Abstract

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Risk, Collateral and Investor Sentiment in Exchange Traded Funds

by

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Abstract

The exchange-traded funds (ETFs) have become very popular investment vehicles among individual and institutional investors alike. However, the international agencies, amongst which the IMF and the Financial Stability Board, have warned about the potential instability issues that may arise from ETFs. This thesis contributes to our understanding of these funds. First, we describe and quantify an overlooked risk behind direct investment into ETFs. Second, we design tools and methodology to improve investment guarantees. Finally, we explore the determinants of ETF investor choices.

The first chapter presents a framework to study counterparty risk and provide empirical estimates for a sample of physical and synthetic funds. We find that counterparty risk exposure is higher for swap-based ETFs, but that investors are compensated for bearing this risk.

In the second chapter, we theoretically derive the optimal composition of the collateral portfolio that minimizes the counterparty risk exposure of an ETF investor. Furthermore, we find, for a sample of swap-based ETFs, that the counterparty risk exposure is significantly lower with optimal collateral portfolios than with actual collateral portfolios.

Finally, we extract, in the third chapter, a novel investor sentiment measure from index option prices. Then, we empirically show that investor sentiment is related to flows entering ETFs. We also observe a correction on flows after three weeks, which is consistent with the idea that sentiment-based trading induces a slow market error correction.

Le premier chapitre présente une méthodologie permettant d’étudier et de quantifier le risque de contrepartie auquel les investisseurs dans les ETFs s’exposent. Au moyen d’un échantillon de fonds physiques et synthétiques, nous montrons que l’exposition au risque de contrepartie est supérieure pour les investissements dans les fonds basés sur des swaps mais que les investisseurs sont compensés pour supporter ce risque.

Dans le second chapitre, nous dérivons théoriquement la composition optimale du portefeuille de collatéral qui minimise l’exposition au risque de contrepartie des investisseurs dans un ETF. Nous trouvons, pour un échantillon de fonds synthétiques, que l’exposition au risque de contrepartie est en effet moins importante avec le portefeuille de collatéral optimal qu’avec le portefeuille de collatéral effectivement en usage.

Une nouvelle mesure de sentiment des investisseurs est finalement extraite des prix des options dans le troisième chapitre. Nous montrons que le sentiment des investisseurs est une composante des flux envers les ETFs. Nous observons ensuite une correction sur les flux qui est compatible avec l’idée que les investissements basés sur des informations non-fondamentales sont suivis par une correction du marché.
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3.1 Introduction
Introduction

During the past decade of accommodative monetary policies and low interest rates, investors’ demand for innovative financial products has grown steadily. Financial engineering has been tested to fulfill this demand, by introducing a wide variety of new derivatives tailored to meet the needs of investors in terms of return and risk. However, the increasing complexity of these products makes their risk assessment very difficult, and can lead to potentially catastrophic effects. For instance, the wide use of structured credit products in the 2000s and the misunderstanding of their underlying risks were at the root of the 2007-2008 financial crisis and the subsequent economic turmoil. Nowadays, investors are increasingly turning towards new complex collateralized products, namely the Exchange Traded Funds (ETFs). In order to avoid new financial disturbances, the international agencies, amongst which the IMF and the Financial Stability Board, have warned about the potential instability issues that may arise from ETFs. Although a deep comprehension of these financial innovations is crucial to ensure optimal investor choices and financial system steadiness, there is currently a lack of academic research that covers the open questions regarding the ETFs. Drawing on this, this thesis aims to bring additional knowledge about these new derivative products, such as the ETFs, in view of informing and protecting investors and consequently avoiding social welfare losses. First, we describe and quantify an overlooked risk behind direct investment into ETFs. Second, we design tools and methodology to improve investment guarantees. Finally, we explore the determinants of ETF investor choices.

Given their low fees and the ability to provide exposure to a variety of asset classes, ETFs have become very popular investment vehicles among individual and institutional investors alike. However, an ongoing criticism regarding the ETFs is the complexity to understand into which products the investor money is finally invested. In other words, an investor may end up, under extreme circumstances, with assets of undesirable qualities. As most ETF Funds engage in securities lending or are based on total return swaps, they expose their investors to counterparty risk. In order to ensure some cushion guarantee, collaterals are posted in case of a default of a counterparty. Given the importance of the collateral for investor’s safety, we investigate the reliability of this backup parachute in Chapter 1. Precisely, we aim to assess the quantity, the quality and the dynamics of the collateral used to protect ETF investors. Moreover, as with other risks in finance, the counterparty risk should be priced. Hence, we also consider whether the investors are compensated for taking on this risk. For all these tasks, we construct a new dataset which includes the composition of the holding and the collateral portfolios of more than 200 funds managed by two leading ETF issuers.

As a first step, we describe and quantify the investor counterparty risk exposure. The next question is then: can we improve the protection of the investors regarding the counterparty
risk? In the investor’s view, the posted guaranty must behave according to essential characteristics in order to offer the right protection against a counterparty default. In Chapter 2, we look to construct a set of clear and general rules for generating the optimal collateral. The goal is to choose among appropriate assets the composition of the collateral portfolio that best suites the protection needs of investors. Drawing analogies with the Markowitz portfolio theory, we want to provide a feasible framework and a methodology that would minimize the counterparty risk exposure of the investors. Moreover, we test the performance of our framework using actual data on ETF collateral portfolios.

By driving the market through the selling or buying orders, the investors hold a central place in the research on financial risk. In the perspective of the ETFs, investor choices can be observed in the process of share redemptions (outflows) and share purchases (inflows). This mechanism is important when we consider the risk attached to these products. Indeed, sizeable or unexpected redemptions can result in the funds experiencing difficulties in meeting investors’ requests. As creation/redemption is linked to the risks of ETF investors, it is important to understand the determinants of the investment flows into ETFs. Particularly, in Chapter 3 we proceed to study if the ETF investors are irrational, in other words, if the ETF flows are driven by non-fundamental information. Should this be the case, it would mean additional complexity in the understanding of the ETF market dynamics, which would need to be appropriately taken into account when managing investment risk.
Chapter 1

The Counterparty Risk Exposure of ETF Investors*

Abstract

As most exchange-traded funds (ETFs) engage in securities lending or are based on total return swaps, they expose their investors to counterparty risk. In this paper, we present a framework to study counterparty risk and provide empirical estimates for a sample of physical and synthetic funds. Our findings contradict the allegations made by international agencies about the poor quality of the collateral used by ETFs. Furthermore, we find that the counterparty risk exposure is higher for synthetic ETFs but that investors are compensated for bearing this risk. Using a difference-in-differences specification, we uncover that ETF flows respond significantly to changes in counterparty risk.

1.1 Introduction

With their low fees and ability to provide exposure to a variety of asset classes, exchange-traded funds (ETFs) have become popular investment vehicles among individual and institutional investors alike. The global ETF industry reaches a total of $2,600 billion in assets under management (AUM) in 2014-Q3 and has experienced an average growth of 30% per year for the past ten years (Blackrock, 2014).

ETFs come in two types. In a physical ETF, investors’ money is directly invested in the index constituents in order to replicate the index return. Differently in a synthetic ETF, the fund issuer enters into a total return swap with a financial institution which promises to deliver the performance of the index to the fund (Ramaswamy, 2011). An industry survey by Vanguard (2013) indicates that 17% of the ETFs in the US are synthetic compared to 69% in Europe.† Furthermore, many leveraged and inverse ETFs traded in the world are

* Joint work with Christophe Hurlin, Christophe Pérignon, and Stanley C.H. Yeung. Hurlin is at the University of Orléans, France; Pérignon and Yeung are at HEC Paris, France.
†Synthetic ETFs are less common in the US because (1) some swaps, such as those between affiliated parties, are generally not permitted under the Investment Company Act of 1940 and (2) swap income faces a higher tax rate than the capital gains incurred by transacting in a physical ETF’s underlying securities.
based on synthetic replications.\footnote{Leveraged ETFs provide exposure that is a multiple (2×, 3×) of the performance of the index, whereas inverse ETFs generate the inverse performance of the index. See Tang and Xu (2013).}

One may wonder why synthetic replication was invented. First, it is more convenient and cheaper for the ETF issuer to outsource the index replication rather than dealing itself with dividend flows, corporate events, changes in index composition, or storage for commodities.\footnote{An industry survey by Morningstar (2012) indicates that swap fees are extremely low and can even be zero if the swap is entered with an investment bank from the same banking group.} Second, swap-based replications limit investors' exposure to tracking error risk (Vanguard, 2013). Third, synthetic replication greatly simplifies the tracking of illiquid assets as well as the issuance of inverse funds. Fourth, these swaps constitute a major source of funding for financial institutions and lead to synergies and cost saving with their investment banks which maintain large inventories of equities and bonds. Finally, they may also allow the banks that act as swap counterparty to reduce their regulatory capital by using high risk-weight securities as collateral. This can lead the ETF provider to gain additional performance given the composition of the collateral. Indeed, recent evolutions in banking regulation made equities more costly to hold for banks. The relative unattractiveness of equities constrains banks to unload shares from their balance sheet, either by selling them off or by pledging them out in order to reduce capital consumption. In equilibrium, as all banks are eager to post equities as collateral (instead of cash or government bonds), they accept to pay an extra spread to their counterparties.

In this paper, we empirically study the counterparty risk of ETF investors. Indeed, physical ETF issuers generate extra revenues by engaging in securities lending (Amenc et al., 2012; Bloomberg, 2013). Hence, there is a possibility that the lent securities will not be returned in due time. Furthermore, for synthetic ETFs, there is a risk that the total return swap counterparty will fail to deliver the index return. In order to mitigate counterparty risk, both securities lending positions and swaps must be collateralized. Recently, the Financial Stability Board (2011), the International Monetary Fund (2011), and Ramaswamy (2011) warned about potential financial stability issues that may arise from synthetic ETFs. The latter were accused of being poorly collateralized and allowing banks to engage in regulatory arbitrage by using risky assets as collateral.\footnote{The debate on ETF reached its climax in November 2011 when Laurence D. Fink, chief executive officer of BlackRock (the leading physical ETF issuer), publically criticized the synthetic ETFs issued by Société Générale’s asset management arm, Lyxor (the leading synthetic ETF issuer): “If you buy a Lyxor product, you’re an unsecured creditor of SocGen” (Bloomberg, 2011).}

We define counterparty risk of ETFs as the risk that the value of the collateral falls below the value of the Net Asset Value (NAV) of the fund when the fund counterparty is in default. Then, we study the composition of the collateral portfolios of 54 physical ETFs managed by the largest ETF issuer, iShares, and of 164 synthetic ETFs managed by the fifth largest ETF provider, db X-trackers. For each fund, we know the exact composition of the collateral portfolio every week between July 5, 2012 and November 29, 2012. This is to the best of our knowledge the first time that such a dataset is used in an academic study. The high granularity of our data allows us to study empirically the counterparty risk of ETFs for various asset exposures, regional exposures, and types of replication.

Our analysis of the collateral portfolios of synthetic funds reveals several important features. First, collateral portfolios are well diversified and their value often exceeds the NAV of the fund (the average collateralization is 108.4%). Second, there is a good fit between
the asset exposure of the fund (e.g. equity or fixed income) and the collateral used to secure the swap. This feature is extremely important given the fact that in the case of a default of the swap counterparty, the asset manager would need to sell the collateral in order to meet redemptions from investors. Third, ETF collateral is of high quality. The equities used as collateral mainly come from large, non-financial firms that exhibit good level of liquidity and positive correlation with the index tracked by the fund. Reassuringly, collateralized equities correlate less with the stock return of the swap counterparty than with the NAV. Turning to fixed-income securities, we notice that bonds predominantly have a AAA rating (65.5%). Furthermore, we find that ETF issuers tend to match the duration of the collateral with the duration of the fixed-income index tracked by the fund, which is sound risk-management practice.

In order to quantify the counterparty risk exposure of investors, we propose two original risk metrics: (1) the probability for a fund of not having enough collateral and (2) the magnitude of the collateral expected shortfall. Both measures are computed conditionally on the default of the fund counterparty. Overall we find that the estimated probability of being undercollateralized can be substantial but that the collateral shortfall remains moderate. When contrasting the level of counterparty risk of investors investing in synthetic and physical ETFs, we find that counterparty risk exposure is higher for synthetic funds but that investors are compensated for bearing this risk through lower tracking errors and similar or lower fees. We also show that ETF investors do care about counterparty risk. Using a difference-in-differences approach, we find that there are more outflows from synthetic ETFs after an increase in counterparty risk.

Our paper contributes to the growing literature on the potential financial stability issues arising from ETFs. Ben-David, Franzoni and Moussawi (2014) show that ETF ownership increases stock volatility through the arbitrage trade between the ETF and the underlying stocks and through inflows and outflows. Da and Shive (2013) find a strong relation between measures of ETF activity and return comovement at both the fund and the stock levels. Focusing on the real effects of ETF rebalancing activities, Bessembinder et al. (2014) report no evidence of predatory trading around the time of the Crude Oil ETFs rolls of crude oil futures (see also Bessembinder, 2014). Focusing on leveraged and inverse funds, Bai, Bond and Hatch (2012) find that late-day leveraged ETF rebalancing activity significantly moves the price of constituent stocks (see also Shum et al., 2014, and Tuzun, 2014). Finally, Cheng, Massa and Zhang (2013) show that ETFs strategically deviate from their indices and overweight promising stocks for which the ETF affiliated bank has superior (lending-related) information.

Unlike previous academic studies, we do not focus on the interplay between the ETFs and the assets they track. Instead, our study considers a source of risk for ETF investors that attracted significant attention from regulators, investors, and the media but, so far, little academic research: the counterparty risk of ETFs. Looking at one MSCI Emerging Markets ETF in January 2011, Ramaswamy (2011) shows that the collateral composition of the fund has very little overlap with the composition of the MSCI Emerging Market index itself. Because it is based on one fund only and remains at the aggregated level,

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Ramaswamy’ study does not tell us whether collateral mismatch is a widespread phenomenon. Furthermore, a rigorous study of the collateral of ETFs requires granular data about all collateral securities and, as acknowledged by Ramaswamy (2011) on page 9, “Extracting this information using the International Security Identifying Number (ISIN) provided for each of the collateral assets would be a cumbersome process”. We take on this task in the present paper and reach different conclusions.

We make several contributions to the existing literature. To the best of our knowledge, our study is the first attempt to assess empirically the quality of the collateral for a large and representative sample of ETFs. Overall, our analysis of the quality of the securities pledged by the fund counterparties do not support the allegations made about the overall poor quality of ETF collateral. This, of course, does not mean that the agencies were wrong at the time but we find no support for this claim using more recent data. Our second contribution is methodological: we develop several risk management tools that can be used by asset managers, regulators, and investors to quantify the counterparty risk of an ETF. Our framework allows us to compare the counterparty risk of several ETFs and identify the types of funds that expose the most their investors to counterparty risk. It can also be used to build a collateral portfolio with the lowest possible level of counterparty risk as in Chapter 2.

The rest of our study is structured as follows. Section 1.2 introduces the different types of ETF structures in a common framework and defines counterparty risk for ETFs. In Section 1.3, we present our dataset and discusses our main empirical findings. We conclude our study in Section 1.4.

1.2 ETF Structures and Counterparty Risk

1.2.1 Physical Model

Physical ETFs attempt to track their target indexes by holding all, or a representative sample, of the underlying securities that make up the index (see Figure 1.1, Panel A). For example, if you invest in an S&P 500 ETF, you own each of the 500 securities represented in the S&P 500 Index, or some subset of them. Almost all ETF issuers have the provision in their prospectus for loaning out their stock temporarily for revenue. For instance, iShares recently changed its maximum on-loan limit from 95% of the AUM to 50%.

Securities lending exposes ETF investors to counterparty risk (Amenc et al., 2012). In order to mitigate this risk, short sellers have to post collateral with the ETF issuer. On a given point in time, if we denote by $I_t$, the NAV of the ETF, $\beta_t \in [0, 1]$ the fraction of the securities that are lent, and $C_t$ the value of the collateral per share, a collateral shortfall occurs if:

$$\beta_t I_t - C_t > 0 \quad (1.1)$$

Such a situation can be problematic if the short sellers cannot return to the ETF the borrowed securities in due time, i.e., if some of them default. In that case, the fund will not be able to meet redemption requests from all ETF investors.
1.2. ETF Structures and Counterparty Risk

Figure 1.1: ETF Structures

Panel A: Physical ETF

Panel B: Unfunded Swap ETF

Panel C: Funded Swap ETF

Notes: This figure describes the different cash-flows and asset transfers for three types of ETF structures: the physical ETF (Panel A), the unfunded-swap based ETFs (Panel B) and the funded-swap based ETFs (Panel C). The latter two are synthetic ETFs.

1.2.2 Unfunded Swap Model

First introduced in Europe in the early 2000’s, synthetic ETFs are an interesting variant of physical ETFs. The most commonly used structure for synthetic replications is the unfunded swap model. In this model, the ETF issuer enters into a total return swap with a counterparty, which can either be an affiliated bank from the same banking group or another bank (see Figure 1.1, Panel B). The swap counterparty commits to deliver the return of the reference index and sells a substitute basket of securities to the ETF issuer.
The second leg of the swap consists of the performance of the basket of securities paid by the issuer to the swap counterparty. An important feature of this model is that the ETF issuer becomes the legal owner of the assets and enjoys direct access to them. This means that if the swap counterparty defaults, the ETF issuer can immediately liquidate the assets.\(^6\)

The counterparty exposure of the issuer, or swap value, is measured as the difference between the NAV, \(I_t\), and the value of the substitute basket (per share) used as collateral, \(C_t\). The swap is marked to market at the end of each day and reset whenever the counterparty exposure exceeds a given threshold, \(\theta \in [0,1]\), expressed as a percentage of the NAV:

\[
I_t - C_t > \theta I_t. 
\]  
(1.2)

In the event of a reset, the swap counterparty delivers additional securities for a value of \(I_t - C_t\) to top up the substitute basket. The European directive UCITS sets \(\theta\) to 10%. In other words, the value of the collateral must at least amount 90% of the NAV. However, a comprehensive industry survey conducted by Morgingstar (2012) indicates that most ETF issuers affirm using a \(\theta\) coefficient of less than 10% and some even claim to maintain full collateralization, i.e., \(\theta = 0\). Thus, in the rest of this analysis, without loss of generality, we will set \(\theta\) to zero.

### 1.2.3 Funded Swap Model

In the funded swap model, the ETF issuer transfers investors’ cash to a swap counterparty in exchange for the index performance plus the principal at a future date (see Figure 1.1, Panel C). The swap counterparty pledges collateral assets in a segregated account with a third party custodian. The posted collateral basket is made of securities which come from the counterparty’s inventory and meet certain conditions in terms of asset type, liquidity, and diversification. In practice, appropriate haircuts apply to the assets posted as collateral to account for the risk of value fluctuations and for imperfect correlation between the index and the collateral value. As a consequence, funded-swap based ETFs are expected to be overcollateralized, \(C_t > I_t\).

The swap counterparty exposure is measured by the difference between the NAV and the collateral value, once properly adjusted for haircut. If we denote by \(h\) the haircut, it means that if:

\[
I_t - C_t (1 - h) > 0 
\]  
(1.3)

additional collateral corresponding to \(I_t - C_t (1 - h)\) is requested in order to maintain appropriate collateralization.\(^7\)

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\(^6\)The unfunded swap model is particularly attractive for tax reasons. For instance, it allows ETFs to be eligible to the French tax-friendly share savings plans, Plan d’Épargne en Actions (PEA). In theory, eligible mutual funds must be made up of at least 75% of shares of companies headquartered in a European member state. As a result, an ETF based on an unfunded swap and that tracks an index of Asian or US equities is eligible as long as the substitute basket is made of enough European equities.

\(^7\)If the collateral portfolio is made of several securities with different haircut levels, \(h\) is a weighted-average of these specific haircuts.
1.2.4 Counterparty Risk Metrics

In any ETF structure, the counterparty risk faced by the ETF issuer and ultimately by the investors, corresponds to the risk that the ETF is not being fully collateralized precisely when the counterparty defaults. Indeed, facing a collateral shortfall is only an issue when the counterparty is unable to meet further margin calls. We propose a global framework for all types of ETF structures. For any fund, a collateral shortfall corresponds to:

\[ \beta_t I_t - C_t (1 - h) > 0 \]  (1.4)

where \( \beta_t \in [0, 1] \) and \( h > 0 \) for physical ETFs, \( \beta_t = 1 \) and \( h = 0 \) for unfunded-swap based ETFs, and \( \beta_t = 1 \) and \( h > 0 \) for funded-swap based ETFs.\(^8\) If condition (1.4) is satisfied, additional collateral or reinvestment in the substitute basket is required to reach \( C_t (1 - h) = \beta_t I_t \) (Morningstar, 2012). Denote by \( \Delta_{t+1} \) the collateral shortfall of a given fund at time \( t + 1 \):

\[ \Delta_{t+1} = \beta_t I_{t+1} - C_{t+1} (1 - h) = \beta_t I_{t+1} (1 + r_{i,t+1}) - C_{t+1} (1 - h) \]  (1.5)

where \( r_{i,t+1} \) and \( r_{c,t+1} \) denote the return of the NAV and the return of the collateral portfolio, respectively. Given the information available at time \( t \), the collateral shortfall \( \Delta_{t+1} \) is stochastic since the returns \( r_{i,t+1} \) and \( r_{c,t+1} \) are unknown.

Within this framework, we propose two metrics for counterparty risk. The first metric we define is the probability for a fund of being undercollateralized conditionally on a default of the counterparty:

\[ P_{t+1} = \Pr (\Delta_{t+1} > 0 \mid F_t, D_{t+1}) \]  (1.6)

where \( F_t \) denotes the set of information available at time \( t \) and \( D_{t+1} \) denotes the counterparty default at time \( t + 1 \). This probability measures the likelihood of a collateral shortfall at the end of the next trading day, conditionally on a default of the counterparty, if no adjustment is made on the composition and the amount of collateral used at time \( t \). A second important dimension of counterparty risk is the magnitude of the collateral shortfall or expected collateral shortfall:

\[ S_{t+1} = \mathbb{E} (\Delta_{t+1} \mid \Delta_{t+1} > 0, D_{t+1}) \]  (1.7)

where \( \mathbb{E} \) denotes the expectation conditional on \( F_t \). This metric corresponds to the collateral shortfall a fund is expected to experience conditionally on the default of the counterparty.

1.2.5 Estimation Strategy and the Torino Scale

We estimate the counterparty risk metrics using a simple nonparametric method. We define \( \omega_t = (\omega_{1,t}, ..., \omega_{K,t}) \), the vector of the weights associated with the \( K \) assets that comprise the collateral portfolio of a fund at time \( t \), with \( \sum_{k=1}^{K} \omega_{k,t} = 1 \). Given Equation (1.5), the collateral shortfall at time \( t + 1 \) can be written as:

\[ \Delta_{t+1} = \beta_t I_{t+1} (1 + r_{i,t+1}) - C_{t+1} (1 - h) \left( 1 + \sum_{k=1}^{K} \omega_{k,t} r_{k,t+1} \right) \]  (1.8)

\(^8\)We assume that the swap is not reset if the fund is overcollateralized \( (\beta_t I_t - C_t (1 - h) < 0) \) since this does not induce any counterparty risk.
where $r_{k,t+1}$ is the daily return of the $k$-th collateral security at time $t + 1$. Given the information set $\mathcal{F}_t$, the potential collateral shortfall at time $t + 1$ only depends on $r_{i,t+1}$ and $r_{k,t+1}$ for $k = 1, ..., K$.

One way to estimate the shortfall probability and the expected collateral shortfall is to assume a given distribution for these returns and to derive closed-form expressions for $P$ and $S$. Alternatively, we follow Berkowitz and O’Brien (2002) and consider a series of hypothetical collateral shortfalls:

$$\Delta_\tau = \beta_t I_t (1 + r_{i,\tau}) - C_t (1 - h) \left( 1 + \sum_{k=1}^K \omega_{k,t} r_{k,\tau} \right)$$  

(1.9)

where $r_{i,\tau}$ is the historical daily return of the NAV, for $\tau = 1, ..., t$ and $r_{k,\tau}$ is the daily return of the $k$-th collateral security at time $\tau$. The hypothetical collateral shortfall $\Delta_\tau$ measures the shortfall that would have arisen in the past with the current values of $I_t$, $C_t$, and $\omega_t$ and past returns on the NAV and on the collateral securities. However, the counterparty in our sample never actually defaulted in the past. As a result, we estimate the counterparty risk metrics using past observations from a high counterparty-risk regime, i.e., a period during which the counterparty experienced a sharp increase in its default probability. We define two nonparametric estimators for the probability and the expected collateral shortfall:

$$\hat{P}_{t+1} = \frac{1}{\dim(\Upsilon)} \sum_{\tau \in \Upsilon} I(\Delta_\tau > 0) \times I(\tau \in \Upsilon)$$  

(1.10)

$$\hat{S}_{t+1} = \frac{\sum_{\tau=1}^t \Delta_\tau \times I(\Delta_\tau > 0) \times I(\tau \in \Upsilon)}{\sum_{\tau=1}^t I(\Delta_\tau > 0) \times I(\tau \in \Upsilon)}$$  

(1.11)

where $I(\cdot)$ denotes the indicator function and $\Upsilon$ denotes a high-counterparty risk regime. Note that if $\Delta_\tau$ is a stationary process, these estimators are consistent and asymptotically normally distributed (Chen, 2008).

Finally, we propose an original approach to graphically summarize the counterparty risks of ETFs in a two-dimensional graph which is called a Torino scale. The latter is a tool developed in the mid 1990’s by MIT Earth, Atmospheric, and Planetary Sciences Professor Richard P. Binzel for categorizing asteroids. The scale aims to measure asteroid-specific collision probabilities with the earth and the energy generated by such collisions. Torino scales are useful whenever one needs to jointly display the probability of an event, the "impact", and the effects of this event, the "energy". In our context, the "impact" corresponds to the occurrence of a collateral shortfall in a fund and the "energy" to the magnitude of the collateral shortfall.

The funds that are located in the North-East corner of the Torino scale are the ones that are the most exposed to counterparty risk, as shown in Figure 1.2. The key advantage of this graphic representation is that it permits to easily compare the risk for different ETFs or to monitor the risk of a given fund through time. Hence, it provides a global view of the counterparty risk in the ETF market, which may be of interest for both regulators and investors.
1.3. Empirical Analysis

Our empirical analysis is based on a sample of 218 ETFs with combined AUM of $115.4 billion, which corresponds to more than 6% (respectively 45%) of the total AUM of all ETFs in the world (respectively in Europe) (Blackrock, 2012). An attractive feature of our sample is that it includes both synthetic ETFs issued by db X-trackers and physical ETFs issued by iShares.
Our dataset of synthetic ETFs is made of public data retrieved from the db X-trackers website (www.etf.db.com). Data on physical ETFs have been collected from the iShares website (www.ishares.com). In our analysis, we only consider the funds for which we have a complete history of weekly collateral between July 5, 2012 and November 29, 2012 and at least one year of data for the ETF price and its index.

We see in Table 1.1 that the 164 synthetic ETFs have a combined AUM of $37.9 billion, which accounts to around 30% of the total AUM of all synthetic ETFs in Europe (Vanguard, 2013). Most synthetic funds are based on a funded swap (112 funds vs. 52 funds based on an unfunded swap). However, both types of funds account for comparable AUM ($20.1 billion vs. $17.8 billion). It is also important to notice that a significant fraction of the synthetic funds (30 funds and 5.1% of AUM) are inverse funds that deliver the inverse performance of an index.

In terms of asset exposure, the majority of the synthetic funds track a stock index (74.5% of AUM). Besides equity, the other funds allow investors to be exposed to a variety of asset classes including Government bonds (11% of AUM), treasuries and commercial papers (6.6%), commodities (3.8%), hedge funds (2.2%), credit (0.7%), corporate bonds (0.6%), and currencies (0.3%). In that sense, our sample is representative of the entire ETF industry as the share of equity ETFs is around 70% and these of fixed-income funds, commodity funds, and currency funds are 18%, 11%, and 0.3%, respectively (Blackrock, 2012). Furthermore, half of the sample funds track European indices (79 funds out of 164 and 58.9% of AUM) whereas the remaining funds replicate the returns of some World indices (22.3%), Asian-Pacific indices (9.4%), or North-American indices (7.2%).

As shown in Table 1.1, the sample of physical ETFs is more than twice larger than the synthetic one ($77.5 billion vs. $37.9 billion of AUM). Similar to the synthetic dataset, physical funds mainly track equity indices (around 70% of AUM) and Government bond indices (around 10%). However, corporate bond funds represent close to 20% of the aggregate AUM of physical funds. Furthermore, the physical fund sample is primarily made of funds that track European indices (47.6% of AUM) and world indices (24.9%).

### 1.3.1 A First Look at the Collateral Portfolios

Allegations were recently made about the overall poor quality of ETF collateral. For instance, the Financial Stability Board (2011, page 4) states: "As there is no requirement for the collateral composition to match the assets of the tracked index, the synthetic ETF creation process may be driven by the possibility for the bank to raise funding against an illiquid portfolio [...] the collateral basket for a S&P 500 synthetic ETF could be less liquid equities or low or unrated corporate bonds in an unrelated market."

To formally test for the validity of these allegations, we collect for each sample fund the composition and the value of its collateral portfolio with a weekly frequency between July 5, 2012 and November 29, 2012. The collateral data have been retrieved from the db X-trackers and iShares websites but because the websites keep no historical data, we had to download the collateral data for each fund, every week over our sample period. Then for each security used as collateral, we obtain its historical daily prices from Datastream.\footnote{For bonds, we use the returns of the bond index that best matches the attributes of the bonds.}
## Table 1.1: Summary Statistics on ETFs

<table>
<thead>
<tr>
<th></th>
<th>Synthetic</th>
<th>Funded</th>
<th>Unfunded</th>
<th>Long</th>
<th>Inverse</th>
<th>Physical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of ETF Funds</td>
<td>164</td>
<td>112</td>
<td>52</td>
<td>134</td>
<td>30</td>
<td>54</td>
</tr>
<tr>
<td>AUM ($ Mio)</td>
<td>Total</td>
<td>37,927</td>
<td>20,122</td>
<td>17,805</td>
<td>36,011</td>
<td>1,916</td>
</tr>
<tr>
<td>Asset Exposure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td>74.5% (111)</td>
<td>85.5% (100)</td>
<td>61.9% (11)</td>
<td>74.0% (90)</td>
<td>82.2% (21)</td>
<td>70.1% (38)</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>11.0% (24)</td>
<td>2.2% (2)</td>
<td>20.8% (22)</td>
<td>10.8% (21)</td>
<td>13.1% (3)</td>
<td>10.0% (8)</td>
</tr>
<tr>
<td>Money Markets</td>
<td>6.6% (4)</td>
<td>-</td>
<td>14.0% (4)</td>
<td>7.0% (4)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Commodities</td>
<td>3.8% (2)</td>
<td>7.2% (2)</td>
<td>-</td>
<td>4.0% (2)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hedge Funds Strategies</td>
<td>2.2% (6)</td>
<td>3.9% (3)</td>
<td>0.3% (3)</td>
<td>2.3% (5)</td>
<td>0.4% (1)</td>
<td>-</td>
</tr>
<tr>
<td>Credits</td>
<td>0.7% (9)</td>
<td>-</td>
<td>1.6% (9)</td>
<td>0.5% (4)</td>
<td>4.3% (5)</td>
<td>-</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>0.6% (3)</td>
<td>-</td>
<td>1.4% (3)</td>
<td>0.7% (3)</td>
<td>-</td>
<td>19.9% (8)</td>
</tr>
<tr>
<td>Currencies</td>
<td>0.3% (4)</td>
<td>0.6% (4)</td>
<td>-</td>
<td>0.4% (4)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi Assets</td>
<td>0.3% (1)</td>
<td>0.6% (1)</td>
<td>-</td>
<td>0.3% (1)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Geographic Exposure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>58.9% (79)</td>
<td>29.4% (41)</td>
<td>92.4% (38)</td>
<td>57.4% (54)</td>
<td>84.1% (25)</td>
<td>47.6% (33)</td>
</tr>
<tr>
<td>World</td>
<td>22.3% (41)</td>
<td>37.0% (38)</td>
<td>5.5% (3)</td>
<td>23.6% (40)</td>
<td>1.1% (1)</td>
<td>24.9% (10)</td>
</tr>
<tr>
<td>Asia-Pacific</td>
<td>9.4% (28)</td>
<td>17.6% (24)</td>
<td>0.2% (4)</td>
<td>9.9% (27)</td>
<td>1.3% (1)</td>
<td>8.4% (8)</td>
</tr>
<tr>
<td>North America</td>
<td>7.2% (14)</td>
<td>11.9% (7)</td>
<td>1.9% (7)</td>
<td>6.8% (11)</td>
<td>13.5% (3)</td>
<td>19.1% (3)</td>
</tr>
<tr>
<td>Rest of the World</td>
<td>2.2% (2)</td>
<td>4.1% (2)</td>
<td>-</td>
<td>2.3% (2)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: This table presents some summary statistics for synthetic ETFs (columns 1-5) and physical ETFs (column 6). For synthetic ETFs, the statistics are also presented separately for funded-swap based ETFs, unfunded-swap based ETFs, long ETFs, and inverse ETFs. The table displays the total number of funds, the combined assets under management (AUM) in USD million, the value-weighted fraction of AUM by asset and geographic exposures, along with the number of funds in parentheses. The sample period is July 5, 2012 - November 29, 2012.
### Table 1.2: Size and Turnover of Collateral Portfolios

<table>
<thead>
<tr>
<th></th>
<th>Synthetic</th>
<th>Funded</th>
<th>Unfunded</th>
<th>Long</th>
<th>Inverse</th>
<th>Physical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collateral Value ($ Mio)</td>
<td>40,939</td>
<td>23,083</td>
<td>17,856</td>
<td>38,706</td>
<td>2,233</td>
<td>6,237</td>
</tr>
<tr>
<td>% of securities lent</td>
<td>All</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.47%</td>
</tr>
<tr>
<td>Collateralization</td>
<td>All</td>
<td>108.4%</td>
<td>114.6%</td>
<td>101.3%</td>
<td>107.9%</td>
<td>115.4%</td>
</tr>
<tr>
<td>Equities</td>
<td>109.6%</td>
<td>115.7%</td>
<td>99.9%</td>
<td>109.0%</td>
<td>117.8%</td>
<td>109.7%</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>102.8%</td>
<td>100.7%</td>
<td>103.1%</td>
<td>102.8%</td>
<td>102.8%</td>
<td>109.3%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>105.4%</td>
<td>-</td>
<td>105.4%</td>
<td>105.4%</td>
<td>-</td>
<td>106.7%</td>
</tr>
<tr>
<td>Number of Collateral Securities</td>
<td>All</td>
<td>3,299</td>
<td>3,014</td>
<td>1,141</td>
<td>3,253</td>
<td>2,511</td>
</tr>
<tr>
<td>Average Number of Collateral Securities per ETF Fund</td>
<td>All</td>
<td>81</td>
<td>110</td>
<td>18</td>
<td>90</td>
<td>43</td>
</tr>
<tr>
<td>Equities</td>
<td>109</td>
<td>117</td>
<td>35</td>
<td>120</td>
<td>58</td>
<td>430</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>14</td>
<td>13</td>
<td>16</td>
<td>15</td>
<td>9</td>
<td>114</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>12</td>
<td>-</td>
<td>12</td>
<td>12</td>
<td>-</td>
<td>233</td>
</tr>
<tr>
<td>Turnover</td>
<td>All</td>
<td>34.0%</td>
<td>47.4%</td>
<td>5.0%</td>
<td>33.5%</td>
<td>36.3%</td>
</tr>
<tr>
<td>Equities</td>
<td>43.9%</td>
<td>46.5%</td>
<td>18.8%</td>
<td>42.1%</td>
<td>51.3%</td>
<td>61.3%</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>1.3%</td>
<td>0.5%</td>
<td>1.4%</td>
<td>1.3%</td>
<td>1.6%</td>
<td>70.3%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>2.4%</td>
<td>-</td>
<td>2.4%</td>
<td>2.4%</td>
<td>-</td>
<td>66.3%</td>
</tr>
</tbody>
</table>

Notes: This table presents some summary statistics on the size and turnover of the collateral portfolios of synthetic ETFs (columns 1-5) and physical ETFs (columns 6). For synthetic ETFs, the statistics are also presented separately for funded-swap based ETFs, unfunded-swap based ETFs, long ETFs, and inverse ETFs. The table displays the collateral value in USD million, the fraction of AUM loaned on the securities lending market (only for physical ETFs), the value-weighted average level of collateralization (collateral value/AUM), the total number of collateral securities, the average number of collateral securities, and the average turnover. The latter is defined as the ratio between the number of different securities, as defined by their ISIN, that enter or exit the collateral portfolio between two dates divided by the total number of collateral securities on both dates. Results are broken down by asset exposure: equities, government bonds, and corporate bonds. The sample period is July 5, 2012 - November 29, 2012.
Table 1.3: Types of Collateral Securities

Panel A: Collateral Securities of Synthetic ETFs

<table>
<thead>
<tr>
<th>Type of Collateral Securities</th>
<th>Equity</th>
<th>Government Bonds</th>
<th>Corporate Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Collateral Securities</td>
<td>2,591</td>
<td>490</td>
<td>218</td>
</tr>
<tr>
<td>ETF Asset Exposure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>74.9%</td>
<td>19.7%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Equity</td>
<td>92.5%</td>
<td>2.7%</td>
<td>4.8%</td>
</tr>
<tr>
<td>Government Bonds</td>
<td></td>
<td>96.5%</td>
<td></td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>-</td>
<td>100%</td>
<td>-</td>
</tr>
<tr>
<td>Others</td>
<td>40.8%</td>
<td>48.8%</td>
<td>10.4%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geographic Origin of the Collateral Securities</th>
<th>Europe</th>
<th>Asia-Pacific</th>
<th>N. America</th>
<th>R. World</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETF Geographic Exposure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>66.0%</td>
<td>17.5%</td>
<td>16.3%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Europe</td>
<td>71.8%</td>
<td>13.9%</td>
<td>13.9%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Asia-Pacific</td>
<td>56.0%</td>
<td>24.1%</td>
<td>19.8%</td>
<td>0.1%</td>
</tr>
<tr>
<td>North America</td>
<td>58.3%</td>
<td>22.1%</td>
<td>19.5%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Rest of the World</td>
<td>58.1%</td>
<td>25.5%</td>
<td>16.3%</td>
<td>0.1%</td>
</tr>
<tr>
<td>World</td>
<td>58.8%</td>
<td>21.7%</td>
<td>19.4%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Panel B: Collateral Securities of Physical ETFs

<table>
<thead>
<tr>
<th>Type of Collateral Securities</th>
<th>Equity</th>
<th>Government Bonds</th>
<th>Corporate Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Collateral Securities</td>
<td>4,893</td>
<td>234</td>
<td>-</td>
</tr>
<tr>
<td>ETF Asset Exposure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>97.2%</td>
<td>2.8%</td>
<td>-</td>
</tr>
<tr>
<td>Equity</td>
<td>98.8%</td>
<td>1.2%</td>
<td>-</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>90.1%</td>
<td>9.9%</td>
<td>-</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>90.8%</td>
<td>9.2%</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geographic Origin of the Collateral Securities</th>
<th>Europe</th>
<th>Asia-Pacific</th>
<th>N. America</th>
<th>R. World</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETF Geographic Exposure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>18.2%</td>
<td>50.8%</td>
<td>30.9%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Europe</td>
<td>19.4%</td>
<td>46.3%</td>
<td>34.2%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Asia-Pacific</td>
<td>18.6%</td>
<td>49.0%</td>
<td>32.3%</td>
<td>0.1%</td>
</tr>
<tr>
<td>North America</td>
<td>15.1%</td>
<td>54.3%</td>
<td>30.5%</td>
<td>0.1%</td>
</tr>
<tr>
<td>World</td>
<td>18.8%</td>
<td>53.8%</td>
<td>27.3%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Notes: This table presents some summary statistics on the securities used as collateral for synthetic ETFs (Panel A) and physical ETFs (Panel B). It displays the number of collateral securities per type of collateral and the value-weighted average percentage of collateral that is held in equity, governments bonds, and corporate bonds, respectively. It also presents for each type of ETF geographic exposure, the value-weighted percentage of collateral that comes from Europe, Asia-Pacific, North America, and Rest of the World, respectively. The government bond category also includes supranational bonds, government guaranteed bonds, government agency bonds, and German regional government bonds. The corporate bond category also includes covered bonds. The sample period is July 5, 2012 - November 29, 2012.

In Table 1.2, we see that the aggregate size of all collateral portfolios is equal to $40.9 billion for synthetic ETFs, which indicates that, on average, the funds included in our analysis are overcollateralized (AUM is $37.9 billion). For a given synthetic fund, the value-weighted average level of collateralization is 108.4%. However, the level of overcollateralization is higher for funded-swaps based ETFs (114.6%) and for inverse funds (115.4%). The result for funded swaps comes from the fact that haircuts are applied to
Chapter 1. The Counterparty Risk Exposure of ETF Investors

the value of the pledged collateral. Differently, no haircut applies for unfunded swaps as, in this case, the asset managers purchase some securities from its swap counterparty, i.e., the substitute basket. In total, there are 3,299 different securities that are used as collateral in the synthetic ETF sample, which leads to 81 collateral securities per fund on average. We notice that the number of securities is much higher for equity (around 100 securities per fund) than for fixed-income funds (10 to 20 securities per fund).

The situation is fairly different for physical funds as only a fraction of the AUM needs to be collateralized, namely the part that is loaned out. On any given day, a typical fund lends 7.5% of its AUM on the securities lending market but the maximum value in our sample is 94.3% (the average of the maximum lending ratios is 18.5%). We also notice that securities lending is more important in Government bond funds (17.2%) than in equity funds (6.5%) or corporate bond funds (6%). Similar to synthetic funds, physical funds are also overcollateralized, with collateral value accounting for 109.1% of the values of the lent securities. The level of diversification of the collateral portfolios is even higher for physical funds as they include hundreds of different collateralized securities (355 on average).

We also report significant time variations in the composition of the collateral portfolios of ETFs. For a given fund, we define the turnover of the collateral portfolio as the ratio between the number of different securities, as defined by their ISIN, that enter \((n^+_{t,t+1})\) or exit \((n^-_{t,t+1})\) the collateral portfolio between \(t\) and \(t+1\) divided by the total number of collateral securities \((N_t)\) on both dates:

\[
\text{turnover}_{t,t+1} = \frac{n^+_{t,t+1} + n^-_{t,t+1}}{N_t + N_{t+1}}. \tag{1.13}
\]

For synthetic (physical) funds, more than a third (two thirds) of the pledged collateral in a given fund changes from one week to the next (see bottom of Table 1.2). Changes through time are particularly strong for equities in synthetic funds and for Government bonds in physical funds.

1.3.2 The Match between Index and Collateral

One of the most persistent criticisms addressed to ETFs is the fact that the collateral may not be positively correlated with the index tracked by the ETF. Indeed, when the correlation is negative, the hedge provided by the collateral is less efficient: if the index return is large and positive when the fund counterparty defaults, the value of the collateral shrinks and a collateral shortfall mechanically arises. To look at this issue empirically, we compare the index tracked by the ETF and the securities included in the collateral portfolio. In Panel A of Table 1.3, we notice that for synthetic funds most of the collateral is made of equities: when measured in value, equities account for around 75% of the collateral vs. 20% for Government bonds and 5% for corporate bonds. The most important finding in this panel is that there is a good match between the index tracked and the collateral as 92.5% of equity ETFs are backed with equity and 96.5% of Government bonds ETFs are collateralized with Government bonds. The match is also pretty good for funds that track European indices as 71.8% of the collateral are made of
1.3. Empirical Analysis

securities issued by European firms. The matching scores between exposure and issuers are significantly lower for ETFs tracking Asian-Pacific and North-American indices.

The situation for physical ETFs in Panel B of Table 1.3 contrasts sharply with the one of synthetic ETFs. Indeed, iShares mainly use equities as collateral as they account for 97.2% of the posted collateral. As a result, the match between the index and the collateral is very high for equity funds (98.8%) but much lower for Government funds (9.9%) and corporate funds (0%). Another strong feature of the collateral used by iShares is the predominant role played by equities issued by Asian (almost exclusively Japanese) companies, which account for more than half of the value of the posted collateral.

Figure 1.3: Correlation between ETF Return and Collateral Return

Notes: This figure presents the nonparametric kernel smoothing probability density function of the estimated correlation coefficients between the return of the NAV and the return of the collateral portfolio. The estimation is based on all weekly combinations of NAV and collateral returns.

To understand the predominant role played by European collateral, which we dub "collateral home bias", one needs to understand the origin of the pledged collateral. Indeed, these securities come from the books of the swap counterparty, typically a large financial institution. In our sample, the swap counterparty is Deutsche Bank and as a result, its books predominantly include securities issued by local firms held for investment purposes, market making, or other intermediation activities.
Table 1.4: Regression Analysis of the ETF-Collateral Correlation

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Physical</th>
<th>Synthetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(AUM)</td>
<td>-0.025</td>
<td>0.009</td>
<td>-0.040</td>
</tr>
<tr>
<td></td>
<td>(-0.77)</td>
<td>(0.16)</td>
<td>(-1.10)</td>
</tr>
<tr>
<td>Funded Swap</td>
<td>-0.444***</td>
<td></td>
<td>-0.215</td>
</tr>
<tr>
<td></td>
<td>(-3.36)</td>
<td></td>
<td>(-0.91)</td>
</tr>
<tr>
<td>Inverse</td>
<td>-3.019***</td>
<td></td>
<td>-3.102***</td>
</tr>
<tr>
<td></td>
<td>(-15.45)</td>
<td></td>
<td>(-17.15)</td>
</tr>
<tr>
<td>Collateralization</td>
<td>0.003*</td>
<td>-0.049</td>
<td>0.004**</td>
</tr>
<tr>
<td></td>
<td>(1.79)</td>
<td>(-1.32)</td>
<td>(2.41)</td>
</tr>
<tr>
<td>log(# Securities)</td>
<td>0.076*</td>
<td>0.077</td>
<td>0.063**</td>
</tr>
<tr>
<td></td>
<td>(1.82)</td>
<td>(1.44)</td>
<td>(2.55)</td>
</tr>
<tr>
<td>Equity Fraction</td>
<td>0.010</td>
<td>0.0169*</td>
<td>0.002*</td>
</tr>
<tr>
<td></td>
<td>(1.48)</td>
<td>(1.95)</td>
<td>(1.94)</td>
</tr>
<tr>
<td>Europe Fraction</td>
<td>0.005***</td>
<td>0.006***</td>
<td>0.003***</td>
</tr>
<tr>
<td></td>
<td>(3.74)</td>
<td>(2.70)</td>
<td>(4.44)</td>
</tr>
<tr>
<td>Asset Exposure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government Bonds</td>
<td>-0.247</td>
<td>-2.359***</td>
<td>-0.029</td>
</tr>
<tr>
<td></td>
<td>(-0.40)</td>
<td>(-10.39)</td>
<td>(-1.00)</td>
</tr>
<tr>
<td>Money Markets</td>
<td>-1.143*</td>
<td></td>
<td>-1.379***</td>
</tr>
<tr>
<td></td>
<td>(-1.72)</td>
<td></td>
<td>(-4.47)</td>
</tr>
<tr>
<td>Commodities</td>
<td>-0.932***</td>
<td></td>
<td>-0.866***</td>
</tr>
<tr>
<td></td>
<td>(-4.24)</td>
<td></td>
<td>(-3.81)</td>
</tr>
<tr>
<td>Hedge Funds Strategies</td>
<td>-0.109</td>
<td></td>
<td>-0.329</td>
</tr>
<tr>
<td></td>
<td>(-0.20)</td>
<td></td>
<td>(-0.76)</td>
</tr>
<tr>
<td>Credits</td>
<td>0.462</td>
<td></td>
<td>0.0149</td>
</tr>
<tr>
<td></td>
<td>(0.56)</td>
<td></td>
<td>(0.03)</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>-1.428***</td>
<td>-2.019***</td>
<td>-1.161***</td>
</tr>
<tr>
<td></td>
<td>(-4.22)</td>
<td>(-8.21)</td>
<td>(-3.76)</td>
</tr>
<tr>
<td>Currencies</td>
<td>-1.278***</td>
<td></td>
<td>-1.271***</td>
</tr>
<tr>
<td></td>
<td>(-4.01)</td>
<td></td>
<td>(-3.96)</td>
</tr>
<tr>
<td>Multi Assets</td>
<td>-0.887***</td>
<td></td>
<td>-0.415***</td>
</tr>
<tr>
<td></td>
<td>(-5.03)</td>
<td></td>
<td>(-4.05)</td>
</tr>
<tr>
<td>Geographic Exposure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>World</td>
<td>0.004</td>
<td>0.087</td>
<td>-0.072</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.43)</td>
<td>(-0.59)</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>-0.855***</td>
<td>-1.417***</td>
<td>-0.827***</td>
</tr>
<tr>
<td></td>
<td>(-6.68)</td>
<td>(-8.42)</td>
<td>(-6.45)</td>
</tr>
<tr>
<td>North America</td>
<td>0.198</td>
<td>0.518</td>
<td>-0.125</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(1.10)</td>
<td>(-1.05)</td>
</tr>
<tr>
<td>Rest of the World</td>
<td>-0.424</td>
<td></td>
<td>-0.409</td>
</tr>
<tr>
<td></td>
<td>(-1.64)</td>
<td></td>
<td>(-1.57)</td>
</tr>
<tr>
<td>Observations</td>
<td>4,543</td>
<td>1,065</td>
<td>3,478</td>
</tr>
<tr>
<td>R²</td>
<td>0.647</td>
<td>0.738</td>
<td>0.756</td>
</tr>
</tbody>
</table>

Notes: This table reports the parameter estimates obtained by regressing the correlation coefficient between the ETF return and the return of the collateral portfolio on a series of ETF-specific variables. The estimation is based on all week-fund observations. The explanatory variables are the level of the ETF asset under management (in log), a dummy variable that takes the value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes the value of 1 if the ETF is an inverse ETF and 0 otherwise, the level of collateralization of the fund, the number of securities in the collateral portfolio (in log), the fraction of equities in the collateral portfolio, as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by GLS with fund random effects and robust standard errors. We transform the explained variable, \( \ln(\frac{\text{ETF}_t - \text{Fund}_t}{\text{ETF}_{t-1} - \text{Fund}_{t-1}}) \), to ensure it remains between -1 and 1. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.
1.3. Empirical Analysis

We present in Figure 1.3 the distributions of the correlation between the return of the ETF and the return of its collateral portfolio.\footnote{When the returns of one or several securities included in a collateral portfolio are missing, we apply the following rule: (1) when the cumulative weight of the missing securities exceeds 5%, we do not compute the portfolio return on this particular date and (2) when the cumulative weights of the missing securities is equal or less than 5%, we compute the portfolio return using all available returns (with weights properly rescaled to sum to 100%). When computing historical correlations between a collateral security and another security, we impose a minimum of 20 days during which both returns are available.} We clearly see that most correlations are positive (the average correlation is 0.305 for synthetic funds and 0.468 for physical funds), which confirms the close connection that exists between the pledged collateral and the index tracked by the ETF. However, for synthetic ETFs, the distribution is bimodal with significant mass associated with negative correlations. Looking more closely at our sample of funds reveals that most negative correlations are associated with inverse ETFs (the average correlation for inverse funds is $-0.583$).\footnote{When computing the ETF-collateral correlations for inverse ETFs we systematically use the short index and not the regular index. For instance, for the db X-tracker on the S&P 500 SHORT index, we use the latter and not the S&P 500 index.} Differently, the distribution obtained for physical ETFs is much more tilted toward positive values, which is consistent with the fact that this sample contains no inverse funds.

We regress the correlation between the returns of the ETF and its collateral portfolio on a series of firm-specific variables using a panel linear specification with individual effects.\footnote{In order to guarantee that the estimated correlation remains within the $[-1, 1]$ range, the dependent variable is defined as $\ln \left( \frac{1 + \text{corr}_{i,t}}{1 - \text{corr}_{i,t}} \right)$. We also estimated a binary specification (panel logit model with random effects) for the sign of the correlation and obtained qualitatively similar results.} We show in Table 1.4 that the fractions of equities and of European securities in the collateral portfolio are positively and significantly associated with the ETF-collateral correlation. Differently, inverse funds and funds that track commodities, currencies, corporate bonds, and money market funds tend to have a lower correlation with their collateral. This low correlation is consistent with the fact that these asset classes are typically more difficult to include in a collateral portfolio.

1.3.3 Equities and Bonds Used as Collateral

To get a better sense of the type of securities used as collateral, we conduct an in-depth analysis of all equities, and then of all bonds. Because they attracted most critics from the media and regulators, we focus in this sub-section only on synthetic ETFs. We start with equities in Panel A of Table 1.5 and show that most equities used as collateral are issued by large, European, non-financial firms. Furthermore, collateralized equities exhibit good liquidity on average, with an average bid-ask spread of 0.21% and an average daily trading volume that corresponds to 4.67% of the market capitalization (see Panel B). Another reassuring finding is the fact that, on average, the pledged equities have a higher beta, or conditional beta, with respect to the ETF than with respect to the stock return of Deutsche Bank, which is the swap counterparty for all ETFs (see Panel C). The average beta is 60% higher when computed with the index than with the swap counterparty and its distribution has more mass on any value greater than 0.4. Using collateral securities that correlate strongly with the counterparty or even worse, are issued by the counterparty, would be inefficient as their value would go to zero in the case of a default of the counterparty.
Table 1.5: Equities Used as Collateral

Panel A: Equity Issuer

<table>
<thead>
<tr>
<th>Region</th>
<th>Europe</th>
<th>Asia-Pacific</th>
<th>N. America</th>
<th>R. World</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>58.5% (752)</td>
<td>21.6% (1,365)</td>
<td>19.6% (469)</td>
<td>0.3% (5)</td>
</tr>
<tr>
<td>Industry Classification</td>
<td>Industrial</td>
<td>Financial</td>
<td>Utility</td>
<td>Transportation</td>
</tr>
<tr>
<td></td>
<td>76.6% (2,047)</td>
<td>11.4% (346)</td>
<td>10.4% (127)</td>
<td>1.6% (71)</td>
</tr>
<tr>
<td>Market Capitalization</td>
<td>Micro-Cap</td>
<td>Small-Cap</td>
<td>Mid-Cap</td>
<td>Large-Cap</td>
</tr>
<tr>
<td></td>
<td>1.4% (30)</td>
<td>9.3% (1,634)</td>
<td>9.9% (404)</td>
<td>79.4% (523)</td>
</tr>
</tbody>
</table>

Panel B: Liquidity

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>median</th>
<th>st.dev.</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Daily Spread</td>
<td>0.21%</td>
<td>0.20%</td>
<td>0.48%</td>
<td>0.01%</td>
<td>12.73%</td>
</tr>
<tr>
<td>Average Daily Volume</td>
<td>4.67%</td>
<td>0.44%</td>
<td>21.83%</td>
<td>0.01%</td>
<td>115.15%</td>
</tr>
</tbody>
</table>

Panel C: Dependence

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>median</th>
<th>st.dev.</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta ETF</td>
<td>0.48</td>
<td>0.45</td>
<td>0.70</td>
<td>-1.83</td>
<td>3.04</td>
</tr>
<tr>
<td>Swap Counterparty</td>
<td>0.30</td>
<td>0.28</td>
<td>0.28</td>
<td>-0.06</td>
<td>1.11</td>
</tr>
<tr>
<td>Conditional Beta ETF</td>
<td>0.50</td>
<td>0.47</td>
<td>0.74</td>
<td>-3.05</td>
<td>4.14</td>
</tr>
<tr>
<td>Swap Counterparty</td>
<td>0.35</td>
<td>0.33</td>
<td>0.30</td>
<td>-0.46</td>
<td>1.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Beta</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ETF</td>
<td>7.5%</td>
<td>11.3%</td>
<td>25.3%</td>
<td>21.4%</td>
<td>13.4%</td>
</tr>
<tr>
<td>Swap Counterparty</td>
<td>0.4%</td>
<td>36.6%</td>
<td>38.3%</td>
<td>19.3%</td>
<td>4.2%</td>
</tr>
<tr>
<td>Conditional Beta ETF</td>
<td>9.2%</td>
<td>9.1%</td>
<td>22.9%</td>
<td>20.9%</td>
<td>14.4%</td>
</tr>
<tr>
<td>Swap Counterparty</td>
<td>0.2%</td>
<td>23.9%</td>
<td>40.2%</td>
<td>25.3%</td>
<td>8.8%</td>
</tr>
</tbody>
</table>

Notes: This table presents some summary statistics on the equities included in the collateral portfolios of synthetic ETFs. Panel A displays the value-weighted percentage of collateral equities by region, industry, and size of the issuer, along with the number of different equities in parentheses. We use the following definitions for size groups: Micro-Cap: below $100 million; Small-Cap: $100 million-$4 billion; Mid-Cap: $4 billion-$10 billion; Large-Cap: Over $10 billion. These ranges were selected to match the average market capitalization of the MSCI World Index of the respective categories. The size figures are as of November 29th, 2012. Panel B displays value-weighted statistics about the average daily percentage bid-ask spread and the average daily volume in percentage of the market capitalization. For each security, the percentage spread and volume are winsorized at the top 1%. Panel C displays value-weighted statistics about the beta coefficient ($\beta_{i,j}$) and conditional beta coefficient ($\beta_{i,j}|r_j < 0$) of the collateral equities with respect to the ETF return and to the swap counterparty return (Deutsche Bank stock return). We compute the conditional betas by using only days during which the index return or the swap counterparty return is negative. The lower part of Panel C presents summary statistics on the distribution of the betas and conditional betas, weighted by the equity value. In Panels B and C, the sample period is between January 1, 2007 and December 31, 2012.

When we look at the bond part of the collateral portfolio in Table 1.6, we notice that bonds predominantly have European issuers (88.3%), which is again consistent with the existence of a home bias (see Panel A). The fraction of European bonds is higher for corporate bonds (96.6%) than for Government bonds (86.0%). Note that the lower fraction of European Sovereign bonds may be due to the fact that our sample period overlaps with the Eurozone crisis of 2012. Turning to bond ratings in Panel B, we see that 65.5% of...
the bonds have a AAA rating. In our sample, the fraction of bonds with at least a AA rating is 84.5% for Government bonds and 64.9% for corporate bonds. Speculative-grade ratings account for 0.4% of the bonds used as collateral and undefined ratings for 1.3%. Our results drastically contrast with those of Ramaswamy (2011) that were based on a single equity ETF. He reports that 8.7% of the bonds are rated AAA, 13.1% at least AA, and 38% are unrated (vs. 65.5%, 80.3%, and 1.3%, respectively, in our sample), which seems to indicate that the fund selected by Ramaswamy is not representative of the entire ETF industry.

Table 1.6: Bonds Used as Collateral

<table>
<thead>
<tr>
<th>Panel A: Bond Issuer</th>
<th>Europe</th>
<th>N. America</th>
<th>Asia-Pacific</th>
<th>R. World</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond Type</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>88.3%</td>
<td>7.1%</td>
<td>4.5%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Gov. Bonds</td>
<td>86.0%</td>
<td>8.6%</td>
<td>5.4%</td>
<td>-</td>
</tr>
<tr>
<td>Corp. Bonds</td>
<td>96.6%</td>
<td>2.0%</td>
<td>1.3%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Panel B: Rating</td>
<td>AAA</td>
<td>AA</td>
<td>A</td>
<td>BBB</td>
</tr>
<tr>
<td>Bond Type</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>65.5%</td>
<td>14.8%</td>
<td>17.2%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Gov. Bonds</td>
<td>75.2%</td>
<td>9.3%</td>
<td>15.3%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Corp. Bonds</td>
<td>30.4%</td>
<td>34.5%</td>
<td>23.9%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Bond Issuer</td>
<td>Europe</td>
<td>66.0%</td>
<td>12.6%</td>
<td>19.2%</td>
</tr>
<tr>
<td></td>
<td>North America</td>
<td>93.9%</td>
<td>0.4%</td>
<td>0.1%</td>
</tr>
<tr>
<td></td>
<td>Asia-Pacific</td>
<td>19.8%</td>
<td>69.9%</td>
<td>3.5%</td>
</tr>
</tbody>
</table>

Panel C: Maturity

<table>
<thead>
<tr>
<th>Bond Type</th>
<th>&lt;1Y</th>
<th>1-3Y</th>
<th>3-5Y</th>
<th>5-7Y</th>
<th>7-10Y</th>
<th>&gt;10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>16.4%</td>
<td>24.4%</td>
<td>14.7%</td>
<td>6.7%</td>
<td>14.7%</td>
<td>23.1%</td>
</tr>
<tr>
<td>Gov. Bonds</td>
<td>11.7%</td>
<td>19.2%</td>
<td>13.3%</td>
<td>8.5%</td>
<td>18.1%</td>
<td>29.2%</td>
</tr>
<tr>
<td>Corp. Bonds</td>
<td>33.4%</td>
<td>43.0%</td>
<td>19.6%</td>
<td>0.5%</td>
<td>2.3%</td>
<td>1.2%</td>
</tr>
<tr>
<td>Bond Issuer</td>
<td>Europe</td>
<td>17.4%</td>
<td>25.3%</td>
<td>14.7%</td>
<td>6.4%</td>
<td>13.2%</td>
</tr>
<tr>
<td></td>
<td>North America</td>
<td>4.5%</td>
<td>21.7%</td>
<td>18.8%</td>
<td>8.0%</td>
<td>16.7%</td>
</tr>
<tr>
<td></td>
<td>Asia-Pacific</td>
<td>17.9%</td>
<td>14.4%</td>
<td>12.4%</td>
<td>17.4%</td>
<td>28.1%</td>
</tr>
<tr>
<td>Index Maturity</td>
<td>Short</td>
<td>21.6%</td>
<td>49.5%</td>
<td>23.7%</td>
<td>3.3%</td>
<td>1.0%</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>10.7%</td>
<td>8.9%</td>
<td>22.7%</td>
<td>34.0%</td>
<td>23.7%</td>
</tr>
<tr>
<td></td>
<td>Long</td>
<td>-</td>
<td>-</td>
<td>7.5%</td>
<td>7.6%</td>
<td>84.9%</td>
</tr>
</tbody>
</table>

Notes: This table presents some summary statistics on the bonds included in the collateral portfolios of synthetic ETFs. Panel A displays the value-weighted percentages of collateral bonds by region, along with the number of different bonds in parentheses. Panel B presents the value-weighted percentages of collateral bonds by bond rating, for different bond types and issuers. The n/a category corresponds to unrated bonds. Panel C displays the value-weighted percentages of collateral bonds by bucket of maturity, for different bond types and issuers. Short, Medium, and Long refer to funds that track a bond index with, respectively, a short maturity (less than 3 years), a medium maturity (between 3 and 10 years), and a long maturity (more than 10 years). The sample period is July 5, 2012 - November 29, 2012.

The maturity spectrum of the bonds within the collateral portfolios covers a wide range of maturities from less than a year to more than 10 years (see Panel C). Overall, 40.8% of the bonds used as collateral have a maturity of less than 3 years and 23.1% have a maturity of more than 10 years. Government bonds and North-American bonds are more on the long
side whereas the maturity of corporate bonds remains almost exclusively below 5 years. Interestingly, we find that the duration of the collateral matches well with the duration of the fixed-income index tracked by the fund. Indeed, ETFs that track an index with a maturity below 3 years mainly have collateralized bonds with a maturity less than 3 years (71.1%). The match is even stronger for funds that track medium maturity indices (3-10 years), with 89.3% of the collateralized bonds within the 3-10 year maturity bucket, and for funds that track long term indices (>10 years), for which 84.9% of the collateralized bonds also have a maturity of more than 10 years.

1.3.4 Counterparty Risk Analysis for Synthetic ETFs

We estimate for each of the 164 synthetic ETFs its counterparty risk measures, ACR. The latter is obtained by multiplying the probability of the fund being undercollateralized and its expected collateral shortfall. Both risk measures are estimated non-parametrically with a one-day horizon. As explained in Section 1.2.5, we estimate these risk metrics from past

Figure 1.4: Estimated Counterparty Risk for Synthetic ETFs

Notes: This figure presents a Torino scale that displays, for all synthetic ETFs, their probability $P$ of having a collateral shortfall (y-axis, range = 0-100%), their expected collateral shortfall $S$ (x-axis, range = 0-35%), and their Aggregate Counterparty Risk $ACR = P \times S$. We contrast ETFs that are based on funded swaps to those based on unfunded swaps.
Table 1.7: Counterparty Risk of Synthetic ETFs

<table>
<thead>
<tr>
<th></th>
<th>ACR</th>
<th>ACR</th>
<th>ACR</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(AUM)</td>
<td>0.093</td>
<td>0.112</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>(0.72 )</td>
<td>(1.01)</td>
<td>(1.10 )</td>
</tr>
<tr>
<td>Funded Swap</td>
<td>-1.514***</td>
<td>-2.464***</td>
<td>-2.525***</td>
</tr>
<tr>
<td></td>
<td>(-3.07 )</td>
<td>(-3.53)</td>
<td>(-3.95 )</td>
</tr>
<tr>
<td>Inverse</td>
<td>0.560</td>
<td>0.708</td>
<td>1.107**</td>
</tr>
<tr>
<td></td>
<td>(1.24 )</td>
<td>(1.55)</td>
<td>(2.44 )</td>
</tr>
<tr>
<td>Collateralization</td>
<td>-0.092***</td>
<td>-0.091***</td>
<td>-0.091***</td>
</tr>
<tr>
<td></td>
<td>(-3.57 )</td>
<td>(-3.55)</td>
<td>(-3.53 )</td>
</tr>
<tr>
<td>log(# Securities)</td>
<td>-0.167</td>
<td>-0.116</td>
<td>-0.189</td>
</tr>
<tr>
<td></td>
<td>(-1.22 )</td>
<td>(-0.85)</td>
<td>(-1.45 )</td>
</tr>
<tr>
<td>Equity Fraction</td>
<td>0.012**</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(2.41 )</td>
<td>(-0.68)</td>
<td>(-0.63 )</td>
</tr>
<tr>
<td>Europe Fraction</td>
<td>-0.006**</td>
<td>-0.006***</td>
<td>-0.005**</td>
</tr>
<tr>
<td></td>
<td>(-2.57 )</td>
<td>(-2.61)</td>
<td>(-2.51 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Control Dummy Variables</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Exposure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geographic</td>
<td>Control Dummy Variables</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Exposure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1,310</td>
<td>1,310</td>
<td>1,310</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.373</td>
<td>0.417</td>
<td>0.528</td>
</tr>
</tbody>
</table>

Notes: This table reports the parameter estimates obtained by regressing the aggregate counterparty risk measure \( ACR \) on a series of ETF-specific variables. The estimation is based on all week-fund observations for all synthetic ETFs. The explanatory variables are the level of the ETF asset under management (in log), a dummy variable that takes the value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes the value of 1 if the ETF is an inverse ETF and 0 otherwise, the level of collateralization of the fund, the number of securities in the collateral portfolio (in log), the fraction of equities in the collateral portfolio, the fraction of European securities in the collateral portfolio, as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by GLS with fund random effects and robust standard errors. We transform the explained variable, \( \ln(y + 1) \), to ensure it remains non-negative. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.

observations that belong to a high swap counterparty-risk regime, namely a 2-month period around the bankruptcy of Lehman Brothers (September 1, 2008 - October 31, 2008). During this period, the default probability of the swap counterparty, Deutsche Bank, implied from CDS spread got multiplied by three compared to its pre-crisis level.

In Figure 1.4, we display for all synthetic funds both the average shortfall probability and expected collateral shortfall, expressed as a percentage of the NAV. For each fund, the risk metrics are averaged across time. The main result in this figure is that counterparty risk exposure varies a lot across funds. Overall, in a high counterparty-risk regime, the probability of experiencing a collateral shortfall is high but the expected shortfall often remains below 10% of the NAV. The high shortfall probability is due to the high market
volatility during our estimation period, which affects both the indices tracked by the ETF and the value of the collateral. When contrasting funds based on the type of swap they rely on, we observe that counterparty risk exposure is on average higher for unfunded-swap funds.

We complement our analysis by regressing the counterparty risk measures, $ACR$, on fund characteristics. We see in Table 1.7 that ETFs based on funded swaps and funds with high levels of collateralization or fractions of European collateralized securities exhibit on average a lower level of counterparty risk. Furthermore, inverse funds tend to expose their investors to more counterparty risk.\textsuperscript{14} This latter result is consistent with our previous findings regarding the negative relationship between the values of the collateral and the index for inverse funds.

### 1.3.5 Benchmarking with Physical ETFs

![Diagram showing estimated counterparty risk for physical and synthetic ETFs](image-url)

**Figure 1.5: Estimated Counterparty Risk for Physical and Synthetic ETFs**

Notes: This figure presents a Torino scale that displays, for all sample ETFs, their probability $P$ of having a collateral shortfall (y-axis, range = 0-100%), their expected collateral shortfall $S$ (x-axis, range = 0-35%), and their Aggregate Counterparty Risk $ACR = P \times S$. We contrast physical ETFs and synthetic ETFs (both funded-swap and unfunded-swap ETFs).

\textsuperscript{14}Note that if we simply compare long synthetic ETFs to inverse synthetic ETFs on a Torino scale (see Figure 1.7 in Appendix A1), it is difficult to draw the same conclusion.
We conduct a similar risk assessment for physical ETFs as they also expose their investors to counterparty risk through securities lending. It is indeed possible that the short sellers who borrow securities from the ETF issuer fail to return them in due time. We contrast counterparty risk estimates for physical and synthetic ETFs in Figure 1.5. We see that the difference in risk exposure is striking: counterparty risk exposures are several orders of magnitude higher for synthetic ETF investors.

Table 1.8: Counterparty Risk of Physical and Synthetic ETFs

<table>
<thead>
<tr>
<th></th>
<th>ACR</th>
<th>ACR</th>
<th>ACR</th>
<th>ACR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synthetic ETF</td>
<td>1.312***</td>
<td>2.162***</td>
<td>2.807***</td>
<td>2.846***</td>
</tr>
<tr>
<td></td>
<td>(5.81)</td>
<td>(5.53)</td>
<td>(7.04)</td>
<td>(7.02)</td>
</tr>
<tr>
<td>log(AUM)</td>
<td>0.030</td>
<td>0.041</td>
<td>0.040</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.42)</td>
<td>(0.41)</td>
<td></td>
</tr>
<tr>
<td>Funded Swap</td>
<td>-1.165***</td>
<td>-1.950***</td>
<td>-1.949***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.67)</td>
<td>(-3.91)</td>
<td>(-4.02)</td>
<td></td>
</tr>
<tr>
<td>Inverse</td>
<td>0.454</td>
<td>0.650</td>
<td>0.831*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.96)</td>
<td>(1.39)</td>
<td>(1.81)</td>
<td></td>
</tr>
<tr>
<td>Collateralization</td>
<td>-0.091***</td>
<td>-0.091***</td>
<td>-0.091***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.63)</td>
<td>(-3.61)</td>
<td>(-3.61)</td>
<td></td>
</tr>
<tr>
<td>log(# Securities)</td>
<td>-0.062**</td>
<td>-0.060**</td>
<td>-0.066**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.19)</td>
<td>(-2.17)</td>
<td>(-2.39)</td>
<td></td>
</tr>
<tr>
<td>Equity Fraction</td>
<td>0.005*</td>
<td>0.002</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.88)</td>
<td>(1.13)</td>
<td>(1.22)</td>
<td></td>
</tr>
<tr>
<td>Europe Fraction</td>
<td>-0.002**</td>
<td>-0.002**</td>
<td>-0.002**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.30)</td>
<td>(-2.34)</td>
<td>(-2.30)</td>
<td></td>
</tr>
</tbody>
</table>

Asset Exposure Control Dummy Variables: No No Yes Yes

Geographic Exposure Control Dummy Variables: No No No Yes

Observations 2,056 2,056 2,056 2,056
R^2 0.135 0.413 0.472 0.521

Notes: This table reports the parameter estimates obtained by regressing the aggregate counterparty risk measure ACR on a series of ETF-specific variables. The estimation is based on all week-fund observations. The explanatory variables are a dummy variable that takes the value of 1 if the ETF is synthetic and 0 otherwise, the level of the ETF asset under management (in log), a dummy variable that takes the value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes the value of 1 if the ETF is an inverse ETF and 0 otherwise, the level of collateralization of the fund, the number of securities in the collateral portfolio (in log), the fraction of equities in the collateral portfolio, the fraction of European securities in the collateral portfolio, as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by GLS with fund random effects and robust standard errors. We transform the explained variable, ln(y + 1), to ensure it remains non-negative. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.

We extend the multivariate regressions presented in Table 1.7 by considering jointly the synthetic and physical ETFs, while using the ETF type as an extra control variable.
Consistent with Figure 1.5, the results shown in Table 1.8 indicate that synthetic ETFs display a higher level of counterparty risk exposure. We also note that our conclusions about the other main drivers of counterparty risk remain robust in this extended sample: counterparty risk exposures is lower for funded swaps, and funds with higher collateralization and more European collateralized securities. Furthermore, the negative coefficient associated with the number of securities is now significant at the 5% level.

1.3.6 Tradeoff between Risk and Performance

We have seen that synthetic ETFs’ investors tend to be more exposed to counterparty risk than physical ETFs’ investors. A natural question is whether the former investors are compensated for bearing this additional risk. We answer this question by considering three important dimensions of an ETF: its costs, its tracking error and its tracking difference. In particular, we formally show that synthetic ETFs are as cheap or cheaper and display better performance (i.e., lower tracking error) than physical ETFs.\(^{15}\) As a result, synthetic funds’ investors are compensated for bearing this additional risk by getting a superior performance for the same price.

We conduct this tests in two ways. We first consider the entire sample of synthetic and physical funds. In Panel A of Table 1.9, our unconditional tests reveal no clear difference between the fees charged by the different types of funds.\(^{16}\) The average fee for physical ETFs are 44 bps vs. 43 bps for synthetic ETFs. However, we find major differences in the tracking error of these funds: the average tracking error is 96 bps for physical ETFs and 13 bps for synthetic ETFs. Another interesting result is the much higher tracking error for funds that pays dividends (\textit{Distributing}), which indicates that a major source of tracking error for funds is the way dividends are handled and passed through to investors. We also find a larger average tracking difference for physical funds compared to synthetic ETFs: -17 bps versus -45 bps, respectively. However, the heterogeneity of the ETF asset exposures between the samples of physical and synthetic ETFs may render the comparison between funds difficult.

Second, we implement a matching method in which we only consider funds that track exactly the same index (i.e., same index provider, same asset class, same treatment of the dividends). We have been able to identify 13 pairs of synthetic ETF and perfectly equivalent physical ETF. We then compare their respective fees, tracking errors and tracking differences in the Panel B of Table 1.9. Consistent with the previous results based on the whole sample, we find that synthetic ETFs outperform their physical counterparts in terms of tracking errors yet charging similar fees. Moreover, the average tracking difference is now 13 bps for the synthetic ETFs and -30 bps for the physical ETFs. This would mean that on average the synthetic funds outperform their respective benchmarks in terms of return.

\(^{15}\)We define the tracking error as the annualized volatility of the difference between the daily returns of the ETF and the daily returns of the index, while the tracking difference is defined as the expected value of this difference.

\(^{16}\)For an empirical analysis of the fees of active and passive (including ETFs) funds in the world, see Cremers et al. (2015).
1.3. Empirical Analysis

Table 1.9: Fees, Tracking Errors and Tracking Differences

<table>
<thead>
<tr>
<th>Panel A: Entire Sample</th>
<th>All</th>
<th>Physical</th>
<th>Synthetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.43 (0.40)</td>
<td>0.44 (0.40)</td>
<td>0.43 (0.45)</td>
</tr>
<tr>
<td>Capitalizing</td>
<td>0.43 (0.45)</td>
<td>0.42 (0.33)</td>
<td>0.43 (0.45)</td>
</tr>
<tr>
<td>Distributing</td>
<td>0.44 (0.40)</td>
<td>0.44 (0.40)</td>
<td>0.44 (0.50)</td>
</tr>
<tr>
<td>Tracking Errors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.36 (0.04)</td>
<td>0.96 (0.72)</td>
<td>0.13 (0.03)</td>
</tr>
<tr>
<td>Capitalizing</td>
<td>0.06 (0.03)</td>
<td>0.93 (0.82)</td>
<td>0.04 (0.03)</td>
</tr>
<tr>
<td>Distributing</td>
<td>0.84 (0.67)</td>
<td>0.96 (0.71)</td>
<td>0.60 (0.48)</td>
</tr>
<tr>
<td>Tracking Differences</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>-0.37 (-0.29)</td>
<td>-0.17 (-0.14)</td>
<td>-0.45 (-0.39)</td>
</tr>
<tr>
<td>Capitalizing</td>
<td>-0.52 (-0.50)</td>
<td>-0.42 (-0.22)</td>
<td>-0.53 (-0.40)</td>
</tr>
<tr>
<td>Distributing</td>
<td>-0.13 (-0.18)</td>
<td>-0.15 (-0.10)</td>
<td>-0.09 (-0.25)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Matched Sample</th>
<th>Total</th>
<th>Physical</th>
<th>Synthetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.39 (0.35)</td>
<td>0.39 (0.40)</td>
<td>0.39 (0.35)</td>
</tr>
<tr>
<td>Capitalizing</td>
<td>0.40 (0.34)</td>
<td>0.40 (0.33)</td>
<td>0.41 (0.35)</td>
</tr>
<tr>
<td>Distributing</td>
<td>0.37 (0.35)</td>
<td>0.39 (0.40)</td>
<td>0.35 (0.35)</td>
</tr>
<tr>
<td>Tracking Errors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.36 (0.08)</td>
<td>0.60 (0.21)</td>
<td>0.11 (0.04)</td>
</tr>
<tr>
<td>Capitalizing</td>
<td>0.20 (0.07)</td>
<td>0.38 (0.11)</td>
<td>0.03 (0.03)</td>
</tr>
<tr>
<td>Distributing</td>
<td>0.64 (0.42)</td>
<td>1.01 (0.47)</td>
<td>0.26 (0.08)</td>
</tr>
<tr>
<td>Tracking Differences</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>-0.09 (-0.21)</td>
<td>-0.30 (-0.27)</td>
<td>0.13 (-0.16)</td>
</tr>
<tr>
<td>Capitalizing</td>
<td>-0.37 (-0.25)</td>
<td>-0.38 (-0.22)</td>
<td>-0.36 (-0.28)</td>
</tr>
<tr>
<td>Distributing</td>
<td>0.44 (-0.12)</td>
<td>-0.15 (-0.32)</td>
<td>1.03 (-1.00)</td>
</tr>
</tbody>
</table>

Notes: This table displays the average (and median) fees, tracking errors and tracking differences in percentage points. In Panel A, we consider all sample ETFs, physical ETFs, and synthetic ETFs. In Panel B, we only consider pairs or physical and synthetic ETFs that track exactly the same index (same index provider, same asset class, and same treatment of the dividends). We distinguish funds that pay out dividends to their investors (Distributing) from those that do not (Capitalizing). Following industry practice, we remove some outliers likely due to misaligned data or other data errors (Morningstar, 2013). Fees correspond to total expense ratios and have been collected on November 29, 2014. Tracking errors are defined as the annualized standard-deviation of daily differences between the daily returns of the fund NAV and index. Tracking differences are the expected value of the daily differences between the daily returns of the fund NAV and index. They are computed using two-year of daily returns covering the period November 29, 2010 - November 29, 2012.

In Table 1.10, we complement these unconditional results by running multivariate regressions for, in turn, fees, tracking errors and tracking difference. We find that the coefficient associated with the synthetic ETF’s dummy variable is negative and significant for both the fees and the tracking errors. Interestingly, we uncover that inverse funds tend to charge higher fees and funds that distribute dividends exhibit larger tracking errors. Differently, tracking error decreases with fund size. In addition, we do not find evidence that tracking differences are larger for the synthetic funds. Indeed, the coefficient associated with the synthetic ETF’s dummy variable is positive but not statistically significant for the tracking differences.\textsuperscript{17}

\textsuperscript{17}We also conduct an alternative analysis to include the individual risk. Table 1.12 in Appendix A.2
Table 1.10: Regression Analysis of Fees, Tracking Errors and Tracking Differences

<table>
<thead>
<tr>
<th></th>
<th>Fees</th>
<th>Tracking Errors</th>
<th>Tracking Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synthetic ETF</td>
<td>-0.090***</td>
<td>-0.344***</td>
<td>0.346</td>
</tr>
<tr>
<td></td>
<td>(-4.25)</td>
<td>(-4.49)</td>
<td>(1.48)</td>
</tr>
<tr>
<td>Distributing</td>
<td>-0.025*</td>
<td>0.279***</td>
<td>0.260</td>
</tr>
<tr>
<td></td>
<td>(-1.75)</td>
<td>(4.97)</td>
<td>(1.05)</td>
</tr>
<tr>
<td>log(AUM)</td>
<td>-0.003</td>
<td>-0.030***</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>(-1.25)</td>
<td>(-3.48)</td>
<td>(1.33)</td>
</tr>
<tr>
<td>Funded Swap</td>
<td>0.026</td>
<td>-0.078</td>
<td>-0.095</td>
</tr>
<tr>
<td></td>
<td>(1.27)</td>
<td>(-1.54)</td>
<td>(-0.72)</td>
</tr>
<tr>
<td>Inverse</td>
<td>0.085***</td>
<td>0.027</td>
<td>-0.649***</td>
</tr>
<tr>
<td></td>
<td>(5.63)</td>
<td>(1.09)</td>
<td>(-5.80)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Observations</th>
<th>184</th>
<th>202</th>
<th>202</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>0.809</td>
<td>0.674</td>
<td>0.304</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports the parameter estimates obtained by regressing the fees, tracking errors and tracking differences on a series of fund-specific variables. The estimation is based on the cross-section of all physical and synthetic ETFs. The explanatory variables are a dummy variable that takes a value of 1 if the ETF is synthetic and 0 otherwise, a dummy variable that takes a value of 1 if the fund pays out dividends to its investors (Distributing), the level of the ETF asset under management (in log), a dummy variable that takes a value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes a value of 1 if the ETF is an inverse ETF and 0 otherwise, as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by OLS with a constant term and robust standard errors. We transform in the two first columns the explained variable, ln(y + 1), to ensure it remains non-negative. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.

1.3.7 Do ETF Investors Care about Counterparty Risk?

We now turn to testing whether ETF flows are sensitive to changes in counterparty risk. We envision that it could be the case for two main reasons. First, a significant fraction of ETF trading is made by institutional investors, which are perceived as sophisticated investors, and are able to withdraw funds quickly when they are not comfortable with the risk they face. A recent example is the run of institutional investors on money market funds following the bankruptcy of Lehman Brothers on September 15, 2008 (Schmidt, Timmermann and Wermers, 2014). Second, the creditworthiness of the swap counterparty can be monitored in real time in the CDS market.

While our sample period starts in 2007, we believe that most investors were not aware presents the results of regressions in which the ACR is added as an exogeneous variable. We cannot conclude that the investors are compensated for the individual counteparty risk. Indeed, the coefficient for the ACR variable is positive (fees), negative (tracking errors) or not statistically significant (tracking differences).
of the counterparty risk concerns before 2011. Indeed, as mentioned in the introduction, the debate on the counterparty risk of synthetic started during the first half of 2011 when some allegations were made by international agencies and trenchant criticisms by industry leaders (see footnote 4). To identify the exact timing, we searched for articles in the financial press that include the words "synthetic ETF" (from the Factiva database) as well as the number of queries on Google including the keywords "synthetic ETF" (from Google Trends) between 2006 and 2014. As shown in Figure 1.6, both proxies for market awareness remained at, or close to, zero until 2011 and then jumped to almost 300 articles and to a Google Search Index of 100.

Figure 1.6: Market Awareness of Counterparty Risk Concerns

Notes: This figure displays the annual number of articles that include the words "synthetic ETF" in the financial press (left axis) and the relative number of queries on Google including the keywords "synthetic ETF" (right axis). The amount of global media coverage is assessed using the Factiva database and the worldwide Google search figures are from Google Trends.

Our setting allows us to cleanly identify, with a difference-in-differences approach, the effect of counterparty risk on synthetic ETF outflows before and after 2011. Since we do not observe flows directly, we follow Frazzini and Lamont (2008) and Barber, Huang and Odean (2015) and infer flows from fund asset value and returns:

\[ flow_{i,t} = NAV_{i,t} \cdot \#shares_{i,t} - NAV_{i,t-1} \cdot \#shares_{i,t-1} \cdot (1 + r_{i,t}) \] (1.14)

where \( r_{i,t} \) is the return of the NAV between time \( t-1 \) and \( t \). By doing so, the performance of the fund is not taken into account as we only capture share redemptions (outflows) and share purchases (inflows). Then, we run the following difference-in-differences panel regression:

\[ outflow_{i,t} = \alpha_i + \beta_1 \cdot Risk_{t-1} \cdot Post_t + \beta_2 \cdot Risk_{t-1} + \beta_3 \cdot Post_t + \beta_4 \cdot Perf_{i,t} + \varepsilon_{i,t} \] (1.15)

where \( outflow_{i,t} \) is either a dummy variable that takes a value of one if there is an outflow between \( t-1 \) and \( t \) and zero otherwise (Probit model) or the log of the absolute outflow.
and zero if there is an inflow (Tobit model). Risk$_{t-1}$ is a dummy variable equal to one if the CDS spread of the swap counterparty at time $t - 1$ is greater than the 75th percentile of its distribution, Post$_t$ is a dummy variable that takes a value of one after June 2011, and Perf$_{i,t}$ is the return of the index tracked by the fund between $t - 2$ and $t$.

Table 1.11: Impact of Counterparty Risk on Fund Flows

<table>
<thead>
<tr>
<th></th>
<th>Outflows</th>
<th>Outflows</th>
<th>Outflows</th>
<th>Outflows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk × Post</td>
<td>0.227**</td>
<td>0.189**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.51)</td>
<td>(2.54)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Syn × Risk × Post</td>
<td></td>
<td></td>
<td>0.140***</td>
<td>0.911***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.84)</td>
<td>(3.48)</td>
</tr>
<tr>
<td>Risk</td>
<td>-0.115</td>
<td>-0.082</td>
<td>-0.042</td>
<td>-0.147</td>
</tr>
<tr>
<td></td>
<td>(-1.48)</td>
<td>(-1.29)</td>
<td>(-1.13)</td>
<td>(-0.75)</td>
</tr>
<tr>
<td>Post</td>
<td>0.305***</td>
<td>0.224***</td>
<td>0.246***</td>
<td>1.077***</td>
</tr>
<tr>
<td></td>
<td>(6.51)</td>
<td>(5.87)</td>
<td>(8.66)</td>
<td>(7.19)</td>
</tr>
<tr>
<td>Perf</td>
<td>-0.005***</td>
<td>-0.006***</td>
<td>-0.010***</td>
<td>-0.056***</td>
</tr>
<tr>
<td></td>
<td>(-2.83)</td>
<td>(-3.45)</td>
<td>(-6.72)</td>
<td>(-7.28)</td>
</tr>
<tr>
<td>Syn</td>
<td></td>
<td></td>
<td>-0.101*</td>
<td>-2.217***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-1.95)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-5.56)</td>
</tr>
</tbody>
</table>

Sample | Equity Synthetic | All Synthetic | All Funds | All Funds

Observations | 5,257 | 7,672 | 11,082 | 11,082

Notes: This table reports the parameter estimates obtained by regressing an outflow variable on a dummy variable that takes a value of one if the 5-year CDS of the swap counterparty, Deutsche Bank, is greater than the 75th percentile of its distribution and zero otherwise (Risk), a dummy variable that takes a value of one after June 2011 and zero otherwise (Post), the return of the index tracked by the fund (Perf), and a dummy variable that takes a value of one if the fund is synthetic and zero otherwise (Syn). The estimation is based on all month-fund observations between August 2008 and December 2013. In columns 1-3, we estimate a Probit model where the explained variable takes a value of one if the monthly flow is negative and zero otherwise. In column 4, we estimate a Tobit model where the explained variable is the log of the absolute outflow. All models are estimated with fund random effects. We display t-statistics in parentheses.

We estimate a Probit specification of Equation (1.15) using monthly data on all synthetic funds over the period August 2008-December 2013 and report the results in first two columns of Table 1.11. In this difference-in-differences identification, we compare synthetic funds in high-risk states after June 2011 ("treated") to synthetic funds in low-risk states as well as synthetic funds before June 2011 ("control"). Our main finding is that the estimated $\beta_1$ parameter is positive and significant which suggests that, once investors became aware of this additional source of risk in 2011, high counterparty risk has triggered more outflows from synthetic ETFs. This result is robust across asset exposures as shown in columns 1 and 2 (equity only vs. all exposures). Interestingly, we also report a negative relationship between the performance of the index tracked by the fund and the probability of observing an outflow, which is consistent with standard findings from the mutual fund literature about return-chasing investors (Chevalier etEllison, 1997). Furthermore, the probability of having an outflow increased significantly for synthetic
funds after counterparty risk concerns became public information ($\beta_3 > 0$ and significant). While our findings are based on a single issuer, they turn out to be consistent with a recent trend in the entire industry. According to figures from consultancy ETFGI, the size of the synthetic segment of the European ETF industry has experienced continuous growth since 2006 and then shrank dramatically in 2011 with a $23$ billion drop in AUM.

We obtain similar results when we consider a larger control group which also includes physical funds (column 3). We extend Equation (1.15) by including an extra dummy variable ($Syn$) for synthetic funds which is also interacted with $Risk$ and $Post$. In column 3, the estimated parameter ($\beta_1$) associated with the interaction terms (now triple) remains positive. The fact that this parameter remains positive indicates that the results in the first two columns are unlikely to be due to omitted factors (e.g., market stress during European debt crisis of 2012) that would impact outflows after June 2011. Indeed, such factors should also impact outflows for physical ETFs, which are now part of the control group. We also notice that the probability to face outflows is lower for synthetic ETFs, which is consistent with the fact that our sample period includes the take-off of the synthetic industry that has triggered some important transfers from physical to synthetic funds. Another robustness check we consider is to estimate the treatment effect on the magnitude of the flows. In column 4, we estimate a Tobit model using the absolute outflows as our dependent variable and the sign of the coefficients remain unchanged. Finally, we re-estimate our four specifications by using the level of the CDS spread as the $Risk$ variable as well as measuring index performance between $t-1$ and $t$. In all cases, we obtain qualitatively similar (unreported) results.

1.4 Conclusion

How safe is the backup parachute of ETFs? To answer this question, we study the collateral portfolios of ETFs that are based on swaps or engage in securities lending, hence exposing their investors to counterparty risk. Overall, our results do not support the allegations made by international agencies about the poor quality of the collateral used to produce ETFs and about the disconnect between the index tracked and the collateral. Funds in our sample tend to be overcollateralized and the collateral is mainly made of equities issued by large firms or highly-rated bonds. Furthermore, the collateral of equity funds are mainly made of equities and the duration of the collateral matches well with the one of the bond index tracked by the ETF.

Our results on collateral quality concur with Louis Brandeis’ saying that “sunlight is said to be the best of disinfectants”. Indeed, scrutiny by the media, regulators, and investors following the criticisms of the ETF industry by international agencies lead to improved disclosure by ETF providers about their replication technology and collateral holdings. We show that increased transparency pressured industry participants to improve standards and practice on collateral management.

We find some heterogeneity in the counterparty risk exposure of ETF investors. Exposure to counterparty risk is shown to be higher for funds that track commodities or currencies and for inverse ETFs which deliver the inverse performance of the underlying index. We find that the counterparty risk exposure is higher for synthetic ETFs but that investors are compensated for bearing this risk. Using a difference-in-differences specification, we
uncover that ETF flows respond significantly to changes in counterparty risk, which sug-
gests that investors closely monitor their counterparty risk exposure. Our findings on
the importance of counterparty risk for ETF investors have been corroborated by a re-
cent change in business models for several large ETF issuers. Long time proponents of
synthetic ETFs, Lyxor and db X-trackers both decided to switch some of their largest
funds to physical replication as well as to launch several new direct replication offerings
(Financial Times, 2014).
Appendix A

A1. Counterparty Risk for Inverse ETFs

Figure 1.7: Estimated Counterparty Risk for Long and Inverse Synthetic ETFs

Notes: This figure presents a Torino scale that displays, for all synthetic ETFs, their probability $P$ of having a collateral shortfall ($y$-axis, range = 0-100%), their expected collateral shortfall $S$ ($x$-axis, range = 0-35%), and their Aggregate Counterparty Risk $ACR = P \times S$. We contrast long synthetic ETFs to inverse synthetic ETFs.
A2. Individual Risk and ETF Performances

Table 1.12: Regression Analysis of Fees, Tracking Errors and Tracking Differences with ACR

<table>
<thead>
<tr>
<th></th>
<th>Fees</th>
<th>Tracking Errors</th>
<th>Tracking Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synthetic ETF</td>
<td>-0.129***</td>
<td>-0.285***</td>
<td>0.835</td>
</tr>
<tr>
<td></td>
<td>(-4.51)</td>
<td>(-3.03)</td>
<td>(1.65)</td>
</tr>
<tr>
<td>Distributing</td>
<td>-0.020</td>
<td>0.131**</td>
<td>0.977*</td>
</tr>
<tr>
<td></td>
<td>(-0.97)</td>
<td>(1.95)</td>
<td>(1.83)</td>
</tr>
<tr>
<td>log(AUM)</td>
<td>-0.003</td>
<td>-0.025***</td>
<td>-0.034</td>
</tr>
<tr>
<td></td>
<td>(-0.85)</td>
<td>(-2.97)</td>
<td>(-1.03)</td>
</tr>
<tr>
<td>Funded Swap</td>
<td>0.099***</td>
<td>-0.154*</td>
<td>-0.222</td>
</tr>
<tr>
<td></td>
<td>(3.29)</td>
<td>(-1.86)</td>
<td>(-0.93)</td>
</tr>
<tr>
<td>Inverse</td>
<td>0.053***</td>
<td>-0.016</td>
<td>-0.626***</td>
</tr>
<tr>
<td></td>
<td>(2.77)</td>
<td>(-0.59)</td>
<td>(-3.90)</td>
</tr>
<tr>
<td>ACR</td>
<td>0.0005***</td>
<td>-0.0011**</td>
<td>-0.0016</td>
</tr>
<tr>
<td></td>
<td>(2.67)</td>
<td>(-2.07)</td>
<td>(-1.09)</td>
</tr>
</tbody>
</table>

Asset Exposure
Control Dummy Variables: yes yes yes

Geographic Exposure
Control Dummy Variables: yes yes yes

Observations: 79 92 92
R²: 0.849 0.801 0.406

Notes: This table reports the parameter estimates obtained by regressing the fees, tracking errors and tracking differences on a series of fund-specific variables. The estimation is based on the cross-section of all physical and synthetic ETFs. The explanatory variables are a dummy variable that takes a value of 1 if the ETF is synthetic and 0 otherwise, a dummy variable that takes a value of 1 if the fund pays out dividends to its investors (Distributing), the level of the ETF asset under management (in log), a dummy variable that takes a value of 1 if the ETF is based on a funded swap and 0 otherwise, a dummy variable that takes a value of 1 if the ETF is an inverse ETF and 0 otherwise, the aggregate counterparty risk (ACR), as well as a dummy variable for each asset exposure and geographic exposure. In all columns, the model is estimated by OLS with a constant term and robust standard errors. We transform in the two first columns the explained variable, ln(y + 1), to ensure it remains non-negative. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.
Bibliography


Chapter 2

Optimal Collateral Portfolios for Exchange-Traded Funds*

Abstract

We theoretically derive the optimal composition of the collateral portfolio that minimizes the counterparty risk exposure of an exchange-traded fund (ETF) investor. In particular, we derive the efficient collateral frontier and show how to pick the optimal collateral portfolio. The optimal weights can be expressed as a function of those of the Markowitz’s minimum-variance portfolio but they also reflect the correlations between the collateral securities and the index tracked by the fund. For a sample of swap-based ETFs, we find that the counterparty risk exposure is significantly lower with optimal collateral portfolios than with actual collateral portfolios.

2.1 Introduction

Collateral is often seen as the most efficient safeguard against default risk. It consists of cash or assets that a counterparty transfers to secure a loan or another financial contract. If the counterparty does not provide the promised payments, the collateral receiver can seize the cash or the assets to recoup his losses. As a result, efficient collateralization requires that the value of the pledged assets increases with the promised cash-flow in the situation when the counterparty defaults. Examples of financial contracts that require collateral posting include loans, derivative securities (forwards, futures, options, swaps), securities lending agreements, and repurchase agreements.

In this paper, we theoretically show how to design an optimal collateral portfolio that aims to minimize the counterparty risk exposure of an ETF investor. As shown in Chapter 1, the latter is exposed to counterparty risk when the fund enters into a swap with an investment bank to replicate the performance of an index or when the fund lends some of its assets on the securities lending market. In both cases, the fund requires some collateral to be posted to mitigate counterparty risk concerns.

* Joint work with Christophe Hurlin and Christophe Pérignon. Hurlin is at the University of Orléans, France; Pérignon is at HEC Paris, France.
We derive an efficient frontier for collateral portfolios and show how to pick the optimal one that leads to the smallest counterparty risk exposure. The optimal weights can be expressed as a function of those of the Markowitz’s minimum-variance portfolio but they also reflect the correlations between the collateral securities and the expected cash-flows. Using collateral data for a sample of total return swaps initiated by a leading ETF issuer, we contrast the levels of counterparty risk generated by, respectively, the actual and the optimal collateral portfolios. We achieve the right mix of collateral posted from a universe of more than 1,500 eligible securities. Overall, we find that optimizing collateral portfolios leads to significantly lower counterparty risk exposures.

This paper is, to the best of our knowledge, the first one to model the optimal composition of collateral portfolios. This topic has recently gained growing interest as a significant fraction of the posted collateral combines various asset classes and not just Treasuries. For instance, Nyborg (2015) shows that securities issued by sovereign entities only account for less than 15% of the securities “repo-ed” at that European Central Bank (ECB). Eberl and Weber (2014) report that, since the outbreak of the financial crisis, the number of eligible asset types at the ECB grew hundredfold. Similarly, a recent industry survey covering major banks, buy-side firms, central counterparties (CCPs), and custodians indicates that 97% of the surveyed participants accept as collateral some government bonds, 56% corporate bonds, 55% covered bonds, 50% equities, 38% asset-backed securities, 32% money market funds, and 16% gold (Sapient, 2014). Furthermore, in Chapter 1, we show for a sample of equity index funds engaging in securities lending that the posted collateral is 100% equity.

This recent shift toward non-sovereign debt collateral is a consequence of increased demand for collateral (Singh, 2013). The later is due to recent regulatory changes, such as mandatory central clearing for derivatives, as well as to the migration of most interbank lending to the secured segment of the money market following the failure of Lehman. For instance, the Euro money market survey conducted by the European Central Bank (2014) indicates that unsecured (respectively secured) borrowing decreased by 80% (respectively increased by 25%) between 2007 and 2014.

Our study on collateral management combines (1) portfolio optimization tools developed to build large efficient portfolios (Jagannathan and Ma, 2003, and Ledoit and Wolf, 2004) and (2) default risk mitigation techniques. By doing so, we complement the academic literature on the role of collateral on financial markets. Duffie (1996) shows that the specialness in repo markets should raise the price of the underlying security by the present value of the saving in borrowing costs in repo markets – a prediction supported empirically by Jordan and Jordan (1997). Using data on repurchase agreements by primary securities dealers, Bartolini et al. (2011) show that the collateral value of Treasuries, as captured by its general collateral repo rates, jumps during periods of liquidity needs. They find that variations in collateral values explain a significant fraction of changes in short-term yield spreads. In the model of Brunnermeier and Pedersen (2009), collateral can lead to margin spirals as losses on one asset increases both volatility and collateral requirements, which in turn induces market participants to reduce their positions and further reinforces the selling pressure.

This study also contributes to the literature on default risk mitigation techniques. This topic has grown in importance since the recent regulation changes that mandate central clearing for standardized derivatives (US Department of Treasury, 2009; European Union,
2.2 Optimal Collateral Portfolio: Theory

2.2.1 Counterparty Risk Exposure

The counterparty risk of ETFs is defined as the risk that the value of the collateral falls below the value of the Net Asset Value (NAV) of the fund, when the fund counterparty is in default. The origin of the counterparty risk depends on the ETF structure.

**Physical ETFs** attempt to track their target indexes by holding all, or a representative sample, of the underlying securities that make up the index (see Figure 1.1 in Chapter 1, Panel A). For example, if you invest in an S&P 500 ETF, you own each of the 500 securities represented in the S&P 500 Index, or some subset of them. Almost all ETF issuers have the provision in their prospectus for loaning out their stock temporarily for revenue. For instance, iShares recently cuts its maximum on-loan limit from 95% of the AUM to 50%. Securities lending exposes ETF investors to counterparty risk (Amenc et al., 2012). In order to mitigate this risk, short sellers have to post collateral with the ETF issuer. On a given point in time, if we denote by $I_t$, the NAV of the ETF (that depends on the value of the tracked index), $\beta_t \in [0, 1]$ the fraction of the securities that are lent, $C_t$ the value of the collateral portfolio per share and $h$ the haircut, the collateral shortfall is defined as:

$$\Delta_t = I_t \beta_t - C_t (1 - h).$$  

(2.1)

A situation in which $\Delta_t > 0$ can be problematic if the short sellers cannot return to the ETF the borrowed securities in due time, i.e., if some of them default. In that case, the fund will not be able to meet redemption requests from all ETF investors.

**Synthetic ETF (unfunded swap model):** First introduced in Europe in the early 2000’s, synthetic ETFs are an interesting variant of physical ETFs. The most commonly used structure for synthetic replications is the unfunded swap model. In this model, the ETF issuer enters into a total return swap with a counterparty, which can either be an
affiliated bank from the same banking group or another bank (see Figure 1.1, Panel B). The swap counterparty commits to deliver the return of the reference index and sells a substitute basket of securities to the ETF issuer. The second leg of the swap consists of the performance of the basket of securities paid by the issuer to the swap counterparty. An important feature of this model is that the ETF issuer becomes the legal owner of the assets and enjoys direct access to them and there is no haircut in this case \((h = 0)\). The swap is marked to market at the end of each day and reset whenever the counterparty exposure exceeds a given threshold, \(\theta \in [0, 1]\), expressed as a percentage of the NAV. So, the counterparty risk exposure is:

\[
\Delta_t = I_t (1 - \theta) - C_t. \tag{2.2}
\]

In the event of a reset, the swap counterparty delivers additional securities for a value of \(I_t - C_t\) to top up the substitute basket. The European directive UCITS sets \(\theta\) to 10%. In other words, the value of the collateral must at least amount 90% of the NAV. However, a comprehensive industry survey conducted by Morningstar (2012) indicates that most ETF issuers affirm using a \(\theta\) coefficient of less than 10% and some even claim to maintain full collateralization, i.e., \(\theta = 0\). Thus, in the rest of this analysis, without loss of generality, we will set \(\theta\) to zero.

**Synthetic ETF (funded swap model):** In the funded swap model, the ETF issuer transfers investors’ cash to a swap counterparty in exchange for the index performance plus the principal at a future date (see Figure 1.1, Panel C). The swap counterparty pledges collateral assets in a segregated account with a third party custodian. The posted collateral basket is made of securities which come from the counterparty’s inventory and meet certain conditions in terms of asset type, liquidity, and diversification. In practice, appropriate haircuts apply to the assets posted as collateral to account for the risk of value fluctuations and for imperfect correlation between the index and the collateral value. Then, the swap counterparty exposure is measured as:

\[
\Delta_t = I_t - C_t (1 - h). \tag{2.3}
\]

Here, we propose a global framework for all types of ETF structures.

**Definition 1 (counterparty risk exposure)** For any ETF fund, the counterparty risk exposure is measured by:

\[
\Delta_t = \beta_t I_t - C_t (1 - h) \tag{2.4}
\]

where \(\beta_t \in [0, 1]\) and \(h > 0\) for physical ETFs, \(\beta_t = 1\) and \(h = 0\) for unfunded-swap based ETFs, and \(\beta_t = 1\) and \(h > 0\) for funded-swap based ETFs.

By convention, if \(\Delta_t \leq 0\), the value of the collateral portfolio, adjusted by the haircut, is larger (over-collateralization) or equal (full collateralization) than the value of the investment and there is no counterparty risk exposure. On the contrary, if \(\Delta_t > 0\), the collateral basket falls short of the value of the investment and additional collateral is required in order to reach \(C_t (1 - h) = \beta_t I_t\) (Morningstar, 2012).
2.2.2 Counterparty Risk Measure

We define the counterparty risk as the risk of a position of not being properly collateralized in the case of the default of the counterparty. A situation in which $\Delta_t > 0$ has no consequence for the investor, except if there is a simultaneous default by the counterparty. Indeed, facing a collateral shortfall is only an issue when the counterparty is unable to meet further margin calls. Alternatively, if a default occurs when $\Delta_t = 0$ or $\Delta_t < 0$, it has no consequence for the investor.

A counterparty risk measure, denoted by $\rho$, is a statistic that quantifies the level of counterparty risk exposure associated to the ETF. This statistic is defined over the conditional distribution of $\Delta_t$, given a default of the counterparty. Let $(\Omega, \mathcal{F}, P)$ be a probability space, and $\Delta_t : \Omega \rightarrow \mathbb{R}$ the counterparty risk exposure. Denote by $D_t$ a binary variable defined over $\Omega$ and equal to one if the counterparty defaults and zero otherwise. We assume that there exists a valid conditional probability measure $Q : \mathcal{F} \rightarrow [0, 1]$ such that for all $\delta \in \mathcal{F}$, $\Pr (\delta | D_t = 1) = Q (\delta)$. Formally, $\rho$ is defined as a function defined on the space $L^\infty (\Omega, \mathcal{F}, Q)$, where $Q$ is the conditional probability measure given the counterparty default. This function implicitly encodes the preferences of the investor over the universe of the counterparty risk exposures, conditionally to the default of the counterparty. We assume that this function satisfies the main conditions that define a risk measure, namely the normalization, monotonicity, and translation invariance axioms.

- Normalization: $\rho (0) = 0$.
- Monotonicity: if $\delta_1, \delta_2 \in \Omega$ and $\delta_1 | D_t = 1 \leq \delta_2 | D_t = 1$ a.s., then $\rho (\delta_1) \leq \rho (\delta_2)$. 
- Translation invariance: if $\alpha \geq 0$ and $\delta \in \Omega$, then $\rho (\delta + \alpha) = \rho (\delta) + \alpha$.

The first axiom states that of there is no counterparty risk exposure, the risk measure is normalized to zero. The monotonicity implies that if a collateral portfolio 1 implies less exposure than a portfolio 2 (almost surely), this portfolio is preferred by the investor and the risk measure for 1 is smaller than for 2. Finally, the translation invariance\(^{18}\) indicates that if the exposure increases from a fixed amount $\alpha$, then the risk measure also increases from the same amount.

Various counterparty risk measures can be used in practice such as the Value-at-Risk, the expected collateral shortfall or the Aggregate Counterparty Risk (ACR) defined in Chapter 1. The counterparty risk measures are not necessarily coherent (Artzner et al., 1999) since the sub-additivity property is not required. Indeed, the split of a collateral portfolio into two sub-portfolios is impossible and the diversification principle (or the combination of two collateral portfolios) has no interpretation in this context.

We assume that the counterparty risk measure depends on the two first moments of the conditional distribution of the counterparty risk exposure.

\(^{18}\)In general for a given risk measure, the translation invariance is denoted as $\rho (r + \alpha) = \rho (r) - \alpha$, since adding a fixed revenue $a$ to the return of the portfolio induces a decrease of the risk measure. In our case, adding a fixed amount to the exposure increases the counterparty risk.
Chapter 2. Optimal Collateral Portfolios for Exchange-Traded Funds

Assumption 1 (A1) The counterparty risk measure $\rho$ can be expressed as a function of the two moments $\mu_\Delta = \mathbb{E}(\Delta_t | D_t = 1)$ and $\sigma_\Delta^2 = \mathbb{V}(\Delta_t | D_t = 1)$:

$$\rho = f(\mu_\Delta, \sigma_\Delta)$$

where the functional form $f(.)$ depends on the conditional distribution of $\Delta_t$.

This assumption can be viewed as an implicit assumption on the conditional distribution of $\Delta_t$, restricting this distribution to belongs to the family of the location-scale distributions. For instance, if $\Delta_t$ has a normal conditional distribution, the expected collateral shortfall and the ACR can be expressed as simple functions of $\mu_\Delta$ and $\sigma_\Delta$:

$$S_t = \mathbb{E}(\Delta_t | \Delta_t > 0, D_t = 1) = \mu_\Delta + \sigma_\Delta \lambda\left(\frac{\mu_\Delta}{\sigma_\Delta}\right)$$ (2.6)

$$ACR_t = \Pr(\Delta_t > 0 | D_t = 1) \times S_t = \mu_\Delta \Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) + \sigma_\Delta \phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right)$$ (2.7)

with $\lambda(y) = \phi(y) / \Phi(y)$ is the inverse Mills ratio, $\Phi(y)$ denotes the cdf of the standard normal distribution, and $\phi(y)$ is the pdf of the standard normal distribution. The derivation of Equations (2.6) and (2.7) are provided in Appendix B.1.

Under this assumption, there exists a domain for which the risk measure increases with both the mean $\mu_\Delta$ and the standard deviation $\sigma_\Delta$ of this distribution.

Assumption 2 (A2) The counterparty risk measure $\rho$ satisfies:

$$\frac{\partial \rho}{\partial \mu_\Delta} > 0 \quad \frac{\partial \rho}{\partial \sigma_\Delta} > 0.$$ (2.8)

The intuition of this assumption is obvious. If the risk measure depends on the two first moments of the conditional distribution of the counterparty exposure, then it increases with the mean and the variance of this distribution since the risk increases with the realization of $\Delta_t$. This assumption is satisfied for all the location scale distribution and the normal distribution in particular (see Appendix B.2 for the normal case).

### 2.2.3 Definition of the Optimal Collateral Portfolio

We show in this section how to construct an optimal collateral portfolio that aims to protect ETF investors against counterparty risk. The collateral portfolio shall be mutually agreed upon by both the collateral provider (the counterparty) and the collateral receiver (the ETF issuer). The process that leads to the optimal collateral portfolio can be divided into three steps.

First, the counterparty and the ETF issuer have to determine a set of eligible securities. In practice, the securities pledged as collateral directly come from the inventory of the

---

19This allows us to select the optimal collateral portfolio through a mean-variance approach (Markowitz, 1959). The mean-variance approach leads exactly to the optimal portfolio when conditional distribution of the shortfall is normal or when the utility function is quadratic. Otherwise, it provides an accurate approximation of the optimal portfolio (Levy and Markowitz, 1979; Kroll et al., 1984).
2.2. Optimal Collateral Portfolio: Theory

counterparty, which includes securities held for investment purposes, market making, underwriting, or other intermediation activities. As a result, there is no need for the collateral provider to purchase any new securities to meet collateral requirements. In general, when choosing the securities to be pledged, the collateral provider aims to reduce its regulatory capital by transferring high risk-weight securities and/or to minimize the opportunity cost of holding collateral. Such securities include those with relatively low fees on the securities lending market; those with relatively low collateral value in the repo market (Bartolini et al., 2011); those that are not eligible as collateral for central-bank credit operations; and securities for which the demand is low on the secondary market (Brandt and Kavajecz, 2004). On the receiver side, only collateral with sufficient tradability will be admitted. For instance, securities that are not listed, issued by small firms, with wide bid-ask spreads, or low market depth may not be accepted as collateral. Similarly, the collateral receiver may prevent unrated bonds from being included in the collateral portfolio. This interaction between the provider and the receiver of collateral leads to the determination of a set of $K$ eligible securities that need to be allocated.

Second, both parties have to determine the level of collateralization. At the end of day $t$, the value of the collateral portfolio $C_t$, adjusted by the haircut $h$, is determined by the fraction $\beta_t$ of the NAV $I_t$ which has to be collateralized and the desired level of collateralization $\alpha$:

$$C_t = \frac{\alpha \beta_t I_t}{(1 - h)} \text{ with } \alpha \geq 1.$$  \hspace{1cm} (2.9)

If $\alpha = 1$, the fund is fully collateralized at time $t$. When $\alpha > 1$ the value of the collateral portfolio, adjusted by the haircut, is larger than the fraction $\beta_t$ of the NAV which has to be collateralized.

Third, given the eligible securities and the level of collateralization on day $t$, the composition of the collateral portfolio is set to minimize counterparty risk on day $t + 1$. This counterparty risk is defined as the risk of being undercollateralized in $t + 1$ given the potential changes in the fund’s NAV and in the value of the collateral securities between $t$ and $t + 1$, conditionally on the default of the counterparty. The collateral shortfall\footnote{Notice that when the fund is fully-collateralized, i.e. $\alpha = 1$, the collateral shortfall in $t + 1$ can be described as a function of the tracking difference between the return of the fund and the return of the collateral:}

$$\Delta_{t+1} = C_t (1 - h) \left( \frac{1 - \alpha}{\alpha} + \frac{r_{i,t+1}}{\alpha} - \sum_{k=1}^{K} \omega_{k,t} r_{k,t+1} \right)$$  \hspace{1cm} (2.11)

where $\omega_{k,t}$ corresponds to the weight of the asset $k$ in the collateral portfolio.

The optimal collateral portfolio corresponds to the portfolio\footnote{For simplicity in the notations, we do not index the weights $\omega$ by $t$.}$^2$ $\omega = (\omega_1, ..., \omega_K)$ that minimizes the counterparty risk measure applied to $\Delta_{t+1}$, denoted $\rho_{t+1}(\omega)$.

**Definition 2** The weights $\omega^* = (\omega_1^*, ..., \omega_K^*)'$ of the optimal collateral portfolio at time $t$
Chapter 2. Optimal Collateral Portfolios for Exchange-Traded Funds

satisfy:

\[
\omega^* = \arg\min_{\omega \in \Theta_t} \rho_{t+1}(\omega)
\]

subject to

\[
\begin{aligned}
\omega \geq 0 \\
e^T \omega = 1
\end{aligned}
\]

where \(\Theta_t\) denotes the set of all the feasible portfolios based on the \(K\) eligible securities.

We impose \(\omega\) to be non-negative since short positions in collateral would be nonsensical. As usual, the sum of the portfolio weights is normalised to one.

In general, the conditional distribution of the shortfall \(\Delta_{t+1}\) is unknown since the joint distribution of the collateral asset’s returns \(r_{k,t+1}\) and the return of the fund’s NAV \(r_{i,t+1}\), is unknown at time \(t\). As a consequence, the functional form of the risk measure \(\rho_{t+1}(\omega) = f(\mu_{\Delta}(\omega), \sigma_{\Delta}(\omega))\) with \(^{22}\) \(\mu_{\Delta}(\omega) = \mathbb{E}(\Delta_{t+1}|D_{t+1} = 1)\) and \(\sigma_{\Delta}^2(\omega) = \mathbb{V}(\Delta_{t+1}|D_{t+1} = 1)\), is also unknown and the optimization program is infeasible. However, under assumptions A1 and A2, it is possible to derive an efficient frontier for the optimal collateral portfolios.

### 2.2.4 Efficient Frontier for Optimal Collateral Portfolios

As for traditional analysis of portfolio management, it is possible to define an efficient frontier for the optimal collateral portfolios. Indeed, for a given level of expected shortfall \(\mu_{\Delta}(\omega) = \gamma\), if a collateral portfolio does not have the minimum shortfall variance, it is always possible to decrease the level of risk by choosing another portfolio with a lower shortfall variance \(\sigma_{\Delta}^2(\omega)\). Therefore the portfolio with the minimum level of risk \(\rho_{t+1}(\omega)\) belongs to the set of portfolios with minimum shortfall variance for all the feasible expected shortfalls \(\gamma\).

Let us define \(\bar{\gamma}\) the expected shortfall of the global minimum variance collateral portfolio, i.e. the portfolio that minimises \(\sigma_{\Delta}^2(\omega)\), and \(\Theta_t\) the set of all the portfolios such that \(\mu_{\Delta}(\omega) \leq \bar{\gamma}\). As the level of risk is decreasing in the expected shortfall, the efficient frontier of optimal collateral portfolios is defined as the set of minimum variance portfolios for all the expected shortfall \(\gamma \leq \bar{\gamma}\).

**Proposition 1** Under assumptions A1 and A2, the efficient frontier for optimal collateral portfolios is given by all the portfolios \(\tilde{\omega}(\gamma) \in \bar{\Theta}_t\) that are solutions of the following optimisation program:

\[
\tilde{\omega}(\gamma) = \arg\min_{\omega \in \bar{\Theta}_t} \sigma_{\Delta}^2(\omega)
\]

subject to

\[
\begin{aligned}
\mu_{\Delta}(\omega) = \gamma \\
\omega \geq 0 \\
e^T \omega = 1
\end{aligned}
\]

Under assumption A1 and A2, the collateral portfolio minimizing the risk measure \(\rho_{t+1}(\omega)\) necessary belongs to the set of portfolios with the minimum variance for all level of

\(^{22}\)For simplicity in the notations, we do not index by \(t + 1\) the two first conditional moments of \(\Delta_{t+1}\).
expected shortfall γ ≤ \( \tilde{\gamma} \). The portfolios with γ < 0 are expected to be over-collateralized in \( t + 1 \), while those with γ = 0 are expected to be full-collateralized. Note that the portfolios on the frontier can have a positive expected shortfall γ > 0 as soon as γ > 0. It means that the ETF may be under-collateralized in \( t + 1 \) on average, if the shortfall variance is small enough.

Both moments \( \mu_\Delta (\omega) \) and \( \sigma^2_\Delta (\omega) \) can be expressed as functions of the moments of the returns of the collateral securities and of the NAV. Define \( r_t = (r_{t,1}, ..., r_{t,K})^T \) the \( K \times 1 \) vector of returns of the collateral securities and \( z_t = (r_{t,1}/\alpha, r_{t,K}^T) \), with:

\[
\mathbb{E} (z_{t+1} | D_{t+1} = 1) = \begin{pmatrix} \mu_i \\ \mu_{(K,1)} \end{pmatrix}
\] (2.14)

\[
\Sigma_z = \mathbb{E} ((z_{t+1} - \mathbb{E} (z_{t+1})) (z_{t+1} - \mathbb{E} (z_{t+1}))^T | D_{t+1} = 1) = \begin{pmatrix} \sigma^2_i \\ \Sigma_i \\ \Sigma_{(K,1)} \\ \Sigma_{(K,K)} \end{pmatrix}
\] (2.15)

where \( \mu_i \) and \( \sigma^2_i \) respectively denote mean and variance of the return of the NAV divided by \( \alpha \), \( z_{i,t} = r_{i,t}/\alpha \). Then, the collateral shortfall in the next day (Equation 2.11) becomes:

\[
\Delta_{t+1} = C_t (1 - h) \left( \frac{1 - \alpha}{\alpha} + z_{i,t+1} - \omega^T r_{t+1} \right)
\] (2.16)

and the corresponding conditional moments are defined by:

\[
\mu_\Delta (\omega) = \mathbb{E} (\Delta_{t+1} | D_{t+1} = 1) = C_t (1 - h) \left( \frac{1 - \alpha}{\alpha} + \mu_i - \omega^T \mu \right)
\] (2.17)

\[
\sigma^2_\Delta (\omega) = \mathbb{V} (\Delta_{t+1} | D_{t+1} = 1) = C_t^2 (1 - h)^2 \left( \omega^T \Sigma \omega + \sigma^2_i - 2 \omega^T \Sigma_i \right)
\] (2.18)

Without loss of generality, we assume that \( C_t (1 - h) = 1 \). Then, under the assumptions A1 and A2, the program in Equation (2.13) can be expressed as:

\[
\hat{\omega} (\gamma) = \arg \min_{\omega \in \mathcal{S}_i} \frac{1}{2} \omega^T \Sigma \omega + \frac{1}{2} \sigma^2_i - \omega^T \Sigma_i
\] (2.19)

subject to

\[
\begin{cases}
\mu_i - \omega^T \mu = \tilde{\gamma} \\
\omega \geq 0 \\
e^T \omega = 1
\end{cases}
\]

with \( \tilde{\gamma} = \gamma - (1 - \alpha) / \alpha \), for all \( \gamma \leq \tilde{\gamma} \). The associated Lagrange function \( f (\omega, \lambda_1, \lambda_2, \lambda_3) \) is:

\[
f (\omega, \lambda_1, \lambda_2, \lambda_3) = \frac{1}{2} \omega^T \Sigma \omega + \frac{1}{2} \sigma^2_i - \omega^T \Sigma_i - \lambda_1 (e^T \omega - 1) - \lambda_2 (\omega^T \mu - \mu_i + \tilde{\gamma}) - \lambda_3 \omega
\] (2.20)

with \( \lambda_1 > 0, \lambda_2 > 0 \) and \( \lambda_{3,i} \geq 0 \) for \( i = 1, ..., K \). To solve this problem, we need to distinguish two cases: one case in which the \( K \) positivity constraints \( \lambda_{3,i} \omega \) are not binding and another case in which there is at least one asset \( i \) for which \( \lambda_{3,i} > 0 \).
Case 1: No positivity constraints. If the positivity constraints are not binding ($\lambda_3 = 0$), the weights of the portfolio that belongs to the efficient frontier can be expressed as a function of the weights of the Markowitz’s mean variance portfolio (without positivity constraint). We define three scalar terms $a$, $b$, and $c$ such that:

$$
a = e^\top \Sigma^{-1} e \quad b = e^\top \Sigma^{-1} \mu \quad c = \mu^\top \Sigma^{-1} \mu
$$

(2.21)

where $e$ is the unit vector.

**Proposition 2** If $\lambda_3 = 0$, the weights of the efficient portfolios are defined as:

$$
\tilde{\omega} (\gamma) = \tilde{\omega}_{MV} + \Sigma^{-1} \Sigma_i^{-1} e + \left( \frac{a \mu^\top - be^\top}{b^2 - ac} \right) \Sigma^{-1} \Sigma_i^{-1} \mu
$$

(2.22)

where $\tilde{\omega}_{MV}$ corresponds to the weights of the Markowitz’s mean variance portfolio:

$$
\tilde{\omega}_{MV} = \left( \frac{b \mu_i - b \gamma - c}{b^2 - ac} \right) \Sigma^{-1} e + \left( \frac{b - a \mu_i + a \gamma}{b^2 - ac} \right) \Sigma^{-1} \mu
$$

(2.23)

and $\gamma = \gamma - (1 - \alpha)/\alpha$.

The proof of Proposition 2 is provided in Appendix B.3. The efficient weights depend on $\Sigma$ and $\Sigma_i$, but they also depend on the vector of expected returns of the collateral securities $\mu$ and on the expected transformed return of the NAV $\mu_i - \tilde{\gamma}$. The latter can be viewed as a target for the expected return of the collateral portfolio, $\omega^\top \mu$. Notice that if the collateral securities and the NAV are independent, $\Sigma_i = 0_{K \times 1}$, then the efficient weights simply correspond to the weights $\tilde{\omega}_{MV}$ of the mean variance portfolio with a target mean $\mu_i - \tilde{\gamma}$.

In the special case where all the collateral securities have the same mean, i.e. $\mu = ke$ with $k \in \mathbb{R}$, the optimizing program (2.19) is independent of $\gamma$ since all the portfolios give the same expected collateral shortfall $\mathbb{E} (\Delta_{t+1} | D_{t+1} = 1)$. Then, we have:

$$
\tilde{\omega} = \arg \min_{\omega \in \Theta_i} \frac{1}{2} \omega^\top \Sigma \omega + \frac{1}{2} \sigma_i^2 - \omega^\top \Sigma_i
$$

subject to

$$
\begin{align*}
\omega &\geq 0 \\
e^\top \omega &= 1
\end{align*}
$$

(2.24)

The solution of this program is obvious and the weights of the collateral portfolio can be expressed as a function of the Global Minimum Variance Portfolio (GMVP).

**Proposition 3** If $\mu = ke$ with $k \in \mathbb{R}$ and $\lambda_3 = 0$, the optimal weights of the collateral portfolio are:

$$
\tilde{\omega} = \Sigma^{-1} \Sigma_i + \left( 1 - e^\top \Sigma^{-1} \Sigma_i \right) \tilde{\omega}_{GMVP}
$$

(2.25)

where $\tilde{\omega}_{GMVP}$ corresponds to the weights of the GMVP:

$$
\tilde{\omega}_{GMVP} = \frac{\Sigma^{-1} e}{e^\top \Sigma^{-1} e}.
$$

(2.26)
The proof of Proposition 3 is provided in Appendix B.4. The optimal weights depend (i) on the variance covariance matrix, $\Sigma$, of the returns of the collateral securities and (ii) on the vector of covariances, $\Sigma_i$, between the returns of these securities and the return of the NAV. Notice that if the returns of the collateral securities and the NAV are independent ($\Sigma_i = 0_{K \times 1}$), the optimal weights simply correspond to those obtained by minimizing the variance of the collateral portfolio, that is to $\tilde{\omega}_{GMVP}$.

**Case 2: Positivity constraints.** When at least one asset $i \in \{1, \ldots, K\}$ is excluded from the optimal portfolio, i.e. for which $\lambda_{3,i} > 0$ and $\omega_i = 0$, there is no closed-form solution for the optimal constrained portfolio. However, it is possible to define an implied global variance-covariance matrix (for both the NAV and the collateral assets returns) $\tilde{\Sigma}_z$ such that the constrained portfolio $\tilde{\omega}(\gamma)$ is the solution of the unbounded program:

$$
\tilde{\omega}(\gamma) = \arg \min_{\omega \in \Omega_{i}} \frac{1}{2} \omega^\top \Sigma \omega + \frac{1}{2} \tilde{\sigma}_i^2 - \omega^\top \Sigma_i
$$

subject to

$$
\begin{align*}
\mu_i - \omega^\top \mu &= \tilde{\gamma} \\
e^\top \omega &= 1
\end{align*}
$$

with

$$
\tilde{\Sigma}_z = \begin{pmatrix}
\tilde{\sigma}_i^2 & \tilde{\Sigma}_{i1} \\
\tilde{\Sigma}_{1i} & \tilde{\Sigma}_{11} \\
\end{pmatrix}
$$

The implied covariance matrix $\tilde{\Sigma}_z$ is a particular shrunk version of the covariance matrix $\Sigma_z$ that depends on the Lagrange coefficients issued from the constrained optimization (Jagannathan and Ma, 2003):

$$
\tilde{\Sigma}_z = \Sigma_z - \lambda_3 e^\top
$$

Furthermore, if we express the ETF as the portfolio of the index constituents, we can then derive an alternative equation for the collateral shortfall. Let $A$ and $B$ be the set of index constituents and the set of eligible collateral securities, respectively. $J$ is the size of the union $A \cup B$. Equation (2.11) for the collateral shortfall at time $t+1$ then becomes:

$$
\Delta_{t+1} = C_t (1 - h) \left( \frac{1 - \alpha}{\alpha} + \sum_{j \in A \cup B} \frac{\delta_{j,t} r_{j,t+1}}{\alpha} - \sum_{j \in A \cup B} \omega_{j,t} r_{j,t+1} \right)
$$

Where $r_{j,t}$ is the return of the $j^{\text{th}}$ asset while $\delta_{j,t}$ and $\omega_{j,t}$ denote the weight of the $j^{\text{th}}$ asset in the index and in the collateral portfolio, respectively. Note that $\delta_{j,t} = 0$ if the security $j$ is not included in $A$ and $\omega_{j,t} = 0$ if $j$ is not included in $B$.

When all the index constituents are available in the eligible collateral set, i.e. $A \subseteq B$, we can rewrite the collateral shortfall $\Delta_{t+1}$. Assuming $C_t (1 - h) = 1$ and defining $v =$

---

23: This implied covariance matrix is positive definite under certain conditions. See Appendix B.5 for a generalized case.
as the weights of the index constituents divided by \( \alpha \), and \( x = \omega - v \) as the vector of the deviations from the index of the weights of the collateral, we obtain:

\[
\Delta_{t+1} = \frac{1 - \alpha}{\alpha} - x^\top R_{t+1}
\]

(2.30)

\[
\mu_\Delta = \mathbb{E}(R_{t+1}) = \frac{1 - \alpha}{\alpha} - x^\top \mu_R
\]

(2.31)

\[
\sigma_\Delta^2 = \mathbb{V}(R_{t+1}) = x^\top \mu_R x
\]

(2.32)

where \( R_{t+1} = (r_{1,t+1}, \ldots, r_{J,t+1})' \) is the vector of returns of the \( J \) securities with \( \mathbb{E}(R_{t+1}) = \mu_R \) and \( \mathbb{V}(R_{t+1}) = \Sigma_R \). The program (2.19) to derive the efficient collateral frontier is now:

\[
\tilde{x}(\gamma) = \arg \min_{x + v \in \Theta_t} x^\top \Sigma_R x
\]

subject to

\[
\begin{align*}
(1 - \alpha)/\alpha - x^\top \mu_R &= \gamma \\
x &\geq -v \\
e^\top x &= (\alpha - 1)/\alpha
\end{align*}
\]

(2.33)

Note that the constraints \( x \geq -v \) correspond to the positivity constraints of the program (2.19). Moreover, adding constraints of the form \( x_l = v_l \) would allow us to take into account the case where some index constituents \( l \) are not available as collateral.

The optimization program (2.33) has an analytical solution when the inequality constraints are not binding. In this case, taking the solution for the deviation \( x \) and solving for the variance of the collateral shortfall leads to an expression for the efficient frontier of the form:

\[
\sigma_\Delta^2 = A \cdot \mu_\Delta^2 + B \cdot \mu_\Delta + C
\]

(2.34)

where \( A, B \) and \( C \) are three constant terms independent of the weights of the collateral portfolios. The frontier is a parabola in the \((\sigma_\Delta^2, \mu_\Delta)\) space.

The case when the ETF is fully collateralized at time \( t \) (\( \alpha = 1 \)) is worthwhile mentioning. In this case, the collateral shortfall can be described as a function of the tracking difference between the return of the fund and the return of the collateral (see footnote 20). It then follows that the program (2.33) is similar to the problem of the portfolio optimization in the presence of a benchmark. Indeed, as in Jorion (2003) or Roncalli (2013), the efficient frontier becomes a straight line in the volatility-mean space:

\[
\sigma_\Delta = D \cdot \mu_\Delta
\]

(2.35)

where \( D \) is a constant term. This result holds as long as all the index constituents are available as collateral securities. The proofs of Equations (2.34) and (2.35) are provided in Appendix B.6.

Moreover, if we assume that the set of eligible securities for the collateral portfolio includes all of the index constituents \( \mathcal{A} \subseteq \mathcal{B} \) and that the ETF is fully collateralized at time \( t \) (\( \alpha = 1 \)), then the optimal collateral portfolio mirrors the index. Thus, an obvious way to set the collateral shortfall to zero is to maintain a perfect match between the collateral portfolio and the index, \( \omega^*_j,t = \delta_{j,t} \) for \( j = 1, \ldots, J \) where \( \omega^*_j,t = \delta_{j,t} = 0 \) when the
security \( j \) is not an index constituent (see Appendix B.7). For a synthetic ETF, this situation corresponds to a hybrid ETF combining some of the features of both synthetic and physical ETFs. Even though the fund is based on a swap, it benefits from a physical replication of the index.

### 2.2.5 Illustration

As an illustration, we consider a simple example with two collateral assets, A and B, \((K = 2)\). We assume that there is no haircut \((h = 0)\) and that there is full collateralisation at time \( t \), i.e. \( I_t = C_t \) and \( \alpha = 1 \). Define the vector of returns \( z_t = (r_{i,t}, r_{A,t}, r_{B,t})^T \), where \( r_{i,t} \) is the return of the fund’s NAV, \( r_{A,t} \) and \( r_{B,t} \) are the returns of the two assets, with:

\[
\mathbb{E}(z_{t+1}) = \begin{pmatrix} 0.3 \\ 0.7 \\ 0.1 \end{pmatrix}
\]

\[
\mathbb{E}((z_{t+1} - \mathbb{E}(z_{t+1})) (z_{t+1} - \mathbb{E}(z_{t+1}))^T) = \Sigma_z = \begin{pmatrix} 2 & 1 & 0.2 \\ 1 & 3 & 0.5 \\ 0.2 & 0.5 & 1 \end{pmatrix}.
\]

In this example, the two collateral assets are positively correlated, but the asset A is more volatile than the asset B and has a larger expected return (0.7 versus 0.1). Furthermore,

Figure 2.1: Efficient Frontier for Collateral Portfolio with Two Assets

Notes: This figure presents the feasible collateral portfolios (in blue) and the efficient frontier (in red) of the collateral portfolios combining two assets, A and B. The minimum variance portfolio M is also displayed. The collateral shortfall mean is represented in the y-axis and the collateral shortfall variance in the x-axis.
both assets are positively correlated with the NAV but the correlation with the first one is larger (0.41) than with the second one (0.35). As a consequence, increasing the weight of the asset A in the collateral portfolio has two opposite effects on the counterparty risk: (1) since A is more risky, it increases the volatility of the shortfall, but (2) since A is more strongly correlated with the NAV and has a higher expected return than asset B, increasing its weight in the portfolio decreases the volatility and the expected value of the collateral shortfall.

We consider a collateral portfolio with a vector of weights $\omega = (\omega_A, 1 - \omega_A)^T$ where $\omega_A \geq 0$ corresponds to the weight of the first security A. Figure 2.1 displays the mean $\mu_\Delta (\omega)$ (y-axis) and the variance $\sigma_\Delta^2 (\omega)$ (x-axis) of all the feasible portfolios, ranging from $\omega_A = 0$ (portfolio B) to $\omega_A = 1$ (portfolio A). Each portfolio is represented by a point. The portfolios that belong to the efficient frontier, defined by the solution of the program (2.13), are colored in red. As previously explained, the efficient frontier correspond to the set of portfolios for which the expected collateral shortfall $\mu_\Delta (\omega)$ is smaller than the expected shortfall $\gamma$ of the minimum variance portfolio (portfolio M).

Figure 2.2: Efficient Frontier for Collateral Portfolio with Three Assets

![Diagram](image)

Notes: This figure presents the feasible collateral portfolios (in blue) and the efficient frontier (in red) of the collateral portfolios combining three assets, A, B and C. The minimum variance portfolio M is also displayed. The collateral shortfall mean is represented in the y-axis and the collateral shortfall variance in the x-axis.

When we consider three collateral assets, the set $\Theta_t$ of all the feasible collateral portfolios can be represented by a surface of couples $(\mu_\Delta (\omega), \sigma_\Delta^2 (\omega))$ and the efficient frontier for the optimal collateral portfolios then corresponds to its lower bound. For instance, consider a third collateral security C with a level of volatility, expected return and correlation
2.2. Optimal Collateral Portfolio: Theory

with the NAV between those of the assets A and B. Define the vector of returns \( z_t = (r_{i,t}, r_{A,t}, r_{B,t}, r_{C,t})^T \) with:

\[
E(z_{t+1}) = \begin{pmatrix} 0.3 \\ 0.7 \\ 0.1 \\ 0.5 \end{pmatrix}
\]

(2.38)

\[
E((z_{t+1} - E(z_{t+1}))(z_{t+1} - E(z_{t+1}))^T) = \begin{pmatrix} 2 & 1 & 0.2 & 0.5 \\ 1 & 3 & 0.5 & 0 \\ 0.2 & 0.5 & 1 & 0.2 \\ 0.5 & 0 & 0.2 & 2 \end{pmatrix}
\]

(2.39)

Figure 2.2 displays all the collateral portfolios \( \omega = (\omega_A, \omega_B, 1 - \omega_A - \omega_B) \) and the efficient frontier. This frontier, obtained as the set of solutions of the program (2.13), corresponds to all the minimum variance collateral portfolios \( \omega \) defined for all the expected shortfall \( \mu_\Delta(\omega) \) smaller than the expected shortfall \( \gamma \) of the minimum variance portfolio (point M).

2.2.6 Optimal Collateral Portfolio

A natural way to identify the optimal collateral portfolio on the efficient frontier is to make an assumption on the (conditional) joint distribution of the returns \( r_{i,t+1} \) and \( r_{k,t+1} \). This distributional assumption can be viewed as an alternative to the specification of the preference of the investors. Indeed, the optimization program defined in Equation (2.13) determine the weights of the optimal portfolios by minimizing a counterparty risk measure \( \rho_{t+1}(\omega) \), which summarises the distribution of the collateral risk exposure \( \Delta_{t+1} \). As a consequence, a distributional assumption is required to determine the optimal collateral portfolio. An alternative optimization program would consist in determining the weights of the collateral portfolio such as to maximize the expected utility of the investor, as in the original Markowitz problem. In this case, an assumption on the preferences of the investor would be required. Both programs give a similar solution as soon as the counterparty risk measure reflects the investor’s preferences, in the sense that if \( \omega_X \succ \omega_Y \), then \( \rho_{t+1}(\omega_Y) > \rho_{t+1}(\omega_X) \).

For a given distributional assumption on \( \Delta_{t+1} \), it is possible to derive an analytical or numerical expression for the collateral risk measure \( \rho_{t+1}(\omega) \). The optimal portfolio \( \omega^* \) can be directly determined as a solution of the optimisation program defined in Equation (2.12). This portfolio necessarily belongs to the efficient frontier of collateral portfolios.

**Definition 3** The optimal collateral portfolio \( \omega^* \) is defined as the portfolio of the efficient frontier associated with the minimum level of counterparty risk:

\[
\omega^* = \arg \min_{\gamma < \gamma} \rho_{t+1}(\tilde{\omega}(\gamma))
\]

(2.40)

The weights of the optimal portfolio \( \omega^* \) corresponds to the point of tangency of the efficient frontier and the counterparty iso-risk curves, defined as follows:
Definition 4 For a collateral risk measure $\rho_{t+1} (\omega)$, a counterparty iso-risk curve corresponds to all pairs of shortfall variance and expected shortfall $(\sigma^2_{\Delta} (\omega); \mu_{\Delta} (\omega))$ that lead to the same level of collateral risk $\bar{\rho} \in \mathbb{R}$:

$$\{ \omega \in \Theta_t : f (\mu_{\Delta} (\omega), \sigma_{\Delta} (\omega)) = \bar{\rho} \}$$

(2.41)

where $\Theta_t$ denotes the set of all the feasible portfolios based on the $K$ eligible securities.

The optimal portfolio $\omega^*$ correspond to the portfolio of the efficient frontier that allows to reach the iso-risk curve associated to the minimum level of risk.

Consider the same example as in Section 2.2.5 and assume that the return vector has a conditional normal distribution. If we consider the ACR as the counterparty risk measure, we have:

$$\rho_{t+1} (\omega) = \mu_{\Delta} (\omega) \Phi \left( \frac{\mu_{\Delta} (\omega)}{\sigma_{\Delta} (\omega)} \right) + \sigma_{\Delta} (\omega) \phi \left( \frac{\mu_{\Delta} (\omega)}{\sigma_{\Delta} (\omega)} \right)$$

(2.42)

with $\lambda (y) = \phi (y) / \Phi (y)$ the inverse Mills ratio. With two assets A and B, the optimal weights $\omega^*$ obtained from the program (2.12), or equivalently from the program (2.40), are $\omega^*_A = 0.77$ and $\omega^*_B = 0.23$. This optimal portfolio corresponds to the tangency point between the efficient frontier and the iso-ACR curve, as displayed in the left plot of Figure 2.3. In the case of three assets (A, B and C), the optimal weights are $\omega^*_A = 0.56$, $\omega^*_B = 0$ and $\omega^*_C = 0.44$. We display in the right plot of Figure 2.3 the efficient frontier, the iso-ACR curve, and the optimal portfolio.

Figure 2.3: Optimal Portfolios

Notes: This figure presents the feasible collateral portfolios (in blue) and the efficient frontier (in red) of the collateral portfolios combining two assets (left plot) and three assets (right plot). The optimal portfolios minimizing the ACR are displayed at the tangency points between the efficient frontiers and the iso-ACR curves (in green). M is the minimum variance portfolio. The collateral shortfall mean is represented in the y-axis and the collateral shortfall variance in the x-axis.
2.2.7 Optimal Collateral Portfolio with a Performance Constraint

In practice, many other types of constraints can be considered in the derivation of the optimal collateral portfolio. One can prevent any issuer from accounting for more than a certain fraction of the collateral. Other constraints can be added on the liquidity of the assets selected for the collateral portfolio or on the collateral portfolio itself. For instance, the optimal weights can be determined under the constraint that the liquidity of the optimal portfolio (measured according to a particular liquidity risk measure) is larger than a given lower bound.

Alternatively, if there are some incentives for a counterparty to use a particular type of collateral (e.g. foreign equities as they lead to particularly high regulatory requirements for banks) or if we take into account the additional revenue due to securities lending, then each type of collateral security has a specific spread. We expand here the collateral portfolio selection by adding this collateral performance dimension. Defining $s = (s_1, \ldots, s_K)^\top$ as the vector of the individual spreads for the $K$ collateral securities, we transform the program (2.12) by imposing that the aggregate spread reaches or exceeds a given threshold $\pi$. In this way, the optimal collateral portfolio $\omega^*$ is given by:

$$\omega^* = \arg \min_{\omega \in \Theta_t} \rho_{t+1}(\omega)$$

subject to

\[
\begin{align*}
\omega &\geq 0 \\
e^\top \omega &= 1 \\
s^\top \omega &\geq \pi
\end{align*}
\]

Moreover, Equation (2.19) which expresses the efficient frontier is rewritten as:

$$\tilde{\omega}(\gamma) = \arg \min_{\omega \in \tilde{\Theta}_t} \frac{1}{2} \omega^\top \Sigma \omega + \frac{1}{2} \sigma_i^2 - \omega^\top \mu_i$$

subject to

\[
\begin{align*}
\mu_i - \omega^\top \mu &= \tilde{\gamma} \\
\omega &\geq 0 \\
e^\top \omega &= 1 \\
s^\top \omega &\geq \pi
\end{align*}
\]

Imposing additional constraints reduces the set of all feasible portfolios $\Theta_t$ and reshapes the efficient frontier. Moreover, it modifies the composition of the optimal portfolio. This alteration can be illustrated taking the example of three assets $A$, $B$, $C$ presented in the section 2.2.5. We now assume that each asset brings additional performance (in basic points) if it is included in the collateral. Furthermore, the ETF manager aims to have a collateral aggregate spread that reaches or exceeds a given threshold $\pi$. Following programs (2.43) and (2.44), Figure 2.4 displays the feasible collateral portfolios, the efficient frontier and the optimal portfolio when $\pi = 7$ and when the spreads for $A$, $B$ and $C$ are of 5 bps, 10 bps and 5 bps, respectively:

$$s = \begin{pmatrix} 5 \\ 10 \\ 5 \end{pmatrix}$$

We also contrast our findings to the set of all feasible portfolios when there is no constraint on the collateral performance. As a result, the set of feasible portfolios in the
constrained optimization is strongly reduced compared to the initial one (unconstrained). For instance, the unconstrained efficient frontier and the optimal portfolio are no longer feasible. The weights of the constrained optimal portfolio are \( w_A^* = 0.38, w_B^* = 0.40 \) and \( w_C^* = 0.22 \). Interestingly, unlike the unconstrained optimum, the asset \( B \) is now a constituent of the optimal portfolio. Thus, adding a performance constraint to the collateral portfolio optimization can lead to some initially undesirable assets to enter in the collateral.

Figure 2.4: Constrained Collateral Portfolios

![Figure 2.4: Constrained Collateral Portfolios](image)

Notes: This figure presents the feasible collateral portfolios (in blue) and the efficient frontier (in red) of the collateral portfolios combining three assets, \( A, B \) and \( C \), when we add a collateral performance constraint. The constrained optimal portfolio is displayed at the tangency points between the efficient frontier and the iso-ACR curve (in green). The unconstrained feasible set of collateral portfolio (in grey) and the unconstrained minimum variance portfolio \( M \) are also displayed. The collateral shortfall mean is represented in the y-axis and the collateral shortfall variance in the x-axis.

2.2.8 Estimation

The efficient frontier for optimal portfolios depends on the covariance matrices of the asset and NAV returns \( \Sigma \) and \( \Sigma_i \), and on the expected returns \( \mu_i \) and \( \mu \). Given the definition of the counterparty risk measure, these moments are conditional on the default of the counterparty. Given the fact that, in our sample, the counterparty of the ETF issuer have never defaulted in the past, these moments have to be estimated with past observations of the returns picked from a high counterparty-risk regime, i.e., a period during which the counterparty experienced a sharp increase in its default probability.
As the number of collateral securities may be larger than the number of dates in the high counterparty-risk regime, the sample covariance matrix suffers from a small sample size problem and ends up being singular. To alleviate this problem, we use the shrinkage estimator of the covariance matrix $\Sigma_z$, proposed by Ledoit and Wolf (2003). This estimator is defined as a weighted average of the sample covariance matrix and a shrinkage target, defined as the variance-covariance matrix issued from a single-factor model, in which all the pairwise correlations are constant. The optimal weight of the shrinkage target, called the shrinkage constant, corresponds to the value that minimizes the expected distance between the shrinkage estimator and the true covariance matrix. The estimators $\mu_i$ and $\mu$ of the expected returns are defined by their empirical counterparts. Finally, given $\Sigma$, $\Sigma_i$, $\mu_i$ and $\mu$, the weights $\tilde{\omega}(\gamma)$, with $\gamma \leq \gamma$, that define the efficient frontier, are determined by solving the program (2.19). The numerical solution with positivity constraints is obtained by using CVX, a package for specifying and solving convex problems (Grant and Boyd, 2008, 2014).

The optimal collateral portfolio can be determined directly by solving the program (2.40), under a conditional distribution assumption on the NAV and collateral asset returns. An alternative consists in using a semi-parametric two-steps approach. The first step consists in determining the efficient frontier for collateral portfolios, based on the estimates $\hat{\Sigma}$, $\hat{\Sigma}_i$, $\hat{\mu}_i$ and $\hat{\mu}$. The second step consists in estimating the level of risk for all these portfolios by using a non-parametric approach and then to determine an estimate of the optimal portfolio $\omega^*$. Indeed, for any collateral portfolio $\omega$, the counterparty risk measure $\rho_{t+1}(\omega)$ can be estimated non-parametrically, i.e., without any assumption on the conditional distribution of the returns. For instance, a sample average of the positive shortfalls or a kernel smoothed version of this estimator (Scaillet, 2004) are two consistent non-parametric estimators of the expected collateral shortfall (Chen, 2008). This estimation is based on a series of hypothetical collateral shortfalls, computed with the weights $\omega$ and the past values of the assets and NAV returns observed during the a high counterparty-risk regime.

As an illustration, we consider the ACR risk measure. Following Berkowitz and O’Brien (2002), we consider a series of hypothetical collateral shortfalls:

$$\Delta_\tau = \beta_i I_t (1 + r_{i, \tau}) - C_t (1 - h) \left(1 + \sum_{k=1}^K \omega_{k,t} r_{k, \tau}\right)$$

(2.46)

where $r_{i, \tau}$ is the historical daily return of the NAV, for $\tau = 1, ..., t$ and $r_{k, \tau}$ is the daily return of the $k$-th collateral security at time $\tau$. The hypothetical collateral shortfall $\Delta_\tau$ measures the shortfall that would have arisen in the past with the current values of $I_t$, $C_t$, and $\omega_t$ and past returns on the NAV and on the collateral securities. We define two nonparametric estimators for the probability and the expected collateral shortfall:

$$\hat{P}_{t+1} = \frac{\sum_{\tau=1}^t I(\Delta_\tau > 0) \times I(\tau \in \mathcal{T})}{\sum_{\tau=1}^t I(\tau \in \mathcal{T})} = \frac{1}{\dim(\mathcal{T})} \frac{\sum_{\tau=1}^t I(\Delta_\tau > 0) \times I(\tau \in \mathcal{T})}{\sum_{\tau=1}^t I(\Delta_\tau > 0) \times I(\tau \in \mathcal{T})}$$

(2.47)

$$\hat{S}_{t+1} = \frac{\sum_{\tau=1}^t \Delta_\tau \times I(\Delta_\tau > 0) \times I(\tau \in \mathcal{T})}{\sum_{\tau=1}^t I(\Delta_\tau > 0) \times I(\tau \in \mathcal{T})}$$

(2.48)

24The intuition of the hypothetical shortfalls is similar to the hypothetical P&Ls used to estimate the VaR (see for instance, Berkowitz and O’Brien, 2002).
Chapter 2. Optimal Collateral Portfolios for Exchange-Traded Funds

where \( I(\cdot) \) denotes the indicator function and \( \Upsilon \) denotes a high-counterparty risk regime. Note that if \( \Delta_\tau \) is a stationary process, these estimators are consistent and asymptotically normally distributed (Chen, 2008). Then an estimator of the ACR can defined as 

\[
\hat{ACR}_{t+1} = \hat{P}_{t+1} \times \hat{S}_{t+1}.
\]

By comparing, the estimates of the ACR for all the portfolios of the efficient frontier \( \tilde{\omega}(\gamma) \) with \( \gamma \leq \tilde{\gamma} \), we can determine the optimal portfolio defined as the portfolio with the minimum estimated ACR. This semi-parametric two-step approach can be viewed as an alternative to the program (2.40).

2.3 Optimal Collateral Portfolio: Empirical Evidence

We implement the methodology presented in the previous section using a sample of total return swaps between a leading asset manager (db-X tracker) and its counterparty, namely Deutsche Bank. Under these swaps, the bank commits to pay to the asset manager the return of a given index and must post collateral to mitigate counterparty risk concerns. On the other hand, the asset manager commits to deliver to the bank the return of the collateral asset, hence the total return swap structure. An important feature of this contract is that the asset manager becomes the legal owner of the assets. This implies that if the swap counterparty defaults, the asset manager can immediately liquidate the assets. As shown in the previous Chapter, such swaps are used by asset managers to produce index-tracking funds based on a synthetic replication and is an important source of funding for the bank providing the swap.

2.3.1 Data

On November 29th 2012, we know the exact composition of the collateral portfolio of four total return swaps: one on the DAX index (equity), one on the Eurostoxx 50 index (equity), one on the iBoxx Global Inflation-Linked index (Treasuries), and one on the iBoxx Sovereigns Eurozone index (Treasuries). These ETFs are four of the largest ETFs provided by db-X tracker. All swaps have the same counterparty, namely Deutsche Bank. As shown in Table 2.1, the assets under management (AUM) of these funds range between $0.628 billion for the iBoxx Sovereigns fund and $8.528 billion for the DAX fund. We also see in Table 2.1 that these swaps are either fully collateralized (\( C_t/I_t = 100\% \)) or almost fully collateralized (\( C_t/I_t > 99\% \)).

<table>
<thead>
<tr>
<th>ETF</th>
<th>AUM</th>
<th>Collateralization</th>
<th>ETF Asset Exposure</th>
<th>ETF Geographic Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX ETF</td>
<td>8,528</td>
<td>99.8%</td>
<td>Equity</td>
<td>Europe</td>
</tr>
<tr>
<td>Euro Stoxx 50 ETF</td>
<td>2,732</td>
<td>99.6%</td>
<td>Equity</td>
<td>Europe</td>
</tr>
<tr>
<td>iBoxx Global Inflation-Linked ETF</td>
<td>1,003</td>
<td>99.2%</td>
<td>Government Bond</td>
<td>World</td>
</tr>
<tr>
<td>iBoxx Sovereigns Eurozone 1-3 ETF</td>
<td>628</td>
<td>100%</td>
<td>Government Bond</td>
<td>Europe</td>
</tr>
</tbody>
</table>

Notes: This table presents some summary statistics for the sample ETFs. The table displays, for each fund, the asset under management (AUM) in USD million, the level of collateralization (collateral value/AUM), as well as the ETF asset and geographic exposure. Data as of November 29, 2012.

25The iBoxx Global Inflation-Linked index covers the major sovereign and quasi-sovereign inflation-linked bond markets of the world. The iBoxx Sovereigns Eurozone 1-3 covers Eurozone-government bonds that have a remaining time to maturity of at least one year and up to three years.
The composition of the collateral portfolios is described in the panel A of Table 2.2. The collateral portfolios of the four ETFs include a total of 81 securities. The set of securities used as collateral for these four ETFs includes 47 equities and 34 government bonds, and there is no corporate bonds. This structure reflects the match between the index which is tracked (equities or treasuries) and the composition of the collateral portfolio (see Chapter 1). The number of posted securities in each portfolio ranges from 9 for the iBoxx Sovereigns Eurozone swap to 31 to the DAX swap.

Besides the 81 securities included in the four collateral portfolios, we also know the identity of another 1,503 securities that are included, on the same day, in the collateral portfolios of another 160 ETFs managed by db-X tracker. However, we omit 67 securities whose data are not available during the high counterparty risk regime (see below). We end up with a pool of 1,517 (= 81 + 1,436) securities that is interpreted as the universe of the set of eligible securities that can be used as collateral (see Section 2.2.3), agreed both by db-X tracker and its Deutsche Bank. As a consequence, the optimal collateral portfolio belongs to the set $\Theta_t$ of all the feasible portfolios $\omega$ based on these $K = 1,517$ eligible securities. Panel B of Table 2.2 displays some characteristics of the set $\Theta_t$. We note that the majority of the used collateral securities are equities: 1,438 out of 1,517 vs. 62 government bonds and 27 corporate bonds. In terms of geographic exposure, most securities are from the Asia-Pacific region, followed by Europe and North America.

Table 2.2: Collateral Portfolios

<table>
<thead>
<tr>
<th>Panel A: Used Collateral Securities</th>
<th>Actual Portfolio</th>
<th>Optimal Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>81</td>
<td>88</td>
</tr>
<tr>
<td>Equities</td>
<td>47</td>
<td>69</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>34</td>
<td>19</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Number of Collateral Securities per Fund</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DAX ETF</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>Euro Stoxx 50 ETF</td>
<td>28</td>
<td>40</td>
</tr>
<tr>
<td>iBoxx Global Inflation-Linked ETF</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>iBoxx Sovereigns Eurozone 1-3 ETF</td>
<td>9</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Eligible Collateral Securities</th>
<th>Europe</th>
<th>Asia-Pacific</th>
<th>North America</th>
<th>R. of the World</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equities</td>
<td>373</td>
<td>882</td>
<td>172</td>
<td>1</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>49</td>
<td>8</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>23</td>
<td>1</td>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: This table presents some summary statistics about collateral portfolios of the sample ETFs (Panel A) and about eligible collateral securities (Panel B). Panel A presents, in contrasting actual and optimal collateral portfolios as of November 29, 2012, the total number of distinct securities used in collateral portfolios as well as the number of distinct equities, government bonds and corporate bonds. Panel A displays also the number of securities used in the collateral portfolio of each sample ETFs. Panel B displays the asset type and the geographic origin of the 1,517 collateral securities eligible on November 29, 2012. The government bond category also includes supranational bonds, government guaranteed bonds, government agency bonds, and German regional government bonds. The corporate bond category also includes covered bonds.

For each collateral security in our sample, we collect from Datastream the historical prices. Because there is, by design, no historical prices for a bond before it is issued, we use the returns of the bond index that best matches the attributes of a given bond: its type
(sovereign vs. corporate), country, rating, and maturity. As explained in the previous Section, we consider the past observations of the returns that belong to a high swap counterparty-risk regime, namely a 2-month period around the bankruptcy of Lehman Brothers (September 1, 2008 - October 31, 2008). Over these two months, the CDS-implied default probability of the swap counterparty, Deutsche Bank, got multiplied by three, and its market capitalization dropped by 50%. All our risk measures are computed using historical returns from this high-risk regime.

### 2.3.2 Efficient Collateral Frontiers and Optimal Portfolios

For each sample swap, we estimate the efficient frontier for collateral portfolios and then pick the optimal portfolio associated to the lowest counterparty risk exposure, as measured by its ACR. We plot in Figure 2.5 the efficient frontier (in red), as well as the actual and optimal collateral portfolio, for the swap on the Euro Stoxx 50 index. Contrary to Section 2.2.5, we do not report the area defined by the set $\Theta_t$ of all the feasible portfolios. We only report the upper (in blue) and lower (in red) frontier of this area. We observe that the actual collateral portfolio do not belong to this frontier. As a consequence, it is possible

**Figure 2.5: Efficient Collateral Frontier for the Euro Stoxx 50 ETF**

Notes: This figure presents the efficient frontier (in red), as well as the actual and the optimal collateral portfolio for the Euro Stoxx 50 ETF, on November 29, 2012. The minimum variance portfolio $M$ is also displayed. The conditional collateral shortfall mean is represented in the y-axis and the conditional collateral shortfall variance in the x-axis. For both the mean and the variance, the shortfall are expressed relative to the NAV of the fund.
to reduce both the mean and the variance of the collateral exposure for the day $t + 1$ and then to reduce the counterparty risk measure, by adopting the optimal portfolio. This Figure clearly illustrates the advantage of our approach in terms of counterparty risk.

Table 2.3: Match between ETFs and Collateral Securities

<table>
<thead>
<tr>
<th>Panel A: Collateral Securities for Actual Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Collateral Securities</td>
</tr>
<tr>
<td>DAX ETF</td>
</tr>
<tr>
<td>Euro Stoxx 50 ETF</td>
</tr>
<tr>
<td>iBoxx Global Inflation-Linked ETF</td>
</tr>
<tr>
<td>iBoxx Sovereigns Eurozone 1-3 ETF</td>
</tr>
<tr>
<td>Geographic Origin of the Collateral Securities</td>
</tr>
<tr>
<td>DAX ETF</td>
</tr>
<tr>
<td>Euro Stoxx 50 ETF</td>
</tr>
<tr>
<td>iBoxx Global Inflation-Linked ETF</td>
</tr>
<tr>
<td>iBoxx Sovereigns Eurozone 1-3 ETF</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Collateral Securities for Optimal Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Collateral Securities</td>
</tr>
<tr>
<td>DAX ETF</td>
</tr>
<tr>
<td>Euro Stoxx 50 ETF</td>
</tr>
<tr>
<td>iBoxx Global Inflation-Linked ETF</td>
</tr>
<tr>
<td>iBoxx Sovereigns Eurozone 1-3 ETF</td>
</tr>
<tr>
<td>Geographic Origin of the Collateral Securities</td>
</tr>
<tr>
<td>DAX ETF</td>
</tr>
<tr>
<td>Euro Stoxx 50 ETF</td>
</tr>
<tr>
<td>iBoxx Global Inflation-Linked ETF</td>
</tr>
<tr>
<td>iBoxx Sovereigns Eurozone 1-3 ETF</td>
</tr>
</tbody>
</table>

Notes: Panel A presents for each fund the value-weighted percentage of actual collateral as of November 29, 2012 that is held in equity, governments bonds, and corporate bonds, respectively. It also presents the value-weighted percentage of actual collateral that comes from Europe, Asia-Pacific and North America, respectively. Panel B presents for each fund the value-weighted percentage of optimal collateral as of November 29, 2012 that is held in equity, governments bonds, and corporate bonds, respectively. It also presents the value-weighted percentage of optimal collateral that comes from Europe, Asia-Pacific and North America, respectively. The government bond category also includes supranational bonds, government guaranteed bonds, government agency bonds, and German regional government bonds. The corporate bond category also includes covered bonds.

In Table 2.2 panel A, we compare actual and optimal optimal portfolios for the four ETFs. We found that the total number of securities included in the collateral portfolios are quite similar (81 in the actual portfolios vs. 88 in the optimal portfolios) but that optimal portfolios are more tilted towards equities rather than toward government bonds. For each fund, the optimal number of securities ranges between 25 and 40 securities, which is comparable in size with actual portfolios. As the optimization is based on a universe of 1,500 securities, these findings illustrates the relative concentration of the
optimal collateral portfolio. The moderate size of the optimal portfolio indicates that our approach leads to realistic and feasible portfolios.

We study in Table 2.3 the match between asset and geographic exposures of the ETFs and the collateral securities. For actual portfolios, the match is perfect in term of asset exposures. In Panel A, we see that the equity ETFs are collateralized at 100% with equities and, similarly, government bond ETFs are only collateralized with government bonds. However, for the optimal portfolios (Panel B), although the match is also perfect for the equity ETFs, the government bond ETFs have a significant fraction of collateral in equity (33% for the iBoxx Global Inflation-Linked ETF and 5% for the Iboxx Sovereigns Eurozone 1-3 ETF). Moreover, optimal collateral portfolios display greater geographic diversification than actual collateral portfolios.

Figure 2.6: Counterparty Risk Exposure of Actual Versus Optimal Portfolios

Notes: This figure presents a Torino scale that displays, for our four swap-based ETFs, their conditional probability $P$ of having a collateral shortfall (y-axis, range = 0-100%), their conditional expected collateral shortfall $S$ relative to the NAV (x-axis, range = 0-2.5%), and their Aggregate Counterparty Risk $ACR = P \times S$, on November 29, 2012. We contrast actual and optimal collateral portfolio for each fund.

In Figure 2.6, we compare the optimal and the actual portfolios for all swaps in a two-dimensional graph, using the conditional probability of facing a collateral shortfall on the vertical axis and the conditional expected collateral shortfall on the horizontal axis. For all swaps, we reach a lower ACR with the optimal collateral portfolios than with
the actual portfolios. Although not a systematic implication of our methodology, we find that both the shortfall probability and the magnitude of the collateral shortfall are lower with the optimal collateral mix. We further quantify the reduction in counterparty risk exposures (ACR) in Table 2.4. We report an important reduction in ACR, between 22% and 46%, and this reduction is larger for equity ETFs.

<table>
<thead>
<tr>
<th>ACR</th>
<th>Actual Portfolio</th>
<th>Optimal Portfolio</th>
<th>Percentage Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX ETF</td>
<td>1.205</td>
<td>0.731</td>
<td>-39%</td>
</tr>
<tr>
<td>Euro Stoxx 50 ETF</td>
<td>1.170</td>
<td>0.635</td>
<td>-46%</td>
</tr>
<tr>
<td>iBoxx Global Inflation-Linked ETF</td>
<td>0.635</td>
<td>0.498</td>
<td>-22%</td>
</tr>
<tr>
<td>iBoxx Sovereigns Eurozone 1-3 ETF</td>
<td>0.052</td>
<td>0.037</td>
<td>-29%</td>
</tr>
</tbody>
</table>

Notes: This table contrasts the aggregate counterparty risk (ACR) for actual and optimal collateral portfolios of our sample ETFs on November 29, 2012. The ACR is the product of the conditional probability of having a collateral shortfall (in percentage) and the conditional expected collateral shortfall (relative to the NAV). The table displays also the percentage change between the actual portfolio ACR and the optimal portfolio ACR.

2.4 Conclusion

Finding the optimal collateral mix for a given financial transaction is a complex task and especially so in an operating environment driven by changing regulations. In this paper, we contribute to the literature on collateral management by deriving expressions for optimal collateral portfolios. First, we define a collateral risk measure that depends on the conditional distribution of the ETF investors’ counterparty risk exposure. Second, assuming that the risk measure increases with both the mean and the variance of the counterparty risk exposure, we determine an efficient frontier on which all optimal collateral portfolios lie. The collateral portfolio that minimizes counterparty risk belongs to this frontier and its weights can be expressed as a function of those of the Markowitz’s minimum-variance portfolio. The weights also reflect the correlations between the collateral securities and the index tracked by the fund. We illustrate our methodology by deriving the optimal collateral portfolios for a sample of swap-based ETFs. Finally, we compare the actual and the optimal collateral portfolios and show that our optimal collateral rules substantially reduce the counterparty risk exposure of investors.

Nowadays, given the growing importance of collateralized financial contracts, setting clear and feasible rules to minimizing the counterparty risk exposure of investors appears essential. Further research could generalize our framework to accommodate more collateralized positions, including options, interest rate swaps, and repurchase agreements.
Appendix B

B1. Expected Collateral Shortfall and ACR under the Normality Assumption.

Under the normality assumption for the conditional distribution of $\Delta_t$, the conditional probability of collateral shortfall and the expected collateral shortfall are given by:

$$
\Pr(\Delta_t > 0 | D_t = 1) = \Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) \tag{B1}
$$

$$
S_t = \mathbb{E}(\Delta_t | \Delta_t > 0, D_t = 1) = \mathbb{E}(\mu_\Delta + \sigma_\Delta z | z > -\mu_\Delta / \sigma_\Delta, D_t = 1) = \mu_\Delta + \sigma_\Delta \lambda\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) \tag{B2}
$$

where $z \sim N(0,1)$ and $\lambda(y) = \phi(y)/\Phi(y)$ is the inverse Mills ratio, $\Phi(y)$ denotes the cdf of the standard normal distribution, and $\phi(y)$ is the pdf of the standard normal distribution.

We can now define the Aggregate Counterparty Risk $ACR$ under the normality assumption as:

$$
ACR_t = \Pr(\Delta_t > 0 | D_t = 1) \times S_t = \Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right)(\mu_\Delta + \sigma_\Delta \phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right)/\Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right)) = \mu_\Delta \Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) + \sigma_\Delta \phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right) \tag{B3}
$$


Expected Collateral Shortfall

If we assume that $\Delta_t$ has a normal distribution, then:

$$
\frac{\partial S_t}{\partial \sigma_\Delta} = \lambda(u) - \sigma_\Delta \lambda(u) (u + \lambda(u)) \frac{\partial u}{\partial \sigma_\Delta} = \lambda(u) \left(1 + u^2 + u\lambda(u)\right) \tag{B4}
$$

with $u = \mu_\Delta / \sigma_\Delta$. Since the inverse Mills ratio is always positive, the sign of $\partial S_t / \partial \sigma_\Delta$ corresponds to the one of $1 + u^2 + u\lambda(u)$. So, the expected shortfall is an increasing function of $\sigma_\Delta$ as soon as:

$$
1 + u^2 + u \frac{\phi(u)}{\Phi(u)} > 0 \tag{B5}
$$

or equivalently when $\Phi(u) + u^2 \Phi(u) + u \phi(u) > 0$. Since that, for a standard normal distribution, we have $\phi(u) = -\Phi(u) u$, this condition becomes $\Phi(u) > 0$. Whatever the value of $\mu_\Delta$ and $\sigma_\Delta$, this condition is always satisfied. As a consequence, the expected
shortfall is always an increasing function of the variance of the collateral shortfall $\sigma_\Delta^2$, whatever the value of its expectation $\mu_\Delta$. Similarly,

$$\frac{\partial S_t}{\partial \mu} = 1 + \frac{\phi'(u) \Phi(u) - \phi(u)^2}{\Phi(u)^2} = 1 - \frac{u\phi(u) + \phi(u)^2}{\Phi(u)^2}$$  \hspace{1cm} (B6)$$

is always positive since $u\phi(u) + \phi(u)^2 < \Phi(u)^2$, $\forall u \in \mathbb{R}$. So, we have:

$$\frac{\partial S_t}{\partial \sigma_\Delta} > 0 \quad \frac{\partial S_t}{\partial \mu} > 0 \quad \forall u \in \mathbb{R}$$  \hspace{1cm} (B7)$$

**Aggregate Counterparty Risk**  If we assume that $\Delta_t$ has a normal distribution, then:

$$ACR_t = \Pr (\Delta_t > 0 | D_t = 1) \times S_t = \mu_\Delta \Phi \left( \frac{\mu_\Delta}{\sigma_\Delta} \right) + \sigma_\Delta \phi \left( \frac{\mu_\Delta}{\sigma_\Delta} \right)$$  \hspace{1cm} (B8)$$

Let us assume that $\Delta_{t+1}$ has a normal distribution, then:

$$\frac{\partial ACR_t}{\partial \mu_\Delta} = \phi(u) + \phi'(u) = \Phi(u) - u\phi(u) > 0 \quad \forall u \in \mathbb{R}$$  \hspace{1cm} (B9)$$

with $u = \mu_\Delta/\sigma_\Delta$. Similarly, we have:

$$\frac{\partial ACR_t}{\partial \sigma_\Delta} = -u^2\phi(u) + \phi(u) - u\phi'(u)$$

$$= -u^2\phi(u) + \phi(u) + u^2\phi(u)$$

$$= \phi(u) > 0 \quad \forall u \in \mathbb{R}$$  \hspace{1cm} (B10)$$

So, we have:

$$\frac{\partial ACR_t}{\partial \mu_\Delta} > 0 \quad \frac{\partial ACR_t}{\partial \sigma_\Delta} > 0 \quad \forall u \in \mathbb{R}$$  \hspace{1cm} (B11)$$

**B3. Proof of Proposition 2**

**Proof.** If $\omega^T \mu - \mu_i = \tilde{\gamma}$ and $\lambda_2 > 0$, the Kuhn-Tucker conditions become:

$$\Sigma \omega - \Sigma_i - \lambda_1 e - \lambda_2 \mu = 0$$  \hspace{1cm} (B12)$$

$$e^T \omega - 1 = 0$$  \hspace{1cm} (B13)$$

$$\omega^T \mu - \mu_i + \tilde{\gamma} = 0$$  \hspace{1cm} (B14)$$

From the first equation, we have:

$$\omega = \Sigma^{-1} \Sigma_i + \lambda_1 \Sigma^{-1} e + \lambda_2 \Sigma^{-1} \mu$$  \hspace{1cm} (B15)$$

Given this expression for $\omega$, the two constraints can be rewritten as:

$$1 - e^T \Sigma^{-1} \Sigma_i = \lambda_1 e^T \Sigma^{-1} e + \lambda_2 e^T \Sigma^{-1} \mu$$  \hspace{1cm} (B16)$$

$$\mu_i - \tilde{\gamma} - \mu^T \Sigma^{-1} \Sigma_i = \lambda_1 e^T \Sigma^{-1} \mu + \lambda_2 \mu^T \Sigma^{-1} \mu$$  \hspace{1cm} (B17)$$
Define three three scalar terms $a, b,$ and $c$ such that:

$$a = e^{\top} \Sigma^{-1} e \quad b = e^{\top} \Sigma^{-1} \mu \quad c = \mu^{\top} \Sigma^{-1} \mu \quad \text{(B18)}$$

The constraints can be expressed as:

$$1 - e^{\top} \Sigma^{-1} \Sigma_i = \lambda_1 a + \lambda_2 b \quad \text{(B19)}$$

$$\mu_i - \bar{\gamma} - \mu^{\top} \Sigma^{-1} \Sigma_i = \lambda_1 b + \lambda_2 c \quad \text{(B20)}$$

Solving for $\lambda_1$ and $\lambda_2$, we have:

$$\tilde{\lambda}_1 = \frac{b (\mu_i - \bar{\gamma} - \mu^{\top} \Sigma^{-1} \Sigma_i) - c (1 - e^{\top} \Sigma^{-1} \Sigma_i)}{b^2 - ac} \quad \text{(B21)}$$

$$\tilde{\lambda}_2 = \frac{b (1 - e^{\top} \Sigma^{-1} \Sigma_i) - a (\mu_i - \bar{\gamma} - \mu^{\top} \Sigma^{-1} \Sigma_i)}{b^2 - ac} \quad \text{(B22)}$$

By substituting $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ in the expression $\omega$, we have:

$$\tilde{\omega} = \Sigma^{-1} \Sigma_i + \left( \frac{b (\mu_i - \bar{\gamma} - \mu^{\top} \Sigma^{-1} \Sigma_i) - c (1 - e^{\top} \Sigma^{-1} \Sigma_i)}{b^2 - ac} \right) \Sigma^{-1} e$$

$$+ \left( \frac{b (1 - e^{\top} \Sigma^{-1} \Sigma_i) - a (\mu_i - \bar{\gamma} - \mu^{\top} \Sigma^{-1} \Sigma_i)}{b^2 - ac} \right) \Sigma^{-1} \mu \quad \text{(B23)}$$

The standard mean variance portfolio with a target mean $\mu_i - \bar{\gamma}$ is the solution of the following program:

$$\min_{\omega} \frac{1}{2} \omega^{\top} \Sigma \omega \quad \text{(B24)}$$

subject to

$$\begin{cases} 
\omega^{\top} \mu = \mu_i - \bar{\gamma} \\
\epsilon^{\top} \omega = 1 \in \mathbb{N}^k
\end{cases}$$

The corresponding optimal solution is:

$$\tilde{\omega}_{MV} = \left( \frac{b \mu_i - b \bar{\gamma} - c}{b^2 - ac} \right) \Sigma^{-1} e + \left( \frac{b - a \mu_i + a \bar{\gamma}}{b^2 - ac} \right) \Sigma^{-1} \mu \quad \text{(B25)}$$

As a consequence, we have:

$$\tilde{\omega} = \tilde{\omega}_{MV} + \Sigma^{-1} \Sigma_i + \left( \frac{ce^{\top} - b \mu^{\top}}{b^2 - ac} \right) \Sigma^{-1} \Sigma_i \Sigma^{-1} e$$

$$+ \left( \frac{a \mu^{\top} - be^{\top}}{b^2 - ac} \right) \Sigma^{-1} \Sigma_i \Sigma^{-1} \mu \quad \text{(B26)}$$

B4. Proof of Proposition 3

Proof. The Lagrange function $f (\omega, \lambda_1, \lambda_3)$ is defined as to be:

$$f (\omega, \lambda_1, \lambda_3) = \frac{1}{2} \omega^{\top} \Sigma \omega + \frac{1}{2} \sigma_i^2 - \omega^{\top} \Sigma_i - \lambda_1 (\epsilon^{\top} \omega - 1) - \lambda_3 \omega \quad \text{(B27)}$$
Appendix B.

If \( \lambda_3 = 0 \), the Kuhn-Tucker conditions become:

\[
\Sigma \omega - \Sigma_i - \lambda_1 e = 0 \tag{B28}
\]
\[
e^\top \omega - 1 = 0 \tag{B29}
\]

From Equation (B28), we have:

\[
\omega = \Sigma^{-1} \Sigma_i + \lambda_1 \Sigma^{-1} e \tag{B30}
\]

Next, multiply both sides by \( e^\top \) and use second equation to solve for \( \lambda_1 \):

\[
e^\top \omega = e^\top \Sigma^{-1} \Sigma_i + \lambda_1 e^\top \Sigma^{-1} e = 1 \tag{B31}
\]
\[
\tilde{\lambda}_1 = \frac{1 - e^\top \Sigma^{-1} \Sigma_i}{e^\top \Sigma^{-1} e} \tag{B32}
\]

and the optimal weights are equal to:

\[
\tilde{\omega} = \Sigma^{-1} \Sigma_i + \left( 1 - e^\top \Sigma^{-1} \Sigma_i \right) \frac{\Sigma^{-1} e}{e^\top \Sigma^{-1} e} \tag{B33}
\]

These weights can be expressed as a linear function of the weights of the Global Minimum Variance Portfolio (GMVP). The GMVP corresponds to the solution of the following optimization problem:

\[
\min_{\omega} \quad \frac{1}{2} \omega^\top \Sigma \omega \tag{B34}
\]
subject to \( e^\top \omega = 1 \) \tag{B35}

The corresponding optimal solution is:

\[
\tilde{\omega}_{GMVP} = \frac{\Sigma^{-1} e}{e^\top \Sigma^{-1} e} \tag{B36}
\]

As a consequence, we have:

\[
\tilde{\omega} = \Sigma^{-1} \Sigma_i + \left( 1 - e^\top \Sigma^{-1} \Sigma_i \right) \tilde{\omega}_{GMVP} \tag{B37}
\]

B5. Mean-Variance Portfolios and Shrunken Covariance Matrix

Following Roncalli (2013), the original minimum variance problem studied in Jagganathan and Ma (2003) can be generalized to the mean-variance case. In our setting, rewriting the program (2.19) gives:

\[
\tilde{\omega} (\gamma) = \arg \min_{\omega \in \mathcal{F}} \quad \frac{1}{2} \omega^\top \Sigma \omega + \frac{1}{2} \sigma_i^2 - \omega^\top \Sigma_i \tag{B38}
\]
subject to \( \begin{cases} C \omega \geq D \\ e^\top \omega = 1 \end{cases} \)
where $C$ and $D$ are two matrices such that:

$$C = \begin{pmatrix} \frac{\mu^\top}{(1,K)} \\ e \\ 0 \end{pmatrix} \begin{pmatrix} (K,1) \\ (K,K-1) \end{pmatrix}$$  \hspace{1cm} (B39)

$$D = \begin{pmatrix} \mu_i - \tilde{\gamma} \\ 0 \end{pmatrix} \begin{pmatrix} (K,1) \end{pmatrix}$$  \hspace{1cm} (B40)

The associated Lagrange function $f(\omega, \lambda_1, \lambda)$ is:

$$f(\omega, \lambda_1, \lambda_2) = \frac{1}{2} \omega^\top \Sigma \omega + \frac{1}{2} \sigma_i^2 - \omega^\top \Sigma_i - \lambda_1 (e^\top \omega - 1) - \lambda^\top (C \omega - D)$$  \hspace{1cm} (B41)

The constraint solution $\tilde{\omega}$ is also the solution to the unbounded program with an implied shrunk covariance matrix $\Sigma^*$ given by:

$$\Sigma^* = \Sigma - C^\top \lambda e^\top + e^\top C$$  \hspace{1cm} (B42)

Roncalli (2013) shows that the matrix $\Sigma^*$ is definite positive if $\lambda_1 \geq \lambda^\top D$ and may be indefinite if $\lambda_1 < \lambda^\top D$.

**B6. Optimization with the ETF expressed as the portfolio of the index constituents**

If the perfect replication of the index is possible using the available collateral securities ($A \subseteq B$) and the inequality constraints are not binding, the associated Lagrange function of the program (2.33) $f(x, \lambda_1, \lambda_2)$ is:

$$f(x, \lambda_1, \lambda_2) = x^\top \Sigma_R x - \lambda_1 (e^\top x - (\alpha - 1)/\alpha) - \lambda_2 ((\alpha - 1)/\alpha + x^\top \mu_R + \gamma))$$  \hspace{1cm} (B43)

The first order conditions become:

$$\Sigma_R x - \lambda_1 e - \lambda_2 \mu_R = 0$$  \hspace{1cm} (B44)

$$e^\top x - (\alpha - 1)/\alpha = 0$$  \hspace{1cm} (B45)

$$(1 - \alpha)/\alpha - x^\top \mu_R - \gamma = 0$$  \hspace{1cm} (B46)

From the first equation, we have:

$$x = \lambda_1 \Sigma_R^{-1} e + \lambda_2 \Sigma_R^{-1} \mu_R$$  \hspace{1cm} (B47)

Define three scalar terms $\hat{a}$, $\hat{b}$ and $\hat{c}$ such that:

$$\hat{a} = e^\top \Sigma_R^{-1} e \hspace{1cm} \hat{b} = e^\top \Sigma_R^{-1} \mu_R \hspace{1cm} \hat{c} = \mu_R^\top \Sigma_R^{-1} \mu_R$$  \hspace{1cm} (B48)

and given the expression for $x$, the two constraints can be rewritten as:

$$\frac{(\alpha - 1)}{\alpha} = \lambda_1 \hat{a} + \lambda_2 \hat{b}$$  \hspace{1cm} (B49)

$$\frac{(1 - \alpha)}{\alpha - \gamma} = \lambda_1 \hat{b} + \lambda_2 \hat{c}$$  \hspace{1cm} (B50)
Appendix B.

solving for $\lambda_1$ and $\lambda_2$, we have:

$$\tilde{\lambda}_1 = \frac{\alpha \gamma \hat{b} + a \hat{b} + \alpha \hat{c} - \hat{b} - \hat{c}}{\alpha(\hat{a} \hat{c} - \hat{b}^2)} \quad (B51)$$

$$\tilde{\lambda}_2 = \frac{\alpha \gamma \hat{a} - \alpha \hat{a} - \hat{a} + \hat{b}}{\alpha(\hat{a} \hat{c} - \hat{b}^2)} \quad (B52)$$

By substituting $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ in the expression for $x$, setting $\gamma = \mu_\Delta$, and then solving for the variance of the collateral shortfall $\sigma^2_\Delta = x^* \Sigma_R x$, we have:

$$\sigma^2_\Delta = A \cdot \mu^2_\Delta + B \cdot \mu_\Delta + C \quad (B53)$$

where $A$, $B$ and $C$ are three constant terms such that:

$$A = \frac{\hat{a}^2 \hat{c} - \hat{b} \hat{b}^2}{\alpha \hat{a} \hat{c} - \hat{b}^2} \quad (B54)$$

$$B = \frac{2\alpha(\hat{a} \hat{b} \hat{c} + \hat{a}^2 \hat{c} - \hat{a} \hat{b}^2 - \hat{b}^3) + \hat{a} \hat{b}(3\hat{b} - \hat{c}) + 2(\hat{b}^3 + \hat{a}^2 \hat{c})}{\alpha(\hat{a} \hat{c} - \hat{b}^2)} \quad (B55)$$

$$C = \frac{\hat{a} \hat{b} + \alpha \hat{a} - \hat{b} - \hat{c})^2 + \hat{c}(\alpha \hat{a} + \alpha \hat{b} - \hat{a} - \hat{b}^2)^2 \ldots}{\alpha^2(\hat{a} \hat{c} - \hat{b}^2)^2} + \frac{2\hat{b}(2\hat{a} \hat{b}^2 + \hat{a} \hat{b} \hat{c} + \hat{a} \hat{c} + \hat{b}^2 + \hat{b} \hat{c}) - 2\hat{a} \hat{b}(\alpha^2 \hat{b}^2) + \alpha \hat{b} \hat{c} + \hat{a} \hat{c} + \hat{b}^2 + \hat{b} \hat{c}}{\alpha^2(\hat{a} \hat{c} - \hat{b}^2)^2} \quad (B56)$$

Then, the collateral efficient frontier is a parabola in the $(\sigma^2_\Delta, \mu_\Delta)$ space.

If, in addition, we assume a full collateralization at time $t$ ($\alpha = 1$), the relationship between the variance and the mean of the collateral shortfall is described by:

$$\sigma^2_\Delta = D \cdot \mu_\Delta \quad (B57)$$

where the constant term $D$ equals:

$$D = \frac{\sqrt{a^2 c - ab^2}}{(ac - b^2)^2} \quad (B58)$$

Thus, the efficient frontier is a straight line in the volatility-mean space.

B7. Hybrid ETF

Proof. Assuming $A \subseteq B$ and $\alpha = 1$, Equation (2.29) for the collateral shortfall becomes:

$$\Delta_{t+1} = C_t (1 - h) \left( \sum_{j \in A \cup B} \delta_{j,t} r_{j,t+1} - \sum_{j \in A \cup B} \omega_{j,t} r_{j,t+1} \right) \quad (B59)$$

An obvious way to set this shortfall at zero consists in choosing the weights $\omega_j$ such that:

$$\omega^*_j = \delta_{j,t} \quad \text{for } j = 1, \ldots, J \quad (B60)$$

Note that $\omega^*_j = \delta_{j,t} = 0$ when the asset $j$ is not a constituent of the index.
Bibliography


Chapter 3
Investor Sentiment and ETF Flows

Abstract

Investor sentiment has been shown to cause deviations with respect to the Efficient Market Hypothesis. In this paper, we extract a novel investor sentiment measure from index option prices. Building on Shefrin (2008) and Barone-Adesi et al. (2014), our investor sentiment index allows, in addition, to have a direct indication of the magnitude and of the direction of the sentiment in a specific market. Then, we empirically show that investor sentiment drives flows in equity exchange-traded funds. We also observe a correction on flows after three weeks, which is consistent with the idea that sentiment-based trading induces a slow market error correction.

3.1 Introduction

According to financial theory, rational investors base their choices on the maximization of their expected utility and apply the Bayes rule to update their beliefs. Assuming a frictionless market with rational investors is enough to verify the Efficient Market Hypothesis (EMH) according to which prices of securities reflect all available information (Fama, 1970). In this case, prices are equal to their fundamental value. The latter corresponds to the discounted sum of expected future cash flows when investors correctly process all available information, and where the discount rate is consistent with a normatively acceptable preference specification (Barberis and Thaler, 2003). Furthermore, having a sufficient number of rational investors engaging in arbitrage guarantees the fundamental value of prices, even in presence of uninformed investors (noise traders) or market friction, see De Long et al. (1990) for instance.

However, over the past three decades, a large number of studies have questioned the validity of the EMH. Theses studies document anomalies with respect to the EMH in terms of, for instance, return predictability, abnormal returns or violations of the law of one price. Hirshleifer (2001) and Barberis and Thaler (2003) provide some excellent reviews

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26 Alternatively, another definition of the EMH is given by Jensen (1978): A market is efficient if it is impossible to make economic profits by trading on the available information.

27 As examples of anomalies to the EMH, we can cite: misdiversification (Coval and Moskowitz, 1999), disposition effect (Shefrin and Statman, 1985), overconfidence (Barber and Odean, 2000) long term re-
of this literature. These anomalies and their persistence contradict the assumptions of error corrections by the rational arbitragers. Behavioral finance aims to understand the anomalies to the EMH and identifies three major channels through which the prices can deviate from their fundamental values. First, investors' preferences may not respect the principle of expected utility maximization. For instance, Kahneman and Tversky (1979) and Tversky and Kahneman (1992) develop a framework called prospect theory, with risk-seeking behavior and loss aversion. A second channel comes from the limits to arbitrage, which can restrain the rational investors from correcting the mispricing by rendering the bets against noise traders costly and risky (De Long et al., 1990 or Shleifer and Vishny, 1997). Gromb and Vayanos (2010) offer a comprehensive survey of the literature on the limits of arbitrage. Finally, the third channel through which we can explain deviations from the EMH is investor sentiment. Baker and Wurgler (2007) define investor sentiment as beliefs about future cash flows and investment risks that are not justified by the facts at hand.\textsuperscript{28}

In this paper, we estimate a new measure of investor sentiment from index option prices and we test whether investor sentiment affects mutual fund flows. Our methodology offers several advantages including directly assessing investor sentiment from actual financial data. Indeed, we derive an explicit measure of (and not a proxy for) investor sentiment without having to conduct costly or difficult surveys of market participants. In addition, it gives flexibility on the choice of the market in which investor sentiment is evaluated and at what frequency it is measured. Then, focusing on the S&P 500 stock market index, we find that the variations of the sentiment are important drivers of the flows towards S&P 500 exchange-traded funds (ETFs) and that the effect reverts in the short term. Indeed, an increase in the level of sentiment triggers additional flows that are corrected for three weeks later. This is consistent with the assumption of mean-reverting investor sentiment trading.

Our paper contributes to two streams of the literature. First, it builds on the literature on investor sentiment that studies the role of irrational behavior in the financial decisions of investors. There are two main approaches to measuring the level of investor sentiment on the market. One direct quantification technique is to conduct a survey of investors or consumers in order to gather information about their beliefs. For instance, the American Association of Individual Investors Survey or the UBS/Gallup Index of Investor Optimism aim to reflect individual investors' prospects. The Livingston Survey analyses the views of institutional investors while the University of Michigan Consumer Sentiment Index focuses on consumers. On the basis of these surveys, we may derive investor sentiment indices but at the cost of possible response biases from survey participants. One can also indirectly assess the investor sentiment using financial proxies. For instance, the closed-end fund discount or the implied option volatility are supposed to incorporate investor sentiment. Baker and Wurgler (2006) and Baker and Wurgler (2007) review a number of these proxies.

\textsuperscript{28}The sentiment comes from uninformed investors and emerges at the aggregate level by impacting securities prices (Shefrin, 2008). Hirshleifer (2001) proposes three sources of non-rational anticipation that drive investor sentiment. First, the limitation of cognitive resources, such as attention, processing power, and memory, makes investors using rules-of-thumb and is a source of systematic errors. Second, the self-deception theory that maintains that individuals are designed to think that they are better than they really are and thus overestimate their own ability. Finally, the emotions when they tend to interfere with rational judgements.

\textsuperscript{versal} (De Bondt and Thaler, 1985), \textit{short term price momentum} (Jegadeesh and Titman, 1993), \textit{mood effects} (Kamstra et al., 2000 or Hirshleifer and Shumway, 2003).
and form an aggregate index of investor sentiment using principal component analysis. In this research, we use an alternative approach that allows us to explicitly derive investor sentiment from securities prices as in Barone-Adesi et al. (2014). This prevents the use of investor surveys or financial proxies and gives maximal flexibility in the choice of the market and of the frequency of measure. It is built on the decomposition of the pricing kernel into a fundamental component and the investor sentiment (Shefrin, 2008).

The empirical research in this field focuses on aggregate stock returns. Using direct measures of sentiment, Lemmon and Portniaguina (2006) find a negative relationship between sentiment and small stock returns. Such result is consistent with the assumption that sentiment primarily affects individual investors and, by consequence, stocks with low institutional ownership. Based on a survey of stock market newsletters, Brown and Cliff (2005) point out that periods of high sentiment, as excessive optimism, are followed by periods of low market return at a two to three year horizon. In other words, they show a mean-reverting pattern in stock prices after a period of overpricing due to investor sentiment. Baker and Wurgler (2006) and Baker and Wurgler (2007) derive an indirect measure of sentiment and find that high periods of sentiment make stocks that are difficult to arbitrage overpriced. These periods of extreme sentiment are followed by abnormally low returns. With the Baker and Wurgler sentiment index, Livnat and Petrovits (2009) study stock price reactions to earning surprises and demonstrate that irrational investors underestimate the news that are contrary to their level of sentiment. For Antoniou et al. (2013), this slow diffusion of signal leads to price momentum. Focusing on a dozen of market return anomalies and using a sentiment measure based on Baker and Wurgler (2007), Stambaugh et al. (2012) show that these mispricings are stronger after periods of high sentiment. Consistent with the assumption that short-sale constraints limit the correction of excessively high prices, they find that overpricing is more persistent than underpricing. Using both surveys and financial proxies, Brown and Cliff (2004) construct a composite sentiment measure and show that past short-term returns are an important determinant of sentiment.

Second, our paper also contributes to the literature on fund flows. Chevalier and Ellison (1997) show that the mutual funds adapt the risk of their holding portfolios in order to get a certain end-of-year performance and satisfy return-chasing investors. Indeed, prior performances of the fund are important determinants of the fund flows. Moreover, Sirri and Tufano (1998) demonstrate that past performance becomes an even more important determinant for those funds that display high marketing efforts, which goes to show that fund investors are affected by search costs. Although fund flows respond negatively to salient fees, Barber et al. (2005) find that operating expenses have no negative effect on flows because of the presence of marketing. Moreover, several studies use mutual fund flows as a proxy for investor sentiment. Warther (1995) find a strong correlation between unexpected cash flows into mutual funds and the aggregate security returns. By analysing inflows and outflows of mutual funds, Frazzini and Lamont (2008) construct a flow-based measure of sentiment and document low future returns for stocks that are experiencing a high sentiment period. Ben-Rephael et al. (2012) approximate sentiment using a shift in the flows between bond and equity funds. They find a mean-reverting pattern in the market excess returns due to sentiment.

To the best of our knowledge, there is little research on the effect of investor sentiment on fund flows. Warther (1995) did not find a correlation between the mutual fund flows and
Chapter 3. Investor Sentiment and ETF Flows

the closed-end fund discounts. Using daily data on three index funds, Goetzmann and Massa (2003) find that bullish financial newsletters are associated with positive inflows. Based on weekly data and investor surveys, Indro (2004) documents positive correlation between fund flows and bullish sentiment. In our paper, we go one step further in this task and use an explicit and market-specific measure of investor sentiment. Based on weekly option and flow data, we find that an increase in the investor sentiment leads first to a rise of the ETF flows and then a decrease three weeks later. This finding is consistent with the assumption of sentiment based trading that induces a slow market error correction.

The rest of our paper is structured as follows. In Section 3.2, we present our methodology used to extract investor sentiment from option prices. In Section 3.3, we conduct an empirical analysis in two steps. First, we estimate an investor sentiment index from index option prices over the period January 2010 - May 2014. Second, we test whether our estimate of market sentiment is a significant driver of fund flows. We summarize and conclude our study in Section 3.4.

3.2 Option Prices Based Sentiment Index

3.2.1 Pricing Kernel and Investor Sentiment

Let \( p_t \) be the price of an asset at time \( t \) that pays a random payoff \( z \) at time \( T \) given the state of the nature \( x_T \). Since the asset price is equal to the expected discounted payoffs, it can be written as:

\[
p_t = \mathbb{E}[M_{t,T}(x_T)z(x_T)]
\]  

where \( M_{t,T} \) is a stochastic discount factor, or a pricing kernel, that incorporates all risk-corrections and time preferences (see Cochrane, 2005). Note that this expectation and all the relationships presented below are conditional on the information available at time \( t \).

In economic terms, \( M_{t,T} \) can be interpreted as the marginal rate of substitution between future and present cash flows. In a rational framework with constant relative risk aversion, the pricing kernel is a monotonic decreasing function of the consumption growth rate. The present value of the payoff in a future high consumption state are indeed relatively less important with respect to the same payoff in a future low consumption state. This is a reflection of decreasing marginal utility with respect to the states of the economy.

If investor sentiment affects asset prices, it should be captured in the pricing kernel \( M_{t,x} \). Shefrin (2008) shows that, with heterogeneous constant relative risk aversion (CRRA) investors, sentiment impacts directly the pricing kernel. Indeed, the investor sentiment arises when investor’s beliefs about the future state of the economy in \( T \) do not correspond to the true density of the state variable. In this case, the logarithm of the pricing kernel \( m_{t,T} = \log(M_{t,T}) \) can be broken down into two building blocks: the logarithm of the pricing kernel in the case of fully rational investors, \( m_{t,T}^e \), and a part that reflects the aggregate errors on investors’ beliefs \( \Lambda_{t,T} \):

\[
m_{t,T}(x_T) = m_{t,T}^e(x_T) + \Lambda_{t,T}(x_T)
\]  

where the last term \( \Lambda_{t,T} \) is called the sentiment function. This function gives, at time \( t \) and for each state of nature, the relative difference in the pricing kernel due to the presence
of sentiment at time $t$ as compared to the pricing kernel under an efficient market. It is a measure of the deviation between the actual market pricing and the rational-efficient pricing in $t$ for an investment horizon of $T - t$. For a CRRA representative investor, Shefrin (2008) shows that the function $\Lambda_{t,T}$ equals:

$$\Lambda_{t,T}(x_T) = \log\left( \frac{P_R(x_T)}{P(x_T)} \right) + \log\left( \frac{\delta_R}{\delta_e} \right)$$

(3.3)

where $P_R$ and $\delta_R$ are the beliefs and the discount factor of the representative investor, respectively, whereas $P$ and $\delta_e$ represent the beliefs and the discount factor when all investors are fully rational, i.e., their beliefs correspond to the true density of the future state of the economy. Note that the discount factors $\delta$ are time-varying but that we omit the time subscript to simplify the notation. Thus, $P$ is the correct or the objective belief that truly represents the future states of the nature. Therefore, the sentiment function arises due to deviations from the efficient market and implies that the aggregate beliefs of all investors $P_R$ do not correspond to the objective beliefs $P$. In other words, the function of sentiment is positive for the states of nature whose probabilities of occurrence are overstated by the representative investor with respect to their objective probability. Consequenly, the payoff in states of nature with a positive level of sentiment function will be inflated by the representative investor. Conversely, negative values of $\Lambda_{t,T}$ lead to an understated payoff. The proofs for Equations (3.2) and (3.3) are provided in Appendix C1.

### 3.2.2 Investor Sentiment Index

The sentiment function $\Lambda_{t,T}$ provides useful information about the actual prices on the market in time $t$ and the beliefs of the market participants for an investment horizon of $T - t$. Assuming that in a market with heterogeneous investors, the representative investor has CRRA preferences, the pricing Equation (3.1) becomes:

$$p_t = E\left[ e^{\Lambda_{t,T}(x_T)} \delta_R g_{t,T}(x_T)^{-\gamma_R} z(x_T) \right]$$

(3.4)

where $g_{t,T}(x_T)$ is the consumption growth rate between $t$ and $T$ given the state of nature $x_T$, and $\gamma_R$ is the coefficient of relative risk aversion of the representative investor (see appendix B1). As for the discount factors, the CRRA coefficient is time-varying but the subscript is omitted to simplify the notation. Once the sentiment function is known, we can identify which states of nature are over- or under-weighted by the representative investor when evaluating $p_t$. However, because $\Lambda_{t,T}$ is defined over the set of all states of nature and can take positive and negative values, it is difficult to directly interpret it. We derive an index of investor sentiment from the sentiment function that allows us to assess both the direction and the magnitude of the impact of sentiment on the market prices. As this index reflects the irrational behavior of investors at the market level, we view this index as an aggregate investor sentiment index.

The investor sentiment index is extracted from the sentiment function, by comparing two different evaluations of the same payoff $z(x_T)$, at time $t$, for all the states of nature in

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29The interpretation focuses on the deviations of the beliefs, since the last term of Equation (3.3) is typically close to zero (Shefrin, 2008).
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Hence, our index is the difference between the market evaluation and the rational evaluation of \( z(x_T) \). The market evaluation is obtained from Equation (3.4), according to which the sentiment directly impacts the price of \( z(x_T) \). The same equation is used to derive the rational pricing, where the sentiment function is set to zero for all states of nature. The index can be interpreted as the additional expected gain or loss, with respect to an efficient market, that the representative investor believes to get in \( T \) by investing in a security with a price \( p_t \) at time \( t \). These deviations in the expected outcome are due to the presence of a non-zero sentiment function. A positive value of the index indicates that the representative investor believes that an investment in the security is more profitable than it would have been under the efficient market evaluation. Therefore, we see positive values of the index as an excessively bullish behavior of the market participant. In the same way, negative values of the market sentiment indicate an excessively bearish behavior. The more extreme the values of the index are, the more the sentiment generates miscalculations of future payoff. Consequently, the sentiment index is equal to zero in an efficient market.

We extract our sentiment measure from the sentiment function. Barone-Adesi et al. (2014) derive three other measures from \( \Lambda_{t,T} \). First, they compare the mean and the variance of the densities \( P_R \) and \( P \). They define optimism as the difference in means, and over-confidence as the difference in variances between the two densities. Then, they compute a general level of sentiment using a distance measure, such as a root mean square error, between the logarithms of the market and the rational pricing kernels. Unlike optimism and over-confidence, our index does not focus on the particular moments of the beliefs densities but rather considers the overall difference between the representative investor’s beliefs and the rational beliefs. Recall that the difference in logarithms of the two pricing kernels is the sentiment function. The distance measure gives the level of sentiment but does not differentiate between bullish or bearish behavior created by the sentiment. Moreover, each point of the sentiment function should be weighted by the probability of occurrence of its state of nature in \( T \) to properly take into account the influence of the sentiment on the general process of pricing the payoffs. Our sentiment index considers the weight of each state of nature \( x_T \) in aggregating the function of sentiment in \( t \) and gives a direct indication of the direction and the magnitude of the sentiment effect on the market.

3.3 Empirical Analysis

In this section, we derive a new market sentiment index using historical financial data. Through different steps leading to the sentiment index, we compare our empirical estimates with the previous academic results. In order to derive the value of the market sentiment index at a given time, we need to estimate successively the empirical pricing kernel and the empirical sentiment function. More precisely, given that there is no consensus on the choice of the state variable, we consider the projection of the pricing kernel on the payoff of a traded asset \( S \) which acts as a proxy for the market portfolio. This projected pricing kernel depends on the realization of \( S_T \) and has the same pricing implications as the original pricing kernel for assets whose payoffs are based on \( S \) (Cochrane, 2005). We focus our analysis on the S&P 500 index and on various S&P 500 ETFs. Firstly, we estimate the empirical pricing kernel and the empirical sentiment function for the period January 2010 - May 2014. Secondly, we derive our investor sentiment index for
this period. Finally, we conduct an analysis of the ETF flows with respect to the change in sentiment.

### 3.3.1 Empirical Pricing Kernel and Sentiment Function

In order to infer the empirical pricing kernel on the S&P 500 index, we download from OptionMetrics the daily prices of European S&P 500 index options between July 1, 2009 and May 31, 2014. In order to have the most reliable prices, we treat our option data as in A"ıt-Sahalia and Lo (1998). We discard options with a price lower than 1/8 or implied volatility of more than 70%. We use an implied futures price \( F_{t,T} \) to replace the prices of less liquid options (i.e. in-the-money options) with the prices implied by the put-call parity at the relevant strike.\(^{30}\) In this way, we get the prices from liquid options only. For our purpose, we keep the prices of call options. This process leads to a set of 881,278 observations from 22,551 call options with an average of 618 daily prices. In average, the options have a time to expiration of 177.29 days and an implied volatility of 21.53%, see Table 3.1. Moreover, we use the historical time series of the S&P 500 index and infer the risk free rate from zero coupon risk free rates. Both are downloaded from Thomson Reuters Datastream.

<table>
<thead>
<tr>
<th>Call Prices ($)</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moneyness</td>
<td>3.29</td>
<td>21.16</td>
<td>-77.37</td>
<td>269.53</td>
</tr>
<tr>
<td>Maturity (days)</td>
<td>177.29</td>
<td>216.57</td>
<td>3</td>
<td>1095</td>
</tr>
<tr>
<td>Implied Volatility (%)</td>
<td>21.53</td>
<td>9.42</td>
<td>6.19</td>
<td>70</td>
</tr>
</tbody>
</table>

Notes: This table reports some summary statistics on the sample of 22,551 S&P 500 call options in the period July 1, 2009 - May 31, 2014 (881,278 daily observations). It displays statistics on the option prices in USD, the option moneyness defined as the strike price over the S&P 500 spot price \((K/S_t-1)\) in percentage, the option maturity as the time to expiration in days and the option implied volatility in percentage.

First, we derive a weekly estimate of the empirical pricing kernel for each Monday between January 2010 and May 2014. Using the fact that the pricing kernel equals the discounted ratio of the risk-neutral and the objective densities of the state variable, we estimate non-parametrically our empirical pricing kernel \( \hat{M}_{t,T} \) by computing the following ratio, as in Ait-Sahalia and Lo (1998) and Ait-Sahalia and Lo (2000):

\[
\hat{M}_{t,T}(S_T) = e^{-r_T} \frac{Q(S_T)}{P(S_T)}
\]

\(^{30}\)The implied futures prices are derived in \( t \) for each maturity \( T \) using the pair put-call that is the closest to at-the-money. Then, the implied futures prices are derived from the parity: \( H(\cdot) + X \exp^{-r_T} = G(\cdot) + F_{t,T} \exp^{-r_T} \) where \( H(\cdot) \) and \( G(\cdot) \) are the call and put prices in \( t \) with a maturity \( T \) and a strike of \( X \), respectively.
where $\hat{Q}$ and $\hat{P}$ are estimates of the risk-neutral density and the objective density for the realization of our market portfolio proxy in $T$, respectively. The risk-free rate between $t$ and $T$ is noted $r_f$.

Following the results of Breeden and Litzenberger (1978) and using a sample of $n$ call options on $S$, we estimate $\hat{Q}$ by computing numerically the discounted second partial derivative of a call option pricing function $H$ with respect to the strike $X$. The call option pricing function is the Black-Scholes formula where the volatility parameter is the non-parametric estimate of the implied volatility. To circumvent the unavailability of the derivative of a call option pricing function $H$ with respect to the strike $X$, we estimate $\hat{Q}$ by computing numerically the discounted second partial derivative of the implied volatility function $H_t$ with respect to $X$ and the strike $X$.

Figure 3.1 displays the empirical pricing kernel $\hat{Q}$ for each Monday between January 2010 and May 2014. To facilitate comparison between periods, we express the different values of the S&P 500 in $T$ relative to their values in $t$: $S_T/S_t$. We see that the empirical pricing kernel is time-varying and decreasing in $S_T/S_t$. However, it does not correspond to the classical stochastic discount factor. Indeed, the empirical pricing kernel is an oscil-

---

31 As in Péron and Villa (2002), the optimal values of the bandwidths $h_{X/F}$ and $h_\tau$ come from Aït-Sahalia and Lo (1998) and are chosen according to these relations:

$$
    h_{K/F} = c_{K/F} \sigma_{K/F} n^{-1/(d+2(q_k/s+m))}
$$

$$
    h_\tau = c_\tau \sigma_\tau n^{-1/(d+2q_\tau)}
$$

where $q_k$ is the number of existing continuous partial derivatives of the function to be estimated with respect to the $k^{th}$ regressor ($k = K/F, \tau$), $m$ is the order of the partial derivative with respect to the $k^{th}$ regressor that we wish to estimate, $d$ is the number of regressors, and $\sigma_k$ is the unconditional standard deviation of the $k^{th}$ regressor. Hence, we set $q_k = m = d = 2$. The parameter $c_k$ is typically of the order of one and Aït-Sahalia and Lo (1998) attests that small deviations from the exact value have no large effects.

32 For instance, the American Association of Individual Investors Survey, the Livingston Survey or the Thomson Reuters/Ipsos Consumer Sentiment Index.
3.3. Empirical Analysis

lating function with increasing parts. This feature means that the representative investor experiences increasing marginal utility with respect to the underlying state variable. Generally, this happens in the state of the economy where the level of $S_T$ is expected to be between 90% and 100% of its level in $t$. Hence, within this range, the marginal utility of the payoff will decrease as the economy declines. In addition, the empirical pricing kernel is largest in the worst states of nature, especially when $S_T/S_t < 0.85$.

Figure 3.1: Empirical Pricing Kernel

Notes: This figure presents the empirical pricing kernel for assets whose payoffs are based on the S&P 500 stock index. The empirical kernel is derived by computing the discounted ratio of the risk neutral density and the objective density of the S&P 500 index where both densities are estimated semi-parametrically on a limited domain for the future value $S_T$. More precisely, we take 250 uniformly spaced gross returns $S_T/S_t \in [0.80;1.20]$. The time-horizon $T - t$ of the pricing kernel is fixed at 6 months. The estimate is conduct each Monday between January 1st, 2010 and May 31, 2014.

This oscillating feature of the empirical pricing kernel is documented in previous studies and is know as the "pricing kernel puzzle". Several studies derive the empirical kernel with S&P 500 as the underlying asset: Ait-Sahalia and Lo (2000) use non-parametric estimators and find, for the year 1993, an average empirical pricing kernel consistent with this puzzle. Using a GARCH specification and non-parametric estimation techniques, Barone-Adesi et al. (2008) and Barone-Adesi et al. (2014) derive estimates of the pricing kernel between 2002 and 2009. Their empirical kernel is time varying, has increasing parts and oscillates. Rosenberg and Engle (2002) conduct a parametric derivation of the pricing kernel for the
period 1991 to 1995 and obtain as result an oscillating function. Theoretically, this pricing kernel shape can be explained by the presence of investor sentiment. Shefrin (2008) shows that the cohabitation of bullish and bearish investors can lead to an oscillating stochastic discount factor. Seeing as the empirical pricing kernel is the result of the aggregation of all participants in the market, it is possible for the representative investor to have an increasing marginal utility even if all of the investors have decreasing marginal utility.

The sentiment function $\Lambda$ provides a comprehensive framework to analyse the effect of the shape of the empirical stochastic discount factor on the prices. Indeed, it allows us to see the difference between the efficient market hypothesis and the actual pricing behavior of the market. Rewriting the relation (3.2) gives a direct interpretation of $\Lambda_{t,T}$:

$$\Lambda_{t,T}(S_T) = \log \left( \frac{M_{t,T}(S_T)}{M_{t,T}^e(S_T)} \right)$$

(3.7)

For each state of nature in $T$, $\Lambda_{t,T}$ is the relative difference between the values of the two pricing kernels. The sentiment function tells us which states of nature are overweighed or underweighed by the market with respect to the EMH. We estimate $\Lambda_t$ following

Figure 3.2: Empirical Sentiment Function

Notes: This figure presents the sentiment function estimated as the regression residuals of the empirical pricing kernel on the efficient pricing kernel, both in logarithm. The regression is run on 250 uniformly spaced gross returns $S_T/S_t \in [0.80;1.20]$, with a time-horizon $T - t$ of 6 months, and is conducted each Monday between January 1st, 2010 and May 31, 2014.
Barone-Adesi et al. (2014) and using the empirical pricing kernel derived above. In a market of CRRA investors, the pricing kernel in the case of fully rational investors is $M_{t,T}^e = \delta_e (S_T/S_t)^{-\gamma_e}$, where $\gamma_e$ is the risk aversion coefficient and $S_T/S_t$ is a proxy for the consumption growth return. Hence, the equation for the logarithm of the pricing kernel (3.2) becomes:

$$m_{t,T}(S_t) = \log(\delta_e) - \gamma_e \log(S_T/S_t) + \Lambda_{t,T}(S_T). \tag{3.8}$$

The estimated sentiment function $\Lambda_{t,T}$ is then obtained as the residual of the OLS regression of the logarithm of the estimated pricing kernel on a constant term and the logarithm of $S_T/S_t$:

$$\log(M_{t,T}) = \theta_0 + \theta_1 \log(S_T/S_t) + \Lambda_{t,T} \tag{3.9}$$

where the belief horizon $T - t$ is fixed at 6 months and the regression is run on the 250 values of the gross return $S_T/S_t \in [0.8; 1.2]$. Thus, for each period $t$, we have an estimate of the sentiment function $\Lambda_{t,T}$ that is defined on the set of all states of nature $S_T$. As a result, we get our empirical sentiment function $\hat{\Lambda}_{t,T}$ in $t$ for belief horizon of $T - t$.

Figure 3.3: Empirical Sentiment Function per Year

Notes: This figure presents the empirical sentiment function separately for each year. The sentiment function is estimated as the regression residuals of the empirical pricing kernel on the efficient pricing kernel, both in logarithm. The regression is run on 250 uniformly spaced gross returns $S_T/S_t \in [0.80; 1.20]$, with a time-horizon $T - t$ of 6 months, and is conducted each Monday between January 1st, 2010 and May 31, 2014. The gross return of the S&P 500 index $S_T/S_t$ is represented in the x-axis.

We can see in Figure 3.2 the empirical sentiment function for each Monday in the period between January 2010 and May 2014. Because the function is non-null, the sentiment is a significant determinant in the evaluation of the market prices and therefore the EMH.
is not verified. The sentiment function is time-varying, meaning that the influence of the sentiment is not the same throughout time.

To better visualize the impact and the dynamics of the sentiment given the states of nature, we analyse $\Lambda_{t,T}$ separately for each year and plot the sentiment function in Figure 3.3. When $\Lambda_{T,t}$ is positive, the respective state of nature is overweighted by the representative investor. We see that between 2010 and 2012, the states of nature of the more extreme events (i.e. periods where $S_T$ deviates more than 10% from its initial value in $t$) are overweighted. Inversely, the events where the index in $T$ is expected to be close to its current value tend to be underweighted. One interpretation is that the representative investor gives a higher probability of occurrence to large fluctuations in the S&P 500 index, relative to the objective occurrence of events. Similarly, the representative investor sets a relatively lower probability to the occurrence of moderate fluctuations. This pattern is less evident in 2013 and seems to reverse in 2014. However, the global influence of the sentiment on the market is difficult to infer from the sentiment function. Indeed, on each period, $\Lambda_{t,T}$ takes in turn positive and negative values. This is the reason why we need a way to summarize the impact of sentiment on the market at a given point in time. Hence, we derive in the next section an investor sentiment index.

Moreover, regression (3.9) allows us to estimate the discount factor and the relative risk aversion coefficient for the case of fully rational investors. We use the estimated coefficients $\hat{\theta}_0$ and $\hat{\theta}_1$ to infer $\hat{\delta}_e$ and $\hat{\gamma}_e$, such that $\hat{\delta}_e = e^{\hat{\theta}_0}$ and $\hat{\gamma}_e = \hat{\theta}_1$. Figure 3.4 presents the estimates for $\delta_e$ (left plot) and $\gamma_e$ (right plot), with their 95% confidence interval, over the sample period. The discount factor estimate fluctuates between 0.83 and 0.98 while the relative risk aversion coefficient is between 3.01 and 7.04. These values are consistent with previous literature (see for instance Frederick et al., 2002 and Barsky et al., 1997).

Figure 3.4: Discount Factor and Relative Risk Aversion Coefficient

Notes: This figure presents the estimates of the discount factor $\hat{\delta}_e$ (left plot) and of the coefficient of relative risk aversion $\hat{\gamma}_e$ (right plot) between January 1st, 2010 and May 31, 2014. We also display, for each estimate, the 95% confidence interval (blue bands).
3.3.2 Investor Sentiment Index

We assess, with given random payoffs $z(S_T)$ and the Equation (3.4), the representative investor evaluation $p_{\text{market},t}$ and the rational evaluation $p_{\text{eff},t}$ of $z(S_T)$ at a time $t$ as:

$$p_{\text{market},t} = \sum_{S_T} \hat{P}(S_T) e^{\hat{\Lambda}_{t,T}(S_T)} \hat{\delta}_e(S_T/S_t)^{-\hat{\gamma}_e} z(S_T) (3.10)$$

$$p_{\text{eff},t} = \sum_{S_T} \hat{P}(S_T) \hat{\delta}_e(S_T/S_t)^{-\hat{\gamma}_e} z(S_T) (3.11)$$

Our market sentiment estimate in $t$ for a belief horizon $T$ compares these two prices and gives the distortion due to irrational beliefs:

$$\lambda_{t,T} = \frac{p_{\text{market},t}}{p_{\text{eff},t}} - 1 (3.12)$$

Hence, we compute directly the investor sentiment index by pricing the payoff $z(S_T) = S_T$ and using the previous equations. Prices $p_{\text{market},t}$ and $p_{\text{eff},t}$ are based on Equations (3.10) and (3.11) where $\hat{\Lambda}_{t,T}$ is the residuals of the regression (3.9). Figure 3.5 represents the time series of our investor sentiment index. The index is computed for each Monday between January 1st, 2010 and May 31, 2014, where the time-horizon $T - t$ of the index equals 6 months. Recall that the value of the index can be interpreted as the propensity

![Graph showing the investor sentiment index between January 10, 2010 and August 14, 2014. The x-axis represents dates from January 10, 2010 to August 14, 2014, and the y-axis represents the index values ranging from -0.12 to 0.06. The index values fluctuate over time, indicating changes in investor sentiment.]
of the representative investor, or the market, to be excessively bullish (if $\lambda_{t,T} > 0$) or excessively bearish (if $\lambda_{t,T} < 0$) regarding the expected realisations of the index $S_T$. At the beginning of our sample period, the level of investor sentiment is negative. Eventually, $\lambda_t$ decreases, indicating lower market expectations of the S&P 500 due to sentiment. It drops further during the winter of 2011 to reach the lowest level in the sample period. Following this drop, the index increases until May 2014. In a nutshell, the market is excessively bearish during the years 2010 to 2012. Afterwards, from the beginning of 2013, the sentiment is positive indicating an excessive bullish behavior of the market participants. Note that the highest levels of sentiment are reached during the last months of our sample period.

Regarding the definition of the investor sentiment index in the Equation (3.12), the values of the sentiment function for the more likely states of nature are the most important drivers of the index. We briefly analyze the shapes of the sentiment function (as in Figure 3.3) with respect to the value of the sentiment index. During the periods of excessive bearish market behavior, namely from 2010-12 when $\lambda < 0$, the empirical sentiment function $\Lambda_{t,T}$ is rather negative for small expected changes in $S_t$. Thus, the sentiment index reflects mostly lower investor probability of the most likely states of nature in $T$ with respect to their true density. On the other hand, higher investor probability for these states of nature correspond to an excessive bullish behavior, as in 2013-2014.

Table 3.2: Correlation Coefficients with Monthly Data

<table>
<thead>
<tr>
<th></th>
<th>$\lambda$</th>
<th>UMCSI</th>
<th>TRI</th>
<th>SSICI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>1</td>
<td>0.64</td>
<td>0.75</td>
<td>0.59</td>
</tr>
<tr>
<td>UMCSI</td>
<td>1</td>
<td></td>
<td>0.72</td>
<td>0.39</td>
</tr>
<tr>
<td>TRI</td>
<td>1</td>
<td></td>
<td></td>
<td>0.33</td>
</tr>
<tr>
<td>SSICI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports the correlation coefficient between several measures of investor sentiment with monthly data: $\lambda$ is the investor sentiment derived in this study; UMCSI the University of Michigan Consumer Sentiment Index, TRI the Thomson Reuters/Ipsos Consumer Sentiment Index and SSICI the State Street Investor Confidence Index. The sample period is January 1st, 2010 - May 31, 2014.

As a comparison with existing measures of investor sentiment, we conduct a correlation study between $\lambda$ and several popular proxies used in the literature. Specifically, we consider four direct measures of investor sentiment: From the Investor’s Intelligence Advisors Sentiment Data that surveys US stock market newsletters and rates them as bullish, bearish or neutral/correction, we construct the Investor’s Intelligence sentiment index ($II$) by taking the percentage of bullish letters over the sum of bullish and bearish letters. The American Association of Individual Investors surveys investors and provides the percentage of individual investors that are bullish, bearish or neutral on the stock market. We take the ratio of the percentage of bullish over the percentage of bullish and
bearish investors in the AAII sentiment measure. Finally, we have two measures focusing on consumers, the University of Michigan Consumer Sentiment Index (UMCSI) and the Thomson Reuters/Ipsos Consumer Sentiment Index (TRI) that survey consumers about their prospects for the general economy. In addition, we use two indirect measures: The State Street Investor Confidence Index (SSICI) that analyses the buying and selling patterns of institutional investors; and a measure of expected volatility relative to current volatility of the S&P 500 index (VOL). This last measure is the ratio of implied S&P 500 option volatility (VIX) over the realized S&P 500 volatility computed using five years of historical data. This measure reflects the relative volatility expectations and can be interpreted as a bearish indicator (see Brown and Cliff, 2004).³³

Table 3.3: Correlation Coefficients with Weekly Data

<table>
<thead>
<tr>
<th></th>
<th>λ</th>
<th>II</th>
<th>AAII</th>
<th>VOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>1</td>
<td>0.52</td>
<td>0.06</td>
<td>-0.16</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>0.41</td>
<td>-0.49</td>
<td></td>
</tr>
<tr>
<td>AAII</td>
<td></td>
<td>1</td>
<td>-0.24</td>
<td></td>
</tr>
<tr>
<td>VOL</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports the correlation coefficient between several measures of investor sentiment with weekly data: λ is the investor sentiment derived in this study; II is the percentage of bullish letters over the sum of bullish and bearish letters issued from the Investor’s Intelligence Advisors Sentiment Data; AAII is the percentage of bullish investors over the sum of bullish and bearish investors issued from the American Association of Individual Investors survey and VOL is a measure of expected volatility relative to current volatility of the S&P 500 index. The sample period is January 1ˢᵗ, 2010 - May 31, 2014.

Because half of these measures are only provided at a monthly frequency (UMCSI, TRI and SSICI), we derive a monthly frequency version of our sentiment index during the sample period. Table 3.2 presents the contemporaneous correlation between our index λ and the UMCSI, the TRI, and the SSICI measures using monthly data. The sentiment measure λ is strongly correlated with all three measures. Note that although all of the correlation coefficients are positive, they are lower for the SSICI, which reflects institutional investor sentiment, and for both the UMCSI and the TRI that focus on consumers. Table 3.3 displays the correlation coefficients between λ and the three other sentiment measures II, AAII and VOL, available at weekly frequency. Interestingly, our sentiment index is positively correlated with II but not with AAII. The negative correlation with VOL is explained by the fact that, in contrast to other measures, VOL is interpreted as a bearish measure. To summarize, the values of the correlation coefficients vary across the different sentiment measures but our sentiment index λ is particularly

³³Unfortunately, as the sentiment measure of Baker and Wurgler (2007) is only available until December 2010, we do not use it in the analysis.
strongly correlated with the sentiment measures issue from consumer surveys (UMSCI and TRI), institutional investor behavior (SSICI) and stock market newsletter (II). We see this feature as reassuring evidence that our index captures some of the most relevant aspects of investor sentiment.

### 3.3.3 ETF Flows and Investor Sentiment

We now investigate whether flows to S&P 500 index ETFs respond to investor sentiment. Due to their low fees and high liquidity, the ETFs offer a cheap and efficient way to get exposure to the stock market. We assume that investors’ interest in a given market is represented by the flows towards ETFs that replicate the index of that specific market. As our sentiment measure represents excessive aggregate bullish or bearish beliefs of investors regarding the S&P 500 index, we expect sentiment to impact the S&P 500 ETFs inflows or outflows.

We collect data on the three largest S&P 500 index ETFs in term of assets under management (AUM), namely the State Street SPDR S&P 500 ETF ($160,160 million of AUM, as of May 31, 2014), the iShares Core S&P 500 ETF ($56,812 million of AUM), and the Vanguard S&P 500 ETF ($18,667 million of AUM). For each ETF, we download from their provider’s websites the daily AUM and the net asset value (NAV) from January 1st, 2010 until May 31, 2014 (except for the Vanguard S&P 500 where the data only start in September 2010). Since we do not observe flows directly, we follow Frazzini and Lamont (2008) and infer flows from fund assets under management and from returns for each Monday of our sample period:

\[
flow_{j,t} = AUM_{j,t} - AUM_{j,t-1} \cdot (1 + r_{j,t})
\]  

(3.13)

where \(r_{j,t}\) is the return of the NAV of the ETF \(j\) between \(t - 1\) and \(t\). By doing so, the performance of the fund is not taken into account as we only capture share redemptions (outflows) and share purchases (inflows). Then, we test if the investor sentiment is a determinant of the ETF flows by studying the cumulated flows that enter or exit the S&P 500 index ETFs. As our sentiment index is not a stationary process, we use the first difference of \(\lambda_t\) to conduct statistical inference. Moreover, in order to remove the influence of economic fundamentals, we follow Baker and Wurgler (2006) and regress \((\lambda_t - \lambda_{t-1})\) on a set of macroeconomic variables.\(^{34}\) The residuals of this regression correspond to our time-series of investor sentiment change, \(\Delta \lambda_t\). Thus, \(\Delta \lambda_t\) shows the direction and the magnitude of the change in excessive expectations regarding the market. For instance, \(\Delta \lambda_t > 0\) means that the market becomes more excessively bullish than during the previous period.

We regress now the cumulated flows issued from the S&P500 ETFs \(\text{Flows}_t\) on the change in sentiment \(\Delta \lambda_t\) and its three first lags. Hence, we test if an increase in the bullish behavior of the market leads to flow variations. Moreover, a set of control variables \(X_t\) are added as explanatory variables. Thus, we consider the following regression:

\[
\text{Flows}_t = \alpha_0 \Delta \lambda_t + \alpha_1 \Delta \lambda_{t-1} + \alpha_2 \Delta \lambda_{t-2} + \alpha_3 \Delta \lambda_{t-3} + \beta X_t + \epsilon_t.
\]  

(3.14)

\(^{34}\)These variables are the growth in industrial production, real growth in durable, nondurable, and services consumption, and growth in unemployment rate. The results are showed in Table 3.5 in Appendix C.2.
### Table 3.4: Aggregate Flows and Sentiment Variation

<table>
<thead>
<tr>
<th></th>
<th>Flows$_t$</th>
<th>Flows$_{t-1}$</th>
<th>Flows$_{t-2}$</th>
<th>Flows$_{t-3}$</th>
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<tr>
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<td>-0.225***</td>
<td>-0.245***</td>
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<td></td>
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<td>(-3.22)</td>
<td>(-3.53)</td>
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<td>(-3.20)</td>
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<tr>
<td>Perf$_t$</td>
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<td>501.1***</td>
<td>584.7***</td>
<td>560.5***</td>
</tr>
<tr>
<td></td>
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<td>(4.38)</td>
<td>(3.69)</td>
<td>(5.07)</td>
<td>(4.75)</td>
</tr>
<tr>
<td>Perf$_{t-1}$</td>
<td>391.5***</td>
<td>485.1***</td>
<td>355.7***</td>
<td>514.1***</td>
<td>552.9***</td>
</tr>
<tr>
<td></td>
<td>(3.48)</td>
<td>(4.32)</td>
<td>(2.66)</td>
<td>(4.00)</td>
<td>(3.71)</td>
</tr>
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<td>Perf$_{t-2}$</td>
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<td>29.47</td>
<td>-68.94</td>
<td>29.36</td>
<td>-9.363</td>
</tr>
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<td></td>
<td>(0.20)</td>
<td>(0.22)</td>
<td>(-0.50)</td>
<td>(0.22)</td>
<td>(-0.06)</td>
</tr>
<tr>
<td>$\Delta \lambda_t$</td>
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<td>180720.1**</td>
<td></td>
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<tr>
<td></td>
<td>(1.98)</td>
<td>(2.05)</td>
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<tr>
<td>$\Delta \lambda_{t-1}$</td>
<td>94479.1</td>
<td>67336.7</td>
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<td></td>
<td>(1.06)</td>
<td>(0.81)</td>
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<tr>
<td>$\Delta \lambda_{t-2}$</td>
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<td>(-0.06)</td>
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<tr>
<td>$\Delta \lambda_{t-3}$</td>
<td>-236838.1***</td>
<td>-226098.4***</td>
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<tr>
<td></td>
<td>(-2.79)</td>
<td>(-2.65)</td>
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<tr>
<td>$\Delta II_t$</td>
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<td>(-0.36)</td>
<td>(-0.61)</td>
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<tr>
<td>$\Delta II_{t-1}$</td>
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<td>(0.80)</td>
<td>(0.75)</td>
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<td>$\Delta II_{t-2}$</td>
<td>12971.0</td>
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<td></td>
<td>(1.58)</td>
<td>(1.50)</td>
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<td>$\Delta II_{t-3}$</td>
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<td>-7394.0</td>
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<td></td>
<td>(-1.21)</td>
<td>(-0.92)</td>
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<tr>
<td>$\Delta AAII_t$</td>
<td>1999.9</td>
<td>788.2</td>
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<tr>
<td></td>
<td>(0.61)</td>
<td>(0.22)</td>
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<tr>
<td>$\Delta AAII_{t-1}$</td>
<td>5293.3</td>
<td>2943.7</td>
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<td>(1.61)</td>
<td>(0.75)</td>
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<tr>
<td>$\Delta AAII_{t-2}$</td>
<td>6559.4**</td>
<td>4182.3</td>
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<td>(2.36)</td>
<td>(1.15)</td>
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<tr>
<td>$\Delta AAII_{t-3}$</td>
<td>7227.0***</td>
<td>8314.5**</td>
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<tr>
<td></td>
<td>(2.69)</td>
<td>(2.51)</td>
<td></td>
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</tr>
<tr>
<td>$\Delta VOL_t$</td>
<td>2072.6</td>
<td>4025.2**</td>
<td></td>
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<tr>
<td></td>
<td>(1.05)</td>
<td>(2.18)</td>
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<tr>
<td>$\Delta VOL_{t-1}$</td>
<td>-59.31</td>
<td>1906.8</td>
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<tr>
<td></td>
<td>(-0.03)</td>
<td>(0.83)</td>
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<tr>
<td>$\Delta VOL_{t-2}$</td>
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<td>4073.6**</td>
<td></td>
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<td></td>
<td>(0.86)</td>
<td>(2.41)</td>
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<tr>
<td>$\Delta VOL_{t-3}$</td>
<td>1997.2</td>
<td>4330.1**</td>
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<td></td>
<td>(1.26)</td>
<td>(2.53)</td>
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</table>

Observations: 228 228 228 228 228
R$^2$: 0.224 0.197 0.215 0.193 0.302

Notes: This table reports the parameter estimates obtained by regressing the total flows issued from the three largest S&P 500 ETFs ($\text{Flows}_t$) on its first lag ($\text{Flows}_{t-1}$), the return of the S&P 500 index between $t$ and $t-1$ ($\text{Perf}_t$) and its three first lags in percentage and these two first lags. In addition, we separately add the change in the investor sentiment index $\Delta \lambda_t$ and its three first lags (column 1), the change in the Investor’s Intelligence sentiment index $\Delta II_t$ and its three first lags (column 2), the change in the American Association of Individual Investors sentiment measure $\Delta AAII_t$ and its three first lags (column 3) and the change in the relative expected volatility $\Delta VOL_t$ and its three first lags (column 4). Finally, all these measures of sentiment are included (column 5). We use weekly observations between January 2010 and May 2014 and the estimations are based on an OLS model with a constant term and robust standard errors. We display t-statistics in parentheses. *** ** * represent statistical significance at the 1%, 5% or 10% levels, respectively.
The control variables included in \( X_t \) are the lagged flow variable \( \text{Flows}_{t-1} \), the return of the S&P 500 index between \( t - 1 \) and \( t \) (\( \text{Perf}_t \)) and its two first lags, i.e. \( X_t = [\text{Flows}_{t-1}; \text{Perf}_t; \text{Perf}_{t-1}; \text{Perf}_{t-2}] \).

We estimate Equation (3.14) using ordinary least squares on a sample of weekly data between January 2010 and May 2014. The results are displayed in the first column of Table 3.4. We find that the estimated coefficient associated with \( \Delta \lambda_t \) is positive and statistically significant at the 95% confidence level. Hence, it appears that inflows to equity funds are contemporaneously correlated with a positive change in the level of investor sentiment. If the market becomes more bullish in \( t \) (\( \Delta \lambda_t > 0 \)), the flows towards ETFs are estimated to be larger, everything else remaining constant. Interestingly, this effect gets reversed after three periods. The flows in \( t \) are negatively correlated with the state of the variable \( \Delta \lambda \) three weeks ago. Note that this result is highly statistically significant and that the variations in flows in \( t \) and in \( t - 3 \), caused by an increase of the bullish sentiment in the market, are of the same magnitude. This pattern of contemporaneous positive correlation followed by a negative correlation three weeks later is consistent with the assumption of sentiment based trading. Indeed, an increase of the level of sentiment first induces excessively high expectations on the S&P 500 index returns and causes immediate additional inflows. This means that the investors react to a rise in the excessive expectations regarding the S&P 500 index by investing more on the S&P 500 ETFs. Then, in a second step, there is a correction of these sentiment-induced investments, which is reflected in the negative coefficient on \( \Delta \lambda_{t-3} \).

Another interesting result in our regression analysis is the fact that index ETF flows significantly respond to past index performance. In Table 3.4, the two coefficients associated with the returns of the index between \( t - 1 \) and \( t \) (\( \text{Perf}_t \)) and between \( t - 2 \) and \( t - 1 \) (\( \text{Perf}_{t-1} \)) are both statistically significant, positive and large. A one-percent increase in the index value between \( t - 1 \) and \( t \) triggers an extra $525 million of additional flows at time \( t \) and more than $391 million at time \( t + 1 \). This finding is consistent with the return-chasing behavior of fund investors documented in Chevalier and Ellison (1997).

For further analysis, we re-estimate Equation (3.14) in Table 3.4 but with the other sentiment measures available at a weekly frequency. Unlike the previous analysis, we find no statistically significant results when we use the difference in the Investor’s Intelligence sentiment index \( \Delta II \) (column 2) and the difference in the relative expected volatility \( \Delta VOL \) (column 4). However, an increase in the American Association of Individual Investors sentiment measure at time \( t \) triggers additional inflows two and three weeks later (column 3). Hence, our sentiment index seems to capture different characteristics of the investor sentiment with respect to the other weekly measures and especially to the AAII index. Recall that our investor sentiment index \( \lambda_t \) is almost not correlated with the AAII measure (Table 3.3).

As a robustness check, we include all of the sentiment measures in the regression (Table 3.4 column 5). Then, we also document additional flows to the ETFs during a positive change in the investor sentiment index \( \lambda_t \) followed by a reduction in the fund flows three weeks later. Thus, this interpretation of sentiment-based investments into ETFs is consistent with the previous results and is statistically significant even when we control for the other measures of sentiment. Moreover, the investor sentiment measure \( \lambda_t \) captures different features than the measures \( II, AAII \) and \( VOL \). This can be due to the fact that \( \lambda_t \) is a measure specific to the S&P 500 market while the other ones are more general. Note
that the effect of the relative volatility $VOL$ becomes significant. This can mean that this explanatory variable can interact with other sentiment measures.

3.4 Conclusion

In this paper, we show that investor sentiment is an important driver of the equity market funds. First, we derive a new investor sentiment measure based on the S&P 500 index from option data. In this regard, we estimate the pricing kernel (or the stochastic discount factor) for assets whose payoffs depend on the S&P 500 index. This empirical kernel is time-variant and oscillates. As a consequence, the representative investor experiences increasing marginal utility with respect to the state of the economy. This result is known as the ”pricing kernel puzzle” and can be explained by the presence of investor sentiment. Furthermore, we derive our investor sentiment index using the empirical pricing kernel. This index is defined as the measure of investor’s excessive expectations on the S&P 500 index returns due to the presence of investor sentiment. After comparing with several other sentiment measures, we consider that our index captures some of the most relevant aspects of investor sentiment. We find that the investors are excessively bearish during the period 2010-2012 and then become increasingly more bullish between 2013-2014.

Second, we investigate whether the investor sentiment is a determinant of the ETF flows and if so, whether it influences the investment choices of market participants. Indeed, ETFs are a good choice to study the impact of sentiment on the investor choices seeing as they are large, liquid and easily accessible. Thus, if sentiment matters, it should impact the ETF flows. Our results confirm previous findings in the literature. Although Warther (1995), using closed-end fund discounts as sentiment proxy, did not find evidence of a connection between fund flows and sentiment, we show, as Goetzmann and Massa (2003) and Indro (2004), a positive and contemporaneous correlation between fund flows and investor sentiment. However, in this paper we use an explicit measure of sentiment and show that the ETF investors react to the variation of the investor sentiment. Moreover, we document new evidence of a mean-reverting investor behavior: that the impact on the flows of investor sentiment gets reversed after a three-week period.
Appendix C

C1. Pricing Kernel with Homogeneous and Heterogeneous CRRA Investors

Let \( J \) be homogeneous and rational agents with time additive utility function. They choose in time \( t \) their present and future consumption plan \( \{ c_j \} \) by maximizing their expected utility. Each individual \( j = 1, \ldots, J \) follows the program:

\[
\max_{\{c_j\}} \mathbb{E}_j[U(\{c_j\})] = \mathbb{E}_j[\sum_{t,x_t} \delta^t u(c_j(x_t))]
\]

subject to \( W_j = \sum_{t,x_t} \nu(x_t)c_j(x_t) \)

for \( t = 1, \ldots, T \). The initial individual wealth is noted \( W_j \) and \( \nu(x_t) \) is the price of an Arrow-Debreu security that pays one if the state of nature \( x_t \) occurs in \( t \) and zero in all the other states. Note that we suppress the actual time-dependence notation. The Lagrange function \( f(c_j, \mu) \) for the maximization program (C1) is:

\[
f = \mathbb{E}_j[\sum_{t,x_t} \delta^t u(c_j(x_t))] - \mu[\sum_{t,x_t} \nu(x_t)c_j(x_t) - W_j]
\]

The first order conditions are:

\[
c_j(x_t) : \frac{P_j(x_t)\delta^t u'(c_j(x_t))}{\nu(x_t)} = \mu
\]

where \( P_j(x_t) \) is the probability that the event \( x_t \) occurs at time \( t \) for the agent \( j \). As all individuals are homogeneous and rational, \( P_j(x_t) = P(x_t) \forall j \) where \( P \) is the true density of the state variable. Moreover, as \( \nu(x_1) = 1 \) we have for all \( t \in [1, T] \)

\[
\frac{P(x_t)\delta^t u'(c_j(x_t))}{\nu(x_t)} = \frac{u'(c_j(x_t))}{1}
\]

Thus, the Arrow-Debreu security prices become:

\[
\nu(x_t) = \delta^t P(x_t) \frac{u'(c_j(x_t))}{u'(c_R(x_t))}
\]

Define a representative investor \( R \) that shares the characteristic features of the homogeneous and rational investors and who prices correctly the Arrow-Debreu securities. Hence:

\[
\nu(x_t) = \delta^t P(x_t) \frac{u'(c_R(x_t))}{u'(c_R(x_1))}
\]

The stochastic discount factor or pricing kernel \( M \) that allows to evaluate any payoff \( z_t \) by the pricing formula \( p = \mathbb{E}[Mz_t] \) equals the ratio \( \nu(x_t)/P(x_t) \) (Cochrane, 2005). Thus \( M \) is:

\[
M(x_t) = \delta^t \frac{u'(c_R(x_t))}{u'(c_R(x_1))}
\]
Appendix C.

Assume that all investors hold constant relative risk aversion (CRRA) preferences (power utility function, $u(c) = c^{1-\gamma}/1 - \gamma$ where $\gamma$ is the relative risk aversion coefficient). The pricing kernel becomes:

$$M(x_t) = \delta_e^t \left( \frac{c_R(x_t)}{c_R(x_1)} \right)^{-\gamma_e}$$  \hspace{1cm} (C8)

Since all investors are rational (i.e. they all have the correct beliefs $P$), we write the discount factor and the CRRA coefficient of the representative investor with a subscript $e$ meaning that the market is efficient. By the condition of market clearing, the consumption demand of the representative investor equals the aggregate supply in any period $t$. Hence, it is possible to rewrite (C6) and (C8) in a function of the growth rate of the consumption $g$ between the initial period and $t$, conditional on the event $x_t$:

$$\nu(x_t) = \delta_e^t P(x_t) g(x_t)^{-\gamma_e}$$  \hspace{1cm} (C9)

$$M(x_t) = \delta_e^t g(x_t)^{-\gamma_e}$$  \hspace{1cm} (C10)

In the case where all agents are rational, homogeneous and CRRA, the logarithm of the pricing kernel is thus:

$$m(x_t) = \log(M(x_t)) = \log(\delta_e^t) - \gamma_e \log(g(x_t))$$  \hspace{1cm} (C11)

Assume now that the CRRA investors are heterogeneous, Shefrin (2008) shows that there exists a representative investor who respects:

$$\nu(x_t) = \delta_R^t P_R(x_t) g(x_t)^{-\gamma_R}$$  \hspace{1cm} (C12)

Thus, the pricing kernel becomes:

$$M(x_t) = \frac{\nu(x_t)}{P(x_t)} = \delta_R^t \frac{P_R(x_t)}{P(x_t)} g(x_t)^{-\gamma_R}$$  \hspace{1cm} (C13)

As in Barone-Adesi et al. (2014), we divide the left-hand side and the right-hand side of this equation by the discount factor $\delta_e^t$ of the representative investor if all investors are rational (i.e. they all have the correct beliefs $P$). In addition, we set $\gamma_R = \gamma_e$ following Shefrin (2008) that determines that the aggregate coefficient of relative risk aversion is not dependent of change in the investors beliefs. Thus, we get:

$$M(x_t) = \frac{\delta_R^t P_R(x_t)}{\delta_e^t P(x_t)} \delta_e^t g(x_t)^{-\gamma_e}$$  \hspace{1cm} (C14)

From this last relation, we can define:

$$e^{\Lambda(x_t)} = \frac{\delta_R^t P_R(x_t)}{\delta_e^t P(x_t)}$$  \hspace{1cm} (C15)

as the transformation to apply to the rational pricing kernel when the representative investor does not have rational beliefs.
To summarize, with heterogeneous CRRA investors, the pricing kernel is a transformation of the rational pricing kernel $M^e(x_t) = \delta_t e^{\lambda(x_t) - \gamma x_t}$:

\[
M(x_t) = e^{\Lambda(x_t) \delta_t g(x_t) - \gamma e_t} = e^{\Lambda(x_t) M^e(x_t)} \tag{C16}
\]

Moreover, the logarithm of the pricing kernel, $m(x_t) = \log(M(x_t))$, breaks down into two building blocks: the logarithm of the pricing kernel in the case of fully rational investors, $m^e$, and a part that reflects the aggregate errors on investor beliefs $\Lambda(x_t)$. Indeed:

\[
m(x_t) = \log(\delta_t) - \gamma \log(g(x_t)) + \Lambda(x_t) = m^e(x_t) + \Lambda(x_t) \tag{C17}
\]

Shefrin (2008) define $\Lambda$ as the sentiment function that gives the relative change in the pricing kernel due to the presence of sentiment, with respect to an efficient market.

### C2. Investor Sentiment and Economic Fundamentals

Table 3.5: Sentiment Change and Macroeconomic Variables

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<th>$\Delta \lambda_t$</th>
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<tr>
<td>$Industrial_t$</td>
<td>-0.263 (-1.13)</td>
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<tr>
<td>$Durable_t$</td>
<td>-0.249*** (-3.34)</td>
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<tr>
<td>$Nondurable_t$</td>
<td>-0.329* (-1.69)</td>
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<tr>
<td>$Services_t$</td>
<td>-0.439 (-0.71)</td>
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<tr>
<td>$Unemployment_t$</td>
<td>-0.001*** (-2.61)</td>
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Observations: 228
R²: 0.125

Notes: This table reports the parameter estimates obtained by regressing the first difference of the investor sentiment index $\Delta \lambda_t = (\lambda_t - \lambda_{t-1})$ on the growth in industrial production ($Industrial_t$), the real growth in durable ($Durable_t$), non-durable ($Nondurable_t$), and services consumption ($Services_t$), and growth in unemployment rate ($Unemployment_t$). We estimate an OLS model with a constant term and robust standard errors. We display t-statistics in parentheses. ***, **, * represent statistical significance at the 1%, 5% or 10% levels, respectively.
Bibliography


