Margining in derivatives markets and the stability of the banking sector

GIBSON BRANDON, Rajna Nicole, MURAWSKI, Carsten

Abstract

We investigate the effects of margining, a widely-used mechanism for attaching collateral to derivatives contracts, on derivatives trading volume, default risk, and on the welfare in the banking sector. First, we develop a stylized banking sector equilibrium model to develop some basic intuition of the effects of margining. We find that a margin requirement can be privately and socially sub-optimal. Subsequently, we extend this model into a dynamic simulation model that captures some of the essential characteristics of over-the-counter derivatives markets. Contrarily to the common belief that margining always reduces default risk, we find that there exist situations in which margining increases default risk, reduces aggregate derivatives' trading volume, and has an ambiguous effect on welfare in the banking sector. The negative effects of margining are exacerbated during periods of market stress when margin rates are high and collateral is scarce. We also find that central counterparties only lift some of the inefficiencies caused by margining.

Reference


DOI : 10.1016/j.jbankfin.2012.10.005
Margining in derivatives markets and the stability of the banking sector

Rajna Gibson, Carsten Murawski

Abstract

We investigate the effects of margining, a widely-used mechanism for attaching collateral to derivatives contracts, on derivatives trading volume, default risk, and on the welfare in the banking sector. First, we develop a stylized banking sector equilibrium model to develop some basic intuition of the effects of margining. We find that a margin requirement can be privately and socially sub-optimal. Subsequently, we extend this model into a dynamic simulation model that captures some of the essential characteristics of over-the-counter derivatives markets. Contrarily to the common belief that margining always reduces default risk, we find that there exist situations in which margining increases default risk, reduces aggregate derivatives' trading volume, and has an ambiguous effect on welfare in the banking sector. The negative effects of margining are exacerbated during periods of market stress when margin rates are high and collateral is scarce. We also find that central counterparties only lift some of the inefficiencies caused by margining.

1. Introduction

For a long time, margining, a mechanism for attaching collateral to derivatives contracts, was considered a panacea to mitigate default risk in derivatives markets (International Swaps and Derivatives Association, 2005). However, in many financial calamities during the past few decades, including the collapses of Metallgesellschaft, Long-Term Capital Management (LTCM), and more recently Bear Stearns, Lehman Brothers, and American International Group (AIG), margining played an ambivalent role. It is not unreasonable to believe that it exacerbated the recent financial crisis. At present, it is an open question as to what the overall effect of margining is in a financial system and in an economy more generally.

In this article, we identify situations in which margining of derivatives, two-way contracts in which both parties are both potential creditors and potential debtors, decreases trading volume, increases default rates and default severity, and reduces welfare in the banking sector. Our analysis shows that margining presents derivatives counterparties and regulators with a delicate trade-off. On the one hand, margins reduce default severity by reducing banks' exposure to the default of their counterparties. On the other hand, margin requirements generate several types of costs, particularly when banks use derivatives for hedging purposes. First, by imposing a funding constraint on banks' trading strategies, margin requirements can limit the number of derivatives contracts traded by a bank and thus can prevent it from implementing its optimal hedging position. Second, increased margin requirements can indirectly constrain a bank's hedging strategy by reducing the number of contracts outstanding of other banks. Moreover, increased margin requirements can reduce the credit quality of a bank's counterparties (that is, increased probability of default and loss-given-default) by constraining the counterparties' hedging strategy.

These considerations lead to the conclusion that margining affects market outcomes through several different direct and indirect channels that may interact in subtle ways, some of them counter-intuitive. Moreover, it may impose negative externalities, that is, negative, indirect effects on other parties within the financial system or the economy, that are not transmitted through prices.

In the remainder of this article, we address the following research question: How do the various margining mechanisms observed in current derivatives markets affect trading volume, default risk, and welfare in the banking sector, in particular, during periods of market stress?

To analyze the effectiveness of margining within the banking sector we present two models. Our first, baseline model is a simple, static equilibrium model in which we develop some basic intuition for the main channels of margining. Subsequently, we present a
dynamic market model that extends the equilibrium model. The dynamic model captures many features of modern derivatives markets so as to analyse the various channels of margining and their interaction in a more realistic setting. However, this model cannot be solved analytically; hence, we evaluate it using simulations.

In the equilibrium analysis, our baseline model, we consider a one-period economy with an incomplete market and two (groups of) risk-averse banks. The banks have opposite endowments in a long-term, illiquid asset and a certain amount of cash. They wish to hedge the risk of their endowment by trading short-term derivatives contracts with each other. Banks maximize the expected utility of wealth by choosing the optimal number of derivatives contracts. Because markets are incomplete, we allow the banks to default.

We then introduce a margin requirement aimed at mitigating default risk associated with the trading of the derivatives contracts. Solving for the banks’ optimal trading strategies, we analyze the impact of margin requirements on their welfare (as measured by their utility of wealth), their default risk, and on the volume of derivatives traded. We find that exogenously imposed margin requirements can be privately and socially sub-optimal. Indeed, using numerical analysis, we find that sometimes a margin requirement of zero is optimal. The negative effects of margining increase as the constraints imposed by the margin requirements on the banks’ optimization problem tighten. More important, our results also suggest that when banks differ in key characteristics that affect the optimal level of the margin requirement, including their probability of default and risk aversion, privately and socially optimal levels of margining may not be the same, which in turn implies that the level of margining in a market will affect not only aggregate welfare but also the relative distribution of welfare.

Subsequently, we extend our baseline model to create a more realistic simulation model of derivatives trading in the banking sector. More precisely, we analyze a market consisting of several heterogeneous banks that face a similar optimization problem as before while assuming that the banking sector is experiencing severely adverse market and credit risk conditions. The latter assumption is made in order to determine how margin requirements affect this banking economy during market crises which are often deemed to represent the market conditions during which collateral is most valuable. In order to make the model more realistic, we calibrate its parameters with actual derivatives market data. We use this model to study the effects of initial margin, variation margin, and a central counterparty on market outcomes.

We find that the introduction of margining, both in the form of initial and variation margin, significantly deteriorates derivatives market liquidity while it increases banks’ default rates and ambiguously affects their welfare when assuming a mean–variance utility function. These results are more pronounced when initial margin levels are strengthened. The simulation results thus support the results obtained with our baseline model regarding the impact of margining on banks’ welfare and derivatives trading liquidity. They further show that, under stress scenarios, tighter margin requirements will even exacerbate banks’ default risk. In all our analyses, initial margin levels are set ex ante and remain constant; that is, we exclude pro-cyclical adverse effects of margining due to increases in margin rates during periods of stress.

Our results are reminiscent of the theory of ‘second best’, according to which the elimination of a market imperfection does not necessarily make an economy better off in the sense that it can exacerbate the negative effects of other market imperfections. We believe that our results explain some of the existing empirical research in this field. Hartzmark (1986) and Hardouvelis and Kim (1995) found that increases in margin rates at major derivatives exchanges led to a decrease in trading volume and open interest.

We also believe that our results explain, at least in part, current events in financial markets such as the collapse of AIG.2 Hence, our results for both the baseline equilibrium and the extended simulation models presented in this article are of interest to public policy makers, especially in light of the recently increased use of margining in over-the-counter (OTC) derivatives markets and its ambiguous effects on welfare in the banking sector. Indeed, we find that almost perfect coverage of counterparty default risk exposure by margining is sub-optimal during periods of market stress.

This finding is relevant to the role of derivatives trading, including credit derivatives trading, in the recent liquidity and credit crisis in the global banking sector.3 Our results also emphasize the significance of the interdependence between different types of risk, such as credit and liquidity risk, suggesting that these risks should ideally be analyzed and managed jointly rather than separately. Therefore, any change in margining policies in financial markets, such as the introduction of a central counterparty, should be considered carefully. At the same time, margining should become a key issue in the design and implementation of financial market policies, as suggested in Turner (2009).

We proceed as follows. In Section 2, we briefly review the literature relevant to this study. In Section 3, we develop our baseline model, a simple equilibrium model to shape the intuition for the various channels of margining. In Section 4, we extend this baseline model to create a more realistic, dynamic simulation model. Finally, Section 5 concludes the study.

2. Literature review

This article focuses mainly on the analysis of the trade-off between the benefits of collateral as a risk mitigation mechanism and its costs. Several strands of the literature have addressed issues related to this topic.

The measurement of default risk in derivatives contracts is inherently more complex than in most other financial contracts. Standard measurement approaches to credit risk often fail in case of derivatives. Duffee (1996) suggests two main reasons for this complexity. First, credit exposure fluctuates with the price of the underlying security. Second, exposures on derivatives contracts are correlated with the probabilities of default. The incorporation of default risk into the valuation of derivatives contracts was first considered by Hull and White (1995) and has since been re-examined by Collin-Dufresne and Hugonnier (2007) and others for a rather broad class of instruments.

Equally challenging as the measurement of default risk is its mitigation. As Swan (2000) reports, market participants have been preoccupied with the development of mechanisms to mitigate default risk in derivatives contracts ever since the inception of modern derivatives markets. Among the first means employed by market participants were appraisals of counterparties and collateral. Over time, rather sophisticated mechanisms evolved,

---

2 On September 16, 2008, the New York Times reported that as a result of adverse market movements and a ratings downgrade, the derivatives counterparties of AIG could ask for up to $10.5 billion in additional collateral in relation to swaps contracts. Because AIG was unable to raise sufficient funds to meet these margin calls and to meet its other counterparty-related obligations, the United States government provided a loan to AIG in exchange for a controlling stake in the company on the grounds that a default of AIG would probably have caused a systemic crisis in derivatives markets and the wider financial system. At the same time, because of rising margin requirements, market liquidity in derivatives markets was drying up, as reported in The Economist on September 20, 2008 (Wall Street’s bad dream). Other recent examples of such (near-)credit events include the cases of Metallgesellschaft in 1993, LTCM in 1998, Amaranth in 2007, as well as ACA in 2008.

3 Although default risk in derivatives contracts has mainly been discussed in relation to credit derivatives, a similar issue exists in most other derivatives markets. The greatest exposures are, so it seems, in the interest-rate swaps market, as reported in The Economist on September 20, 2008 (A nuclear winter?).
including derivatives clearinghouses and central counterparties. A historical account of these mechanisms is provided by Loman (1931), Moser (1994), and Moser (1998).

Margining aims to reduce default risk in derivatives by increasing the minimum delivery rate of a counterparty, thereby reducing credit exposure. As the International Swaps and Derivatives Association (2010) shows, it is widely employed by derivatives traders. Margining is also an essential risk management tool for derivatives clearinghouses, as described in Moody’s Investor Service (1998) and Knott and Mills (2002), among others.

We mentioned earlier that margins impose costs on market participants and thus should be reflected in market activity and in prices. Variation margin changes the cash flow and the value of a futures contract. Cox et al. (1981) argue that the variation margins of futures contracts can therefore be regarded as stochastic dividends. Margins also impose a funding constraint on market participants. It is thus conceivable that margin requirements affect agents’ trading behavior and market activity. Cuoco and Liu (2000) incorporate margin requirements into the portfolio optimization problem of a single agent and computed the effects on portfolio weights, albeit for the trading of primary assets. An adverse effect of margins on market liquidity is noted by Telser (1981). Nevertheless, the theoretical discussion remains inconclusive. While Anderson (1981) argues that opportunity costs of collateral are probably too low for margining to have any effects on liquidity, Kalavathi and Shanker (1991) claim that opportunity costs can in fact be significant in terms of yield foregone.

The effect of margins on prices is analyzed by Brunnermeier and Petersen (2008) in an equilibrium model, albeit for primary assets. They find that under certain conditions margins can have adverse effects on asset demand and supply and can result in downward price spirals. Gärleanu and Pedersen (2011) show that in a world of risk-averse, heterogeneous agents, margin requirements increase securities’ required returns.

Empirical evidence thus suggests that margining can lead to market inefficiencies. Some authors, such as Koepl and Monnet (2010), point out that some of these inefficiencies can be lifted by the establishment of a central counterparty provided by an exchange clearinghouse. If market participants trade through a central counterparty, original bilateral contracts are extinguished and replaced by new contracts with the central counterparty. A central counterparty is typically of very high credit quality, partly because of its stringent risk management approach. Thus, bilateral credit risk of variable quality is replaced by high quality credit risk exposure to the central counterparty. Furthermore, use of a central counterparty allows multilateral netting of contracts. Koepl and Monnet (2010) develop a model in which a central counterparty is necessary to implement efficient trade when trades are time-critical, liquidity is limited and there is limited enforcement of trades. They also show that the efficiency of central counterparties depends on their governance structure.

Jackson and Manning (2006) present a model in which multilateral netting by a central counterparty reduces default risk substantially. This reduction results from the additional netting benefits offered by a central counterparty relative to bilateral netting, the higher dispersion of losses through mutual insurance agreements, often in the form of a default fund, and the diversification of the central counterparty across an array of imperfectly correlated assets.

Another important factor, as Acharya and Bisin (2011) point out, is that central counterparties have the potential to level information asymmetries about each others’ positions in derivatives markets, thereby mitigating potential externalities associated with derivatives trading.

Recent regulation passed in the United States (Dodd-Frank Act, 2010) and Europe stipulates that all sufficiently standard derivatives traded by major market participants must be cleared through regulated central counterparties. Duffie and Zhu (2011), however, show that the introduction of a central counterparty in a previously uncollared derivatives market can reduce netting efficiency and increase counterparty exposure. In the scenario they consider, market participants trade several classes of derivatives with each other on a bilateral basis, and these are netted on a bilateral basis. A central counterparty is introduced providing multilateral netting (netting across counterparties) for one class of derivatives traded. Duffie and Zhu show that in this kind of scenario a central counterparty is effective only if the benefits of multilateral netting in the class of derivatives cleared by the central counterparty are larger than the resulting loss in bilateral netting opportunities across all uncleared derivatives.

Santos and Scheinkman (2001) study the impact of competition and information asymmetry on the equilibrium level of margins set by exchanges and their central counterparties. In their model, the central counterparties set optimal levels of margins when there is no asymmetry in the information available to the central counterparty and its clients. If information is distributed asymmetrically, there are situations in which the margin levels set by central counterparties are suboptimal. The choice of margin levels is affected by exogenous factors, particularly the size of the default penalties incurred by the clients.

Previous work considering the level of the financial system has shown that capital requirements, like margining, do not necessarily have positive effects on the safety and soundness of banks (Van-Hoose, 2007). In fact, at a macro-economic level, capital requirements that are pro-cyclical, as margin requirements tend to be, can exacerbate an economic downturn (Heid, 2007; Repullo and Suarez, 2009). In a related analysis, Castiglionesi and Wagner (2009) show that interbank (liquidity) insurance is only optimal when banks insure up to a certain level of idiosyncratic shocks and that optimality breaks down when banks suffer large shocks. On the other hand, banks may also have a tendency to underinsure because they do not fully internalize the impact that saving each other has on other banks in the financial system.

The effects of margins on market activity have also been investigated empirically. Analyzing changes in the initial margin observed at the Chicago Mercantile Exchange and the Chicago Board of Trade, Hartzmark (1986) finds that increases in the initial margin resulted in a decline of the open interest and of the volume on both exchanges. Similarly, Hardouvelis and Kim (1995) find that an increase in the initial margin for contracts traded on the New York Commodity and Mercantile Exchange and the Chicago Board of Trade reduced the trading volumes of the contracts significantly. Margin requirements also seem to bear price impacts. Johannes and Sundaresan (2007) show that swap rates in over-the-counter markets increase with an increase in the cost of collateral.

3. Baseline model

In this section, we develop a stylized equilibrium model to analyze the effects of margin requirements on banks’ welfare, default
risk, and on their trading volume in derivatives contracts. We consider two banks with symmetric endowments yielding uncertain payoffs. The banks can trade derivatives contracts with each other to hedge their endowments. However, the financial market is incomplete as a result of asymmetric information. To reduce the ‘burden’ created by idiosyncratic uninsurable shocks, we allow banks to default on their obligations from derivatives trading. We then introduce a margin requirement aimed at reducing default risk related to the trading of the derivatives contract. Subsequently, we use the results obtained in this analysis to derive a set of propositions concerning the impact of margin requirements on banks’ welfare and default risk and on aggregate trading volume in the derivatives contract. Finally, we evaluate the effects of margining numerically. Using numerical analysis, we also compute optimal levels of margining.

3.1. Economic setting

We consider an economy consisting of two groups of banks each with an initial endowment of $w_0 > 0$ units of a consumption good. We assume that the banks behave as if they maximized the expected utility of terminal wealth through a common, strictly increasing, strictly concave utility function $u(\cdot)$. Two dates are considered. At date 1, banks can trade in a derivatives contract. At date 2, one of two equally probable observable states of the world, $s_1$ and $s_2$, occurs. In each state, one bank receives an endowment of $x$ units of the consumption good. The other bank ‘typically’ receives a larger endowment $y$. The occurrence of this bad draw is private information and is independent across banks.

Table 1 summarizes the structure of uncertainty and endowments in our economy. We initially assume $y > x > z > 0$.

<table>
<thead>
<tr>
<th>Group</th>
<th>Endowment in $s_1$</th>
<th>Endowment in $s_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x$</td>
<td>$y$ with probability $\pi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$ with probability $1 - \pi$</td>
</tr>
<tr>
<td>2</td>
<td>$y$ with probability $\pi$</td>
<td>$z$ with probability $1 - \pi$</td>
</tr>
</tbody>
</table>

Each bank mirrors the other; the expected aggregate endowment in this economy is therefore constant. We assume that the marginal utility in the state in which a bank receives the certain endowment $x$ is higher than the expected marginal utility in the state in which it receives an uncertain endowment.

**Assumption 1.** $u'(w_x) > \pi u'(w_y) + (1 - \pi)u'(w_z)$, where $w_0 = w_0 + \theta$ with $\theta = x, y, z$. **Assumption 1** suggests that, in equilibrium, a bank would thus buy derivatives contracts that transfer wealth to the state in which it receives the certain endowment $x$.

3.2. Derivatives market, default, and margins

The derivatives contract available for trade between the two banks at date 1 is a forward contract. More precisely, we assume that the forward contract is normalized so that the bank holding a long position in the contract is entitled to receive one unit of the consumption good in state $s_1$ and must deliver one unit of the consumption good in state $s_2$. These assumptions guarantee that, in equilibrium, banks take positions that have negative value in the state in which they face a lottery (endowment $y$ or $z$). However, because the financial market is incomplete, banks are unable to perfectly hedge their endowments. The presence of idiosyncratic uninsurable shocks (receiving $z$ instead of $y$) makes it undesirable to require that banks deliver fully on their commitments because, in this case, they would not be able to transfer much wealth to the state in which they receive endowment $x$. We therefore allow banks to default on their obligations. Default is essentially an institutional arrangement that allows the payoffs to co-vary, albeit imperfectly, with the idiosyncratic shock. We borrow the technology of **Diamond (1984)** and **Dubey et al. (2005)**. That is, we introduce a penalty that is imposed directly on the utility of the individual bank and that is measured in utility units. If at date 2, a bank is committed to deliver an amount $l \geq 0$ in the observed state and delivers $D \geq 0$, it suffers a penalty $\max(l - D, 0)$ in utility units. If the bank delivers fully on its commitments, it suffers no penalty; otherwise, it defaults and incurs a penalty proportional to the amount it fails to deliver. The default penalty reflects a deadweight cost of default, such as a loss of reputation.

We interpret the parameter $z$ as the economy-wide bankruptcy code and assume that banks cannot influence it. Because default is meant to allow for some relief when the idiosyncratic shock $z$ occurs, it is natural to assume that $u'(z) > \lambda$. We also assume that the penalty is not too small.

**Assumption 2.** $u'(w_2) > \lambda > \pi u'(w_y) + (1 - \pi)u'(w_z)$.

**Assumption 2** implies that banks do not expect to default in the state in which they face a lottery, because the probability of receiving $z$ is much smaller than the probability of receiving the larger $y$. However, they will always default when they receive $z$.

In addition, we ensure that banks that obtain $y$ will always deliver in excess of collateral.

**Assumption 3.** $u'(w_y - w_0) < \lambda$.

To ensure a minimum payment in case of default, banks are subject to a margin requirement. The collateral posted to meet margin requirements is covered by the bank’s initial wealth and delivered with probability one to satisfy any claims against the bank. We assume that the margin requirement is such that a bank with a net position of $\kappa$ is required to post at least $|\xi|\Phi$ collateral, where $\Phi > 0$. Banks are allowed to take any positive or negative position in the contract.

We are now ready to describe the banks’ optimization problem.

3.3. Banks’ optimization problem

Suppose that the default parameter is $\lambda > 0$ and that the bank must post collateral of $|\xi|\Phi$ units of the consumption good for every unit of contract traded, with $\Phi > 0$. A bank must decide how many contracts to trade and what fraction of obligations to deliver when its short position has positive value.

We denote the number of derivatives held by bank $i = 1, 2$ by $\xi_i$. Banks face the problem of maximizing the expected utility of terminal wealth by determining the optimal number of derivative assets to buy and the optimal delivery rate. We shall first consider the case of bank 1. The expected utility of bank 1 with endowment

---

5 For the sake of brevity, we will speak of two banks instead of two groups of banks. The model applies not just to banks but to more general situations as well.

6 We borrow the basic set-up from Santos and Scheinkman (2001). Our analysis is different from theirs, however. Whereas they study a market in which all trading is intermediated by an exchange, we study a market in which all trading takes place directly between banks.
\( \theta \in \{x, y, z\} \), given a position \( x^1 \) in the derivative asset and a delivery rate \( D^1(\theta) \), is as follows:\(^7\)

\[
U(x^1, D^1; \Phi) = \begin{cases} 
\frac{1}{2}[\mu(w_\ell(x^1)) - \lambda \max\{(1 - D^1(x)x^1, 0)\}], \\
\frac{1}{2}[\mu(w_u(x^1)) - \lambda \max\{(1 - D^1(y)x^1, 0)\}], \\
\frac{1}{2}[\mu(w_z(x^1)) - \lambda \max\{(1 - D^1(z)x^1, 0)\}],
\end{cases}
\]

with

\[w_\ell(x^1) = w_\ell - \max(-x^1, 0)D^1(x) + \max(x^1, 0)(\pi D^1(y)) + (1 - \pi)D^1(z),\]

\[w_u(x^1) = w_u - \max(x^1, 0)D^1(y) + \max(-x^1, 0)D^1(x),\]

\[w_z(x^1) = w_z - \max(x^1, 0)D^1(z) + \max(-x^1, 0)D^1(x).\]

The above equations define the wealth of bank 1 which trades \( x^1 \) units of the forward contract. For bank 1, the forward has positive value in state \( s_1 \) when it receives endowment \( x \), and it has negative value in state \( s_2 \) (endowment \( y \) or \( z \)). The first equation above reflects the fact that in state 1, bank 1 is entitled to receive one unit of the consumption good from bank 2 if it holds a long position in the contract \( (x > 0) \) and is obliged to deliver one unit if it is short. The actual size of the delivery on a long position is uncertain, however, and depends on bank 2's delivery rate. In state 1, bank 2 receives either \( y \) (with probability \( \pi \)) or \( z \), and it will deliver accordingly. The second and third equations reflect bank 1's wealth in state 2 when it receives \( y \) and \( z \), respectively.

We now consider bank 1's optimization problem:

\[P^1_0(\Phi) : \max_{x^1, D^1} U(x^1, D^1; \Phi),\]

subject to

\[|x^1| \Phi \leq w_0,\]

\[D^1(y), D^1(z) \geq \max(\Phi D^1, 0)\] and \( D^1(x) \geq \max(-\Phi x^1, 0)\). \( \text{(3)} \)

The first constraint reflects the fact that the amount of collateral the bank posts is limited by its initial endowment. The last three constraints ensure that the bank delivers at least the collateral it posted. Note that we assume that there is no consumption. Because the bank's objective function is strictly concave optimized over a convex set, problem \( P^1_0 \) has a unique solution. Let \((x^1, [D(\theta)]_{\theta=x,y,z})\) denote the solution to \( P^1_0 \). The optimization problem of bank 2 is the mirror of that of bank 1 described above. Thus, \( x^2 = -x^1 \), implying that the market clearing condition is automatically satisfied. Further, \( D^1(\theta) = D^2(\theta) = D(\theta) \), justifying the missing superscript.

We show in Lemma 1 (Online Appendix, Section Appendix A) that it is never optimal for bank 1 to go short; that is, \( x^1 \geq 0 \). By symmetry, bank 2 will always take short positions; that is, \( x^2 \leq 0 \). Notice that because \( x^2 \geq 0, D(x) = 0 \). Assumption 2 also guarantees that \( D(z) = |x| \Phi \); that is, a bank receiving \( z \) will default and only deliver the collateral. This justifies writing the generic solution to problem \( P^1_0(\Phi) \) as a pair \((x^1, D)\) in which the first element is the position in the forward contract of each bank and the second element is the fraction of obligations delivered by bank 1 in state \( y \).

Let \( D^* \) denote the optimal delivery rate by bank 1 in state \( y \). If \( D^* \geq 1 \), bank 1 fully delivers, while if \( D^* < 1 \), it delivers only part of its obligations. Then, Lemma 1 allows us to write the wealth of bank 1 as

\[w_\ell(x^1) = w_\ell + x^1(\pi(\mathbb{1}_{[D^* > 1]} + \mathbb{1}_{[D^* < 1]}D^*) + (1 - \pi)\Phi),\]

\[w_u(x^1) = w_u - x^1(\pi(\mathbb{1}_{[D^* > 1]} + \mathbb{1}_{[D^* < 1]}D^*)),\]

\[w_z(x^1) = w_z - x^1 \Phi,\]

with \((x^1, D), i = 1, 2\) denoting the unique solution to Problem \( P^1_0 \).

3.4. The role of margining: analytical results

We now analyze the effect of a change in the level of margin, \( \Phi \), on the optimization problem of the two banks. More precisely, we investigate how a change in the margin level affects the banks’ welfare, their default risk, and their trading volume in derivatives contracts. Because the two banks’ optimization problems are symmetric, we will in the following analysis only consider the case of bank 1. The propositions’ proofs are, for expository purposes, provided in an Online Appendix.

Let us first examine the influence of margin on the bank’s welfare. We define a bank’s welfare as its expected utility. The following proposition states the relation between an increase in the margin requirement, \( \Phi \), and the bank’s expected utility. Let \( f(\Phi) \) denote the value function of bank 1’s optimization problem \( P^1_0(\Phi) \). We then have the following result:

**Proposition 1.** Assumptions 1 and 2 imply that \( df(\Phi)/d\Phi \), the derivative of the value function of bank 1’s optimization problem with regard to the margin requirement \( \Phi \), is negative if and only if

\[
\left(1 - \pi\right) \frac{a^2}{2} \left(\mu(w_x + x(\pi D + (1 - \pi)\Phi)) - \mu(w_x - x\Phi) + \lambda\right) \leq \gamma, \quad (4)
\]

where \( \gamma \) is the Lagrange multiplier of the funding constraint imposed by the margin requirement.

An increase in the margin requirement leads to a higher pay-off in state \( x \) and to a higher payment (but a lower default penalty) in state \( z \), reflected by the terms on the left-hand side of Eq. (4). Thus, Proposition 1 states that an increase of the margin requirement decreases welfare whenever its expected net benefit, in utility terms, is lower than \( \gamma \), the shadow cost of the funding constraint imposed by the margin requirement.

We next analyze the impact of an increase in the margin requirement \( \Phi \) on the size of the bank’s position (trading volume) in the forward contract, \( x^1 \).

**Proposition 2.** Assumptions 1–3 imply that the position of bank 1 in the derivative asset, \( x \), is a decreasing function of the margin requirement, \( \Phi \).

According to Proposition 2, the optimal position in the forward contract, \( x \), decreases when the margin requirement, \( \Phi \), is increased. In other words, the marginal utility of an additional unit of the forward contract decreases with an increasing margin requirement. Both results depend strongly on the assumption that the banks have a (strictly) concave utility function. The expected costs of a higher margin rate (in particular, a higher payment in state \( z \)) are valued more highly by the banks than the expected benefits (in particular, a higher receipt in state \( x \)).

Finally, let us consider the effect of margining on the bank’s default risk. We define default risk as the probability of default multiplied by the loss-given-default. Banks never default in the state in which they receive \( x \), and they always default when they receive \( z \). First, it is important to mention that in the context of this stylized model, the default probability of each bank is independent of the margin requirement; the latter thus only affects the loss-given-default. The loss-given-default is given by \((1 - \Phi)\gamma\), that is, by the fraction of obligations not covered by collateral (by assumption, banks will only deliver the collateral they posted when they...
receive \( z \). Obviously, an increase in \( \Phi \) reduces the loss-given-default per unit of contract, and, because \( x \) is a decreasing function of \( \Phi \), it also reduces total loss-given-default. Thus, the margin requirement reduces loss-given-default in the case in which banks receive \( z \). However, banks can also choose to default in the state in which they receive \( y \). As long as they deliver fully in state \( y \), an increase in the margin requirement \( \Phi \) indeed reduces default risk in the forward contract. If, however, an increase in the margin requirement makes it more desirable for banks to default in state \( y \), such an increase can thus increase overall default risk. We do not, however, elaborate on this particular aspect at this stage. Therefore, in the context of our stylized model, we can state a third proposition.

**Proposition 3.** Assumptions 1 and 2 imply that, given a position \( x \) of bank 1 in the asset, the loss-given-default, \( x(1-\Phi) \), is a decreasing function of \( \Phi \).

### 3.5. Optimal margin rates: numerical illustration

In the remainder of this section, we compute the optimal level of margining in the economy using the framework just presented and perform comparative statics with regard to some key model parameters.\(^8\)

We start by illustrating the effect of a change in the margin requirement on the solution of banks’ optimization problem in Eq. (2) by computing optimal trading volume, delivery rate in the default state, and expected utility. The analysis presented in this section is based on numerical optimization because analytical solutions of most of the problems of interest are intractable.

As highlighted in Section 3.4, the effect of a change in the margin requirement, \( \Phi \), on expected utility depends on the relative benefits and costs in the different states as well as on the shadow cost of the funding constraint imposed by the margin requirement. Specifically, an increase in \( \Phi \) will increase the expected payoff in the state in which the forward contract has positive value by increasing the minimum payment in the state in which the counterparty will default. In the state in which the forward contract has negative value, an increase in the margin requirement will increase the minimum payment in the default state, but this in turn will reduce the default penalty.

To illustrate the overall effect of a change in the margin requirement, we assume an exponential utility function of the form \( u(c; \gamma) = 1 - e^{-c\gamma} \). We set initial wealth to \( w_0 = 0.9 \), wealth gains in the three terminal states to \( x = 3.1 \), \( y = 5.1 \), \( z = 0 \), respectively, and the default penalty to \( j = 0.25 \). We will vary the quality of the bank \( \pi \) from 0.900 to 0.995 and the level of margining, \( \Phi \), from 0 to 1. We choose the parameter values of \( \pi \) and \( \Phi \) such that (i) they are realistic and (ii) Assumptions 1-3 hold.

**Fig. 1.** This figure shows the optimal position in the forward contract, \( x^* \), the optimal delivery rate in state \( y, D^* \), and the expected utility, \( U^* \), as functions of the quality of the bank \( \pi \) for two different levels of the margin requirement, \( \Phi = 0 \) (unconstrained case) and \( \Phi = 1 \) (complete margining), and two different levels of risk aversion, \( \gamma = 1.5 \) (Panels a-c) and \( \gamma = 2.2 \) (Panels d-f).

\(^8\) We are grateful to the anonymous referee who suggested the analyses presented in this section.
(complete margining) and two different levels of risk aversion, \( \gamma = 1.5 \) (Panels a–c) and \( \gamma = 2.2 \) (Panels d–f).

We can see from Fig. 1 Panel a that the optimal position in the forward contract, \( \alpha^* \), increases when the quality of the bank, \( \pi \), increases, that is, when the probability of default \( (1 - \pi) \) decreases. For high levels of \( \pi \) (low probability of default), \( \alpha^* \) is almost one in the unconstrained case \( (\Phi = 0) \). With complete margining \( (\Phi = 1) \), the funding constraint imposed by the margin requirement is binding at high levels of \( \pi \) and \( \alpha^* \) is constrained to 0.9, the initial wealth of the bank, \( w_0 \). As \( \pi \) decreases and default becomes more likely, the loss of utility in the default state imposed by an increase in the margin requirement weighs more heavily, and the drop in \( \alpha^* \) becomes more severe.

The increase in the margin requirement from \( \Phi = 0 \) to \( \Phi = 1 \) also has a negative effect on expected utility \( U^* \) (Panel c) but no effect on the optimal delivery rate \( D^* \) which remains at 1 (Panel b).

Fig. 1 Panels d to f show the results of the same analysis for a higher level of \( \gamma \) \( (\gamma = 2.2) \), that is, for lower risk aversion. As expected, the position in the contract, \( \alpha' \), decreases less as default becomes more likely (\( \pi \) decreases) compared to the case of higher risk aversion, and the effect of an increase in the margin requirement on \( \alpha' \) and \( U^* \) is less pronounced.

So far, we have looked at the effect of a change in the margin requirement \( \Phi \) on the utility of the bank. We now ask which level of margin, \( \Phi^* \), maximizes expected utility, \( U^* \), given a set of parameters; that is, we ask which level of \( \Phi \) is optimal. We compute the optimal level of margining, \( \Phi^* \), for different levels of the quality of the bank, \( \pi \). In particular, we want to know whether the optimal level of margining, \( \Phi^* \), changes when the quality of the bank decreases, that is, when default becomes more likely.

Fig. 2 shows the optimal level of margining, \( \Phi^* \) (Panel a), and the corresponding values of \( \alpha' \) (Panel b), \( D^* \) (Panel d), and \( U^* \) (Panel c) for two different levels of \( \gamma \), the risk aversion coefficient. Panel a shows that \( \Phi^* \) changes as a function of \( \pi \). For very high values of \( \pi \), that is, when default is very unlikely, \( \Phi^* \) is zero. As \( \pi \) decreases, \( \Phi^* \) increases until it reaches 1 (complete margining). Thus, the optimal level of margining is a function of the probability of default (as well as of the other model parameters, of course). When default is very unlikely, the cost of margining outweighs its benefits and a margin requirement of zero is optimal; when default is relatively likely, it is the other way round, and complete margining is optimal.

Fig. 2 also shows that the optimal margin rate \( \Phi^* \) decreases when risk aversion increases \( (\gamma \) decreases). This decrease occurs
because an increase in the margin requirement increases the minimum delivery rate in the default state ($z$), which in turn decreases the default penalty. The resulting net gain in marginal utility in state $z$, relative to the other states, is more pronounced the higher risk aversion is.

To summarize, in this section, we analyzed the impact, in equilibrium, of margining in the case of two identical banks trading with each other. We saw that margin requirements affect welfare through various channels and that the effect of an increase in margin requirements on welfare is not necessarily positive. We found that an increase in margin requirements decreases trading volume and may decrease banks’ welfare whenever the marginal benefits of margining do not cover their costs. Furthermore, we showed that increased margin requirements lower aggregate default risk in such a simple economy. We also found that the level of margining that maximizes expected utility or welfare varies between zero (no margining) and one (complete margining) depending on key model parameters including the probability of default and risk aversion. For example, in our model for banks of high quality and with high risk aversion, a low level of margining is better, whereas for banks of low quality and with low risk aversion, the opposite is the case.

These results suggest that privately and socially optimal levels of margining can diverge when banks differ in the key characteristics considered in the model. Consequently, whenever banks are not symmetric, and, furthermore, whenever we account for the real sector of the economy—which is ignored in this stylized model—the level of margining will affect not only aggregate welfare but also the relative distribution of welfare within the banking system (and, likely, within the rest of the economy). This observation in turn poses a real challenge for regulating or imposing margin rates because the regulator will have to strive for efficiency while considering in parallel that a ‘one size fits all’ margin rule is likely to bias welfare distribution and create competitive disadvantages within the banking sector.

All of the results discussed above suggest that determining the optimal level of margining in a market, such as the over-the-counter derivatives market, is not a trivial problem and that some of the intuition circulated in public, as mentioned at the beginning of the Introduction, that higher margin rates are always better may be misleading.

Although our model reflects some key aspects of margining of derivatives contracts, it is very simple. One of its shortcomings is the symmetry of banks in the economy. Another is the fact that the probability of default is exogenous. Both of these, and other, aspects should be investigated further in future studies.

4. Extended model

Another shortcoming of our analysis so far is that it is static, whereas trading typically takes place in a dynamic environment. In this section, we extend our analysis by investigating the effectiveness of margin requirements in an extended dynamic market simulation model. In this multi-period model, calibrated to actual derivatives market data, banks trade derivatives with each other for hedging purposes in a highly volatile environment. It is at times of market crises that margining is, a priori, expected to be most valuable.

The parameters of the extended model resemble the recent credit crisis that began in August 2007 and reached an unprecedented level of geographic banking contagion during the fall of 2008. In particular, the collapse of AIG, triggered or at least exacerbated by increases in margin requirements following downgrades by the two major rating agencies, is indicative of a situation in which the marginal costs imposed by higher margins are likely to exceed their marginal benefits for a particular firm. In addition to AIG, many other financial institutions with similar derivatives positions suffered from the same problems (note that this statement does not necessarily apply to their counterparties).

4.1. Preliminary considerations

Before we describe our extended model in detail, we point out some characteristics of banks and derivatives markets that are likely to alter the simple conclusions reached by the baseline model regarding the effects of margin requirements on banks’ individual and aggregate default risk.

So far, we have considered the case in which banks’ wealth was always positive; that is, we assumed $w_y > w_z > w_x > 0$. Thus, in the previous setting, banks never defaulted if they abstained from trading the derivatives contract. Default risk was introduced into the economy by banks taking positions in the derivatives contract; that is, it was entirely endogenous. It is more realistic to assume that a bank’s endowment can be negative in some states. In such situations, an increase in the margin requirement can successfully reduce (endogenous) default risk associated with the trading of the derivatives contract, but at the same time, it can increase (exogenous) default risk associated with the endowment. A more realistic scenario of this type is considered in the extended model presented in this section.

Another issue that can affect the results obtained in this section is the heterogeneity of the banking sector. Heterogeneity in either initial wealth $w_y$ or in the probability of the occurrence of state $x$, renders the results previously derived ambiguous. We will consider such heterogeneity in the extended model presented in this section.

An additional factor to consider is that banks generally conduct business with more than one counterparty. The baseline model considered a situation in which two ex ante homogeneous banks or groups of banks traded exclusively with each other. Such a setting potentially underestimates the constraints imposed by margin requirements and the positive correlation among the trading partners’ default risks. However, certain risk mitigation mechanisms, such as re-hypothecation and central counterparties, can reduce the default exposure of a given position in a derivatives contract and at the same time reduce margin requirements. However, neither of these mechanisms can completely eliminate either default risk or (funding) liquidity risk. The issues in relation to default risk and margining addressed in this study will thus prevail in the presence of those more sophisticated mechanisms, albeit to a lesser extent. We will address these other default risk mitigation instruments in more detail when we discuss the simulation results.

Finally, we should recognize that banks trade derivatives in a multi-period environment. By assumption, the derivatives contract that we consider has a value of zero at date 0. Its value then changes over time according to the state of the world. Both counterparties to the contract have an exposure to default risk. This

10 Following the collapse of Lehman on September 25, 2008, the Wall Street Journal wrote the following about the perils of re-hypothecation for Lehman’s customers: “But here is the catch. In the process, a client’s re-hypothecated assets become mixed up with the broker’s, so in the event of a bankruptcy filing, like Lehman’s, clients have to join the bank’s other unsecured creditors at the back of the line to retrieve some of what they are owed.”
exposure can be split into two parts, namely, current and potential future exposure. Current exposure refers to (the positive part of) the current value of the contract. Thus, at any given time, one counterparty (to whom the contract currently has positive value) has positive current exposure, while the other counterparty (to whom the contract currently has negative value) has zero current exposure. Potential future exposure relates to that part of the exposure that might build up between the current date and the expiry date of the contract. In derivatives markets, a margining rule typically consists of two parts. The first part, referred to as the initial margin, is applied at inception of the contract and is meant to cover potential future exposure. It usually remains constant throughout the life of the contract. The size of the initial margin is typically a function of the variance or of some other dispersion measure of the contract's value. Variation margin, on the other hand, offsets current exposure throughout the life of the contract and is generally settled in cash. It is charged periodically—usually daily—to capture the changes in the contract's value. We show in Section 4.4 that variation margin typically reduces the loss-given-default but at the same time can inter-temporally tighten the bank's funding constraint and thus adversely impact its probability of default.

4.2. Hypotheses

Inspired by the baseline model's propositions stated in Section 3.4 and by the above discussion of the complex real issues surrounding the impact of margin requirements on banks' default risk, we now formulate three hypotheses that we shall evaluate with the help of our extended model. In so doing, we will emphasize the role of margin requirements on banks' welfare and default risk and on their derivatives trading activities under extreme market conditions—market stress—while allowing for heterogeneity of these banks, for the partially exogenous nature of their default risk, and for their wealth distribution to be representative of that observed among the top global derivatives players in the world. To test our hypotheses, we will consider different types of definitions of welfare, one based on quadratic utility and one based on a richer utility function that takes into account higher moments.

Our first hypothesis relates margin requirements to banks' welfare. Based on our baseline model and on Proposition 1 defined in Section 3.4, we expect that increasing margin requirements will reduce banks' welfare in states in which the opportunity costs of hedging purposes. Thus, it is primarily the positive dependency between the default probability of a bank and its own—as well as its counterparties'—funding constraints that creates negative externalities for banks facing higher margin requirements.

Recognizing the difficulty of modeling the interactions between the various mechanisms at play that influence the default risk of each bank, we argue as our third hypothesis that the net effect of increased margins on the banks' default risk is indeterminate.

4.3. Extended model set-up

The economic setting used in this multi-period, dynamic model is more general than that used in the stylized bilateral banking model developed in the previous section, but it shares some commonalities with respect to the assumptions made in the previous model. Here, banks also hold an exogenous random position in an illiquid asset and trade another, liquid derivatives contract with each other to hedge the price (market) risk of their initial holding. Some of the banks are under severe stress and, upon delivery of the hedging contract, may default on both the illiquid asset they are endowed with and the hedging contract.

More precisely, in the extended model, banks are exposed to interest rate and default risk exposures through client demand for a bond. Clients may default on their obligations, but banks can only hedge their interest rate risk and do so by trading in an interest rate swap contract with each other in a market. We investigate several different types of margining in order to analyze their effects on banks' trading volume in the swap contract ('market liquidity'), default risk, and wealth.

To do this, we consider an economy with a real sector and a financial sector consisting of N banks indexed by i = 1, . . . , N. At time t = 0, banks are endowed with a certain amount of cash c. The equity of bank i is denoted by Ei. At the beginning of every period t, banks receive a demand for a bond B with maturity t + T0 from the real sector, that is, from their clients. Client demand is uniformly distributed within [−IEi, IEi] with l ∈ R+. Client demand can be positive (lending) or negative (borrowing). Clients might default on their obligations. We assume that the hazard rate is the same for all clients and denote the clients' hazard rate by h∗. The bond traded with each client is either a fixed-rate or a floating-rate bond. For half of the banks (i mod 2 = 1), all lending is in floating-rate bonds, whereas all borrowing is in fixed-rate bonds, and vice versa for the other half. A bond promises to pay an amount B0i(t + 1) at maturity t + 1, but the promise may be broken with a hazard rate h∗. If default occurs at time t, an amount t C0 + 1 is paid at t + 1, conditional upon no prior default.

Client demand exposes the banks to both interest-rate risk and default risk. Default risk cannot be hedged, rendering the market incomplete. Interest rate risk can be hedged by trading in an interest rate swap contract. By entering into a swap contract, a bank agrees to pay the agreed swap rate and to receive the current interest rate (long position) or vice versa (short position). More precisely, by executing at time t a swap contract maturing at time t = T0, the counterparties exchange fixed interest payments at an agreed swap rate, s(t + T0), against floating interest rates. We assume that the swap contract has the same notional principal as the bond, that principals are not exchanged, and that the swap has the same time to maturity as the bond, that is, T0 = T0. The swap contract is subject to counterparty default risk.

Every swap position might be subject to a margin requirement of ϕ ∈ R+ units of cash per contract traded. This (initial) margin has to be posted on the date on which a bank initiates a position. Contracts might also subject to variation margins. In this case, the change in the value of a bank's position between two dates is settled in cash. Bank i, holding xj i, swap contracts at date t − 1, has a variation margin requirement of xj i, j(tj i − S(t − 1, sj i)) + x(ij(tj i − S(t − 1, sj i))) at date t, where S(s) denotes the value of the contract given...
swap rate $s$, and $s$ denotes the settlement price. In other words, the value of the position of the previous period is set to zero (first term), and so is the value of the contracts traded in the current period (second term). The second term is necessary because contracts might be traded at a price different from the current period’s settlement price and therefore already have to be ‘marked to market’ in the period in which they are traded. Thus, variation margin eliminates the current interest rate exposure of a position.

At first, we will assume that the swap contracts are traded bilaterally, that is, directly between banks. The value of a swap contract is zero at initiation and zero at expiry. A swap contract results in periodic cash flows comprising the ‘coupon’ payment (the differential between the current interest rate and the swap rate) and the variation margin (the differential between the previous period’s and the current period’s contract value). Obviously, margining changes the swap’s cash flow pattern. We then introduce various types of margining, including initial margin, variation margin, and a central counterparty to study their effects on market outcomes.

We assume that a bank will always seek to eliminate its exposure to interest rate risk. Whenever a bank enters into a fixed-rate bond, it tries to enter into a swap contract of the same quantity. Importantly, the bank’s positions in the bond and in the swap are constrained by a solvency requirement and a liquidity requirement. The solvency requirement ensures that the bank’s equity is positive at the time when the bank enters into a new position. The liquidity requirement ensures that the bank has sufficient cash to enter into a new position.

To reflect some established features of derivatives markets and hopefully to yield a more realistic insight into the effects of margining, the key model parameters are calibrated to actual market data. First, we capture the high level of concentration among derivatives market participants. Second, we consider that many market participants have increasingly developed significant counterparty exposures in relation to derivatives contracts. Third, we account for the fact that banks mainly pledge cash and, typically, not risky assets such as other derivatives contracts as collateral. Fourth, we refine our analysis of margin requirements by considering the separate effects of introducing initial margin, variation margin, and a central clearing counterparty in the simulated trading environment. The central counterparty enables multilateral netting of derivatives contracts in addition to initial and variation margin. Finally, in keeping with common practice, we will assume that positions with the central counterparty are excluded from the solvency requirement (see above); this is in line with market practice, according to which default risk exposures to a central counterparty tend to have a zero (regulatory) capital requirement. A detailed description of the simulation model is provided in Appendix B of the Online Appendix.

4.4. Simulation results

We now turn to the discussion of the simulation results. We will attempt to answer the following two questions: First, what is the impact of margin requirements on the banks’ welfare, on their default risk, and on derivatives market liquidity in the case of extreme market conditions? Second, what are the additional benefits and/or costs imposed by variation margin and by a central counterparty relative to a simple margining system that only requires agents to post initial margins?

Let us start by describing the ‘base case’, that is, the case in which banks do not trade at all in the swap contract. Over time, banks build up a portfolio of bond positions. Half of the banks have long positions in floating-rate bonds and short positions in the fixed-rate bond (floating-fixed portfolio), and vice versa for the other half (fixed-floating portfolio). Given the interest rate environment in the simulations, banks with a floating-fixed portfolio will experience a net loss, while banks with a fixed-floating portfolio will generate a significant profit. Absent any hedging activities, several banks with a floating-fixed portfolio will default during the simulation.

Table 2 displays simulation results for the base case (BC). It provides seven measures, namely, banks’ average terminal equity, the average volatility of their equity, the skewness and excess kurtosis of their terminal equity, the average probability of default in this simulated economy, the average loss-given-default on the traded swap contract per default occurrence, and, finally, the average volume of swap contracts traded in the economy. The first two measures can, in the simple context of a quadratic utility function, be jointly used to assess banks’ welfare effects induced by margining. However, we shall also look at the third and fourth moment of terminal equity to determine the welfare impact of derivatives hedging when reducing tail risk and/or negative skewness also matter to the banks. The next two measures will be used to study the impact of margining on the default risk associated with the trading of the swap contract, and the last measure will be used to assess the liquidity impacts of alternative margin requirements.

Let us first compare the results of the base case (BC) with those of the case in which banks trade the hedging contract in the absence of default risk mitigation mechanisms (0). We clearly see that hedging is beneficial in this environment for a bank that maximizes quadratic utility because it increases the bank’s expected equity and lowers its volatility. These effects are not trivial. Hedging reduces the standard deviation of terminal equity by 19.3%, and it increases the average terminal equity by 5.2%. For banks that also care about skewness and kurtosis of terminal equity, the impact of hedging on welfare is more ambiguous because it increases both the negative skewness and the fat tails of terminal wealth. The average default rate decreases by 36.4%.

To examine the impact of initial margin, we now compare the results of case (0) and case (IM) in Table 2. Looking at the joint impact of initial margin on the mean and standard deviation of

<table>
<thead>
<tr>
<th>BC</th>
<th>0</th>
<th>IM</th>
<th>IM &amp; VM</th>
<th>CCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_T$</td>
<td>166.0</td>
<td>174.7</td>
<td>169.8</td>
<td>162.2</td>
</tr>
<tr>
<td>$\sigma_E$</td>
<td>0.492</td>
<td>0.397</td>
<td>0.401</td>
<td>0.498</td>
</tr>
<tr>
<td>$\text{Skew}_E$</td>
<td>0.029</td>
<td>0.016</td>
<td>0.009</td>
<td>0.034</td>
</tr>
<tr>
<td>$\text{Kurt}_E$</td>
<td>0.417</td>
<td>0.184</td>
<td>0.197</td>
<td>0.424</td>
</tr>
<tr>
<td>$d$</td>
<td>0.176</td>
<td>0.112</td>
<td>0.140</td>
<td>0.240</td>
</tr>
<tr>
<td>$\text{LGD}$</td>
<td>0.227</td>
<td>9.94</td>
<td>5.74</td>
<td>5.62</td>
</tr>
<tr>
<td>$V$</td>
<td>19.0</td>
<td>15.4</td>
<td>15.4</td>
<td>15.4</td>
</tr>
</tbody>
</table>

11 The value of the swap obviously depends on the hazard rate of the counterparty. Because we assume a single hazard rate across banks at the moment, we drop the reference to the counterparty, i.e., $S_i(t) = S$ for all $i = 1, \ldots, N$.

12 There exist situations in which complete hedging is optimal for banks. Bauer and Ryser (2004) show that this is the case, e.g., in the presence of high asset price volatility. Alternatively, Schrand and Unal (1998) show that banks sometimes engage in risk shifting, i.e., they hedge “homogeneous” (low-margin) risks such as interest-rate risk in order to take on “non-homogeneous” (high-margin) risks such as credit risk.

13 These stylized facts refer to data reported in OCC (2006) and International Swaps and Derivatives Association (2006).

14 Each parameter configuration discussed in the previous section was simulated 100 times. The simulations are computationally very intensive. Variation of the number of runs showed that 100 simulation runs are sufficient to obtain stable results.
terminal wealth, we can conclude that, for the level chosen in the simulations, initial margin requirements lower banks’ welfare, assuming quadratic utility, under extreme market scenarios. We observe both a modest increase in the standard deviation of equity and a significant drop in its average terminal value. However, the impact of margining on banks’ utility is more complex in the case in which higher order moments of the banks’ utility functions are also considered and priced. Indeed, the presence of margins reduces both negative skewness and excess kurtosis of the banks’ terminal wealth, which to some extent counterbalances its negative impact on the first two moments of terminal wealth. An assessment of the net welfare implications of margins will ultimately depend on the trade-offs between the four moments of terminal wealth considered as well as on the level of initial margining retained in the simulations.

Thus, at the level of margining chosen for the simulations, margining seems to decrease welfare for banks with quadratic utility in the stressed market scenarios considered. For a bank with a richer utility function, the level of margin chosen may still allow its benefits to outweigh its costs precisely because we deal with a stressed market environment.

Moreover, regarding market liquidity, we observe a slight drop in the volume of swaps traded when margin requirements are introduced, which is in line with our expectations based on our baseline model.

The relation between margin requirements and default risk is the most difficult to grasp. Remember that in the previously derived theoretical model, higher margin requirements lowered losses given default and thus always reduced exogenous default risk. However, here, we face an environment where multiple heterogeneous banks are hedging with swap contracts while facing both endogenous as well as exogenous default risk. Our conjecture was that the impact of higher margin requirements in such an environment was rather intricate and thus could not be easily signed. The simulation results under extreme market conditions suggest a negative impact of higher margin requirements on both the average loss-given-default and the average probability of default. Indeed, initial margin drastically increases the average loss-given-default.15 This result can be explained by the fact that we consider both endogenous and exogenous default sources under extreme market conditions. Imperfect hedging exposes banks more—not less—to large losses from other fragile institutions that are more likely to default in such a setting. Impairing banks’ ability to hedge not only increases their likelihood to default, it also reduces the equity available for distribution among creditors in default states, thereby increasing the average loss-given-default. Hence, and in contrast to what is often claimed by practitioners in derivatives markets,16 collateral requirements can have spurious and even negative effects on both the recovery value and the default probability of a counterparty, and they can exacerbate economy-wide default risk under severe market conditions.

The introduction of variation margin and its impact on default risk has been briefly discussed in Section 4.1. There, we conjectured that variation margin would reduce the total loss-given-default in the stylized economy considered. This is indeed the case when we turn to the simulated economy. When comparing the average loss-given-default in the IM and the IM & VM cases, we see that in the latter case, it decreases by 42.3%. However, the tightening of the funding constraint associated with variation margin at the same time increases the default probability by 71%, so that the net effect of variation margin is to increase rather than decrease default risk under extreme market conditions. Furthermore, variation margin considerably deteriorates the benefits of trading in the swap contract, by 16.7%. In addition, the welfare effects of variation margin are also negative for banks with quadratic utility, increasing the standard deviation of their wealth by 24.0% and lowering the average value of their wealth by 4.5%. The latter conclusion may not apply to banks with more general utility functions because variation margin positively influences the skewness and excess kurtosis of their terminal wealth, thereby positively affecting their welfare. This significant positive impact on the banks’ welfare results from the fact that extreme downside risk resulting from an adverse market or counterparty movement becomes substantially reduced in magnitude when daily marking-to-market is introduced.

Finally, the introduction of a central counterparty offers, in our set-up, only marginal improvements over variation margin in that it reduces the average loss-given-default by 2.1%.17

4.5. Robustness analysis

In the foregoing analysis, we found that under stressed market conditions, the presence of margining decreases market liquidity and ambiguously affects bank sector welfare, thus corroborating the first two hypotheses provided by the stylized model derived in Section 3. Furthermore, we found that both initial and variation margin amplify the default risk in the simulated economy under extreme market conditions. We would now like to investigate how a change in the level of initial margin rates affects the previous conclusions. More precisely, we consider various levels of initial margins and analyze the effects of an increased initial margin requirement on banks’ welfare and default risk and on swap market liquidity. As before, we compare trading with initial margin (IM) to cases of no trading (BC) and of trading without any risk mitigation mechanisms (0). We subsequently add variation margin (VM) and a central counterparty (CCP).

We set initial margins at 95%, 97%, and 99%-Value-at-Risk of the interest rate swap contract. As shown in Table 3, the effects

### Table 3

This table shows the effects of increases of initial margin on the market outcome. BC denotes the case in which banks do not trade in the swap contract, and 0 denotes the case in which banks do trade in the swap contract but do not employ any risk mitigation mechanisms. In the other cases, banks trade a swap contract with initial margin set at 95% (IM95), 97% (IM97), and 99% (IM99) value-at-risk levels. Case IM95 is the same as case IM in Table 2. The following table provides an overview of empirical results. **

<table>
<thead>
<tr>
<th>Margin Type</th>
<th>BC</th>
<th>IM95</th>
<th>IM97</th>
<th>IM99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>0.016</td>
<td>0.009</td>
<td>0.006</td>
<td>0.001</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.184</td>
<td>0.197</td>
<td>0.205</td>
<td>0.214</td>
</tr>
<tr>
<td>LGD</td>
<td>0.122</td>
<td>0.140</td>
<td>0.156</td>
<td>0.172</td>
</tr>
<tr>
<td>Hedge</td>
<td>19.0</td>
<td>18.6</td>
<td>18.4</td>
<td>18.1</td>
</tr>
</tbody>
</table>

15 In the following, when we refer to loss-given-default, we mean the average loss per default in relation to swap contracts. It does not include losses in relation to bonds traded with clients.

16 Margining is sometimes considered a panacea to cure default risk, as the following quote from the International Swaps and Derivatives Association (2005) illustrates (emphasis added): “The mechanism by which collateral provides benefit is through improvement of the recovery rate. Collateral does not make it more or less likely that a counterparty will default and does not change the value of a defaulted transaction”. See Acharya and Bisin (2011) for a related discussion of externalities of derivatives trading.

17 To a large extent, the latter finding can be explained by the fact that the holdings of swaps contracts are highly concentrated among a small number of large banks, and so is default risk. The introduction of a central counterparty does not lead to substantially higher netting effects and as a result, does not affect our results significantly.
of initial margin persist when its level is increased within the range commonly observed in financial markets. An increase in the level of initial margin reduces trading volume and increases both default rates and default severity. The impact of higher margin rates is clearly detrimental for the welfare of quadratic utility banks, but it is not necessarily so in the more general case in which banks value positive skewness and less fat tails. Indeed, in the latter case, banks implicitly face lower opportunity costs and higher benefits from higher margins in the presence of extreme market scenarios.

We then add variation margin to the initial margin requirement. Variation margin persistently and significantly reduces the welfare of banks that maximize quadratic utility while also reducing market liquidity; however, these effects are less pronounced as initial margin levels increase and the funding constraint becomes tighter. This can be seen when comparing the values displayed in Table 4 with those reported in the previous table. On the contrary, the positive impact of variation margin on the welfare of banks that value positive skewness and lower fat tails diminishes in absolute terms as the level of the initial margin increases and the funding constraint becomes tighter. Finally, a comparison of the default probabilities and losses-given-default presented in the two tables suggests that the negative impact of variation margin on banks’ default risk increases significantly as the level of initial margin increases.

Finally, we introduce a central counterparty. The simulation results for the cases IM & VM & CCP are displayed in Table 5. One would expect a central counterparty to lift at least some of the inefficiencies introduced by initial and variation margins. In the previous setting, variation margin coupled with high initial margins was shown to significantly exacerbate default risk in the swaps market. The introduction of a central counterparty does modestly reduce banks’ expected terminal wealth, but it simultaneously reduces the negative skewness and the fat tails of the banks’ wealth distribution. Thus, we can say that, overall, a central counterparty is likely to positively influence banks’ welfare under extreme market conditions. Moreover, it reduces average losses given default, albeit only modestly, and thus perhaps not sufficiently, to compensate for the adverse effects of variation margin on default risk, especially at very high initial margin levels.

### 4.6. Limitations

The above simulations show that the margining system currently in place in most derivatives markets can cause externalities on banks’ welfare, on market liquidity, and especially on the default risk in the banking sector observed under extreme market scenarios. While we believe that these simulations capture certain key characteristics of derivatives markets, in particular the interaction between market, credit, and liquidity risk, they also bear certain limitations. We now discuss the most important of these limitations.

First, the assumption that the banks’ objective is to completely hedge their exposure to market risk may be too strong. If this assumption were relaxed, the negative impact of default risk mitigation mechanisms might be less pronounced.

Second, we consider a derivatives market consisting of banks who act only as interest rate risk hedgers. In this market, demand and supply are usually not balanced because of the wealth distribution of these banks. In reality, the net excess hedging demand of the banking sector may be lower than in our model, and, furthermore, it may be balanced by third parties acting as speculators (those, however, might be subject to similar constraints). In such
institutions, there is a "netting externality" in that (bilateral) netting of
contracts held with other banks (Acharya and Bisin, 2011). In addition,
the bank potentially reduces the expected payoff of contracts held
with its counterparties reduces the expected payoffs of contracts held with other banks and their clients (Acharya and Bisin, 2011). Finally, there is a margining (or collateral) externality, that is, the increase of margins reduces the credit exposure of one bank but at the same time potentially decreases expected payoffs of the contracts held by the other banks in the system and by other, unsecured creditors. A detailed analysis of such externalities is only possible in a general equilibrium model that takes into account not just the financial sector but also the real economy. Such an analysis is, however, beyond the scope of this study.

5. Conclusion

In this study, we examined the impact of margin requirements on derivatives contracts and on banks' welfare, trading volume, and default risk. Both the baseline and the extended simulation models presented in this study suggest that margin requirements may have a negative impact on banks' welfare, increase their default risk and lower their trading volume in derivatives contracts. These adverse effects are particularly acute when margin rates are high and collateral is scarce. The extended model further illustrates that these effects are most likely to prevail during periods characterized by highly volatile interest rate and credit markets.

In some sense, an increase in margin requirements is then comparable to a bank run in that a bank is forced to deliver cash to its creditors, cash that it does not have at hand.

The relation between margin requirements on one side and trading volume, default rates, losses-given-default, and banks' welfare on the other can, in principle, be tested empirically. Hartzmark (1986), Hardouvelis and Kim (1995) and others investigated the relationship between margin requirements and open interest as well as trading volume for several derivatives exchanges. They found that an increase in margin requirements tends to have adverse effects on both open interest and trading volume. The theoretical model and the simulation results presented in this study offer a potential explanation of these findings. To the best of our knowledge, though, no previous empirical analysis has so far examined the relation between margin requirements and default risk as well as its various constituents.

It should be clear from the analyses presented that the effectiveness of margining should be evaluated not only in terms of its effects on credit exposure but also in terms of its impact on banks' welfare. Of particular interest is the search for default risk mitigation mechanisms that preserve the welfare benefits of margining while reducing its costs. One such mechanism is rehypothecation, which allows market participants to re-use the collateral they receive to serve margin requirements. Rehypothecation can significantly reduce the amount of collateral necessary to cover a given portfolio of contracts. Rehypothecation has two major drawbacks, however. First, it does not completely eliminate the credit exposure of a given position, as described in Section 3. Second, it exposes the collateral posted to default risk, as some hedge funds painfully experienced in the case of Lehman's collapse.

Another mechanism that can be used to alleviate the burden of margining is the introduction of a central counterparty. A central counterparty provides the same benefits as rehypothecation. It typically reduces credit exposure even further and can at the same time further decrease the margin requirement of a given position.

In the context of the model presented in this study, the advantages of a central counterparty manifest themselves mainly through a reduction of the average losses given default; the impact on aggregate default risk remains marginal. Central counterparties, however, provide additional benefits. Most important, they provide...
facilities, such as default funds, to mutualize losses in relation to counterparty defaults, thereby reducing moral hazard. In addition, by centrally processing transactions, they reduce legal and operational risk.

Another important advantage of a central counterparty over other default risk mitigation mechanisms is the centralization of information about market participants’ positions that the central counterparty can channel. This in turn permits centralization of aggregate default risk management, which can be desirable from a macro-economic policy perspective. In OTC markets, the distribution of risk is typically unknown, rendering default risk management cumbersome, particularly in periods of market stress. Schinasi (2006) claimed that these information asymmetries in OTC markets constitute a major threat to the stability of the financial system. Central counterparties thus offer benefits that can be significant in that they can effectively reduce some of these informational asymmetries and potential related externalities. As discussed earlier, central counterparties have the potential to improve transparency of trading positions in a market, which in turn would enable market participants to take this information into account when valuing contracts, thereby internalizing potential externalities (Acharya and Bisin, 2011).

Let us conclude with two final remarks based on this study’s findings. First, the choice and the implementation of alternative default risk mitigation mechanisms in derivatives markets can have a significant impact on the welfare and the risk profiles of banks trading in these markets. Second, in this study, margin rates were set exogenously. This observation leads to the following questions: (1) What is the optimal level of margining for an economy? and (2) How does welfare change if margin rates are determined endogenously as part of the banking system’s optimization problem? We hope that these important questions will be addressed in further research.

Acknowledgements

The manuscript benefited from comments by an anonymous referee as well as those of Charles Calomiris, Enrico Di Giorgi, Hans Geiger, Andrew Green, Bruce Grundy, Philip Haene, Markus Leippoint, David Marshall, John McPartland, Jim Moser, Robert Steigerwald, and Suresh Sundaresan. A previous version of the manuscript was presented at the ANU Summer Camp (2008), the 13th Finisia-MCFS Conference, the 20th Australasian Banking and Finance Conference, the 17th Annual Derivatives Securities and Risk Management Conference at the Federal Deposit Insurance Corporation, the University of Melbourne, the University of Zurich, the Federal Reserve Bank of Chicago, the Annual Meeting of the European Finance Association (2006), FINRISK Research Day (2006), and the conference “Issues Related to Central Counterparty Clearing” at the European Central Bank (2006). We thank the participants at these conferences and seminars for discussions and helpful information. A portion of this research was conducted while the second author was visiting the Graduate School of Business at Columbia University in 2005/2006. Murawski thanks Suresh Sundaresan for his hospitality. This research has been carried out within the project on “Conceptual Issues in Financial Risk Management” of the National Centre of Competence in Research “Financial Valuation and Risk Management” (NCCR FINRISK). NCCR FINRISK is a research program supported by the Swiss National Science Foundation.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jbankfin.2012.10.005.