu U^\dagger \partial_\mu U),
\]

where

\[
U = \exp\left(\frac{2i T^a \pi^a}{v}\right).
\]

We have here denoted, collectively, the generators of table 1 \(T^a\), and the Goldstone bosons \(\pi^\alpha (\alpha = 1, ..., 15)\). We have chosen the normalization \(\text{Tr}(T^a T^a) = 4\pi^2\).

The covariant derivative \(\partial_\mu U\) is given by

\[
\partial_\mu U = \partial_\mu U + W_\mu U - iB_\mu,
\]

where

\[
W_\mu = ig T^a W^a_\mu,
\]

\[
B_\mu = ig T^a B^a_\mu.
\]

In the previous equations \(W^a_\mu\) and \(B^a_\mu\) are the gauge vector fields associated to \(SU(2)_L \times U(1)_Y\). In this model no term containing the gluon field is present in eq. (2.3) (in a technicolor realization the techniquarks are colorless).

We now consider the Yukawa couplings among the ordinary fermions and the \(\pi\). To this purpose it is advantageous to decompose the matrix \(U\) in the following way:

\[
U = \begin{pmatrix}
U_{uu} & U_{ud} & U_{ue} & U_{ua} \\
U_{du} & U_{dd} & U_{de} & U_{da} \\
U_{cu} & U_{cd} & U_{ce} & U_{ca} \\
U_{su} & U_{sd} & U_{se} & U_{sa}
\end{pmatrix}
\]

(2.5)

(The index ud, for instance, refers to the fact that the element \(U_{ud}\) can be thought of as made of the techniquarks \(U_d\) and \(D_u\).) Given this representation of the matrix \(U\), it is straightforward to write the Yukawa couplings which are \(G_W\) invariant. For the case we are considering, we will not provide a complete classification of all possible Yukawa couplings, which instead we will do for the case presented in the next section. Moreover, only the couplings with the quarks of the third generation will be included, namely those with the quarks top and bottom. The generalization to the whole set of fermions belonging to the three known generations presents no difficulty, without being particularly illuminating. We will assume that the Yukawa couplings are given by the following terms:

\[
\mathcal{L}_Y = \frac{1}{2} \left( \bar{f}_R U_{ii}^\dagger t_L + \bar{f}_R U_{ii}^\dagger b_L + h.c. \right)
\]

\[
+ m_4 (\bar{f}_R U_{ii}^\dagger t_L + \bar{f}_R U_{ii}^\dagger b_L + h.c.)
\]

\[
+ m_5 (\bar{f}_R U_{ii}^\dagger t_L + \bar{f}_R U_{ii}^\dagger b_L + h.c.)
\]

\[
+ m_6 (\bar{f}_R U_{ii}^\dagger t_L + \bar{f}_R U_{ii}^\dagger b_L + h.c.).
\]

(2.6)

The parameters \(m_3, m_5, m_7\) and \(m_8\) are here taken to be real. The index \(i\) is the color index. It is immediate to verify that \(\mathcal{L}_Y\) is indeed invariant under \(G_W\). By expanding the matrix \(U\) in powers of \(\pi\), one recovers all the tower of couplings with one, two, \(n\) Goldstones, as needed to properly ensure the required gauge invariance. In particular, from the first term of the expansion, we can read the masses of the top and bottom quarks:

\[
m_t = m_3 + m_5, \quad m_b = m_4 + m_6.
\]

(2.7)

In our approximation, all other fermions are massless. The interaction terms \(\mathcal{L}_G\) and \(\mathcal{L}_Y\) are what we need to evaluate the effective potential.

Before doing this, some comments are in order. Since the low energy effective theory is nonrenormalizable, the one-loop effective potential is divergent. By introducing an ultraviolet cut-off \(\Lambda\), roughly situ-
ated in the TeV range, such divergences can be classified. In general one expects quadratic divergences, logarithmic divergences and finite terms. In the following we will evaluate explicitly only the terms quadratic in the cut-off \( \Lambda \), neglecting the other contributions. The situation is here completely analogous to that of ref. [2]. The remaining contributions can of course be evaluated, but they are expected to be less important than the quadratic ones, when these are present.

The introduction of an ultraviolet cut-off \( \Lambda \) should, ultimately, correspond to the following procedure. Since the terms \( \mathcal{L}_d \) and \( \mathcal{L}_v \) in the effective lagrangian produce one-loop divergences which cannot be absorbed into a redefinition of the couplings contained in \( \mathcal{L}_e + \mathcal{L}_F \), the low-energy effective action must be supplemented with appropriate counterterms \( \mathcal{L}_{ct} \) such that the results of a one-loop computation are finite and coincide with those of the underlying fundamental theory. The counterterms \( \mathcal{L}_{ct} \) are expected to include all those short distance effects which are produced by the integration over the heavy modes of the fundamental theory and which are not contained in \( \mathcal{L}_e + \mathcal{L}_F \). If the scale characterizing the heavy modes is \( \Lambda \), it is natural to assume that the net result from the one-loop amplitude of \( \mathcal{L}_e + \mathcal{L}_F \) and the \( \mathcal{O}(\Lambda) \) counterterms in \( \mathcal{L}_{ct} \) is simply given by the amplitude evaluated with a cut-off at momenta close to \( \Lambda \). Although it is clear that this guess may contain some oversimplification of the complete physical picture, one hopes to be able in this way to extract the main results of the theory.

Particular care must be paid in analyzing the result. To be sure that the potential obtained is not an artefact of the regularization procedure, one must explicitly check that the terms generated in the effective potential are themselves invariant under transformations of \( G_w \) and that they possess the correct chiral behaviour: they must vanish whenever a special choice of parameters in \( \mathcal{L}_e + \mathcal{L}_F \) restores the chiral symmetry \( G \).

We are now in a position to present the result for this toy model. In the Landau gauge we obtain

\[
V_{\text{eff}} = -\frac{3A^2}{8\pi^2} \left( m_3^2 J_1 + m_2^2 J_2 + m_1^2 J_3 + m_0^2 J_4 + m_3 m_2 J_5 + m_2 m_1 J_6 \right). \tag{2.8}
\]

The quantities \( J_i (i = 1, \ldots, 6) \) are given by

\[
\begin{align*}
J_1 &= U_{ud} U_{ud}^* + U_{du} U_{du}^*, \\
J_2 &= U_{ud} U_{ud}^a + U_{da} U_{da}^a, \\
J_3 &= U_{ud} U_{cd}^* + U_{cd} U_{dc}^*, \\
J_4 &= U_{cd} U_{ct}^* + U_{tc} U_{ct}^*, \\
J_5 &= U_{uu} U_{cc}^* + U_{cu} U_{cu}^* + h.c., \\
J_6 &= U_{ds} U_{ts}^* + U_{ds} U_{ts}^* + h.c. \tag{2.9}
\end{align*}
\]

We observe that in this case there is no contribution from \( \mathcal{L}_d \) to the one-loop effective potential \( V_{\text{eff}} \). Neglecting \( \mathcal{L}_F \), all the 15 Goldstone bosons remain massless (at the order \( \Lambda^3 \)). This result agrees with ref. [2] (recall that we are considering the limit \( \Lambda = 0 \) of the model studied there). On the other hand the Yukawa interactions provide for the effective potential given in eq. (2.8). One can easily check that the quantities \( J_i (i = 1, \ldots, 6) \) are invariant under \( G_w \). Moreover, the only choice of parameters which restores the chiral symmetry at the level of \( \mathcal{L}_F \) is \( m_3 = m_4 = m_5 = m_6 = 0 \). We list below the mass eigenstates and the corresponding eigenvalues of the mass-squared matrix, derived from the potential \( V_{\text{eff}} \) of eq. (2.8):

\[
m^2(\pi^a) = 0, \quad a = 1, 2, 3,
\]

\[
m^2(\bar{\pi}^+) = m^2(\bar{\pi}^-) = \frac{3A^2}{8\pi^2} (m_3 m_5 + m_4 m_6),
\]

\[
m^2 \left( \frac{\bar{\pi}^3 - \pi_{3\omega}}{\sqrt{2}} \right) = \frac{6A^2}{\pi^2 v^2} m_3 m_6,
\]

\[
m^2 \left( \frac{\bar{\pi}^3 + \pi_{3\omega}}{\sqrt{2}} \right) = \frac{6A^2}{\pi^2 v^2} m_3 m_5,
\]

\[
m^2 \left( \frac{P_3^0 + P_3^1}{\sqrt{2}} \right) = m^2 \left( \frac{P_3^0 + P_3^1}{\sqrt{2}} \right) = \frac{3A^2}{2\pi^2 v^2} (m_3 + m_5)^2,
\]

\[
m^2 \left( \frac{P_3^0 - P_3^1}{\sqrt{2}} \right) = m^2 \left( \frac{P_3^0 - P_3^1}{\sqrt{2}} \right) = \frac{3A^2}{2\pi^2 v^2} (m_4 + m_6)^2. \tag{2.10}
\]
\[ m^2(P^Z) = m^2(\bar{P}^Z) = \frac{3A^2}{2\pi^2v^2} (m_1^2 + m_2^2 + m_3m_4 + m_5m_6), \]
\[ m^2(P^+) = m^2(\bar{P}^+) = \frac{3A^2}{2\pi^2v^2} (m_3^2 + m_5^2 + m_1m_4 + m_2m_6), \]

where we have introduced the linear combinations
\[ P^Z = \frac{\pi^Z - i\pi^Z}{\sqrt{2}} \]
\[ \bar{P}^Z = (P^Z)^\dagger, \]
and we have defined \( P^Z \) in the standard way.

We note that the would-be Goldstone bosons \( \pi^a \), which are going to be absorbed by the gauge bosons \( W \) and \( Z \), are massless. We also observe a curious change of sign: the minus sign of the left-hand side of eq. (2.8), coming from the fermion loop, has been cancelled by a minus sign coming from the expansion of the invariants \( J \). In this way, for instance, the combinations \((P_1^Z + P_3^Z)/\sqrt{2}\) and \((P_1^Z - P_3^Z)/\sqrt{2}\) acquire a positive mass squared. This situation is opposite to what is usually obtained in computing the dependence of the effective potential from the radial mode. Such a mode, here deliberately omitted from the low-energy spectrum, receives from the fermion loops negative contributions to its squared mass, which tends to destabilize the theory. One can think, as an example, to the Higgs particle of the standard model. Our computation shows that, on the contrary, the angular modes contained in \( U \) will not in general obey the same rule as the radial mode, and they can receive from fermion loops positive contributions to their squared mass. From eq. (2.10) we read the conditions for the stability of the one-loop effective potential (remember that the parameters \( m \), are here taken to be real):
\[ m_3m_5 > 0, \quad m_4m_6 > 0 \]

Such conditions can be easily met by choosing \( m_3 \) and \( m_5 \) of the same sign and similarly for \( m_4 \) and \( m_6 \).

The structure of the mass spectrum of the pseudo-Goldstones can be understood by observing that in the chiral limit one has a \( SU(2)_L \times SU(2)_R \sim O(4) \) which commute with the electric charge. This corresponds to require that these generators commute with the diagonal \( T_3 = (T_{3L} + T_{3R}). \) In \( O(4) \) there is only one such generator: \( K_3 = (T_{3L} - T_{3R}). \) Therefore the general structure of the mass matrix must be
\[ M^2 = A + B K_3 + C T_3. \]  \hspace{1cm} (2.13)

Notice that the \( (\frac{1}{2}, \frac{1}{2}) \) representation of \( SU(2) \times SU(2) \), to which the Goldstone bosons belong, decomposes as \( 1 + 3 \) with respect to \( O(3) \subset O(4) \) and therefore the following sum rules for the masses are implied for all the quadruplet-
\[ \sum m_3^2(T_3 = 0) = \sum m_5^2(K_3 = 0). \]

In the model previously considered, one has for example
\[ m^2(\bar{\pi}^3 - \pi^3) + m^2(\bar{\pi}^3 + \pi^3) = m^2(\bar{\pi}^+) + m^2(\bar{\pi}^-). \]  \hspace{1cm} (2.15)

In the following we collect the allowed range for the various masses:
\[ m^2(\pi^a) = 0, \quad a = 1, 2, 3, \]
\[ m^2(\bar{\pi}^\pm) \leq \frac{3A^2}{4\pi^2v^2} (m_1^2 + m_2^2), \]
\[ m^2(\bar{\pi}^0) \leq \frac{3A^2}{2\pi^2v^2} m_5^2, \]
\[ m^2(\bar{\pi}^3 + \pi^3) \leq \frac{3A^2}{2\pi^2v^2} m_5^2, \]
\[ m^2(\bar{\pi}^3 - \pi^3) \leq \frac{3A^2}{2\pi^2v^2} m_5^2, \]
\[ m^2(\bar{P}^+ + P^+) \leq \frac{3A^2}{2\pi^2v^2} (m_5^2 + m_5^2), \]
\[ m^2(\bar{P}^- - P^-) \leq \frac{3A^2}{2\pi^2v^2} (m_5^2 + m_5^2). \]  \hspace{1cm} (2.16)

As we see from the previous list, the pseudo-Goldstone masses have values which are naturally close to those of the massive fermions of the theory, in this
case the top and the bottom. For numerical illustration just take \( m_{t} = 5 \text{ GeV}, m_{W} = 150 \text{ GeV} \) and \( A = 1000 \text{ GeV} \). One then finds, in this toy model:
\[
m^2(\pi^+) = 0, \quad m^2(\pi^0) \leq (168 \text{ GeV})^2,
\]
\[
m^2\left(\frac{\pi^3 - \pi^0}{\sqrt{2}}\right) \leq (8 \text{ GeV})^2,
\]
\[
m^2\left(\frac{\pi^3 + \pi^0}{\sqrt{2}}\right) \leq (238 \text{ GeV})^2,
\]
\[
m^2\left(\frac{P^0 + P^3}{\sqrt{2}}\right) = (238 \text{ GeV})^2,
\]
\[
m^2\left(\frac{P^3 - P^0}{\sqrt{2}}\right) = (8 \text{ GeV})^2,
\]
\[
m^2(P^0) \leq (238 \text{ GeV})^2,
\]
\[
m^2(P^3) \leq (238 \text{ GeV})^2.
\]

3. Case B. \( G = SU(8) \times SU(8), H = SU(8) \)

We consider here the case of a chiral symmetry \( SU(8) \times SU(8) \), spontaneously broken to the diagonal \( SU(8) \) subgroup. Such a pattern of global symmetries is realized for instance in a popular technicolor model, with technifermions given by a generation \((U, D, N, E)\) of techniquarks and technileptons, with the same quantum numbers as the corresponding ordinary fermions. The spontaneous breaking of \( SU(8) \times SU(8) \) to the diagonal \( SU(8) \)

Table 2

SU(8) generators. Here \( \lambda_{a} \) are the Gell-Mann matrices and \( \xi_{a} \) are three orthonormal vectors in a three-dimensional vector space.

\[
\begin{align*}
T^+_a &= \frac{1}{2} \left( \begin{array}{cc} \tau \otimes \xi^a & 0 \\ 0 & -\tau^a \end{array} \right) \\
T^0 &= \frac{1}{2} \left( \begin{array}{cc} 1 \otimes \xi^0/3 & 0 \\ 0 & -1 \end{array} \right) \\
T^+_0 &= \frac{\sqrt{3}}{2} \left( \begin{array}{cc} 1 \otimes \xi^0/3 & 0 \\ 0 & -1 \end{array} \right) \\
T^3 &= \frac{1}{\sqrt{2}} \left( \begin{array}{cc} \xi \otimes \xi^3 & 0 \\ 0 & 0 \end{array} \right) \\
T^3_0 &= \frac{1}{\sqrt{2}} \left( \begin{array}{cc} \xi \otimes \xi^3 & 0 \\ 0 & 0 \end{array} \right) \\
T^0 &= \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 0 & i\xi \otimes \xi^0 \\ -i\xi \otimes \xi^0 & 0 \end{array} \right)
\end{align*}
\]
\( \mathcal{L}_Y = m_1 \left( \bar{t}_i U^i_{1t} r_1 + \bar{t}_k U^k_{1t} b_1 \right) + \text{h.c.} \) 

\( + m_2 \left( \bar{b}_k U^k_{2t} t_1 + \bar{b}_k U^k_{2t} b_1 \right) + \text{h.c.} \) 

\( + \mu_1 (\bar{r}_k U^k_{3t} c_1 + \bar{r}_k U^k_{3t} s_1) + \text{h.c.} \) 

\( + m_3 \left( \bar{t}_k U^k_{4t} \lambda_1 + \bar{t}_k U^k_{4t} \lambda_2 \right) + \text{h.c.} \)

\( + m_4 \left( \bar{b}_k U^k_{5t} \lambda_1 + \bar{b}_k U^k_{5t} \lambda_2 \right) + \text{h.c.} \)

\( + m_5 \left( \bar{c}_k U^k_{6t} \lambda_1 + \bar{c}_k U^k_{6t} \lambda_2 \right) + \text{h.c.} \)

\( + m_6 \left( \bar{u}_k U^k_{7t} \lambda_1 + \bar{u}_k U^k_{7t} \lambda_2 \right) + \text{h.c.} \)

\( + m_7 \left( \bar{d}_k U^k_{8t} \lambda_1 + \bar{d}_k U^k_{8t} \lambda_2 \right) + \text{h.c.} \)

\( + m_8 \left( \bar{e}_k U^k_{9t} \lambda_1 + \bar{e}_k U^k_{9t} \lambda_2 \right) + \text{h.c.} \)

\( + m_9 \left( \bar{\mu}_k U^k_{10t} \lambda_1 + \bar{\mu}_k U^k_{10t} \lambda_2 \right) + \text{h.c.} \)

\( + m_{10} \left( \bar{\tau}_k U^k_{11t} \lambda_1 + \bar{\tau}_k U^k_{11t} \lambda_2 \right) + \text{h.c.} \)

where the \( \lambda^\alpha \) are the Gell-Mann matrices and all the parameters \( m_i \) and \( \mu_i \) are taken to be real. The apparently strange numeration of the couplings in eq. (3.6) is due to the absence of \( \nu_R \). The missing couplings \( m_5, m_8 \) and \( \mu_3 \) would correspond to Yukawa interactions with \( \nu_R \). From \( \mathcal{L}_Y \) we easily recover the expressions for the fermion masses:

\( m_1 = m_1 + 3 m_5 + \frac{3}{2} m_{11} + m_7 \),

\( m_2 = m_2 + 3 m_8 + \frac{3}{2} m_{12} + m_9 \),

\( m_3 = m_3 + 3 m_{10} \),

\( m_{10} = 0 \). (3.7)

All the other fermions remain massless in this approximation.

Starting from the interaction lagrangian \( \mathcal{L}_{\text{int}} + \mathcal{L}_Y \) we now evaluate the one-loop effective potential. As explained in the previous section we will consider only the terms quadratic in the ultraviolet cut-off \( \Lambda \). In the Landau gauge we obtain

\( V_{\text{eff}} = V_{\text{eff}}^g + V_{\text{eff}}^\gamma \), (3.8)

where

\( V_{\text{eff}}^g = - \frac{3 A^2}{16 \pi^2} \left[ \frac{v^2}{8} \text{Tr} \left( T_3 U^T T_3 U \right) + \frac{\nu^2}{4} \text{Tr} \left( \left( T^3 + \frac{T_3}{\sqrt{3}} \right) U^T \frac{T_3}{\sqrt{3}} U \right) \right] \), (3.9)

and

\( V_{\text{eff}}^\gamma = - \frac{A^2}{8 \pi^2} \left[ (m_1^2 + \frac{3}{2} m_{11}^2 - m_7^2) I_1 \right. \)

\( + (m_2^2 + \frac{3}{2} m_{12}^2 - m_9^2) I_2 + (3 m_5^2 - m_3^2) I_3 \)

\( + (3 m_8^2 + m_4^2) I_4 + (2 m_2 m_5 + 3 m_3^2 + \frac{3}{2} m_2 m_{11}) I_5 \)

\( + (2 m_2 m_6 + 3 m_3^2 + \frac{3}{2} m_{16} m_{12} + m_{10}^2) I_6 \)

\( + (m_7 m_8 + 3 m_5 m_9 + \frac{3}{2} m_{12} m_{12} + m_{10} m_{10}) I_7 \)

\( + (m_7 m_9 + 3 m_5 m_8 + \frac{3}{2} m_{12} m_{12} + m_{10} m_{10}) I_8 \)

\( + (2 m_{11} + \frac{3}{2} m_{11}) I_9 + (2 m_{12} + \frac{3}{2} m_{12}) I_{10} \),

(3.10)

where the quantities \( I_i (i = 1, ..., 10) \) are given by
\[ I_1 = U^\dagger_{w_1} U_{w_1} + U^\dagger_{w_2} U_{w_2}, \]
\[ I_2 = U^\dagger_{w_3} U_{w_3} + U^\dagger_{w_4} U_{w_4}, \]
\[ I_3 = U_{v_1} U^\dagger_{v_1} + U_{v_2} U^\dagger_{v_2}, \]
\[ I_4 = U_{v_3} U^\dagger_{v_3} + U_{v_4} U^\dagger_{v_4}, \]
\[ I_5 = U^\dagger_{w_1} U_{w_1} + U^\dagger_{w_2} U_{w_2}, \]
\[ I_6 = U^\dagger_{w_3} U_{w_3} + U^\dagger_{w_4} U_{w_4}, \]
\[ I_7 = U^\dagger_{v_3} U_{v_3} + U^\dagger_{v_4} U_{v_4} + \text{h.c.}, \]
\[ I_8 = U^\dagger_{v_3} U_{v_3} + U^\dagger_{v_4} U_{v_4} + \text{h.c.}, \]
\[ I_9 = \text{Tr}(U^\dagger_{\alpha_1} \alpha^{\dagger}_{\alpha_1} U_{\alpha_2} \alpha^{\dagger}_{\alpha_2} + U^\dagger_{\alpha_3} \alpha^{\dagger}_{\alpha_3} U_{\alpha_4} \alpha^{\dagger}_{\alpha_4}), \]
\[ I_{10} = \text{Tr}(U^\dagger_{\alpha_5} \alpha^{\dagger}_{\alpha_5} U_{\alpha_6} \alpha^{\dagger}_{\alpha_6} + U^\dagger_{\alpha_7} \alpha^{\dagger}_{\alpha_7} U_{\alpha_8} \alpha^{\dagger}_{\alpha_8}). \]

One can show that these quantities are indeed invariant under $G_w$. The mass eigenstates and eigenvalues are derived expanding at the second order the invariants $I_i$ in the effective potential. The mass spectrum we obtain is

\[ m^2(\pi^a) = 0, \quad a = 1, 2, 3, \]
\[ m^2(\pi^u) = m^2(\pi^d) = \frac{A^2}{2\pi^2v^2}(\rho_1 + \rho_2), \]
\[ m^2(\pi^u - \pi^d) = \frac{A^2}{2\pi^2v^2}(\rho_3), \]
\[ m^2(\pi^u \pm \pi^d) = \frac{A^2}{\pi^2v^2}(\rho_3 + 2, \rho_3 - 2), \]
\[ m^2(\pi^u + \pi^d) = \frac{A^2}{2\pi^2v^2}(3\rho_3 + 2\rho_7 + 3\rho_8), \]
\[ m^2(\pi^u - \pi^d) = \frac{A^2}{2\pi^2v^2}(3\rho_3 + 2\rho_7 + 3\rho_8), \]
\[ m^2(\pi^u + \pi^d) = \frac{A^2}{\pi^2v^2}(\rho_3 + 2\rho_7 + 3\rho_8), \]
\[ m^2(\pi^u - \pi^d) = \frac{A^2}{2\pi^2v^2}(\rho_3 + 2\rho_7 + 3\rho_8). \]

where

\[ \rho_1 = 2m^2_1 + \frac{3}{2}m^2_1 - 2\mu_2^2 + \frac{1}{2}v^2g^2, \]
\[ \rho_2 = 2m^2_1 + \frac{3}{2}m^2_1 - 2\mu_2^2 - \frac{1}{2}v^2g^2, \]
\[ \rho_3 = 6m^2_1, \]
\[ \rho_3 = 4m_1m_5 + 6m^2_1 + \frac{3}{2}v^2g^2, \]
\[ \rho_3 = 4m_1m_5 + 6m^2_1 + \frac{3}{2}v^2g^2, \]
\[ \rho_3 = 2m_1m_7 + 6m_2m_7 + \frac{3}{2}m_5m_1, \]
\[ \rho_3 = 2m_1m_7 + 6m_2m_7 + \frac{3}{2}m_5m_1, \]
\[ \rho_3 = 4m_1m_5 + 6m_2m_5 + \frac{3}{2}m_5m_{12} + 2m_4m_1, \]
\[ \rho_3 = 4m_1m_5 + 6m_2m_5 + \frac{3}{2}m_5m_{12} + 2m_4m_1, \]
\[ \rho_3 = 4m_1m_5 + 6m_2m_5 + \frac{3}{2}m_5m_{12} + 2m_4m_1. \]

As in the SU(4) model the $\pi^a$ remain massless and are absorbed as longitudinal degrees of freedom of $W$ and $Z$. In general all the other pseudo-Goldstone bosons get mass, even those colorless and the mass rule of eq. (2.14) holds for all the multiplets. The stability of the one-loop effective potential requires that the masses given in eq. (3.12) are positive. This fact of course puts some constraints on the Yukawa couplings, but we shall not pursue a general analysis here. As an example, one can satisfy the stability conditions by taking $\mu_2 = 0$ ($i = 1, 2, 4$) and all the $m_i$ of the same sign. We notice that the parameters $\mu_i, i = 1, 2, 4$, give a negative contribution to the squared masses of all color triplets and therefore tend to destabilize the potential. Taking into account only the bosonic contribution to the effective potential one finds that only the colored sector receives mass; putting $Z_2 = 0$ we find the same spectrum as in ref. [2]:

\[ m^2\left(\frac{P_y - P_Y}{\sqrt{2}}\right) = m^2\left(\frac{P_z - P_Y}{\sqrt{2}}\right). \]
\[ m^2(\pi^\pm/2) = m^2 \left( \frac{\pi^\pm + \pi^\mp}{\sqrt{2}} \right) = m^2 \left( \frac{\pi^\pm - \pi^\mp}{\sqrt{2}} \right) \]

\[ = \frac{9A^2}{8\pi^3} g_8^2, \]

\[ m^2(P_5^2) = m^2 \left( \frac{P_5^0 + P_5^1}{\sqrt{2}} \right) = \frac{A^2}{2\pi^3} (g_8^2 + \frac{3}{8} g_9^2), \]

\[ m^2(P_5^3) = \frac{A^2}{2\pi^3} (g_9^2 + \frac{5}{8} g_9^2). \]

(3.14)

To compare with ref. [2] one should make the identification \( 3A^2/2\pi = M^2 \).

Coming to the general case, \( \Sigma \neq 0 \), it is easy to derive the following allowed ranges for the masses of the colorless pseudo-Goldstone bosons:

\[ m^2(\pi^a) = 0, \quad a = 1, 2, 3, \]

\[ m^2(\pi^\pm) \leq \frac{2A^2}{\pi^3 v^2} \left( \frac{1}{2} m_3^2 + \frac{1}{2} m_6^2 + \frac{5}{8} m_1^2 \right), \]

\[ m^2(\pi^+ - \pi^0) \leq \frac{2A^2}{\pi^3 v^2} \left( m_2^2 + \frac{1}{2} m_1^2 \right), \]

\[ m^2(\pi^+ + \pi^0) \leq \frac{2A^2}{\pi^3 v^2} m_1^2. \]

(3.15)

These ranges are obtained by observing that the corresponding square-masses depend on few combinations of the whole set of original parameters. For instance the mass of the pseudo-Goldstone boson \( (\pi^+ + \pi^-)/\sqrt{2} \) is proportional to the product \( m_3 m_6 \) with \( m_3 = m_t \) and \( m_6 = m_t + 3 m_3 + \frac{5}{8} m_1 \), and, at the same time, from eq. (3.7), one has \( m_3 = m_3 + m_b \). We observe that necessary conditions for the presence, in the colorless sector, of terms proportional to the top and the bottom masses are, respectively, a nonvanishing \( m_3 \) and \( m_b \). Numerical values from eq. (3.15) for \( m_t = 1.7 \) GeV, \( m_b = 5 \) GeV, \( m_t = 150 \) GeV and \( A = 1000 \) GeV are

\[ m^2(\pi^a) = 0, \]

\[ m^2(\pi^\pm) \leq (194 \text{ GeV})^2, \]

\[ m^2(\pi^+ - \pi^0) \leq (9 \text{ GeV})^2, \]

\[ m^2(\pi^+ + \pi^0) \leq (274 \text{ GeV})^2. \]

We stress that, barring very specific choices for the initial Yukawa couplings, the natural range for the masses of \( \pi^+ \), \( \pi^- \), and \( (\pi^+ + \pi^-)/\sqrt{2} \) is around the top quark mass, whereas for \( (\pi^+ - \pi^-)/\sqrt{2} \) it is close to the bottom quark mass. The model analyzed so far is probably not realistic and a detailed discussion about its phenomenology would be inappropriate. Nevertheless we note that, as far as the colorless sector is concerned, there is no conflict with the present data, for values of the masses close to the upper limit of the range given in eq. (3.15). We recall that the Z may decay into the two charged combinations \( \pi^+ \) and \( \pi^- \) of \( \pi^+ \) and \( \pi^\mp \) but not into a pair of neutral states [5]. The three-body decays requires again the presence of \( \pi^+ \) and \( \pi^- \). On the other hand the radiative decay of Z into a photon plus the light combination \( (\pi^+ - \pi^-)/\sqrt{2} \) proceeds via an anomalous coupling [6] and has probably a too low rate [7] to be seen at LEP I without high luminosity option.

Finally, just to illustrate our results with an example, we give a numerical estimate of the pseudo-Goldstone boson spectrum, obtained by taking \( \mu = 0, \]

\[ m_1 = m_2 = \frac{1}{2} m_{ttop}, m_3 = m_0 = \frac{1}{2} m_0, m_4 = 0 \]

for \( i 
eq 1, 2, 7, 9, A = 1 \text{ TeV}, \alpha_0 = 0.12 \) and \( m_t = 150 \text{ GeV} \):

\[ m^2(\pi^a) = 0, \quad a = 1, 2, 3, \]

\[ m^2(\pi^\pm) = \frac{A^2}{\pi^3 v^2} (m_3^2 + m_6^2) = (194 \text{ GeV})^2, \]

\[ m^2(\pi^+ - \pi^0) = \frac{2A^2}{\pi^3 v^2} m_2^2 = (9 \text{ GeV})^2, \]

\[ m^2(\pi^+ + \pi^0) = \frac{2A^2}{\pi^3 v^2} m_1^2 = (274 \text{ GeV})^2, \]

\[ m^2(\pi^\pm + \pi^\mp) = \frac{A^2}{4\pi^3 v^2} (m_3^2 + m_6^2 + \frac{5}{8} v^2 g_9^2) \]

\[ = (426 \text{ GeV})^2, \]

\[ m^2(\pi^\pm + \pi^\mp) = \frac{A^2}{2\pi^3 v^2} (m_1^2 + \frac{5}{8} v^2 g_9^2) \]

\[ = (437 \text{ GeV})^2, \]

(3.16)
\[ m^2 \left( \frac{\pi^0 \! - \! \pi^0}{\sqrt{2}} \right) = \frac{A^2}{2\pi^2 v^2} \left( m_b^2 + \frac{3}{4} v^2 g_s^2 \right) \nonumber \\
= (415 \text{ GeV})^2, \nonumber \\
\]
\[ m^2 \left( \frac{P^0_\perp + P^0_\parallel}{\sqrt{2}} \right) = \frac{A^2}{2\pi^2 v^2} \left( 4m_t^2 + v^2 g_s^2 + \frac{1}{2} v^2 g^2 \right) \nonumber \\
= (392 \text{ GeV})^2, \nonumber \\
\]
\[ m^2 \left( \frac{P^0_\perp - P^0_\parallel}{\sqrt{2}} \right) = \frac{A^2}{2\pi^2 v^2} \left( 4m_b^2 + v^2 g_s^2 + \frac{1}{2} v^2 g^2 \right) \nonumber \\
= (280 \text{ GeV})^2, \nonumber \\
\]
\[ m^2(P^0_\perp) = \frac{A^2}{2\pi^2 v^2} \left( 3m_t^2 + m_b^2 + v^2 g_s^2 - \frac{1}{2} v^2 g^2 \right) \nonumber \\
= (363 \text{ GeV})^2, \nonumber \\
\]
\[ m^2(P^0_\parallel) = \frac{A^2}{2\pi^2 v^2} \left( m_t^2 + 3m_b^2 + v^2 g_s^2 + \frac{1}{2} v^2 g^2 \right) \nonumber \\
= (316 \text{ GeV})^2. \quad (3.16 \text{ cont'd}) \nonumber \\
\]

To conclude this section we comment on the parameters \( \epsilon_1 \) and \( \epsilon_2 \), or \( T \) and \( U \) defined in refs. [8,9]. We observe that a large isospin splitting, as obtained in the colorless sector, could in principle give rise to a sizeable contribution to \( \Delta \rho = \epsilon_1 \) and \( \epsilon_2 \). We have computed the one-loop effect of the pseudo-Goldstone bosons \([9]\) to \( \epsilon_1 \) and \( \epsilon_2 \) using the same cut-off \( A = 1 \text{ TeV} \). The relevant contributions come from the color triplets and from the colorless sector. The result is that they generally give a sizeable, negative contribution to \( \epsilon_1 \), which is quadratic in the pseudo-Goldstone masses, while the contribution to \( \epsilon_2 \) is negligible. For the mass spectrum given in eq. (3.16) we obtain a contribution to \( \epsilon_1 \) of about \(-0.003\) to be compared to the usual standard model top contribution equal to 0.007. In general the overall effect of the pseudo-Goldstone loops is to modify the upper limit on the top mass coming from the SM radiative corrections. We find that depending on the actual value of the cut-off \( \Lambda \) the negative contribution from the pseudo-Goldstone bosons can compensate that of the SM or even become the dominant one.

4. Conclusions

In theories of dynamical electroweak symmetry breaking without scalar fields, the spontaneous breaking of a large initial global symmetry \( G \) leads to pseudo-Goldstone bosons in the low-energy spectrum. The masses of such states are in direct relations to the possible sources of an explicit breaking of \( G \). Among these sources, the mechanism which gives rise to masses to the known fermions is of primary relevance. By considering the low-energy effective theory, where the mechanism of fermionic mass generation is encoded in the allowed Yukawa couplings, we have provided a framework for an explicit and quantitative evaluation of the pseudo-Goldstone mass spectrum. Our approach proceeds through the evaluation of the one-loop effective potential in the leading approximation, where counterterms of the desired symmetry properties are generated. The results show that, in general all pseudo-Goldstone bosons receive masses from fermion loops, even those which remain massless when only the gauge bosons corrections come into play. The stability conditions of the effective potential have solutions in large portions of the parameter space: the contributions to pseudo-Goldstone masses are quite often positive, contrary to what happens, for instance, with the ordinary Higgs of the standard model. Those states which stay massless when Yukawa couplings are neglected, tend to acquire masses close to the values of the heaviest fermions of the low-energy theory, namely the top and the bottom quarks. To facilitate the comparison with the existing literature we have developed our formalism only for two specific examples. However the underlying mechanism and the conclusions presented here are expected to be general properties of a whole class of theories.

References