Children's number processing is context dependent

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Children’s number processing is context dependent

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The aim of this paper was to test the hypothesis of a context dependence of number processing in children. Fifth-graders were given two numbers presented successively on screen through a self-presentation procedure after being asked either to add or subtract or compare them. We considered the self-presentation time of the first number as reflecting the complexity of the encoding for a given planned processing. In line with Dehaene’s triple-code model, self-presentation times were longer for additions and subtractions than for comparisons with two-digit numbers. Alternative interpretations of these results in terms of more cognitive effort or more mental preparation in the case of addition and subtraction than comparison are discussed and ruled out.

Keywords: Arithmetic; Mental representations; Numerical cognition.

In our daily life, the numbers we have to deal with obviously differ by the format in which they are presented (e.g., number words, digits, patterns of dots on dice) and by the nature of the task they are involved in. Moreover, it has been suggested that they differ by the format of representation in which they are encoded and mentally manipulated. The most specified and precise theory relying on the complex encoding hypothesis is probably Dehaene’s triple-code model of number processing (1992, 2001; Dehaene & Cohen, 1995, 1997). According to this model, numerical information can be mentally manipulated in three different representation formats: an analogical quantity or magnitude code, an auditory-verbal code, and a visual Arabic code. Each activity involving numbers would rely on one of these specific codes (Dehaene, 2001; Dehaene & Cohen, 1997). Magnitude comparison would rely on an analogical representation of numbers, or, in other words, on a language-independent spatial representation of numbers on a mental line. Their quantitative meaning would be retrieved and positioned on a left–right oriented mental number line without transcoding into a verbal code. On the contrary, this transcoding would be necessary for the resolution of simple...
operations such as $7 + 2$ or $4 \times 5$. Indeed, in Dehaene’s model, the network-like organisation for arithmetic facts takes the form of tables that are stored as verbal associations (e.g., “seven plus two is nine”) in an auditory verbal word frame. Whereas activities such as magnitude comparison or simple operation solving rely on a simple and direct transcoding (i.e., in an analogical format and in a verbal format for comparison and retrieval respectively), multidigit number operations, such as the addition $37 + 22$, would require more complex processes and the coordination of several representational codes. According to Dehaene and Cohen (1995, 1997; Cohen & Dehaene, 2000), such problems are solved via the mental manipulation of a spatial image of the operation in the Arabic notation. The authors qualify this manipulation as a “semantic elaboration” because “it requires a good understanding of the quantities involved in the original problem” (Dehaene & Cohen, 1995, p. 102). Naturally, solving these problems requires a large amount of controlled attention to go carefully through a series of steps to reach the answer. Brain imaging studies show indeed that the prefrontal areas of the brain that control nonautomatic activities are activated for such complex calculations (Roland & Friberg, 1985). Accordingly, it has been shown that operand manipulations via mental decompositions are required to achieve complex calculations (Lemaire & Arnaud, 2008; Thevenot, Barrouillet, & Fayol, 2001). In our example, participants could decompose 37 and 22 into $30 + 7$ and $20 + 2$ respectively, add 20 to 30, temporarily store this intermediary result (50), then compute or retrieve $7 + 2$, and finally add 9 to 50. Therefore, in Dehaene’s model, multidigit calculations involve the mental manipulation of spatial image of the operation in Arabic notation as well as verbal representations, which are necessary to retrieve intermediate results in long-term memory or to browse the numerical chain. If Dehaene is correct in assuming that there is a variety of representations of numbers related to specific uses and tasks, then the processing of numbers should be systematically affected by the nature of the task in which these numbers are involved.

It is exactly what we have shown (Thevenot & Barrouillet, 2006) by measuring the time that adults took to read and store two-digit numbers after being informed whether they will be subsequently involved in an addition, a subtraction, or a comparison task. Participants were presented with series of three numbers that successively appeared on a computer screen. They were informed about the precise nature of the task that they had to perform by a letter (A for addition, S for subtraction, and C for comparison), which appeared on screen before the first number. In the addition condition, the participants were asked to decide whether or not the third number presented was the sum of the two numbers previously displayed. In the subtraction condition, they had to decide whether this third number corresponded to the difference between the two first. Finally,
in the comparison condition, they had to decide whether or not this third number fell between the two previous numbers. The letter and the three numbers were successively displayed using a self-presentation procedure. We showed that the self-presentation times of the first number were longer for two-digit additions and subtractions than for comparisons. We assumed that the self-presentation time of the first number was indicative of the complexity of the sole encoding process, because no other processing could be applied before the presentation of the second number. A simple and direct transcoding was required for comparison (i.e., analogical code) but the coordination of two representational codes was required for two-digit addition or subtraction (i.e., verbal and visual codes), hence the longer encoding times for the former than for the latter.

Then, in accordance with Dehaene’s model, the processing of numbers is affected by the nature of the task in which these numbers are involved. Surprisingly, to our knowledge, the role of context on children’s number processing had never been tested in cognitive psychological research. However, the question whether their number representation show the same properties as adults’ number representations is important in light of current debates over the developmental origins of number sense. According to the number sense theory (Dehaene, 1992; Dehaene & Cohen, 1995), understanding and manipulating numerical quantities is part of human biological evolutionary heritage. This assumption is supported by the fact that specific cerebral circuits seem to be associated with the representation and acquisition of knowledge about numerals. Indeed, brain imaging studies (Dehaene, Spelke, Stanescu, Pinel, & Tsivkin, 1999, for example) and neuropsychological dissociation observations (Delazer & Benke, 1997, for example) have permitted researchers to speculate that ventral occipitotemporal sectors of both hemispheres are involved in the visual Arabic number form, that the left perisylvian areas are implicated in the verbal representations of numbers and that the intraparietal areas of both hemispheres are involved in the analogical quantity representation. Then, multiple brain areas contribute to the cerebral processing of numbers and their coordination emerges during child development. As stated by Dehaene (2001, p. 27):

the hypothesis developed in the number sense is that all children are born with a quantity representation which provides the core meaning of numerical quantity. Exposure to a given language, culture, and mathematical education leads to the acquisition of additional domains of competence such as a lexicon of number words, a set of digits for written notation, procedures for multi-digit calculation, and so on. Not only must these abilities be internalized and routinized; but above all, they need to be coordinated with existing conceptual representations of arithmetic. The constant dialogue, within the child’s own brain, between linguistic, symbolic, and
To sum up, children’s new numerical abilities develop through practice and emerging mental representations are articulated with early conceptual representations. As a consequence, as soon as children are able to store arithmetic facts in long-term memory and are able to perform complex calculations (i.e., via nonretrieval procedures), the way numbers are processed should be similar in adults and children. More precisely and within our problematic, exactly as in adults, fifth-graders’ number processing should be affected by the nature of the task to be performed. Therefore, in this study, we expected to replicate the results we obtained with adults: Encoding processes in children should be faster when numbers are involved in a comparison, which, according to the triple code model requires a simple analogical coding, than when the same numbers are involved in complex additions and subtractions, which require the coordination of the Arabic and verbal codes in order to be mentally manipulated. We chose to study 10-year-old children because it was crucial for us that our participants could solve two-digit additions and subtractions using the same strategies as adults, that is through decomposition, computing, and retrieval. Even if retrieval strategies are observed early for very simple addition problems (i.e., operands up to 5), it is only in fifth-grade that the use of retrieval strategies extent to all simple addition problems (i.e., operands between 0 and 9) (LeFevre & Kulak, 1994; Lemaire, Barret, Fayol, & Abdi, 1994).

**METHOD**

**Participants**

Thirty fifth-grade children from Swiss elementary school took part in this experiment (mean age = 10.7 years, \( SD = 7 \) months). Parental permission and the child’s own consent were obtained.

**Materials and procedure**

Sixteen pairs of numbers between 10 and 30 were chosen randomly in the set of 43 pairs that fit the following constraints. Their sum never exceeded 39 and their difference was larger than 4. The numbers, their sums and differences never ended with a 0. These constraints were respected in order to minimise the probability of addition and subtraction solving by retrieval. Each of these 16 pairs of numbers was presented in the addition, subtraction, and comparison conditions resulting in 48 experimental trials.
presented randomly. In the “addition” condition, children had to decide whether or not a third number corresponded to the sum of the two first, whereas in the “subtraction” condition they had to decide if this third number corresponded to their difference. Finally, in the “comparison” condition, they had to decide whether this third number fell between the two numbers previously presented. In each condition, half of the trials elicited a “Yes” response. For additions and subtractions, the number eliciting a “No” response was constructed by adding or subtracting 1 from the correct answer to avoid solution by approximation rather than genuine calculation (Ashcraft & Battaglia, 1978). For comparisons, this number corresponded in half of the cases to the smaller operand minus 1 and in the other half to the larger plus 1 (see Appendix). Before the experimental session, two training trials in each condition were presented involving pairs of numbers that differed from those used in the experiment, but with the same constraints.

Each trial began with a letter indicating the task to be performed: “A” for addition, “S” for subtraction, and “C” for comparison. The participant was instructed to replace the letter by the first number by pressing the spacebar on the keyboard when she was ready. This procedure was introduced by Thevenot and Barrouillet (2006) to ensure that the self-presentation time of the first operand was not contaminated by any mental preparation to the forthcoming task and reflected only encoding time. In this previous study, the pattern of presentation times of the first operand was exactly the same when the signal for the task to be performed was either self-presented or displayed on screen for a fixed time of 1 s, strongly suggesting that mental preparation is terminated when participants press the key for presenting the first number. The participant removed this number from the screen and displayed the second, and then the third number by pressing the same key again. When the third number was displayed on screen, the participant was asked to give her answer (yes or no) by pressing one of two labelled keys on the keyboard. There was no mention of speed to respond when reading the operands but the participants were informed that the self-presentation times were recorded by the computer. This information was given to justify the fact that they had to keep their fingers on the labelled keys throughout the experiment.

The experiment was carried out using a PC and controlled by E-Prime software. The self-presentation times of the three numbers as well as the accuracy of the participants’ answers were recorded by the computer.

RESULTS

Rate of correct answers in the problem-solving task

Three children who did not understand the task in the comparison condition (i.e., two children systematically failed to give a “no” answer when the target
was inferior to the smaller number of the pair and one child systematically gave a “no” answer when the target was in fact lying between the two given numbers) were eliminated from the analyses, which were therefore conducted on the data of 27 children. A 3 (task: addition, subtraction, and comparison) × 2 (proposed answer: true and false) ANOVA with the two factors as repeated measures was performed on the rate of correct answers in the problem-solving task (Table 1). The effect of the task was significant, $F(2, 52) = 10.44$, $MSE = 0.02$, $p < .001$. Addition was associated with higher rate of correct answers (0.94) than subtraction (0.80), $F(1, 26) = 26.17$, $MSE = 0.02$, $p < .001$. or comparison (0.88), $F(1, 26) = 4.36$, $MSE = 0.02$, $p < .05$, and comparison was more often solved successfully than subtraction, $F(1, 26) = 5.08$, $MSE = 0.03$, $p < .04$. The effect of the proposed answer was also significant, $F(1, 26) = 8.11$, $MSE = 0.02$, $p < .009$: When the proposed answer was true, the rate of correct answers was lower (0.84) than when it was false (0.90). The significant interaction between the two factors, $F(2, 52) = 9.86$, $MSE = 0.01$, $p < .001$, revealed that the last effect was observable only for subtraction, $F(1, 26) = 15.03$, $MSE = 0.03$, $p < .001$, but not for addition ($F < 1$) or for comparison ($F < 1$).

**Mean self-presentation times of the letter indicating the task to be performed**

When children correctly reject a false answer, it is impossible to know if it is because they reached the correct result or because they reached an incorrect result that does not correspond to the proposed target. Therefore, the following analyses were carried out on the succeeded trials requiring a “yes” answer (i.e., hits). A one-way ANOVA was performed on the mean self-presentation times of the first letter indicating the task. The effect of the task was significant, $F(2, 52) = 3.92$, $MSE = 104859$, $p = .05$. Self-presentation times were similar in the addition (1600 ms) and in the comparison (1531 ms) conditions ($F < 1$). However, they were longer in the subtraction (1743 ms) than in the comparison condition, $F(1, 26) = 5.66$, $MSE = 106639$, $p < .05$.

<table>
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<tr>
<th>Additon</th>
<th>Subtraction</th>
<th>Comparison</th>
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<td>$M$</td>
<td>$SD$</td>
<td>$M$</td>
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<td>True</td>
<td>0.93</td>
<td>0.08</td>
</tr>
<tr>
<td>False</td>
<td>0.94</td>
<td>0.09</td>
</tr>
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</table>
$p < .03$, and longer in the subtraction than in the addition condition, $F(1, 26) = 4.14, \text{MSE} = 93392, p = .05$ (Table 2).

Mean self-presentation times of the first number

A one-way ANOVA performed on the mean self-presentation times of the first number revealed a significant effect of the task, $F(1, 26) = 6.13, \text{MSE} = 130920, p < .005$, by participants, and $F(2, 30) = 10.14, \text{MSE} = 48194, p < .001$, by items. Exactly as we observed in adults (Thevenot & Barrouillet, 2006), self-presentation times were shorter in the comparison (1720 ms) than in the addition (1937 ms), $F(1, 26) = 4.77, \text{MSE} = 132472, p < .04, F(2, 15) = 21.15, \text{MSE} = 35258, p < .001$, and the subtraction condition (2061 ms), $F(1, 26) = 13.10, \text{MSE} = 119663, p < .002, F(2, 15) = 15.86, \text{MSE} = 45444, p < .002$. However, the difference between addition and subtraction was not significant, $F(1, 26) = 1.48, \text{MSE} = 140624, p = .23, F(2) < 1$ (Table 2). Out of the 27 children of this study, 16 of them exhibited the expected pattern (i.e., longer self-presentation times of the first number both in the addition and subtraction conditions than in the comparison condition), $p < .002$ (binomial test).

Mean self-presentation times of the second number

A one-way ANOVA performed on the mean self-presentation times of the second number revealed a significant effect of the task, $F(1, 26) = 29.58, \text{MSE} = 2316663, p < .001$, by participants, and $F(2, 30) = 94.07, \text{MSE} = 366401, p < .001$, by items. Self-presentation times were longer for subtraction (4647 ms) than for addition (3776 ms), $F(1, 26) = 4.06, \text{MSE} = 2520312, p = .05, F(2, 15) = 6.59, \text{MSE} = 405776, p < .03$, longer for subtraction than for comparison (1557 ms), $F(1, 26) = 40.51, \text{MSE} = 3181622, p < .001, F(2, 15) = 119.07, \text{MSE} = 519839, p < .001$, and longer

\begin{table}[h]
\centering
\caption{Mean self-presentation times of the letter and the numbers in ms (and standard deviations) as a function of the task}
\begin{tabular}{lcccc}
\hline
 & \text{Addition} & & \text{Subtraction} & & \text{Comparison} \\
 & $M$ & $SD$ & $M$ & $SD$ & $M$ & $SD$ \\
\hline
Letter & 1600 & 646 & 1743 & 667 & 1531 & 425 \\
First number & 1937 & 729 & 2061 & 698 & 1720 & 701 \\
Second number & 3776 & 1802 & 4647 & 2763 & 1557 & 495 \\
Third number & 2553 & 2750 & 4160 & 4982 & 2720 & 1647 \\
\hline
\end{tabular}
\end{table}
for addition than comparison, $F(1, 26) = 53.28$, $MSE = 1248056$, $p < .001$, $F(1, 15) = 223.71$, $MSE = 173589$, $p < .001$ (Table 2).

Mean self-presentation times of the third number

A one-way ANOVA was performed on the mean self-presentation times of the third number. The effect of the task was marginally significant by participants, $F(2, 52) = 2.94$, $MSE = 7164907$, $p = .06$, and significant by items, $F(2, 30) = 36.28$, $MSE = 1054561$, $p < .001$. Self-presentation times did not differ significantly in the addition (2553 ms) and in the comparison (2720 ms) conditions, both $F1$ and $F2 < 1$. The analysis by participants did not reveal any differences between the subtraction (4160 ms) and comparison conditions, $F(1, 26) = 2.19$, $MSE = 12764500$, $p = .15$, but the analysis by items revealed a significant difference, $F(2, 30) = 46.63$, $MSE = 1279356$, $p < .001$. Finally both analyses revealed that self-presentation times of the third number were longer in the subtraction than in the addition condition, $F(1, 26) = 7.03$, $MSE = 4957180$, $p < .02$, $F(2, 30) = 40.38$, $MSE = 1362941$, $p < .001$ (Table 2). As it can be noted in Table 2, standard deviations were very high in the addition and subtraction conditions. This was mainly due to one child who took more than 24 s to take his decision in the condition subtraction and 14 s in the condition addition. Conversely, his or her self-presentation times of the second number were shorter than those of the other children. Although the main strategy used by children was to achieve the calculations as soon as the second number was presented on screen, this particular child probably waited until the presentation of the third number before engaging in calculations. However, it has to be noted that all the patterns of results described here, and particularly the pattern associated with mean self-presentation times of the first number, were not affected when this child was discarded from the analyses.

DISCUSSION

This paper shows that, exactly as in adults, the way children process numerical information is affected by the nature of the task in which numbers are involved: It is quicker for fifth-graders to encode numbers when they are involved in a comparison, which requires a simple analogical coding in Dehaene’s triple code model, than when they are involved in complex addition and subtraction, which require the coordination of the Arabic and verbal codes in order to be mentally manipulated. To our knowledge, this study is the first one underlining the role of context in numerical processing in children. This original result, and the fact that both children and adults
show a context-dependent number processing, are in line with Dehaene’s number sense theory. According to this theory, new mental representations of numbers are integrated and articulated to earlier conceptual representations, which are common to all individuals. Once the different circuits are integrated, there would be no reason for the resulting complex system to be different in children and adults. Accordingly, beyond differences in response times reflecting an increase in processing efficiency, the pattern of encoding times observed in children perfectly fitted adults’ encoding times reported by Thevenot and Barrouillet (2006).

Though the results were in line with our expectations, it could be argued that the differences in the self-presentation times of numbers were not due to differences in the encoding process per se, but to processes such as additional mental preparation or cognitive efforts put into the more difficult tasks, namely complex addition and subtraction. If it were the case, our results would not give direct evidence for Dehaene’s triple code model and would also be compatible with models that postulate the sole existence of abstract representations providing the basis for any subsequent number processing (McCloskey, 1992, for example). It turns out that, in the present experiment, the complexity of the number mental representation required to execute the task is not confounded with the complexity of the task. Indeed, for children, mental subtraction problems are more difficult than mental addition problems. Studies that directly compare children performance in addition and subtraction are scarce. Yet, Barrouillet, Mignon, and Thevenot (2008) investigated the strategies used by third-graders in solving the 81 elementary subtractions that are the inverses of the one-digit additions with addends from 1 to 9 and compared their results to the ones obtained on the 81 elementary additions studied by Barrouillet and Lépine (2005). The authors showed that subtractions were significantly more difficult than additions. The same result is obtained in this study with older children. Indeed, the rate of errors in the problem-solving task was higher for the subtraction condition than for the addition and comparison conditions. Moreover, self-presentation times of the second and third numbers were longer in the subtraction condition than in the addition and comparison conditions, which attest that it took longer for children to reach the correct answer and solve the problem when a subtraction rather than an addition or a comparison had to be performed. Then, in our experiment, the complexity of the representation required to perform a complex calculation is not confounded with the objective difficulty of the task: Even if subtraction is objectively more difficult than addition, self-presentation times of the first number involved in such activities is the same in both conditions.

However, although our results cannot be interpreted in terms of objective difficulty of the task, one could argue that, especially if we consider that
no feed-backs were given to children in our experiment, it is the subjective difficulty of the task that explains our results. Nevertheless, the results obtained on the self-presentation times of the letter indicating the nature of the task to be performed preclude this other alternative interpretation: The mean self-presentation time of the letter “S” for subtraction was longer than the mean self-presentation times of the letters “A” and “C” for addition and comparison, respectively. By giving the participants the opportunity to control the presentation time of the letter that indicated the nature of the subsequent task, we gave them the possibility to control the time required for their mental preparation. Undoubtedly, the mental preparation duration is related to the experienced (i.e., subjective) difficulty of the task. Then, even if subtraction is subjectively more difficult than addition, self-presentation times of the first number involved in such activities is the same in both conditions. Therefore, neither the objective nor the subjective difficulty of the task can account for our result.

Another possible explanation of our results has been proposed by Campbell and Metcalfe (2008). They demonstrated that processing time for numerals can be affected by the following or the previous task. More precisely, they suggest that task switching (i.e., switching back to encoding after performing addition, subtraction, or comparison on the preceding trial) could account for our results because switch cost is greater after difficult task compared to easier task (Allport, Styles, & Hsieh, 1994; Campbell, 2005). As a consequence they argue that “Thevenot and Barrouillet paradigm could be influenced by factors not directly related to encoding the operands for the purpose of the planned operation” (pp. 234–235). However, in this experiment, as well as in Thevenot and Barrouillet (2006) and contrary to Campbell and Metcalfe’s studies, the trials were presented to participants in a completely random order. Therefore, there was no more probability for a given trial to be preceded by a specific condition (i.e., addition, subtraction, or comparison) than another condition. Consequently, asymmetrical switch costs across trials could not account for our results.

In conclusion, the present study is the first one showing evidence that 10-year-old children already present a context-dependent number processing: The time they need to encode numerical information depends on the complexity of the numerical mental representation required to solve the problem. Moreover, all our results converge on the fact that our paradigm permits to apprehend factors that are directly related to encoding processes per se rather than other factors such as mental preparation, cognitive preparation, or switch costs.
REFERENCES


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