Thesis

Single photon entanglement: from foundations to applications

BRUNO, Natalia

Abstract

Optical quantum systems are ideal for testing quantum mechanical laws, but they also find many uses into possible applications to our everyday life related to quantum information science, such as quantum key distribution (QKD) or, in general, quantum communication. Nonlinear and quantum optics and their applications both to quantum communication and to tests of fundamental physics are described in this Thesis. In particular, we have developed and characterised heralded single photon sources at telecommunication wavelengths (1550 nm), based on nonlinear interactions of the second order between electric fields in dielectric materials. This is the starting point for the preparation of single photon entangled states, that we used as a resource for heralded amplification of photonic qubits, with a direct application in Device Independent Quantum Key Distribution (DIQKD) protocols, but also for testing fundamental questions in quantum physics, demonstrating the presence of entanglement in a state that counts a macroscopic number of particles, 500 photons.

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Heralded single photon entanglement: from foundations to applications

Thèse

présentée à la Faculté des Sciences de l’Université de Genève pour obtenir le grade de Docteur ès Sciences, mention physique

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Le Doyen

N.B. - La thèse doit porter la déclaration précédente et remplir les conditions énumérées dans les "informations relatives aux thèses de doctorat à l'Université de Genève".
Ai miei nonni
To my grandparents
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Abstract

At the time of the early formulation of quantum theory, many predictions and observed phenomena seemed to lead to contradictory conclusions. Nowadays we have a good understanding of quantum theory, and thanks to the experimental results of the last decades we have confirmed its validity. Optical quantum systems are ideal for testing quantum mechanical laws, but they also find many uses into possible applications to our everyday life related to quantum information science, such as quantum key distribution (QKD) or, in general, quantum communication.

During my thesis work, I used nonlinear and quantum optics and their applications both to quantum communication and to tests of fundamental physics. I developed and characterized heralded single photon sources at telecommunication wavelengths (1550 nm), based on nonlinear interactions of the second order between electric fields in dielectric materials. This allowed me to generate and manipulate quantum states of light, such as single photon entangled states. Entanglement is a quantum mechanical phenomenon that directly arises from the superposition principle, and we can say that it is both a fundamental feature and a resource for applied quantum physics.

I used single photon entanglement as a resource for heralded amplification of photonic qubits, where thanks to a quantum teleportation with a gain we are able to compensate losses in a communication channel. This amplifier finds a direct application in Device Independent Quantum Key Distribution (DIQKD) protocols, where the violation of a Bell inequality is required in order to establish a secret key between two parties, without any knowledge about the measurement devices, which can be treated as black boxes.

I exploited single photon entanglement also as a starting point for testing fundamental questions in quantum physics. In particular, I tested the validity of quantum theory at the macroscopic scale. In this regime, decoherence and limited measurement precision play an important role in preventing us from directly observing quantum effects in our every day life. By displacing a vacuum-single photon superposition in the phase space, I experimentally demonstrated the presence of entanglement in a state that counts a macroscopic number of particles, 500 photons.
Résumé

Lors de la première formulation de la théorie quantique, de nombreuses prédictions et les phénomènes observés semblaient conduire à des conclusions contradictoires. Aujourd’hui, nous avons une bonne compréhension de la théorie quantique, et grâce aux résultats expérimentaux des dernières décennies, nous avons confirmé sa validité. Les systèmes quantiques optiques sont idéaux pour tester les lois de la physique quantique et ils trouvent aussi de nombreuses applications dans notre vie quotidienne liées à la science de l’information quantique, telles que la distribution de clé quantique (QKD) et de façon plus général, la communication quantique.

Au cours de mon travail de thèse, j’ai utilisé l’optique non linéaire et quantique pour des applications à la fois liées à la communication quantique et à des expériences de physique fondamentale. J’ai développé et caractérisé des sources de photons uniques annoncées aux longueurs d’onde des télécommunications (1550 nm), basées sur des interactions non linéaires du second ordre entre les champs électriques dans les matériaux diélectriques. Ces outils m’ont permis de générer et de manipuler des états quantiques de la lumière, tels que les états intriqués de photons uniques. L’intrication est un phénomène de physique quantique qui découle directement du principe de superposition. Il est à la fois un élément fondamental et une ressource pour la physique appliquée.

J’ai utilisé l’intrication de photons uniques comme une ressource pour l’amplification annoncée de qubits photoniques où, grâce à une téléportation quantique avec un gain nous sommes capables de compenser les pertes dans un canal de communication. Cette amplification trouve une application directe dans les protocoles de Device Independent Quantum Key Distribution (DIQKD). Dans ce protocole la violation d’une inégalité de Bell est nécessaire afin d’établir une clé secrète entre deux parties, sans aucune connaissance sur les appareils de mesures, qui peuvent être traités comme des boîtes noires.

J’ai aussi exploité l’intrication des photons uniques comme un point de départ pour tester des questions fondamentales de physique quantique. Plus particulièrement, j’ai testé la validité de la théorie quantique à l’échelle macroscopique. Dans ce régime, la décohérence et l’imprécision de la mesure jouent un rôle important en nous empêchant de l’observer directement dans notre vie de tous les jours. En déplaçant une superposition entre le vide et un photon unique dans l’espace des phases, j’ai démontré expérimentalement la présence d’intrication dans un état qui compte un nombre macroscopique de particules, 500 photons.
# Contents

Aknowledgments .......................................................... iii
My thesis explained to my sister ......................................... 1

**Introduction** ................................................................ 5

1 **How to create heralded single photons** ......................... 9
   1.1 Nonlinear optical processes ...................................... 10
   1.2 Joint spectrum and spectral purity ............................ 13
   1.3 Statistics of down converted light ............................ 15
   1.4 Two photon interference on a beam splitter ............... 19
   1.5 A source of spectrally and spacially pure photons ....... 20

2 **Heralded amplification of photons** ............................... 25
   2.1 Qubit amplification and the detection loophole .......... 26
   2.2 Ralph and Lund’s noiseless linear amplifier ............... 27
   2.3 Principle of the heralded amplification of photons ........ 28
   2.4 Taking into account experimental imperfections ........ 29
   2.5 Experimental implementation and full characterisation of a single photon heralded amplifier .................. 31
   2.6 Heralded single photon amplification without post selection .......... 33
   2.7 Heralded amplification of qubits ............................ 36
   2.8 Experiment ....................................................... 38
   2.9 Discussion and conclusion ...................................... 41

3 **Displacing single photon entanglement in the macroscopic domain** ............................. 45
   3.1 From micro to macro ............................................. 46
   3.2 Experimental implementation .................................. 48
   3.3 Results ............................................................ 55
   3.4 Discussion and conclusion ...................................... 55

4 **General conclusion and outlook** ................................ 59

Bibliography / Bibliographie .............................................. 65
Appendix 75

List of publications 77

List of publications 77
1 Heralded photon amplification for quantum communication . . . . . 79
2 A complete characterization of the heralded amplification of photons 84
3 Displacement of entanglement back and forth between the micro and macro domains . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 94
4 Generation of tunable wavelength coherent states and heralded single photons for quantum optics applications . . . . . . . . 105
5 Pulsed source of spectrally uncorrelated and indistinguishable photons at telecom wavelengths . . . . . . . . . . . . . . . . . . . 111
6 High efficiency coupling of photon pairs in practice . . . . . . . . 120
7 Simple, pulsed, polarization entangled photon pair source . . . . . 132
8 Heralded amplification of photonics qubits . . . . . . . . . . . . . 137
My thesis explained to my sister

This is an introduction for my sister, but also for my parents, my aunt and my friends who always wonder about what I do.

To have an idea of what I worked with during my thesis, the first thing to know is what interference is. I think that the concept of interference is quite easy to catch from our everyday experience. Just picture yourself throwing stones in a lake: you will see a circular wave forming around the point where the stone fell, and if while it is propagating you throw another stone you will see another circular wave superposed to the first, creating some pretty pattern on the surface. Figure 0.1 shows what happens at the surface of an ocean when many waves interfere.

![Interference of water waves in the Atlantic Ocean.](image)

An example that is closer to what I am going to talk about is the interference between electromagnetic waves. When you listen to the radio and you hear a strange noise, people usually say "there is interference". But what does that mean? Two different radio stations emit radio waves, which are electromagnetic waves as much as the light that we can see. Sometimes these waves will sum up together, sometimes they won’t, and the result will not be a very nice symphony, but rather a confuse noise.

As an example, we can imagine only two waves interfering, like in Young’s experiment (see Figure 0.2), where a plane wave passing through a double slit results into an interference pattern. This is really similar to what happens between circular waves created by two stones that fall close to each other in the water.

You can easily see this kind of interference shining a laser pointer on a hair, which is also equivalent to a double slit.

As a matter of fact, we can repeat the same experiment with a beam of particles, like electrons, and we will obtain the same interference pattern. The only way to
describe such phenomena is to assume that we can also describe electrons with waves like we do with light.

These waves are called wavefunctions, and they are waves of probability that describe a property of a physical system (position, velocity, energy of a particle, for example). Moreover, they interfere exactly like light waves. The interference between two wavefunctions is known as quantum interference, or the superposition principle.

Quantum interference also has as a consequence a very peculiar phenomenon, called entanglement. The concept of entanglement was introduced by Erwin Schrödinger, and in the same years (1935/1936) Einstein, Podolsky and Rosen started a long debate about completeness of quantum theory. These were among the best scientists of their time, and yet they had troubles in accepting the fact that quantum mechanics changed the concept of reality of a physical observable property. Superposition of two wavefunctions involving, for example, two photons, creates correlations in the result of independent measurements on the two particles. These correlations persist even at a long distance, because they actually have nothing to do with the distance between the particles. In simple words: if I have two balls, one white and one black, and I give one to you at random, when you will look at the color of your ball and you find white, you’ll know that my ball is black even if, in the meanwhile, I went to the Moon. The only difference between the game with balls and quantum mechanics is that in quantum mechanics the color of your ball is not defined until you look at it. Only in 1964, John Bell wrote an inequality that depends on these correlations and that is satisfied by classical systems while violated by quantum mechanics, showing with this that reality and locality are actually contradictory concepts in quantum mechanics. The first experiments that proved the validity of quantum mechanics through the violation of a Bell inequality were done in the 80s, and since then every experiment that has been done on this subject always confirmed the same result.

Another thing that we know is that also light is composed of particles: they are called photons. It was Einstein who showed it, when he discovered the photoelectric
effect, which gave him a Nobel prize in 1921 (no, it was not for the theory of Relativity!). The photoelectric effect describes the fact that when a particle of light shines on a metal, if it has enough energy, one electron will jump out of its orbital. If one single photon hits a semireflective surface, the two possibilities of being reflected or transmitted will interfere. In particular, the situation in which no photon is reflected and one is transmitted interferes with the opposite case, when one photon is reflected and none is transmitted. Measuring the number of photons on each side of the surface is equivalent to looking at the color of the ball in the previous example. This is single photon entanglement (mentioned in the title), and it is the basis of most of the experiments done during my thesis.

![Figure 0.3: Single photon interference.](image)

The aim of this thesis is to see how we can use quantum interference of photons in experiments that have also some practical application. One application is based on the fact that using quantum systems for encoding and manipulating information can bring many advantages. Quantum interference allows one, for example, to input into a computer many quantum bits in a superposition, to be elaborated all at the same time. This is the strength of quantum computation, which you may have heard of. However, the information generated in a quantum computer can also be encoded in some physical medium to be transmitted far away and processed by another computer. In this thesis I will not talk about quantum computers, but rather about quantum communication. Light has always been a good carrier of information, and its quantum properties make it even more useful than you might think.
Introduction

Due to the progress of the last decades in miniaturization of electronic components, we can remarkably reduce the size of a calculator, increasing at the same time its computational power. However, quantum mechanics tells us that there is a limit in size below which quantum effects dictate the dynamics, and the classical laws that govern the functioning of our smartphones are no longer valid. Since the early formulation of quantum theory, we know that the limit in which quantum mechanics tends to classical laws is equivalent to the limit in which the Plank constant $\hbar = \frac{h}{2\pi}$, ($h \simeq 6,626 \cdot 10^{-34} J \cdot s$) is small with respect to the action of the system. To make an analogy with optics, one can see this limit when going from the evolution of electric and magnetic fields through Maxwell’s equations (waves) to the propagation of light along rays (linear trajectories determined by Fermat’s principle, or least time principle) in the limit in which the wavelength of the radiation is very small compared to the other distances involved: $\lambda \to 0$. In the same way we can think of $\hbar \to 0$ as the limit in which the solution of the Schrödinger’s equation is replaced by a trajectory that is determined by the principle of least action.

Therefore, a different kind of hardware has become more interesting: a machine that exploits quantum theory in a more direct way, to simulate Nature, as suggested by R. P. Feynman (Feynman 1982). This was the dawn of quantum computation and quantum communication, intended to use quantum effects to manipulate information and exchange it between parties. The main advantage of quantum computation over the classical one is the superposition principle, which allows the introduction of the concept of a qubit. Being generic superpositions of two quantum states, qubits represent one of the main advantages of quantum versus classical computation, in which a bit can only have two possible values, 0 and 1.

The strength of quantum superposition doesn’t lie only in the qubit but also in the presence of entanglement. The concept of entanglement was introduced by Schrödinger as the characteristic trait of quantum mechanics (Schrödinger 1935), and it generated a long and famous debate starting from the work of Einstein, Podolsky and Rosen in 1935 (Einstein et al. 1935). Quantum mechanics was revolutionising the concepts of locality and reality. Only in 1964 John Bell solved the problem (Bell 1964), defining the concept of non locality in quantum mechanics with a mathematical formalism. Later on, J. F. Clauser, M. A. Horne, A. Shimony

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1In relativity, the principle of locality implies that two space like separated observables must commute, here by locality we mean, in a more general view, that systems are influenced only by objects that are within their light cone.
and R. A. Holt proposed a practical implementation of Bell’s idea to experimentally test quantum mechanics, through the violation of a CHSH inequality (Clauser et al. 1969). Every experiment\(^2\) done until the present day that follows Bell’s idea confirmed the validity of quantum theory (Aspect et al. 1981, 1982; Clauser, Shimony 1881; Clauser 1976; Franson 1989; Freedman, Clauser 1972; Fry 1973; Fry, Thompson 1976; Kwiat et al. 1993; Tittel et al. 1998).\(^3\)

Nonlocal correlations among distinct physical systems, independent on their reciprocal distance, allow operations that are otherwise inaccessible, because there is no analogous phenomena in classical physics. Many quantum protocols are based on entanglement, e. g. quantum teleportation (Bennett et al. 1993), quantum key distribution (Ekert 1991), measurement based quantum computation (Raussendorf, Briegel 2001), and so on.

Nowadays, information can be encoded in many physical systems, such as trapped ions (Wineland et al. 2003), Josephson junctions (Makhlin et al. 1999), energy-time (Brendel et al. 1999), polarisation (Knill et al. 2001) or orbital angular momentum (Allen et al. 2003) of photons (not to cite all the possibilities). Some of these, atomic or superconductive qubits, are advantageous for quantum simulation and implementation of quantum algorithms, others are better suited for quantum communication and cryptography, e. g. discrete and continuous variable (Andersen et al. 2014; Cerf et al. 2007) optical states.

In particular, optical encoding has an enormous advantage over the other systems: thanks to our complete understanding of quantum electrodynamics, optical systems are the easiest to manipulate and measure both in classical and in quantum regimes. Photons have been the most useful instrument for fundamental tests of quantum mechanics and proof of principle experiments. Moreover, they are the best carrier of information, mainly at telecommunication wavelengths, where the losses in single mode fiber are minimal. This makes them necessary building blocks for quantum communication applications.

The main goal of my research has been to develop experimental tools, based on light sources at telecom wavelengths and fiber optics components, that are useful both for investigating fundamental aspects of quantum theory and for advances in quantum communication (QC). In particular, the two main experimental works carried on during this thesis work are both based on heralded single photon entanglement.

Photon pairs generated in parametric processes can be entangled, and this entanglement has been exploited in many experimental applications, even though it is not heralded. In the absence of deterministic single photon sources, heralding entanglement is a key resource for quantum communication, as we will see during

\(^2\)Most of the first experimental violations of Bell inequalities were realized with photonic entangled states

\(^3\)with the exception of (Pipkin 1978)
this Thesis. The most straightforward way of preparing heralded entanglement is sending a heralded single photon $|1\rangle$ on a beam splitter. The resulting state is a single photon delocalized over two spatial modes, the outputs $a$ and $b$ of the beam splitter. We name this state **heralded single photon entanglement**, and it can be written in the following way:

$$\frac{1}{\sqrt{2}} \left( |0\rangle_a |1\rangle_b + e^{i\phi} |1\rangle_a |0\rangle_b \right).$$

(0.1)

This state is very simple, but for some time it was not easily accepted as entangled by all the scientific community (Enk 2005) even though its non local correlation have been proved using homodyne detection (Hardy 1994; Hessmo et al. 2004; Tan et al. 1991).

![Figure 0.4: A heralded single photon on a beam splitter results into a single photon delocalized over two spatial modes, reflected and transmitted.](image)

In the second chapter (2), I will show how heralded single photon entanglement can be used as a resource for heralded teleportation with a gain, commonly named heralded amplification of photonic qubits. Photonic qubits are usually encoded in degrees of freedom such as polarization, time bin, orbital angular momentum, or frequency. The experimental realization of a time bin qubit amplifier will be discussed in this Thesis. I will also present the heralded amplification of photons in a non post selected scheme.

In the third chapter (3) goes back to fundamental questions. Heralded single photon entanglement can be used as a starting point to generate a micro-macro entangled state, which is a state that involves a large number of photons. This is done by displacing in the phase space one of the two modes occupied by the photon. This experiment shows the presence of entanglement in a macroscopic state of light. The number of photons involved in the system is of the order of $10^3$, large enough to be easily seen by the human eye (provided that the wavelength is in the
visible spectrum). If one uses a *classical detector* that can only resolve large photon number differences, the two macroscopic components of the state can be efficiently distinguished. This makes the state analogous to an optical Schrödinger cat state.
1 How to create heralded single photons

Haud igitur penitus pereunt quaecumque videntur, 
quando alit ex alio reficit natura nec ullam 
rem gigni patitur nisi morte adiuta aliena. 
(Titus Lucretius Carus, De rerum natura vv. 262-264)

Engineering and building sources of single photons is becoming an ever harder 
task, as the demands that these sources need to meet are becoming increasingly 
challenging. For some applications, single photons are required to be on-demand, 
tunable and/or with a specific bandwidth, e. g. in quantum communication, where 
narrowband single photons at telecom wavelengths are required, possibly in a deter-
ministic source. Research in this field has found several different approaches, due to 
the need of single photons for implementations in quantum information sciences and 
in quantum communication applications. Various solutions have been considered 
in the last few decades, such as deterministic sources based on single atoms (Kurz 
et al. 2013), color centers in diamond (Kurtsiefer et al. 2000; Lohrmann et al. 2011), 
quantum dots (Gregersen et al. 2010; Stock et al. 2011), or probabilistic, but her-
alded, sources which exploit parametric down conversion (SPDC) (Fasel et al. 2004; 
Krapick et al. 2013) in media with nonlinear optical susceptibility $\chi^{(2)}$ or four wave 

In this chapter I give an overview of the two main heralded single photon sources 
that I developed, characterized and used during my thesis work, which are both 
based on SPDC. After a general description of nonlinear optical interactions, I anal-
yse more in detail the joint spectral distribution of down converted photons and its 
relation to their spectral purity, I show how it is possible to characterise the number 
of Schmidt modes via measuring the second order autocorrelation function, and how 
the indistinguishability can be quantified by exploiting two-photon interference. In 
the final section, I present a source of heralded single photons at telecom wave-
lengths, based on SPDC in a bulk periodically poled Potassium Tytanyl Phospate 
(PPKTP) crystal, optimised in order to have high heralding efficiency and high 
purity at the same time.

$^1$Therefore, visible objects don’t dissolve completely, as nature creates things from other things 
and it does not allow anything to be born if not from another’s death.
1.1 Nonlinear optical processes

Nonlinear processes take place when, in the presence of an electric field, the dipole moment density $P(r, t)$ of a dielectric exhibits a nonlinear dependence on the driving field, which can be expanded as a power series in the regime of weak electric field $E(r, t)$:

$$P(r, t) = \frac{\epsilon_0}{\omega_0} \left( \chi^{(1)} E(r, t) + \chi^{(2)} E^2(r, t) + \chi^{(3)} E^3(r, t) + \ldots \right) = P^L(r, t) + P^{NL}(r, t)$$

(1.1)

where $\epsilon_0$ is the permittivity of the vacuum, $\chi^{(1)}$ is the linear susceptibility, $\chi^{(2)}$ and $\chi^{(3)}$ are the second and third order susceptibilities, and so on. $P(r, t)$ can therefore be rewritten as the sum of a linear term plus a term that depends non linearly on the electric field. In the following, we consider only non linear effects of the second order, related to $\chi^{(2)}$, such as sum and difference frequency generation (SFG and DFG, respectively) and spontaneous parametric down conversion (SPDC). These effects happen to appear only in materials without an inversion center (non centrosymmetric) and on all surfaces, where the lattice symmetry is broken, since all the even terms of the susceptibility vanish when there is inversion symmetry. Due to vacuum fluctuations, polarizability of the vacuum also leads to nonlinear optical effects, but with very low probability. In the materials that we consider, such as Lithium Niobate (LN) and Potassium Titanly Phosphate (KTP), the $\chi^{(1)}$ is of the order of unity, while $\chi^{(2)}$ is the order of $10^{-11} \text{m/V}$.

1.1.1 Nonlinearities of the second order

The second order nonlinear susceptibility is a tensor of rank 3 that describes the second order polarization dependence on the fields and the symmetries of the material. If we write the electric field as a sum of plane monochromatic waves propagating along the $z$ axis:

$$E(z, t) = \sum_n E(\omega_n) e^{i(k_n z - \omega_n t)} + \text{c.c.}$$

(1.2)

then the second order nonlinear polarization can also be written as

$$P^{(2)}(z, t) = \epsilon_0 \chi^{(2)} \sum_{nm} (E(\omega_n)e^{i(k_n z - \omega_n t)} + \text{c.c.})(E(\omega_m)e^{i(k_m z - \omega_m t}) + \text{c.c.})$$

(1.3)

and from this expression one can derive the terms that are responsible for the different second order interactions, such as SFG ($\omega = \omega_n + \omega_m$), second harmonic generation ($\omega = 2\omega_n$, SHG), DFG ($\omega = \omega_n - \omega_m$).

In order to ensure constructive interference between all the plane waves involved in the sum, the phase that the fields acquire during the propagation in the nonlinear material has to be matched with the generated field at each point, therefore some
1.1 Nonlinear optical processes

Phase matching conditions have to be satisfied, which summarizes by fixing the phase mismatch at 0:

$$\Delta k = k_{nm} - k_n - k_m = 0$$  \hspace{1cm} (1.4)

where $k_i = k(\omega_i) = n(\omega_i)\omega_i/c$ and $k_{nm} = k(\omega_n + \omega_m)$ in the case of SFG, for example. If one exploits birefringence to fulfill this conditions, there can be different configurations:

- Type I phase matching, in which the two lower frequencies have the same polarisation, perpendicular to the third one;
- Type II phase matching, in which the two lower frequencies have orthogonal polarisations, one perpendicular and one parallel to the third one.

Phase matching can be achieved for any wavelength in materials that have a non-zero phase mismatch, with the aid of Quasi Phase Matching (QPM), a technique in which the nonlinear coefficient sign is periodically inverted with a poling period $\Lambda$, inscribed in the crystal. This is done in order to compensate the dispersion and allow the cancellation of the phase mismatch:

$$\Delta k = k_{nm} - k_n - k_m + \frac{2\pi}{\Lambda} = 0.$$  \hspace{1cm} (1.5)

This means that birefringence is no longer necessary and the three involved fields can have the same polarisation, corresponding to the Type 0 phase matching.

1.1.2 Spontaneous parametric down conversion

Parametric down conversion is a second order nonlinear process that involves three fields: a pump at frequency $\omega_p$ and two generated fields called signal ($\omega_s$) and idler ($\omega_i$). The other processes mentioned above consist of two input fields that can generate a third field with sum or difference of the two initial frequencies, satisfying energy conservation and phase matching conditions.

When a laser beam impinges upon a medium with second order nonlinearity fulfilling the phase matching conditions, it gives rise to parametric down conversion, and the Hamiltonian of this process is:

$$H_I = \hbar \chi^{(2)} \zeta \int d\omega_s d\omega_i d\vec{k}_s d\vec{k}_i d\vec{k}_p S(\omega_s, \omega_i, \vec{k}_s, \vec{k}_i, \vec{k}_p) \times a^\dagger(\omega_s, \vec{k}_s) a^\dagger(\omega_i, \vec{k}_i) a(\omega_p, \vec{k}_p) + \text{h.c.}$$  \hspace{1cm} (1.6)

where $\zeta$ is a constant that depends on the pump electric field, and

$$S(\omega_s, \omega_i, \vec{k}_s, \vec{k}_i, \vec{k}_p) = \epsilon(\omega_s, \omega_i) \phi(\omega_s, \omega_i, \vec{k}_s, \vec{k}_i, \vec{k}_p)$$  \hspace{1cm} (1.7)
is the joint spectral amplitude (JSA), $a^\dagger(\omega_s, \vec{k}_s)$, $a^\dagger(\omega_i, \vec{k}_i)$ and $a^\dagger(\omega_p, \vec{k}_p)$ are the creation operators of the signal and idler fields, and of the pump field, respectively, while $\epsilon(\omega_s, \omega_i)$ and $\phi(\omega_s, \omega_i, \vec{k}_s, \vec{k}_i, \vec{k}_p)$ correspond to the pump spectral envelope and the phase matching function, which is defined in the following.

For a Sech$^2$-shaped pump pulse, such as a pulse generated in a mode-locked laser, the envelope is:

$$
\epsilon(\omega_s, \omega_i) \propto \text{sech}\left[ \frac{(\omega_i + \omega_s - \omega_p)}{3\pi} \cdot \frac{2\log(2 + \sqrt{3})}{\Delta\omega_p} \right] \tag{1.8}
$$

with $\Delta\omega_p$ the full width at half maximum of the pump pulse in frequency. In the Gaussian approximation, this becomes:

$$
\epsilon(\omega_s, \omega_i) \propto \exp\left( \frac{(\omega_i + \omega_s - \omega_p)^2}{4\sigma_p^2} \right) \tag{1.9}
$$

with $\sigma_p = \Delta\omega_p/2\sqrt{2\ln2}$ and $\omega_p$ the central wavelength of the pump. This approximation is considered valid throughout the thesis, since the pump laser used for parametric down conversion in the presented works is a Titanium-Sapphire mode locked laser at 780 nm with picosecond pulses. By simulating the JSA it can be observed that the deviations of the gaussian approximation from the exact theoretical model can be considered small. Moreover, a different pump shape and pump width gives a different JSA, and this effect can be used to shape the down converted photons.

The phase matching function $\phi(\omega_s, \omega_i, \vec{k}_s, \vec{k}_i, \vec{k}_p)$ for a crystal of a length $L$ depends on the direction of $\vec{k}_{i,s,p}$ and on the poling period (Ljunggren, Tengner 2005). However, in the approximation of collinear propagation we can consider only the $z$ direction, and the function is given by:

$$
\phi(\omega_s, \omega_i, k_s, k_i, k_p) \propto \text{sinc}\left[ \frac{L}{2} \left( k_i(\omega_i) + k_s(\omega_s) + \frac{2\pi}{\Lambda} - k_p(\omega_p) \right) \right]. \tag{1.10}
$$

Again, we consider the Gaussian approximation:

$$
\phi(\omega_s, \omega_i) \propto \exp\left( -\alpha^2 \frac{\Delta k^2 L^2}{4} \right), \tag{1.11}
$$

with $\alpha = 0.439$ and $\Delta k = k_i(\omega_i) + k_s(\omega_s) + \frac{2\pi}{\Lambda} - k_p(\omega_p)$. For very small deviations from the central wavelengths of the pump $\omega_{p,0}$ and of the emitted photons $\omega_{i,0}$ and $\omega_{s,0}$, we have $\Delta k = (\omega_i - \omega_{i,0})k_i' + (\omega_s - \omega_{s,0})k_s' - (\omega_p - \omega_{p,0})k_p'$, where $k_j' = \frac{\partial k_j(\omega_j)}{\partial \omega_j} |_{\omega_j,0}$ is the group velocity dispersion. By using these approximations, the joint spectral
1.2 Joint spectrum and spectral purity

According to the previous section, the state generated via SPDC can be described as

\[ |\psi\rangle \propto |0\rangle + \sqrt{p} \int d\omega_s d\omega_i S(\omega_s, \omega_i) |\omega_s, \omega_i\rangle + O(p), \]  \hspace{1cm} (1.14)

where \( p \ll 1 \) can be seen as the pair creation probability (Laiho et al. 2009).

Given \( S(\omega_s, \omega_i) \) defined in equation 1.13 we can write the density matrix of the two photon state as:

\[ \rho = \int d\omega_s d\omega_i d\omega'_s d\omega'_i S(\omega_s, \omega_i) S^*(\omega'_s, \omega'_i) |\omega_s, \omega_i\rangle \langle \omega'_s, \omega'_i| \]  \hspace{1cm} (1.15)

then we can quantify the purity of the state of one photon with the Schmidt number \( K \) (Ekert, Knight 1995; Schmidt 1907), defined as \( \frac{1}{K} = \text{Tr} [\rho_{s,i}^2] = P_{s,i} \) (where \( \rho_{s,i} = Tr_{i,s} [\rho] \) is the partial trace of the density matrix over the signal/idler photon):

\[ P_{s,i} = \sqrt{1 - \frac{c_{si}^2}{c_sc_i}} \]  \hspace{1cm} (1.16)
1 How to create heralded single photons

Experimentally, in order to estimate $P$ it is necessary to measure the parameters $c_s, c_i, c_{si}$, and this can be done measuring the joint spectral probability distribution $|S(\omega_s, \omega_i)|^2$, or joint spectral intensity (JSI). One way of measuring the JSI consists of placing a tunable filter on each of the twin photons after splitting them (when the phase matching is type II, a polarization beam splitter deterministically separates the photon pairs) and detect the transmitted part. Only correlated photons give a coincidence count. A schematic of the setup is represented in Figure 1.1.

Figure 1.2 shows the theoretical prediction and the experimental measurement of the JSI for photon pairs produced in a periodically poled LN crystal (PPLN) pumped by a 780 laser in continuous wave regime (Figure 1.2.a,b) and pulsed regime (Figure 1.2.c,d). In the CW case, one expects perfect correlation in the wavelengths of the twin photons. If one looks at the contour of the JSI in the x-y plane, it’s an ellipse rotated at $45^\circ$. In the pulsed case the pump laser has a broader spectrum, and changing the coefficients $c_s, c_i$ and $c_{si}$ has the effect of rotating the ellipse.

In order to generate pairs of photons in a separable state, i. e. $K = 1$, the angle of the ellipse with respect to the x-y axis should be zero, and this corresponds to having $c_{si} = 0$, therefore:

$$\alpha^2 L^2 \sigma_p^2 (k'_p - k'_s)(k'_p - k'_i) = -1.$$  \hspace{1cm} (1.17)

To satisfy this equation, the condition $k'_s < k'_p < k'_i$ must be fulfilled in the nonlinear medium. In the approximations considered above, this cannot be true in every nonlinear medium, e. g. it is not valid in a PPLN crystal. In other words, equation 1.17 requires that one of the two photons would travel faster than the pump beam in the nonlinear medium, while the other would be slower. This is allowed in PPKTP, where $(k'_p - k'_s)(k'_p - k'_i) = 2.63 \times 10^{-2} \text{ m}^{-1} \text{ GHz}^{-1}$. Thus, for a 3 cm long crystal with a poling period of 47.8 $\mu$m a purity close to one should be achievable with a pump bandwidth of 467 rad$^{-1}$ GHz, which corresponds to a pulse duration with FWHM = 2.6 ps. Furthermore, for having more than one source of pure photons that are also indistinguishable between each other in all the combinations one needs to have pure photons in a particular configuration of the joint spectrum, the one in which the photons are both factorable and with the same bandwidth. In this case
1.3 Statistics of down converted light

Let’s consider the simple case in which the pump is in a single mode, and the integral over the frequency is not necessary:

\[ H_I = \hbar \chi^{(2)} (\zeta a_s^\dagger a_i^\dagger + \text{c.c.}) \]  

(1.18)

The output state is then a two-mode squeezed state:

\[
|\psi\rangle = e^{iH_I t} |0\rangle = \text{sech}(|\zeta| \chi^{(2)} t) \sum_{n=0}^{\infty} \tanh^n(|\zeta| \chi^{(2)} t) |n\rangle_s |n\rangle_i \\
= \sqrt{1 - p} \sum_{n=0}^{\infty} p^{n/2} |n\rangle_s |n\rangle_i
\]

(1.19)

where \( p = \tanh^2(|\zeta| \chi^{(2)} t) \) is proportional to the probability of creating a photon pair in the regime of weak electric field \( (p \ll 1) \). It’s now easy to see that the probability of having \( n \) photons in the signal beam is \( P(n) = (1-p)p^n \), which can be expressed

Figure 1.2: (a),(b)/(c),(d): Simulated and measured JSI of pairs of photons at 1560 nm generated in a SPDC process, in which 780 nm laser in CW/pulsed regime pumps a 2 cm long PPLN crystal with type II phase matching condition.

the projection of the joint spectral distribution on the \( \omega_s - \omega_i \) plane is close to a circle.

1.3 Statistics of down converted light

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|\psi\rangle = e^{iH_I t} |0\rangle = \text{sech}(|\zeta| \chi^{(2)} t) \sum_{n=0}^{\infty} \tanh^n(|\zeta| \chi^{(2)} t) |n\rangle_s |n\rangle_i \\
= \sqrt{1 - p} \sum_{n=0}^{\infty} p^{n/2} |n\rangle_s |n\rangle_i
\]

(1.19)

where \( p = \tanh^2(|\zeta| \chi^{(2)} t) \) is proportional to the probability of creating a photon pair in the regime of weak electric field \( (p \ll 1) \). It’s now easy to see that the probability of having \( n \) photons in the signal beam is \( P(n) = (1-p)p^n \), which can be expressed
as a thermal distribution $P(n) = \mu^n / (1 + \mu)^{n+1}$ with $\mu = p/(1-p)$ as a mean number of photons. From this, one can calculate the second order autocorrelation function:

$$g^{(2)} = \frac{\langle a^\dagger a^\dagger aa \rangle}{\langle a^\dagger a \rangle^2} \approx \frac{2P(2)}{P(1)^2} = 2 + O(p) \quad (1.20)$$

From this simple example, one can see that if we can select a single mode out of the down converted photons, the statistical distribution of the photons is thermal (Bose-Einstein). When we have several modes\(^2\) it is possible to show that the distribution tends to a Poissonian distribution. The multimode character is due to the fact that the coherence time of the generated photons is shorter than the pump coherence time, hence it is shorter than the uncertainty over the pair creation time. Therefore, being able to measure the statistical distribution can be a way to estimate the number of modes. In a Hanbury Brown and Twiss-type (HBT) experiment (see Figure 1.3), where the light source that one wants to study is sent on a beam splitter (BS), counting the coincidences between two detectors placed at the two output of the BS directly gives the $g^{(2)}(t)$, and at $t = 0$ one expects to find $g^{(2)}(0) = 2$ for thermal statistics (eq. 1.20 ), $g^{(2)}(0) = 1$ for Poissonian statistics (i.e. multimode or coherent state).

![Figure 1.3: Schematic setup for a Hanbury Brown and Twiss experiment.](image)

Assuming that one measures in the regime of small pair production and a low transmission efficiency, we can see that the $g^{(2)}(t)$ can be approximated as:

$$g^{(2)}(0) = \frac{2P(2)}{P(1)^2} = \frac{\langle I_2(0)I_1(0) \rangle}{\langle I(0) \rangle^2} \simeq 1 + \frac{1}{K} \quad (1.21)$$

where $K$ is the average number of Schmidt modes. Let’s calculate explicitly the intensity of the light in the signal mode $\langle I(0) \rangle$ and the mean square intensity

\(^2\)In most of the cases we can talk about spectral modes, as the spatial mode can be easily selected using a SM fiber.
\[ |\psi\rangle = |00\rangle + \sqrt{p} F_1 + \frac{p}{2} F_1^2 + O(p^{3/2}) \]  
(1.22)

\[ F_1 = \int d\omega_s d\omega_i S(\omega_s, \omega_i) |\omega_s, \omega_i\rangle \]  
(1.23)

\[ \langle I(0) \rangle = \int_{-\Delta T/2}^{\Delta T/2} dt \langle \psi | a_s^\dagger(t) a_s(t) |\psi\rangle \]  
(1.24)

\[ a_s^\dagger(t) = \frac{1}{2\pi} \int_0^\infty d\omega_s f_s(\omega_s) e^{i\omega_s(t-t_0)} a_s^\dagger(\omega_s) \]  
(1.25)

where \( f_{s,i}(\omega_{s,i}) \) is the spectral amplitude of a filter which can eventually be used for example to select one mode. This is a factor in front of the JSA, and a typical Gaussian filter can be described as:

\[ f_{s,i}(\omega_{s,i}) = F_{s,i} \exp \left[ -\frac{(\omega_{s,i} - \omega_0)^2}{4\sigma_{s,i}^2} \right] \]  
(1.26)

where \( \sigma_p \) (\( \sigma_{s,i} \)) is the half width at \( 1/e^2 \) (standard deviation)\(^3\). \( F_{s,i} \) includes the normalisation factor and the transmission of the filter. Adding a filter, the coefficients of the JSA become:

\[ c_s = \frac{1}{4} \left( \frac{1}{\sigma_p^2} + \alpha^2 L^2 (k_p' - k_s')^2 + \frac{1}{\sigma_s^2} \right) \]  

\[ c_i = \frac{1}{4} \left( \frac{1}{\sigma_p^2} + \alpha^2 L^2 (k_p' - k_i')^2 + \frac{1}{\sigma_i^2} \right) \]  

\[ c_{si} = \frac{1}{2} \left( \frac{1}{\sigma_p^2} + \alpha^2 L^2 (k_p' - k_s')(k_p' - k_i') \right) \]  

\[ \Rightarrow \ S(\omega_s, \omega_i) = \frac{1}{N} \exp \left[ -c_s \Delta\omega_s^2 - c_i \Delta\omega_i^2 - c_{si} \Delta\omega_s \Delta\omega_i \right]. \]  
(1.27)

Where \( N = 2\pi/\sqrt{c_s c_i - c_{si}^2} \) is a normalisation factor such that \( \int d\omega_s d\omega_i |S(\omega_s, \omega_i)|^2 = 1 \). Therefore, the intensity of the signal mode is:

\[ \langle I(0) \rangle = p \int d\omega_s d\omega_i |S(\omega_s, \omega_i)|^2 = p + O(p^2) \]  
(1.28)

Here we used the assumption that the resolution time \( \Delta T \) of the detector is shorter than the spacing between two pulses (which means being able to resolve the different pulses), but it is long compared with the coherence times associated

\(^3\)Normally we measure \( \Delta \lambda \), the FWHM of the spectrum, then the standard deviation is \( \sigma = \pi e \Delta \lambda/\lambda^2 \sqrt{\ln(4)} \).
with the pulse length and the filters. We also used the orthogonality conditions: 
\[ a(\omega)|\omega'\rangle = \delta(\omega - \omega')(0) \] and 
\[ \langle \omega | \omega' \rangle = \delta(\omega - \omega'). \]

For the mean square intensity we take into account the term for double pair creation, because it is the first non-zero term when we apply two annihilation operators:

\[
\langle I_2(0)I_1(0) \rangle = \int dt_1dt_2 \langle \psi | a^\dagger_{s_1}(t_1)a^\dagger_{s_2}(t_2)a_{s_1}(t_1)a_{s_2}(t_2) | \psi \rangle
\]

(1.29)

using the relations:

\[
a_{s_1}(\omega_{s_1})a_{s_2}(\omega_{s_2}) |\omega_{s_1},\omega_{s_2}\rangle = (\delta(\omega_{s_1} - \omega_{s_2})\delta(\omega_{s_1} - \omega_{s_2}') + \\
\delta(\omega_{s_1} - \omega_{s_2}')\delta(\omega_{s_1} - \omega_{s_2})) |00\rangle
\]

(1.30)

\[
\langle \omega_{i_1},\omega_{i_2}' | \omega_{i_2}'',\omega_{i_2}''' \rangle = \delta(\omega_{i_1} - \omega_{i_2}'')\delta(\omega_{i_1} - \omega_{i_2}''') + \\
\delta(\omega_{i_1} - \omega_{i_2}'')\delta(\omega_{i_1} - \omega_{i_2}'''')
\]

(1.31)

and using the fact that \[ \int dt \exp [ i(t)(\omega - \omega') ] = 2\pi \delta(\omega - \omega'), \]
we have

\[
\langle I_2(0)I_1(0) \rangle = \langle I(0) \rangle^2 + p^2 \frac{2\pi}{N^2 c_s c_i (c_s c_i - c_s^2)} + O(p^4)
\]

(1.32)

then the second order autocorrelation function is:

\[
g^{(2)}(0) = \frac{\langle I_2(0)I_1(0) \rangle}{\langle I(0) \rangle^2} = 1 + \frac{(c_s c_i - c_s^2)}{c_s c_i (c_s c_i - c_s^2)}
\]

(1.33)

\[
= 1 + \frac{1 - c_s^2}{c_s c_i} = 1 + \mathcal{P}.
\]

Here we obtained, with no surprise, the same expression for the purity that we have found in eq. (1.16), under the same assumptions. The difference is now that we are looking only at the signal photon, so it doesn’t make sense to include the filter on the idler photon. Indeed, if one photon is pure, the other is also pure because the wave function becomes factorable.

In the case of the PPLN source described in section 1.2, in which the high correlation in the energy of the down converted indicates a multimode emission, we can look at the autocorrelation function of one of the two photons, e.g. the signal photon, in a HBT experiment, described in Figure 1.3. The histogram of coincidences between the detectors placed at the two outputs of the BS is measured for different filter bandwidths, see Figure 1.4. The measurement is done in the pulsed regime, where we can consider that a single mode is defined by the width of the laser pulse, approximately 80 GHz, which is equivalent to 0.65 nm at telecom wavelengths. We can see that indeed only the measurement done with a filter narrower than the pump one can see that the \( g^{(2)} \) approaches the value of 2, revealing the thermal statistics of the selected mode.
1.4 Two photon interference on a beam splitter

Figure 1.4: Left: Histograms of coincidences between detectors D1 and D2 at the two outputs of the beam splitter, for different filtering, measured for a PPLN crystal pumped by a pulsed laser in a HBT type experiment. The darkest histogram corresponds to the statistics of the down converted photon without filtering, the other histograms, from dark to light blue, correspond to a measurement taken with a 1.3 nm, 0.5 nm and 0.2 nm filter, respectively placed before the 50/50 beam splitter. The histograms are shifted with respect to each other so that they can be distinguished. Right: Purity as a function of the filter bandwidth, the black curve is the theoretical prediction, blue points are estimated using the ratio between the peak at delay zero between two detectors after a BS, and the side peaks in the coincidence histograms.

1.4 Two photon interference on a beam splitter

In 1987, Hong, Ou and Mandel (HOM) first introduced two photon interference as a powerful tool to measure subpicosecond time intervals (Hong et al. 1987a), which is otherwise very demanding if not impossible with the present detection technology. We can model an ideal balanced beam splitter using the creation operators of the
input modes $a^\dagger$, $b^\dagger$ and the output modes $c^\dagger, d^\dagger$ as follows:

$$a^\dagger = \frac{c^\dagger + id^\dagger}{\sqrt{2}} \quad b^\dagger = \frac{c^\dagger - id^\dagger}{\sqrt{2}}$$  \hspace{1cm} (1.34)$$

then an input state of the form $a^\dagger b^\dagger |00\rangle$ becomes:

$$a^\dagger b^\dagger |00\rangle \rightarrow \left( \frac{c^\dagger + id^\dagger}{\sqrt{2}} \right) \left( \frac{c^\dagger - id^\dagger}{\sqrt{2}} \right)$$

$$= \frac{1}{2} \left( c^\dagger c^\dagger - ic^\dagger d^\dagger + id^\dagger c^\dagger + d^\dagger d^\dagger \right) |00\rangle$$  \hspace{1cm} (1.35)$$

where, if the two particles are indistinguishable in all degrees of freedom, the terms $c^\dagger d^\dagger$ and $d^\dagger c^\dagger$ cancel, leading to a final state of the form $\frac{1}{2}(|02\rangle + |20\rangle)$. This means that if we are able to measure the coincidence counts in the modes $c$ and $d$ while varying one degree of freedom (e.g. the difference in arrival time on the BS), we can observe an interference dip. The coincidence rate depends on the symmetry of the JSA under exchange idler↔signal, which quantifies the indistinguishability of the twin photons (Hong et al. 1987a):

$$R_c(\Delta t) = \frac{1}{2} - \frac{1}{2} \int d\omega_s d\omega_i S(\omega_s, \omega_i) S^*(\omega_i, \omega_s)e^{i(\omega_s - \omega_i)\Delta t}.$$  \hspace{1cm} (1.36)$$

Using equation 1.13, we can find the expression (Hong et al. 1987b):

$$R_c(\Delta t) = \frac{1}{2} \left( 1 - I e^{-\Delta t/(2\sigma^2)} \right) \quad \sigma = \alpha \frac{L}{2} (k'_s - k'_i),$$  \hspace{1cm} (1.37)$$

where $I = I(L, \sigma_p, k'_s, k'_i, k'_p)$ is the indistinguishability of the two photons, which gives directly the visibility of the dip, and $\sigma$ is the correlation time of the two photons.

In general, the signal and idler photons are not indistinguishable. For example, Figure 1.6 shows the spectra of the two photons at 1560 nm generated in a PPLN crystal pumped by a pulsed laser at 780 nm, which are clearly asymmetric, and thus not completely indistinguishable in energy, at least after projecting the spatial mode into a Gaussian mode with a single mode fiber. Spectral filtering is then necessary in order to remove the distinguishability, as one can see in Figure 1.6, which shows the HOM dip as a function of the delay between the two photons for different kinds of filtering.

1.5 A source of spectrally and spatially pure photons

In this section, I briefly describe the generation of indistinguishable photon pairs at telecom wavelengths based on a type-II SPDC process in a periodically poled
1.5 A source of spectrally and spacially pure photons

Figure 1.6: Left: spectra of the signal (blue line) and idler (green line) photons produced via SPDC in a 2 cm long PPLN crystal. Right: Hong Ou Mandel dip between filtered twins photons: the visibilities are 0.53, 0.93 and 0.99 for the gray, magenta and blue curve, which correspond to measurements without filtering, with a 1.3 nm filter and with a 0.2 nm filter.

potassium titanyl phosphate (PPKTP) crystal. To study this source, we build a double-pass setup in order to have the possibility to create independent pairs of photons and finally measure the visibility of the interference of uncorrelated photons.

Figure 1.7 shows the experimental setup. A Ti-Sapphire picosecond mode-locked laser at 772 nm is injected into a 3 cm long PPKTP bulk crystal in two opposite directions. The photon pairs are emitted in these two directions and then deterministically separated using PBSs. All four photons are generated around 1544 nm and coupled into single mode fibres with an efficiency of (90 ± 4)%.

If we consider the JSA without any Gaussian approximation, i.e. taking into account the fact that the shape of the pump pulse is a hyperbolic secant, the crystal geometry and the wavevector orientations, we can access all of the correlations for the signal and idler photon in wavelength, wavevector, as well as joint wavelength-wavevector basis (Guerreiro et al. 2013). Taking into account the real pump pulse shape allows us to estimate correctly the purity in frequency.

Moreover, a main change at this point is due to the fact that we have to consider different wavevector orientations, as the pump laser is in general focused at the center of the crystal. Focusing introduces correlations in wavelength and wavevector for the emitted photon pairs. Eliminating these correlations cannot be done simply using the fact that coupling into single mode fiber (SMF) acts like a Gaussian spatial filtering. As a matter of fact, we need to find the optimal beam waist for the pump in order to maximize the spectral purity of the photons collected into single mode fibers.

Looking at the energy-wavevector correlations in Figure 1.8a, we see that these
1 How to create heralded single photons

Figure 1.7: a) Experimental setup. A 772nm ps-pulsed laser pumps a periodically poled potassium titanylphosphate (PPKTP) crystal in a double-pass configuration. Dichroic mirrors (D) separate the telecom and visible wavelengths. A variable delay is placed before the second passage of the pump laser through the crystal. The generated photon pairs are first collimated by a set of lenses (L) and then separated with polarising beam-splitters (PBS) and coupled into single mode fibres. Three characterisation techniques are shown: b) the joint spectral intensity; c) the second order autocorrelation function $g^{(2)}(\tau)$, and d) Hong-Ou-Mandel (HOM) interference.

have to be reduced in order to have an efficient coupling, while increasing the signal-idler wavevector correlation (Guerreiro et al. 2013). Thus, from this simulation we can study the influence of the spatial modes of the fields on the spectral purity and on the coupling efficiencies. Intuitively, we can see that to maximise the correlation between the wavevectors the pump beam should have a single wavevector, i.e. it should be a plane wave.

We can define a focusing parameter $\xi$ as the ratio between the half length of the crystal and the Rayleigh range of the beam $z_R$ ($\xi = L/2z_R$), and to minimize this value the waist of the pump beam should be as large as possible. In the experimental setup, we set a waist of 296 $\mu$m for the pump laser in the center of the crystal, which corresponds to a focusing parameter of 0.0425.

Once we fix this beam waist, we can extract the spatial correlations between the two photons after coupling in a single mode fiber, as a function of the collection waist. Figure 1.8a shows the spectral purity of the two photons and the spatial-spectral purity, the photon pair spectral purity and the photon’s spatial-spectral purity, as a function of the photon collection waist. For the experimental realisation,
1.5 A source of spectrally and spatially pure photons

Figure 1.8: (a) Spatial-spatial and spectral-spatial purity as a function of the collection waist. (b) Second order autocorrelation function measured for different values of the pump bandwidth.

A collection waist of 187 µm (ξ = 0.212) was chosen to find a good compromise between spectral purity and coupling efficiency. It is the spatial correlation generated by a plane wave pump which leads to this high probability of heralding a photon in the mode of an optical fiber, by projecting the first. Minimizing the spectral-spatial and spectral correlations at the same time, we can avoid the loss associated with typical filtering approaches (as we used for the PPLN source), therefore increasing the total heralding efficiency.

As sketched in Figure 1.7(b),(c),(d), and explained in the previous sections, we can use three techniques to estimate the purity of the photons produced by this source: the second order autocorrelation function (Figure 1.7(c)), the JSI (Figure 1.7(b)) and the HOM interference between uncorrelated photons (Figure 1.7(d)).

First, we can measure the second order autocorrelation function $g^{(2)}(\tau)$ in a HBT experiment, shown in Figure 1.8b as a function of the pump bandwidth. This first measurement is used to set the value of the pump bandwidth, in this case we find an optimal value for FWHM=0.33 nm, with $g^{(2)}(0) = 1.91 \pm 0.04$, corresponding to a purity of $P = 91\% \pm 4\%$.

The JSI can be reconstructed using two tunable narrowband filters, followed by APDs and a coincidence measurement setup. The result, as well as the theoretical prediction, is shown in Figure 1.9. By fitting the experimental data with a two dimensional Gaussian function we obtain a purity of $91\% \pm 3\%$. The side-peaks typical of a squared sinc function are attenuated as an effect of the spatial filtering with single mode fibres, given that the collection waist is optimised to reduce wavelength-wavevector correlations and at the same time improve the spectral
1 How to create heralded single photons

purity.

As with the JSI and the second order autocorrelation function we measured
the single mode character of the emitted photons, the Hong Ou Mandel interfer-
ence between two independent signal photons has a visibility that is only limited
by the purity, while the interference between a signal and an idler photons also
gives us information about the indistinguishability. A HOM dip is measured using
signal-signal and signal-idler photons generated in opposite directions, making two
independent sources, and the result is shown in Figure 1.10. In both cases we obtain
net visibilities of $91 \pm 2\%$, indicating that the photons are almost single mode and
that they are perfectly indistinguishable.
2 Heralded amplification of photons

Noiseless linear amplification was first proposed in 2009 by T.C. Ralph and A.P. Lund (Ralph, Lund 2009), with the purpose of amplifying coherent states without any added noise in a non-deterministic, but heralded, scheme. This is done by exploiting quantum teleportation based on single photon entanglement, which allows one to herald the teleportation of any state with a certain gain on the probability of having a photon in output with respect to the vacuum, and since it relies on interference, this can be done using only linear optical elements. This idea has found several different implementations, either by exploiting polarisation modes (Ferreyrol et al. 2010; Xiang et al. 2010) or using techniques such as single photon addiction and subtraction (Fiurášek 2009; Zavatta et al. 2011). In 2010, Gisin et al. (Gisin et al. 2010) showed that this scheme can be easily extended to discrete variables, i.e. used for heralded amplification of polarisation qubits or, in general, photonic qubits. The most straightforward application of this idea can be found in Device Independent Quantum Key Distribution (DI-QKD), where the violation of a Bell inequality is required in order to establish a secret key between two parties, without any knowledge about the devices, which can be treated as black boxes. To guarantee the security of DI-QKD protocols, it is necessary that the violation of the Bell inequality is free from any loophole, and photon amplification would in principle allow one to overcome loss and as such allows for the necessary 82.8% efficiency to be regained.

After explaining the motivations for photonic qubit amplification, I briefly describe the principle of heralded amplification of single photons, taking into account the effect of imperfections of sources and detectors on the output state. In the second section I show the experimental characterisation of a heralded single photon amplifier and in the last section I present the realisation of a time bin qubit amplifier implemented in a fiber based setup, using telecom wavelength and efficient superconducting detectors. Finally, I show the first realisation of a non-postselected heralded single photon amplifier.
2.1 Qubit amplification and the detection loophole

The realisation of a loophole free Bell test is a key element in DI-QKD for ensuring the security of a shared key without assumptions on the internal working of the devices. In particular, in long distance quantum communication, where it is easy to close the locality loophole\(^1\), the detection efficiency loophole (Pearle 1970) plays an important role.

Losses are not only present in the quantum channel, one should also consider other sources of loss such as the coupling of the photons from the source to the fibre and the efficiency of the photo detectors. Efficient coupling of photons into single mode fiber can be achieved with bulk sources, as shown in the previous chapter (Bruno et al. 2014; Guerreiro et al. 2013) and with waveguides (U'Ren et al. 2004), opening the possibility for completely integrated quantum circuits with high efficiency. Concerning the detector efficiency, big steps forward have been done since the introduction of superconducting nanowires, which have been shown to have high efficiencies in the telecom range.

Even with the very recent promising results, loss in optical fiber still puts a limit on long distance quantum communication: considering the low attenuation loss of optical fibers at telecom wavelengths, tipically 0.2 dB/km, one can easily calculate that the transmission in an optical fiber is

\[
t_f(x) = 10^{-\frac{0.2}{10}x} \quad \rightarrow \quad t_f(10\text{km}) \approx 0.63
\]

while the required value for DI-QKD is 82.8\%, when considering the CHSH inequality.

![Figure 2.1: Transmission in a SMF.](image)

Usually, when performing Bell tests, one evokes the fair sampling assumption: the set of detected particles is a fair set. In DI-QKD, this assumption cannot be valid, since a hypothetical eavesdropper can actually use the lost events to trick Alice and Bob. One typical example is that the eavesdropper can force the two measurement devices to have a result only when their settings follow a particular scheme. In this case, heralding the presence of a photon can allow the distant receiver to know when a photon has arrived, and to choose the measurement setting only after the heralding occurs. After the heralded amplification, the detection

\(^1\)This requires the two parties to be spacelike separated, so they can not influence each other measurement results.
efficiency will only depend on the efficiencies of Alice and Bob’s measurement devices, which can be optimised with present day technology.

2.2 Ralph and Lund’s noiseless linear amplifier

The fact that amplification of optical fields always comes with a price is certainly comforting, given that this doesn’t violate fundamental principles such as the uncertainty principle (Arthurs, Goodman 1988) and the no-cloning principle (Wootters, Zurek 1982). Indeed, it is easy to show (Ralph, Lund 2009) that a unitary operator that amplifies a coherent state, satisfying the relation $[\hat{a}, \hat{a}^\dagger] = 1$:

$$\hat{T}|\alpha\rangle = c|g\alpha\rangle \quad |g| > 1, |c| = 1 \quad (2.2)$$

is not possible. Consider the transformation $\hat{T}\hat{a}$ acting on a coherent state, where $\hat{a}$ is the annihilation operator for the field:

$$\hat{T}\hat{a}|\alpha\rangle = \alpha|g\alpha\rangle \quad (2.3)$$

$$= \hat{T}\hat{T}^\dagger|\alpha\rangle \quad (2.4)$$

$$= \hat{T}\hat{T}^\dagger|g\alpha\rangle \quad (2.5)$$

this implies that $|g\alpha\rangle$ is an eigenstate of the operator $\hat{b} = \hat{T}\hat{a}\hat{T}^\dagger$ with eigenvalue $\alpha$, therefore $\hat{b} = \hat{a}/g$ and $[\hat{b}, \hat{b}^\dagger] = 1/g^2$, but $[\hat{b}, \hat{b}^\dagger] = T^\dagger[a, \hat{a}^\dagger]T = T^\dagger T = 1$, which leads to a contradiction.

For some applications like quantum communication, quantum key distribution, entanglement distillation with continuous variables (CV) and metrology, the unavoidable noise that comes with amplification represents a significant limitation (Andersen et al. 2014). However, Ralph and Lund showed non deterministic, but heralded, amplification of optical fields can be achieved without noise. Single photon entanglement can be used as a resource for a noiseless linear amplifier (NLA), in a scheme that generalizes the quantum scissors (Pegg 1998). The scheme consists of splitting the initial coherent state into $N$ modes, interfering them with an ancilla prepared in an entangle state, and then coherently recombining the $N$ outputs.

Since its proposal, the NLA for CV systems has found many implementations (Ferreyrol et al. 2010; Xiang et al. 2010; Zavatta et al. 2011) together with full theoretical analysis for applications in EPR entanglement distillation (J. et al. 2014). Alternative schemes for simulating the effect of heralded amplification in post-selection with a measurement based scheme have been proposed and experimentally realised ( Chrzanowski et al. 2014).
2.3 Principle of the heralded amplification of photons

As already mentioned, a heralded photon amplifier acts on its input state, reducing the weight of the vacuum component with respect to the single photon one. In the case of discrete variable (DV), the protocol is conceptually very simple.

Figure 2.2 shows the concept. The probability $P_{in}$ of having an input photon can be measured as in Figure 2.2.a. A beam splitter with variable transmission $t$ is used to prepare an ancillary (aux) photon in an entangled state distributed on two spatial modes, which is used for the teleportation. A successful Bell state measurement (BSM) on the input qubit and the reflected mode heralds the output state: this consists of a balanced beam splitter followed by a single photon detector.

Let’s consider an input state that is in a mixture between the vacuum $|0\rangle$ and a photon $|1\rangle$, with weights depending on the losses the photon went through. More precisely, naming $P_{in}$ the transmission of a quantum channel, the state of a single photon sent throughout it becomes

$$\rho_{in} = (1 - P_{in})|0\rangle\langle 0| + P_{in}|1\rangle\langle 1|.$$

Figure 2.2: Scheme of the experimental setup. a) First, one should measure the probability $P_{in}$ of having an input photon; b) then, the amplification is performed combining the input state with the reflected output of a non balanced beam splitter with splitting ratio $t/(1-t)$, a click in detector D1 heralds the presence of a photon in the transmitted arm; c) to check that the process is coherent, we can imagine to save the "lost" part of the input state and recombine it with the output to observe interference fringes.

Assuming that the input photon and the auxiliary photon are completely indistinguishable in all degrees of freedom and the detector resolves the number of
photon component with respect to the vacuum has changed, and it depends on the beam splitter transmission \( t \). We can define the gain of the amplifier as the ratio between input and output probability:

\[
G(t) = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{g^2(t)}{1 - P_{\text{in}} + g^2(t)P_{\text{in}}} = \frac{t}{(1-t)(1-P_{\text{in}}) + tP_{\text{in}}},
\]

where \( g^2(t) = t/(1-t) \). Clearly, in the ideal case, the value of the gain obtained when using a maximally entangled state as a resource (obtained with \( t = 1/2 \)), should be \( G(t = \frac{1}{2}) = 1 \), i.e. it is simply a teleportation of the input state. Only for \( t > \frac{1}{2} \) we expect to have a positive gain, greater than 1. We will see that in real experimental conditions, with non photon number resolving (PNR) detectors and losses in the setup, this is not necessarily the case.

For infinite losses (\( P_{\text{in}} \to 0 \)) the gain tends to \( t/(1-t) \), which is infinity as \( t \to 1 \). On the contrary, the heralding probability is inversely proportional to the losses, which means that there is a limit in which the amplifier does not represent an advantage anymore.

To check the coherence of photon amplification, we can imagine to save the "lost" part of the input state and recombine it with the output on a beam splitter to observe single photon interference fringes. (Figure 2.2.c).

### 2.4 Taking into account experimental imperfections

We want to estimate and measure the gain of the photon amplifier. To do that, it is useful to look at the experimental setup with all the imperfections that affect the amplification: transmission losses, non-unitary detection efficiency and non photon number resolving detectors. If we define \( P_a \) as the probability of having an ancillary photon, considering an efficiency \( \eta \) for the detector used in the BSM, the output state of the amplifier can be written as:

\[
\rho_{\text{out}} = \frac{P_0|0\rangle\langle 0| + P_1|1\rangle\langle 1|}{P_0 + P_1}
\]

where

\[
\begin{align*}
P_0 &= (1 - P_{\text{in}})P_a(1-t)\eta + P_{\text{in}}(1 - P_a) \eta + P_{\text{in}}P_a(1-t)(2\eta - \eta^2) \\
P_1 &= P_{\text{in}}P_a t \eta,
\end{align*}
\]
and as a consequence the gain $G(t) = P_{\text{out}}/P_{\text{in}}$ will be:

$$G(t) = \frac{P_a t}{\eta P_{\text{in}} P_a - \eta P_{\text{in}} P_a + P_{\text{in}} + P_a}$$  \hspace{1cm} (2.10)$$

Figure 2.3 shows the gain of the photon amplification as a function of the beam splitter’s transmission $t$. For perfect coupling and detection efficiency, the gain reaches the maximum possible value, which is limited by the fact that the probability of having a photon in output can not be bigger than one: $G_{\text{max}} = 1/P_{\text{in}}$ (see eq. 2.7). For imperfect detection efficiency at the level of the BSM, the curve doesn’t change much, while the dependence on the coupling efficiency of the auxiliary photon is almost linear, as expected.

Having non PNR detectors is accounted in the term $P_{\text{in}} P_a (1 - t)(\eta^2 - 2\eta)$ in $P_0$, that describes the case in which the input photon is not lost but the ancilla is reflected with probability $(1-t)$. If the detector cannot resolve the photon number, it will count an event given by two photons with probability $(1-(1-\eta)^2) = (2\eta-\eta^2)$, and this increases the probability of heralding the vacuum. The result is that for $t = 1/2$, which in the ideal case corresponds to a teleportation, the protocol reduces to a coherent attenuation.
To characterize the behaviour of a single photon heralded amplifier as a function of the input probability and of the beam splitting ratio $t/(1 - t)$, we will consider a simpler experimental setup, that exploits polarisation modes instead of spatial modes, as in (Kocsis et al. 2013). A more detailed explanation can be found in reference 2 (see list of publications). Figure 2.4 shows the two schemes. The standard photon amplifier consists of two beam splitters, one for the preparation of the single photon entanglement and the other one for the BSM. With polarisation modes we can use half waveplates followed by polarisation beam splitters to choose the splitting ratio ($\text{HWP}_1$ and $\text{PBS}_1$), the input probability ($\text{HWP}_2 + \text{PBS}_1$) and to check the coherence without any need for stabilizing the phase between the "lost" part of the state and the teleported one, which go through the same path but in two orthogonal polarisation modes. On the contrary, the scheme shown in 2.2.c requires phase stabilization, as it involves two separated spatial modes.
Heralded amplification of photons

Figure 2.5: Experimental gain (left) and visibility (right) vs losses and $t$. The measurements are taken scanning the angle of HWP$_1$ to change $t$ and HWP$_2$ to change the losses of the input photon. The insets show the theoretical predictions, in good agreement with the experimental result.

For simplicity, in this experiment, the two photons belong to the same pair, generated via SPDC in a PPLN crystal, previously described in this Thesis. See (Bruno et al. 2013) for more experimental details.

This scheme allows us to easily reconstruct the theoretically predicted curves, as shown in Figure 2.5 on the left. The gain increases as the transmission increases, due to the fact that we are amplifying the state more and more. However, for a fixed value of $t$, the gain also increases as the losses of the input state increase. As explained above, the maximum gain will tend to infinity as the input probability tends to zero, since they are inversely proportional.

Looking at Figure 2.4 one can see that the lost part of the input photon is actually transmitted by PBS$_1$, so it goes through the same path as the output photon, but with an orthogonal polarisation. To combine the two modes, we can use a half waveplate followed by a PBS ($HWP_3 + PBS_2$), observe fringes and calculate the visibility. This will give a measure of the fidelity of the heralded amplified photon.

The visibility (Figure 2.5, right) decreases as the losses and the transmission are not balanced, as one expects when looking at a Mach Zehnder interferometer with unbalanced intensities in the two arms. As the measured fringes match the theoretical prediction, we can consider them a proof of coherence. Only when the intensity in the two interfering modes are balanced we can use the visibility as a measure of fidelity.

The probability of having a photon once a successful BSM is announced, is shown in Figure 2.6 as a function of the input loss, for eleven different values of $t$. The upper horizontal axis shows the equivalent of fiber length in km, corresponding to each amount of loss. The probability is renormalized by the coupling of the auxiliary photon into SMF and by the efficiency of the detector, to show the intrinsic performance of the amplifier itself. The transmission of the optical components doesn’t prevent the output probability from reaching the required value of 82.8%. 
2.6 Heralded single photon amplification without post selection

Heralded single photon amplification was realised (during this thesis work) in a fiber based setup in (Osorio et al. 2012) and fully characterised in bulk optics and polarisation modes (Bruno et al. 2013), but in both of these experiments the input and the auxiliary photons were twin photons coming from the same pair in a SPDC process. This means that a successful amplification can be obtained only after post selection on a coincidence count, because the two single photons are not independently heralded. For a heralded single photon amplifier without post selection, two independent heralded single photon sources are needed, which shall also be indistinguishable in all degrees of freedom. This is experimentally challenging, since at least one of the two indistinguishable heralded single photons, the ancilla, should also have a very high coupling in order to guarantee a high gain. The SPDC source based on a bulk PPKTP crystal presented in section 1.5 is a very good candidate for performing such kinds of experiments: it produces photons at telecom wavelengths, the single (spectral and spatial) mode character of the photons ensure a high coupling efficiency, while the pulsed aspect helps the synchronisation.

Another important aspect that must be taken into account is related to the detection scheme: on one side, on a practical level, commercial APDs have a maximum efficiency of 25% with some kHz of dark counts and a dead time of the order of the µs, which limits the rates of four fold coincidences counts. On the other side, which
Figure 2.7: Complete experimental setup for non post-selected single photon heralded amplification.

is more important on a fundamental level, even after amplifying the probability of having a photon, violating a Bell inequality free from any loophole is not an achievable task with a low detection efficiency. A solution to this problem is to use superconducting nanowire single photon detectors (SNSPD), which have been shown to have high efficiencies at telecom wavelengths and low recovery times, together with low dark count rates. In this work, we used MoSi based SNSPDs with an efficiency of $\sim 70\%$ at 1550nm and a few hundred dark counts per second at $T < 2.5\text{K}$.

With a source of two pure and independent heralded single photons and efficient detectors, non post selected HPA can be performed. The setup is depicted in Figure 2.7: a PPKTP crystal produces the two photon pairs in a double pass configuration, then one photon is sent through a channel with variable transmission and the other acts as the ancilla. The gain and the output probability $P_{out}$ are measured as a function of the input probability $P_{in}$, and the result is shown in Figure 2.8.

This result represents the first unconditional heralded amplification (when the gain reaches unity, we can talk about teleportation) of a Fock state qubit (Figure 2.9). Indeed, if we consider a heralded single photon prepared, using a beam splitter, into an entangled state over two spatial modes:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$$

(2.11)

and send one arm through a fiber at a distance, amplifying the state after the lossy channel via a HPA allows one to retrieve the original entangled state. Quantum correlations between two separated spatial modes of the electromagnetic field have been shown via interference (Lombardi et al. 2002) and homodyne measurements (Hessmo et al. 2004). With the possibility of violating a Bell inequality based
2.6 Heralded single photon amplification without post selection

on vacuum-one photon qubits, the feasibility of the unconditional amplification of a qubit in this basis represents a promising step forward towards DI-QKD and quantum communication at a distance.

Unfortunately, this result shall be taken as yet another proof of principle: the output probability shown in Figure 2.8 doesn’t surpass the 25%. This is due to the fact that the experimental setup was effectively optimised for a heralded qubit amplification, which will be presented in the next section. Therefore, the limitation on $P_{\text{out}}$ comes only from the losses on the heralded ancilla, which, for future experiments, can be easily overcome.

Figure 2.8: Top: gain of the heralded single photon amplifier, with $t = 0.9$. Bottom: Output probability for the heralded single photon amplifier. Solid lines are calculated taking in account the measured transmission losses, dashed lines are calculated assuming $P_a = 0.9$.

Figure 2.9: Schematic setup for a Fock state qubit amplifier where the input qubit is part of a vacuum-one photon entangled state over two spatial modes.
2.7 Heralded amplification of qubits

The idea of a heralded photon amplifier can be extended to qubits, by repeating the amplification stage for each of the two involved modes. Figure 2.10 shows the scheme of a polarization qubit amplifier as an example.

![Figure 2.10: Scheme of the polarization qubit amplifier (right) compared with the single photon amplifier (left). The qubit amplifier can be thought as a doubled photon amplifier, as two orthogonal polarization modes are coherently amplified in the same setup. A heralded single photon is prepared in a polarization qubit, then sent through a lossy channel. Heralded qubit amplification is performed exploiting two ancillary photons, prepared in an entangled state on an unbalanced beam splitter. A successful BSM heralds the presence of a qubit in the output mode.](image)

The principle of heralded qubit-amplification can be summarized as follows. A heralded single photon is prepared in a time-bin qubit of the form

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}} (|s_v\rangle + e^{i\phi}|\ell_H\rangle)$$

(2.12)

(where $s_v/\ell_H$ means short, vertical/long, horizontal) and then sent through a channel with transmission $P_{in}$. Its state will become a statistical mixture between one photon carrying the qubit and the vacuum state:

$$\rho_{in} = (1 - P_{in})|00\rangle\langle 00| + P_{in}|\psi_{in}\rangle\langle \psi_{in}|.$$  

(2.13)

We call it $\rho_{in}$ as this will be the input state of the amplifier. A pair of auxiliary photons (ancillae) in the product state $|s_v\rangle|\ell_H\rangle$, which will result in an entangled state when sent on a beam splitter with transmission $t$, is needed to perform the amplification. The qubit amplification consists of a teleportation protocol with a gain $G$, achieved via a BSM between the mode in input and ancilla (see Figure 2.10)
which heralds the output state in a mixture between the vacuum and the initial qubit, up to a unitary transformation, with different relative weights:

\[
\rho_{\text{out}} = (1 - G(t)P_{\text{in}})|00\rangle\langle 00| + G(t)P_{\text{in}}|\psi_{\text{in}}\rangle\langle \psi_{\text{in}}|
\]

(2.14)

\[
G(t) = \frac{t}{(1 - P_{\text{in}})(1 - t) + tP_{\text{in}}}
\]

(2.15)

This equations describe the ideal case, with lossless auxiliary photon source and detectors with unit efficiency which resolve the number of photons. Even though there have been good results both in efficiently heralding of single photons produced in spontaneous parametric down conversion and in highly efficient detectors (superconducting) at telecom wavelengths, the lack of deterministic single photon sources and of PNR detectors must be taken into account when comparing experimental data with theoretical predictions. If one takes into account non PNR detectors with efficiency \(\eta\) and a probability \(P_{a}\) of having an ancilla (for simplicity, we consider \(P_{a}\) to be the same for both auxiliary photons), the output probability can be calculated considering all the possible events that result in a succesful BSM:

1. the input qubit enters the amplifier with probability \(P_{\text{in}}\), the two auxiliary photons are present with probability \(P_{a}^{2}\), one is reflected \((1 - t)\) and the other is transmitted \((t)\) at the unbalanced beam splitter, therefore a coincidence in the BSM occurs with probability \(\eta^{2}\), with a factor of 1/2 due to the fact that half of the times the photons are indistinguishable and they interfere on the 50/50 beam splitter, compensated by a factor of 2 due to the fact that this events can occur in two configurations (i. e. the short photon is transmitted and the long is reflected or viceversa); the probability of this event is the product of all the terms: \(P_{\text{in}}P_{a}^{2}(1 - t)t\eta^{2}\);

2. the input qubit is lost with probability \(1 - P_{\text{in}}\), but the two auxiliary photons are both reflected, giving a coincidences which is accounted as a BSM, with a total probability \((1 - P_{\text{in}})P_{a}^{2}(1 - t)^{2}\eta^{2}\);

3. the case in which the input qubit enters the amplifier, but one of the two ancillae is lost with probability \(1 - P_{a}\) while the other is reflected, also gives a coincidence with probability \(P_{\text{in}}P_{a}(1 - P_{a})(1 - t)\eta^{2}\);

4. finally, \(P_{\text{in}}P_{a}^{2}(1 - t)^{2}\eta^{2}(2 - \eta)\) is the probability of the event in which the three photons are all directed to the BSM, and because of the fact that the detector don’t resolve the number of photons, one of the two detectors will click with probability \(1 - (1 - \eta)^{2}\), announcing a succesful BSM which, however, heralds the presence of the vacuum in the output mode, decreasing the effective gain.

When considering all these events, the output probability of the amplifier is
written as:
\[ P_{\text{out}} = G(t) P_{\text{in}} \]
\[ = P_{\text{in}} P_a^2 (1 - t) t \eta^2 / \left( P_{\text{in}} P_a^2 (1 - t) t \eta^2 + (1 - P_{\text{in}}) P_a^2 (1 - t)^2 \eta^2 + P_{\text{in}} P_a (1 - P_a) (1 - t) \eta^2 + P_{\text{in}} P_a^2 (1 - t)^2 \eta^2 (2 - \eta) \right) . \] (2.16)

The gain is defined as the ratio between the probability of having a photon in the output given a successful BSM, \( P_{\text{out}} \), and the probability of having a qubit in the input mode, \( P_{\text{in}} \):
\[ G(t) = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_a t}{P_a (1 - t)(1 - P_{\text{in}} \eta) + P_{\text{in}}} . \] (2.17)

### 2.8 Experiment

For the experimental realisation of a heralded qubit amplifier, one needs to produce three pure and indistinguishable heralded single photons: one qubit and two auxiliary photons. The three photons have to be in a pure state in order to interfere. Therefore, this experiment would require to have six photons. In addition, losses should be minimised due to the fact that the gain is sensitive to the local loss of Bob’s auxiliary source\(^2\). Six photon experiments have been already successfully performed (Lu et al. 2007), but in order to demonstrate qubit amplification it’s only necessary to use four photons and post select on a four-fold coincidence event.

For this purpose, the SPDC source based on a PPKTP crystal described in the previous chapter (1.5) is an optimal choice, since it produces four pure single photons at telecom wavelength, with high coupling efficiency. In the following, we will see the experimental realisation of a fiber based time-bin qubit amplifier (a paper is in preparation). The complete setup is shown in Figure 2.11. Furthermore, for this work, MoSi superconducting nanowire single photon detectors (SNSPD) were used, with an efficiency of 70% at \( T < 2.5K \) and fast recovery time (~80 ns).

A heralded single photon can be prepared in a time bin qubit state of the form shown in eq. 2.12 with the aid of a 50/50 beam splitter and a fiber delay to differentiate the long and short arms, which can then be recombined in the same mode using a PBS. In this way, the two temporal modes are encoded in two polarisation modes, which is useful for practical reasons in order to perform the BSM with no additional loss. Moreover, having two polarisation modes allows us to change the phase between long and short paths after the interferometer, so that the same interferometer can be used for the analysis, to avoid active stabilisation of the setup.

As we can work in polarisation, in order to be able to distinguish two out of four Bell states in a BSM with linear optics, two PBS followed by two detectors

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\(^2\)For simplicity, we can consider that Alice is the one who sent the input qubit and Bob wants to amplify it in order to violate a Bell inequality at a distance.
Figure 2.11: Complete experimental setup for the amplification of time bin qubits. A heralded single photon is prepared in a time-bin qubit and subsequently sent through a lossy channel. The amplification is heralded by a 3-fold coincidence given by a click in the detector D1, which heralds the input photon, followed by D2 and D3, heralding its teleportation. 4-fold coincidence with the output photon in detector D4 are recorded. We use MoSi SNSPDs with 70% detection efficiency at 1550nm, at a temperature $T < 2.5$ K. Two polarisation modes that accompany the two temporal modes, short and long, allow one to separate the $\ell$ and $s$ components in the analysis without further loss. A temporal delay introduced in one of the outputs of the BSM allows one to project the input modes onto two Bell states ($\Psi^\pm$) using two detectors instead of four. The analysis is performed in the same interferometer used for the preparation, with the aid of a fibred circulator. In this way, active stabilisation of the phase is not needed. The relative phase in the input qubit is controlled after the interferometer, taking again advantage of the two polarisation modes, using a variable liquid crystal retarder.

Each (one per output) are necessary, resulting in four detectors (see Figure 2.4). Another solution, depicted in Figure 2.11, is to introduce a delay on one output of the 50/50 BS, splitting the two polarisations with PBSs and then combine the paths with horizontal polarisations and the paths with vertical polarisation using two additional PBSs. At this point, only two detectors are necessary: one for each polarisation. The arrival times of the photons in the detector are two, due to the fiber delay added on one output of the BS. This allows us to use two detectors instead of four.

The amplification is heralded by a 3-fold coincidence given by a click in the detector D1, which heralds the input photon, followed by D2 and D3, heralding a successful BSM. The probability of having a qubit $P_{in}$ is measured dividing the number of coincidences between detectors D1 and D2 by the number of counts.
in detector D1. The output probability $P_{\text{out}}$ is given by the four fold coincidence between the trigger D1, a successive BSM heralded by a coincidence in D2 and D3, and a click in D4, divided by the three fold heralding coincidence D1-D2-D3. To measure the fidelity of the output state, defined as $F = \langle \psi_{\text{in}} | \rho_{\text{out}} | \psi_{\text{in}} \rangle$, we project the output state on the input.

### 2.8.1 Results

![Figure 2.12: Four fold coincidences as a function of $\Delta \phi$. The red curve corresponds to $\Psi^-$, and the visibility is $0.98 \pm 0.02$. The blue curve corresponds to the other possible event, $\Psi^+$, and the visibility is $0.93 \pm 0.02$. The difference in visibility is due to small polarisation dependent loss in the setup, while some birefringence caused a little phase shift, which can be joined to the unitary transformation needed to retrieve the qubit at the output. The coincidence rate is of 10 counts/minute for each of the two curves. For this measurement $t = 0.7$.](image)

Figure 2.12 shows the fringes observed in the four fold coincidence counts when varying the phase between long and short. The possible outputs of the BSM are two: the red curve corresponds to the case in which the output of the BSM is $\Psi^-$, while the blue curve, with opposite phase, corresponds to $\Psi^+$. This is because the output $\Psi^+$ requires a $\sigma_z$ operation in order to retrieve the initial qubit state. The fringes represent the overlap of the output state with the input, which defines the fidelity.

The gain of the amplifier was measured as a function of $P_{\text{in}}$ for two different values of $t$: 0.7 and 0.9, and the result is shown in Figure 2.13(top), together with the theoretical prediction. The output probability $P_{\text{out}}$, i.e. the probability of having a photon in output once a successful BSM is heralded, is plotted in Figure 2.13(bottom) as a function of $P_{\text{in}}$. This is the most important quantity to look at. Indeed, the gain gives us a measure of how much the qubit component was amplified, while
2.9 Discussion and conclusion

To enable DI-QKD, it is necessary to have an output probability that allows one to close the detection loophole: 82.8%, represented by a green line in Figure 2.13.

It is good to remind at this point that the limit of 0.828 for closing the detection loophole is related to the CHSH inequality.

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Figure 2.13: **Top**: Gain as a function of the input probability $P_{\text{in}}$. The red and blue curves correspond to $t = 0.7$ and $t = 0.9$, respectively. Black corresponds to $t = 0.99$. Full lines take in account the measured transmission loss for the ancillae, dashed lines are calculated assuming the best case scenario in which the probability of having an ancilla is $P_{\text{a}} = 0.9$. **Bottom**: Output probability as a function of the input probability $P_{\text{in}}$. Red, blue and black correspond to $t = 0.7$, $t = 0.9$ and $t = 0.99$, respectively. Full lines take in account the measured transmission loss for the ancillae, dashed lines are calculated assuming the best-case scenario in which the probability of having an ancilla is $P_{\text{a}} = 0.9$. The colored area indicates where the gain becomes higher than 1, i.e., one actually has amplified the input state. Above the green line, the detection loophole is closed. Error bars get larger as the function describing $P_{\text{out}}$ approaches a critical point in the limit of $P_{\text{in}} \rightarrow 0$.

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3It is good to remind at this point that the limit of 0.828 for closing the detection loophole is related to the CHSH inequality.
For this value, in the ideal situation (black dashed line) the amplifier represents a real advantage for DI-QKD, but only in the best-case scenario in which the auxiliary source emits photons with $P_a = 0.9$, and the detectors are non PNR. However, there are some relaxations on this limit, for example it has been proven that when using a non maximally entangled state the required efficiency goes down to 66.7% (Eberhard 1993), or to 50% when optimizing the number of measurement settings (Wilms et al. 2008).

For the time-bin qubit amplifier, the fibre based setup at telecom wavelengths, the absence of narrow spectral filtering and the high coupling of the photons into SMF are key elements for an efficient qubit amplification protocol. Even though the total transmission of the ancillae is still 29.6%, representing the main limit on the performances of the device, this is not due to the amplifier itself, i.e. the unbalanced beam splitter together with the BSM. Indeed, the main source of loss for the ancillae is a combination of polarisation beam splitters and a delay line needed to synchronize the arrival times of the photons at the amplifier. Improving the transmission of the ancilla is a challenging but realistic task (Guerreiro et al. 2013). Moreover, these losses could be significantly reduced in an integrated optical circuits (Martin et al. 2012) which would ideally contain both the auxiliary sources and the unbalanced beam splitter, and possibly even detectors for the BSM.

A modification in the scheme proposed in (Pitkanen et al. 2011) allows one to improve the gain, eliminating the case in which two auxiliary photons are reflected. However, this scheme is more demanding from an experimental point of view, since it requires to interfere the two auxiliary photons after the unbalanced beam splitter such that they merge in the same mode. While this approach has advantages, it would be far more practical in an integrated chip implementation where the phase could be more easily controlled. Figure 2.14 shows a possible experimental implementation of this scheme, for the amplification of polarisation qubits: the

$$|\psi\rangle_{in} = \alpha |H\rangle + \beta |V\rangle$$

$$|H\rangle \otimes |V\rangle$$

Figure 2.14: Scheme of the qubit amplifier proposed by (Pitkanen et al. 2011)
auxiliary photons are prepared in the state $|H\rangle \otimes |V\rangle$ and sent on the unbalanced beam splitter. A half wave plate at $22.5^\circ$ turns the polarisation in the diagonal basis, and when followed by a PBS it is equivalent to a HOM interference:

$$|H\rangle \otimes |V\rangle \longrightarrow |D\rangle \otimes |A\rangle = \frac{1}{2} (|H\rangle \otimes |H\rangle - |V\rangle \otimes |V\rangle)$$ (2.18)

where $|D/A\rangle = (|H\rangle \pm |V\rangle)/\sqrt{2}$ are the diagonal and antidiagonal linear polarisations. So, when the two auxiliary photons are both reflected at the unbalanced beam splitter, the HOM interference prevents them from heralding the vacuum state by simulating a successful BSM. Since it is necessary to split the two ancillae and then recombine them, this method requires the stabilisation of the phase between the two paths.

Finally, the same setup allowed us to implement, for the first time, the unconditional heralded teleportation and amplification of a single photon in a fiber based scheme at telecom wavelengths. A crucial importance in this experiment should be attributed to the PPKTP source of pure photons and to the high efficiency and low recovery time of the MoSi SNSPDs.
3 Displacing single photon entanglement in the macroscopic domain

There are well defined rules to go from quantum systems to their classical analog, in the limit of $\hbar$ small with respect to the action. In general, this corresponds to an increase in size, passing from a microscopic to a macroscopic system. However, macroscopicity is not always synonymous of classical: consider Bose Einstein condensation, where a large number of particles cooled down to their ground state may exhibit quantum properties, or superconductivity. With present day technology we are able to study these kinds of macroscopic quantum systems, but probing quantum theory in the macroscopic domain using much simpler tools like linear optics is fascinating and it represents an interesting challenge.

In this chapter, I report on an experiment in which entanglement is shown in a system involving a large number of photons, large enough to be seen by the human eye. Single photon entanglement is again the starting point of this experiment: a photon is delocalized over two spatial modes, one of which is then displaced in the phase space. As a result, the state is a superposition of two orthogonal components populated by a large number of photons. This system is analogous to an optical Schrödinger cat state: using a classical detector that can only resolve large photon number differences, the two macroscopic components can be efficiently distinguished. This is explained in the first section. In the second section, I describe the experimental implementation: light sources, preparation and analysis. Finally I will present and comment on the results.

Pictorial representation of single photon entanglement and micro-macro entanglement.
Credits to Lavinia Parlamenti Photographer and Emmy.
No animals were harmed in the making of this work.
3.1 From micro to macro

One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter, there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer that shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts. It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself, it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

Erwin Schrödinger, Die gegenwärtige Situation in der Quantenmechanik (The present situation in quantum mechanics), Naturwissenschaften

Erwin Schrödinger seemed skeptical about the possibility of having a macroscopic object in a quantum state, even though it is thanks to their quantum mechanical behaviour that the constituents of light and matter make up every macroscopic object in the universe that surrounds us. What Schrödinger, and Einstein together with him, refused to accept is the fact that reality could be represented by a superposition state. In Bose Einstein condensates and, in general, in superfluid or superconductive materials\textsuperscript{1}, the quantum description of the system may refer to many microscopic effects that coherently add up on a macroscopic scale. In that sense, the detection of a photon by a SNSPD (used for some experiments in this thesis) is itself an indirect observation of the quantum-to-classical transition of a macroscopic, despite its quite small size, system. The boundary between quantum and classical behaviour is simply the critical temperature of the superconductor. Here I name \textit{classical} a system whose description doesn’t need quantum mechanics.

A different question is whether a single macroscopic degree of freedom can be prepared in a quantum superposition (Leggett 1980; M. et al. 1987). For this, one needs a clear definition of what \textit{macroscopic} means. In Schrödinger’s Gedankenexperiment, there is a \textit{microscopic} state that describes the possibility that one atom decays or none does, which via a local unitary evolution becomes a \textit{micro-macro} superposition:

\[
\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad \rightarrow \quad \frac{1}{\sqrt{2}}(|0\rangle|\text{alive cat}\rangle + |1\rangle|\text{dead cat}\rangle)
\]

where the microscopic state of the atom that decays or not is now entangled with the health state of the cat. This state has two main features: it involves a large

\textsuperscript{1}Bose Einstein condensates may manifest both superfluidity and superconductivity.
3.1 From micro to macro

number of particles (the cat) and the two components are classically distinguishable. Sekatski et al. (Sekatski et al. 2012) proposed a simple experiment that allows to create a Schrödinger cat-like state with these features using accessible tools like single photons, coherent states and linear optics.

Following the proposal in (Sekatski et al. 2012), let’s consider a single photon delocalized over two spatial modes:

\[ |\psi\rangle_{ab} = \frac{1}{\sqrt{2}}(|0\rangle_a|1\rangle_b + |1\rangle_a|0\rangle_b) \]  

Then, one of the two modes is displaced in the phase space, acquiring a large number of photons. This is done applying a displacement operator \( D_a(\alpha) = e^{\alpha a^\dagger - \alpha^* a} \), where \( a^\dagger, a \) are the creation and annihilation operators of the field on the displaced mode. In practice, this means interfering one mode of the delocalized photon and a coherent state \( |\alpha/\sqrt{r}\rangle \) on a beam splitter with reflectivity \( r \), as depicted in Figure 3.1 (Paris 1996).

![Figure 3.1: Scheme for the generation of micro macro entanglement starting with an entangled single photon, which is subsequently displaced in the phase space.](image)

This is a local unitary transformation that changes the number of particles, but not the amount of entanglement, and the state becomes:

\[ |\psi\rangle_{ab} = \frac{1}{\sqrt{2}} D_a(\alpha) (|0\rangle_a|1\rangle_b + |1\rangle_a|0\rangle_b) = \frac{1}{\sqrt{2}} (|\alpha\rangle_a|1\rangle_b + D_a(\alpha)|1\rangle_a|0\rangle_b). \]  

This state is a quantum superposition delocalized over two optical modes, that can be for example two separated optical fibers, and that involves a microscopic part on one mode, meaning one photon or none, and a macroscopic part, the size of which depends on the value of \( |\alpha|^2 \), on the other mode, in two possible orthogonal states: a coherent state and a displaced single photon (\( \langle 0|D_a^\dagger(\alpha)D_a(\alpha)|1\rangle = 0 \) since \( D_a(\alpha) \) is unitary).

These two states have very close mean number of photons, but their distributions in the photon number are quite different: in general, the variance of the number operator when applied to a displaced number state \( D_a(\alpha)|n\rangle \) is:

\[ \langle (\Delta n)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2 = (2n + 1)|\alpha|^2, \]
so the variance of a coherent state is $|\alpha|^2$, while the variance of the displaced single photon we is three times bigger, $3|\alpha|^2$ (Oliveira et al. 1990). One way of discriminating these two states is based on measuring the photon number with a threshold detector, and then one could distinguish the two states with a maximal probability of $\sim 0.74$. If one looks at the state in the diagonal basis, the distribution are even more distinct, see Figure 3.2.

\[ P(n) \]

Figure 3.2: Photon number distribution for the coherent state (purple) and the displaced single photon (blue) in the \(|\{0\}, |1\rangle\rangle\) (left) and \(|\{0\} + |1\rangle, |0\rangle - |1\rangle\rangle\) (right) basis.

In (Sekatski et al. 2014), a work done in our group in Geneva, Sekatski et al. proposed a criterion that defines whether a quantum superposition is macroscopic, based on the possibility of discriminating the two macroscopic components (dead and alive cat) with a classical measurement. In the case of an optical state, the measurement is done with a photon counter with limited resolution. Comparing an optical superposition with the superposition of two Fock states differing by $N$ photons, one can quantify the minimum amount of noise that can be tolerated in order to distinguish the two components.

In order to reveal entanglement in the described optical system, a simple way proposed in (Sekatski et al. 2012) is to displace the state back to the single photon level, i.e. where we have well defined measurements of entanglement, via an operation $D_a^\dagger(\alpha) = D_a(-\alpha)$. Then one can measure a lower bound the concurrence\(^2\)

\[ C > V(p_{01} + p_{10}) - 2\sqrt{p_{00}p_{11}} \]

(3.5)

where $V$ is the single photon interference visibility, $p_{ij}$ is the probability of having $i$ and $j$ photons in the modes A and B (see Figure 3.1), respectively.

### 3.2 Experimental implementation

We can describe the experimental setup in three main steps: the light sources, the state preparation and the state analysis.

\(^2\)A positive concurrence indicates the presence of entanglement in states who live in a bidimensional Hilbert space.
3.2 Experimental implementation

3.2.1 Heralded single photon source

For the heralded single photon source (HSPS) a pulsed laser at wavelength 780 nm, 76 MHz repetition rate, pumps a 2 cm PPLN crystal generating a pair of photons at telecom wavelength by SPDC, with type II quasi-phase matching conditions. The two photons are then separated by a PBS and coupled into single mode fibers with 50% heralding efficiency. The strong spectral correlation of our photons doesn’t allow us to work without any filtering, but we can choose to filter only the heralding photon down to 26 GHz (narrow enough to remove the correlation). In this way we have the trigger (signal, 1563 nm) photon in a pure spectral state, therefore the idler photon (1557.5 nm) is heralded in a spectral mode with the same degree of purity, and with the pump’s bandwidth (80 GHz). With this method the losses on the heralded photon are minimized.

Figure 3.4: Left: Joint spectrum of the photons generated in the PPLN crystal. Right: spectrum of the post-selected single mode, after placing a narrowband filter on the trigger photon.

3.2.2 Coherent state source

For the displacement, we need interference with a coherent state that is in the same mode as the HSP. This kind of interference has historically always been realized
doubling the frequency of a laser and then down-convert it and using the same laser as a local oscillator (Lvovskiy et al. 2013a; Pittman, Franson 2003; Pittman et al. 2005; Rarity et al. 2005). Moreover, the use of femtosecond pulses allows one to have a broad spectrum, making it easier to filter both the photon and the local oscillator to eliminate any remaining distinguishability. In our case, we want to avoid filtering on the HSP, as it introduces losses that inevitably perturb the vacuum-single photon entangled state. For this reason, the scheme for the generation of the coherent state is slightly different than the common one.

The coherent state source is based on the same pump laser used in the HSP source, which impinges on a PPLN crystal together with a CW laser at the same polarization and wavelength as the signal photon in the HSPS source. Emission of pulsed light at 1557.5 nm, with 80 GHz bandwidth, is stimulated by difference frequency generation and coupled into a SMF fiber. Figure 3.5(Right) shows the second order autocorrelation function of the generated state as a function of the power in the CW seed laser, for a fixed pump power. The measurement is done with a narrowband filter placed before the beam splitter, in order to remove the other modes generated by the parametric process, which decrease the value of the $g^{(2)}(0)$. One can see that when the laser is off, the only contribution comes from SPDC light which exhibits thermal statistics, while increasing the power the statistics tends to a Poissonian distribution. The measured $g^{(2)}(0) = 1$ confirms that the state has a poissonian statistics. As the process is stimulated, we can write the second order autocorrelation function as follows:

$$g^{2}(0) = 1 + \frac{N_{SP}}{KN_{SP} + N_{ST}},$$

where $N_{SP}$ and $N_{ST}$ are the number of photon per mode emitted by spontaneous emission and the number of photons emitted by stimulated emission, $K$ is the
number of Schmidt modes of the down converted photons. When only one mode is stimulated, \( N_{ST} \) is given by \( N_{ST} = N_{s}N_{SP} \) (Polyakov, Migdall 2009; Sanguinetti et al. 2012), with \( N_{s} \) corresponding to the number of photons in the seed field. So, we obtain:

\[
g^2(0) = 1 + \frac{1}{N_{s} + \mathcal{K}}. \tag{3.7}
\]

This can also be seen from Figure 3.6, where the spectrum of the generated light is plotted for different values of the power of the CW pump.

Figure 3.6: Left: spectrum of the DFG light for different powers of the CW pump. Right: intensity of the DFG light as a function of the pump power. The power of the DFG is not linear with the seed power, this can be due to a saturation of the detectors, and a small effect of pump depletion, typical for powers above 50mW.

Figure 3.7: Left: spectrum of the HSP. Right: spectrum of the DFG.

To prove the indistinguishability of the two sources (wavelength, polarization, spatial and spectral mode) we can measure a HOM dip (Hong et al. 1987a; Pittman, Franson 2003; Pittman et al. 2005; Rarity et al. 2005) on a 50/50 beam splitter. Figure 3.7 shows that the spectrum of the HSP and of the DFG are identical, single mode fibers select a single spatial mode and polarisation controllers allow us to
eliminate distinguishability in polarisation. When the two states are equal in all
the degrees of freedom, the visibility is limited by the statistics of the sources, in
particular by the probability of having two photons coming from the same source,
which can give a coincidence count. Assuming that the probability of having three
or four photons is negligible, the interference visibility is given by:

\[
V_{\text{max}} = \frac{\mathcal{O} P_{1,a} P_{1,b}}{P_{1,a} P_{1,b} + P_{0,a} P_{2,b} + P_{2,a} P_{0,b}},
\]

(3.8)

where \( P_{n,j} \) is the probability to have \( n \) photon in the input arm \( a \) or \( b \) and \( \mathcal{O} \) quantifies the overlap between the two photons over all the observables. For a
cohherent state \( |\alpha\rangle \) the probability distribution is given by a Poissonian:

\[
P_{n,\alpha} = e^{-|\alpha|^2} |\alpha|^{2n} \frac{1}{n!},
\]

(3.9)

and for the HSP by\(^3\)

\[
\begin{aligned}
P_{0,\text{HSP}} &= 1 - (P_{1,\text{HSP}} + P_{2,\text{HSP}}) \\
P_{1,\text{HSP}} &= t + 4(t - 1)tP_1 \\
P_{2,\text{HSP}} &= 2t^2P_1,
\end{aligned}
\]

(3.10)

where \( P_1 \) and \( t \) are the probabilities to emit a pair and the photon’s transmission
through the system, respectively. These equations are valid if we assume a low
detection efficiency for the heralding photons.

Figure 3.8 shows the experimental results and the theoretical prediction for mean
number of photons \( |\alpha|^2 = 0.05 \) and \( P_1 = 0.05 \) and 0.01. The theoretical prediction
fits perfectly with the experimental measurement, which proves the indistinguisha-
bility between the coherent state and the HSP.

### 3.2.3 Preparation and analysis

The state is prepared by sending the single photon on a 50/50 beam splitter (see
Figure 3.9). After that, the displacement operator \( \mathcal{D}(\alpha) \) is applied, combining
one of the two modes (which we call mode \( B \)) with the coherent state on a highly
transmissive beam splitter (90/10). On this second coupler a HOM dip is measured
to find and compensate the temporal delay between the two sources. The upper
bound is given by:

\[
V_{\text{max}} = \frac{P_{1,a} P_{1,b}}{\frac{r^2 + t^2}{2rt} P_{1,a} P_{1,b} + P_{0,a} P_{2,b} + P_{2,a} P_{0,b}}.
\]

(3.11)

The maximum visibility obtainable with a 90/10 beam splitter is 22%, and we
measure \( V = 23\% \pm 4\% \). This measurement is very important for the experiment,

\(^3\)the probabilities are conditioned on the detection of a heralding photon
3.2 Experimental implementation

Figure 3.8: HOM dip obtained for two different values of the probability of photon pairs emission for the HSPS. The blue and black data points correspond to $P_1 = 0.05$ and 0.01, respectively. The mean number of photons per pulse in the coherent state $|\alpha|^2$ is fixed at 0.05. The curves represent the theoretical prediction for two photons with a bandwidth of 80 GHz and a maximum visibility given by the equation (3.8) for $\mathcal{O} = 1$. The error bars are given by the photon counting statistics, with acquisition times of 120 s per point for $P_1 = 0.01$ and 20 s for $P_1 = 0.05$, resulting in smaller error bars for the former.

because it proves that the photon and the displaced vacuum are in the same mode and that they interfere, which is a necessary if we want to obtain a displaced single photon. This is very important, since it is the only proof that we are actually working with a state that contains a large number of photons. Note that what follows would have given the same result if the coherent state and the HSP were in different modes.

In the third part of the setup we find the state analysis stage (see fig. 3.9). This part starts with a displacement of the state to the single photon level, which is done in practice by applying an inverse displacement operator $D(-\alpha)$ on the path $b$. In order to do this we need to combine the beam with a coherent state which has exactly opposite phase with respect to the first one, and the same average number of photons $|\alpha|^2$. This operation requires an interferometer with high visibility and phase stability. The maximum visibility can be achieved with a perfectly balanced interferometer, but this does not allow us to actively stabilize it, since the phase difference is 0 for every wavelength. A good solution is the introduction of a different dispersion in one of the two arms. This is achieved by introducing a free space path (\(\sim 1cm\)) in one arm, compensated (at the wavelength of the HSP) by fibre in the other arm. Scanning the wavelength of a reference laser, we observe a full fringe every 400 nm, from which we estimate a path length difference of 20 \(\mu\)m. The achieved visibility after stabilisation is 99.98810(8)% with a monochromatic laser and 99.985(2)% with our coherent state, which has a shorter coherence length. The


Figure 3.9: Complete experimental setup. **SOURCES:** a pulsed laser at 780 nm pumps two PPLN crystals for the generation of a heralded single photon $|1\rangle$ via SPDC and a coherent state $|\alpha\rangle$ via DFG. The single photon is heralded in a single mode by filtering the trigger photon with a narrowband filter ($f$). Then, the photon and the coherent state are coupled into SMF. The HSP is injected in a Mach Zehnder interferometer to generate MICRO entanglement. On arm A, the HSP and the CS are combined on a 90/10 BS to implement a displacement operator, to generate MACRO entanglement. A second displacement with opposite phase is then performed on the same path, with the aid of an actively stabilized interferometer. In the **ANALYSIS,** a system of PBSs and half waveplates (HWP) are used to switch between measuring the number of photons in arms A and B and observing single photon interference fringes.

Figure 3.10: HOM dip between HSP and DFG on a 90/10 beam splitter, used for the displacement.
visibility in the pulsed regime is mainly limited by the path length difference. To set the phase of $\alpha$ in the inverse displacement the interferometer is locked on the slope of an interference fringe of the reference laser at 1460 nm. The interferometer is also thermally stabilized.

Finally, a free space path composed by a PBS, HWP and a second PBS is used to choose between recombining (HWP at $\pi/8$) or not (HWP at 0) the two modes. The two output are then sent on two avalanche-photo-diodes (APD) with 25% detection efficiency, with a 3 ns gate. The external trigger comes from a third APD, which is detecting the signal photon of the HSP source.

### 3.3 Results

As it is clear from equation 3.5, we need to measure the single photon interference visibility $V$, and the probabilities $p_{ij}$ of having $i$ photons in one mode of the entangled state and $j$ photons in the other. Recall, a positive concurrence indicates the presence of entanglement in the state.

To measure the visibility we scan the interference fringes using a piezo in the arm $a$. Varying $|\alpha|^2$ between 0 and 600 photons per pulse, the visibility goes from 0.96 to 0.65. This is due to the fact that the path-length difference is not optimally compensated and the interferometer is not actively stabilized. The experimental results are shown in Figure 3.11. The probabilities $p_{01}, p_{10}, p_{11}, p_{00}$ are measured setting the HWP to 0. $p_{01}$ and $p_{10}$ are given by the counts in the two arms divided by the number of triggers, while $p_{11}$ is given by the coincidences divided by the number of triggers and $p_{00} = 1 - p_{01} - p_{10} - p_{11}$. The measured probabilities are shown in Figure 3.12, top. As the size of the displacement increases, $p_{01}$ and $p_{11}$ increase, while $p_{10}$ stays constant, as expected, since we perform the two displacements with a residual noise only in one mode. From these measured quantities, we estimate a positive concurrence for $|\alpha|^2$ up to 550 photons per pulse (Figure 3.12, bottom).

### 3.4 Discussion and conclusion

The presented experiment shows the presence of entanglement in a state that involves up to 500 photons. The fact that entanglement is preserved at the macroscopic level is due to the unitarity of the displacement operator. Using homodyne measurements, it is possible to show correlations in the quadratures without re-displacing back to the single photon level (Lvovsky et al. 2013b). In this experiment, we do not detect directly the macroscopic state. In the experiment, we (reasonably) assumed that the HOM interference between the single photon and the coherent state is a measure of their overlap, and that this doesn’t change when
Figure 3.11: Top: Single photon interference visibility as a function of $|\alpha|^2$. As the size of the displacement increases, the visibility decreases linearly, due to the limited control over the relative phase between the two displacements. Bottom: example of one interference fringe measured to estimate the visibility by fitting experimental data.
3.4 Discussion and conclusion

Figure 3.12: Top: probabilities $p_{01}, p_{10}, p_{11}$ as a function of $|\alpha|^2$. Bottom: measured upper bound on the concurrence as a function of $|\alpha|^2$. 
we increase the mean number of photons $|\alpha|^2$. A measurement of the difference in variance of the two states, $D(\alpha)|1\rangle$ and $|\alpha\rangle$, however, using a detector with a limited resolution in the photon number, would give us a more direct proof of the displacement operation.

Even though all the setup is thermally stabilized, small phase fluctuations are unavoidable. The ability with which we can control the phase of the second displacement $D_\alpha(-\alpha)$ is crucial, as it fixes the maximal value of $|\alpha|^2$ for which we can still detect entanglement. Assuming a phase noise with a Gaussian distribution, this is given by the interferometric visibility $V = e^{-\sigma/2}$ (Minář et al. 2008). When measuring the photon number, since the phase information is completely erased, it can be seen as its conjugate variable, and the stability is irrelevant. However it is absolutely important to control it when dealing with homodyne measurements. For this reason, experimentally, the local oscillator and the state to be measured usually follow the same optical path with orthogonal polarisation modes. This was in an experiment very similar to the presented one, performed in Calgary (Lvovsky et al. 2013b), where homodyne detection was used to prove the distinguishability of the two macroscopic components by measuring the quadratures. However, in that work, the displacement is done simply by rotating a waveplate on the common path of the photon and the local oscillator polarization modes. It is good to underline that we can say that the interference really happens only when one projects the two modes on the same polarization. On the contrary, regarding the experiment done in Geneva, using well separated spatial modes is more demanding from an experimental point of view, and there is the need of stabilizing a phase. Nevertheless, in this way every unitary operation is well defined.

Phase noise can be seen as a form of decoherence for our state: indeed, phase fluctuations are related to fluctuations in temperature due to the imperfect isolation of the experiment, i.e., interaction with the environment. Another way of seeing it is to consider the second displacement as a noisy displacement, which will result into an imperfect measurement. Therefore, the inability of detecting entanglement for a state containing more than 550 photons on average is due to our (nevertheless high) measurement precision.

In analogy with the critical temperature in superconductivity, the fact that the possibility of detecting a quantum feature depends on the phase fluctuation (or on the noise of the measurement) defines some sort of phase transition from quantum to classical: if the noise is too high, we can only see the classical behaviour. Below a certain critical value of noise, we can see the quantum properties.
4 General conclusion and outlook

The final test of any physical theory lies, of course, in experiment.
R. P. Feynman, PhD thesis, Conclusion, p. 74 (1942)

The aim of this thesis work was to use single photon entanglement as a resource, both for applications in quantum communication and to probe controversial aspects of quantum theory. In particular, the question I wanted to address was, in the first place, whether we can exploit single photon entanglement at telecom wavelength to enable long distance quantum communication. In the third part of my thesis though, I showed an experiment that aimed to probe fundamental questions, which are the basics for possible future applications. The two experiments presented in chapters 2 and 3 are deeply different, albeit they are both based on heralded single photon entanglement. However, in the global picture, the two works show the importance of research both in foundations and in applications of physics.

Nonetheless, it is the beauty of the physics of interactions between light and matter that is at the origin of all the experiments I performed. Nonlinear optics is the basis for the development of the building blocks for tests of fundamental theories and implementation of applied physics: in this case, photons. I worked with heralded single photons generated via SPDC, and thanks to the remarkable advances of the last years in material science, I've been able to develop and characterize different kinds of single photon sources.

In chapter 1 I have presented the techniques that are part of my daily laboratory work, which consist of studying the statistics of the light sources I work with and in optimizing every degree of freedom based on what is required for a certain experiment. The apex of my work in this field was the realization of a photon pair source based on a PPKTP crystal, in which the photon pairs have two important characteristics:

1.a the photons are generated at telecommunication wavelengths and in pulsed, but narrowband, regime;

1.b they are in a spectral separable state with spectral-spatial correlations, which allow high coupling efficiency\(^1\).

\(^1\)the coupling efficiency is the probability of detecting a photon conditioned on the detection of its sister photon.
In regard to point 1.a, working with telecommunication wavelengths is relevant for quantum communication. In our case, the photons are degenerate and indistinguishable, and even though sources of pure heralded single photons where the heralding photon is at another wavelength (e.g., 800 nm, where detection reaches very good efficiencies at room temperature) have been demonstrated, having both photons at telecom wavelengths can represent an advantage for some applications. One can imagine, for example, to send both photons far from the source in an optical fiber to reduce the losses in optical fiber while maintaining the same total distance between the two parties. Moreover, the fact that superconducting detectors can achieve high efficiencies at telecom makes the wavelength of the heralding photon a matter of taste (and, for the moment, fundings or good collaborations). Having a pulsed pump laser is necessary for generating spectrally pure photons, however, it can also be helpful for synchronization. It’s good to underline, however, that synchronization of picosecond pulses at a distance is a challenging task, due to the need of actively stabilizing a long fiber at a distance with interferometric techniques and feedback loops. Point 1.b is very important when low loss, together with high interference visibility, is required. This happens typically in experiments with many photons: the spectral purity ensures high visibility without need of filtering (hence eliminating filtering losses), while the spectral-spatial correlations allow us to optimize the coupling efficiency of the photons up to 90%. The work is published in 5.

In a wider perspective, one has to remember that photon sources based on SPDC have a limitation which lies in the probabilistic nature of spontaneous processes. On one side, producing pairs allows one to herald the presence of a photon, and this can be done with very high efficiency both in bulk crystals and in waveguides. On the other, there is a probability of emitting more than one pair that makes a heralded single photon not perfectly a single photon, therefore, in absence of photon number resolving detectors, it has to be kept small (Christ, Silberhorn 2012). Other solutions are represented by photons emitted by single atoms (Kurz et al. 2013), color centers in diamond (Kurtsiefer et al. 2000; Lohrmann et al. 2011), quantum dots (Gregersen et al. 2010; Stock et al. 2011). These kinds of sources have for the moment other limitations, such as the need of cryogenic temperatures, difficulty in the collection of the emitted photons, distinguishability between independent sources. These are however only technical difficulties, in contrast with the fundamental limitation of probabilistic effects.

In chapter 2 I discuss the main research line of my thesis work: the amplification of photonic qubits (see 1, 2, 8) for DIQKD. I have shown a full characterization of the heralded amplification of photons, studying the performance of amplification as a function of the losses in the input state, the overall transmission and the detection efficiency. The final goal of my work in this topic can be summarized in two main points:
2.a achieve heralded amplification of qubits;
2.b implement a fiber based setup for applications in quantum communication.

Heralded amplification of qubits was done using time bin qubits in a post-selected scheme, but also in a non-postselected scheme using vacuum-single photon (or Fock state) qubits. Teleportation of photonic qubits has been demonstrated in postselected schemes in 1997 (Boschi et al. 1998; Bouwmeester et al. 1997) and recently in a non postselected scheme (Takeda et al. 2013), taking advantage of a hybrid technique that uses continuous variable states. In our case, when setting the gain to unity, we have achieved non postselected teleportation of a Fock state qubit, in addition we can perform amplification of the one photon component with respect to the vacuum. Interestingly, Fock state qubits could also be used for quantum communication, since single photon entanglement can also be exploited to violate a Bell inequality at a distance (Monteiro et al.). In this framework, the unconditional amplification of a heralded single photon shown in this Thesis is an important step forward for QC.

In the works 1 and 8, a completely fiber-based scheme together with sources of photon pairs at telecom wavelengths makes the implementation of the photonic qubit amplifier adapted for quantum communication (point 2.b). In 8, there are two main limitations in the setup, which prevent us from claiming that the work represents a real advantage for DIQKD. The first is related only to the time bin qubit amplification scheme: amplification takes place only if conditioned on the detection of an output photon, i.e. with post selection. For the amplification of Fock states, this problem does not appear. The second and more important problem is related to the coupling efficiency of the auxiliary photons. This due to the fact that after coupling into single mode fiber (∼80%), a series of delay lines and polarizing beam splitters (see the setup in Figure 2.11) is below 40%, fixing an upper bound for the output probability that is then decreased by the transmission of the unbalanced beam splitter $t = 90\%$ and finally by the limited detection efficiency (∼70%), giving an upper bound of $P_{\text{max}}^{\text{out}} = P_b t \eta$ of approximately 20%. However, improvements of this setup are possible: with a coupling efficiency of 90%, achievable with our source, and a beam splitter with transmission $t = 99\%$, a detection efficiency of 93% is required for the violation of the CHSH inequality free from the detection loophole. This kind of efficiency has been demonstrated for telecom wavelengths in fibre coupled SNSPDs (MarsiliF. et al. 2013).

An interesting perspective has to be found in the possibility of realizing in practice a device for photon amplification in an optical circuit completely integrated on a chip. Indeed, due to the recent progress in realization of integrated circuits (Krapiec et al. 2013; Martin et al. 2012; Miller 1969; Politi et al. 2008, 2009; Spring et al. 2013; Tanzilli et al. 2012), one can considerably reduce the size of devices that until now were covering an entire optical table. For a realistic implementation of the qubit amplifier on chip, the required components are (at least):
• a source for the generation of the auxiliary photon;
• an unbalanced beam splitter, with a transmission \( t \) that in principle can be fixed, but it could be varied using interferometric techniques;
• two integrated detectors (Sprengers et al. 2011) or pigtailed followed by fiber coupled SNSPDs for the Bell state measurement;
• two inputs for the qubit and a synchronisation signal, and one pigtailed output to connect directly to a SNSPD, or to the setup required for a Bell test.

This would be a useful tool for possible realization of DIQKD in a practical scenario, where a heralded single photon delocalized over two distinct optical modes (fibers) can be amplified after a lossy channel in a heralded way. Given the state of the art of research in this field, and the fact that more and more research teams are starting to be interested in the subject, I think I can safely say that an integrated circuit is a promising option for the next step in photonic qubit amplification.

Another kind of amplification of single photon entangled states was discussed in chapter 3, where what was amplified is actually the number of photons involved in the state. This, as we saw, has nothing to do with the usual concept of amplifiers, neither with the photon amplifier discussed above. We showed how, displacing one of the entangled modes in phase space, we can increase the number of particles yet maintaining the same degree of entanglement. The trick lies in the fact that the displacement operation is unitary. An advantage of this unitarity is that it can be inverted, allowing us to detect entanglement with well established techniques in the single photon regime.

The main questions that we have asked ourselves during this work are, purely for the sake of symmetry, two:

3.a what is a macroscopic quantum state? Is our state quantum and macroscopic?

3.b besides the fundamental interest, is our macroscopic quantum state useful for something?

The concepts of macroscopic classical state and microscopic quantum state are well defined. A microscopic and classical state is harder to imagine, but we can do it. When thinking of a macroscopic quantum state, some contradictions arise. Going
back to Schrödinger’s Gedankenexperiment, the image of the two possibilities for
the cat to be alive and dead interfering is quite disturbing. Why? One reason is
that we don’t really know how the death of the cat is defined, is it dead when one
area of its brain stops working? Is it when its heart that stops beating? Is it when
every single cell of its body is dead? So we need to define two orthogonal states,
imagining that there is a sharp separation between the cat being alive or dead.
For this reason, it is maybe better to think of our optical cat. There is no real
agreement in the scientific community about what an optical cat is, and on what
is the best criterion to define a macroscopic quantum state (De Martini, Sciarrino
2012; Sekatski et al. 2014). The experiment by De Martini et al. (De Martini et al.
2008) in which one photon from a polarization entangled pair is phase-covariantly
cloned, originated a long debate about whether there is entanglement or not in this
state. Another example is given by a superposition of two coherent states, the most
classical quantum state that we can imagine, with opposite phases: $|\alpha\rangle + |\alpha\rangle$. In
this case, the two components are well separated in the phase space. Optical kitten
states of this kind have been realized, even if populated by a relatively low mean
number of photons ($n=2$ in (Ourjoumtsev et al. 2007)). The state that we analysed
involves two states that are orthogonal, but that nearly overlap in the phase space.
As a matter of fact, the mean number of photons in a coherent state is $|\alpha|^2$, in a
displaced single photon is $|\alpha|^2 + 1$. However, we have seen that these states are
distinguishable in the photon number distribution because of a factor of three of
difference in the variance of the latter with respect to the former. Starting from this
we can distinguish the two states using a threshold detector, or a detector that has
a limited resolution in photon number and is coarse grained by thermal noise. The
distinguishability of the two states has been proven with homodyne measurement
by Lvovsky et al. (Lvovsky et al. 2013b). Moreover, comparing the state with a
superposition of two Fock states it is possible to define an equivalent size of the
state, and we find that for $|\alpha|^2 = 500$ the state is as macroscopic as a superposition
between two Fock states that differ by about 40 photons (Sekatski et al. 2014).
For sure, since the state involves two orthogonal components populated by a large
number of particles, this makes it a good candidate for a macroscopic quantum
state. However, we can notice that the number of e-bits does not increase with the
size of the system, and neither does the amount of entanglement.

In the conclusion of chapter 3 we also discussed how phase noise can affect the
micro-macro entangled state. It can be considered as decoherence (Zurek 2003), as
the interferometer is coupled to the environment, or as noise in the measurement
apparatus. In some cases, these two aspects can be distinguished and are well
separated: in general, we cannot reverse the effect of decoherence, but we can reverse
the effect of a noisy measurement. In our case, there is a correspondence
between the two: reversing the effect of decoherence is done by measuring the
variation in the intensity of a stabilization laser (photon number, conjugate variable
of the phase), and applying a unitary transformation (corresponding to a phase) to
compensate the action of the environment. The limit on our ability in observing quantum effects is directly related to how well we compensate the action of the environment.

This consideration can be a bridge to the second question I posed. This (as well as the definition of macroscopic) is a rather open question, although we can give some comments about it. The high sensitivity of this state to phase fluctuations can be useful for precision measurements, finding applications in metrology like NOON states or in detection of gravitational waves, like squeezed states. The perspective of mapping this state onto a massive object is also interesting. It would lead to a light-matter entangled state. The option which has been considered in our group in Geneva was to map the state into a quantum memory: it would in principle allow one to store the macroscopic state in many excitations of the atoms in the memory, hence many excitations of one single collective mode, entangled with a single photon in the other mode. This would also allow one to test decoherence models, but the main limitation for the moment is due to the low efficiency of quantum memories, which inevitably destroy the vacuum-single photon superposition.

In the course of this thesis work I have had the possibility of working with the foundations of non linear optics, which are the basis for the generation of the main tool I used for my experiments: heralded single photon entanglement. I have shown how this can be a realistic application for quantum communication and at the same time a powerful tool for testing the foundations of quantum theory. This path is somehow a paradigm of what I have learned about research: starting from the well established foundations, research is the process that allows a little step forward, firstly in the applications of the well known results, and secondly, but with equal importance, in the search for new knowledge. I liked to end with an open question, as a reminder that there is always room for finding out new and fantastic facets of Nature.
Bibliography


Ma, T., Zhou, Q., Zhang, W., Huang, Y., Cui, X., Lu, M. (2013). “1.5 μm orthogonally polarized dual-output heralded single photon source based on op-


Appendix
List of publications


Publications

1 Heralded photon amplification for quantum communication
Heralded photon amplification for quantum communication

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Heralded noiseless amplification based on single-photon sources and linear optics is ideally suited for long-distance quantum communication tasks based on discrete variables. We experimentally demonstrate such an amplifier, operating at telecommunication wavelengths. Coherent amplification is performed with a gain of $G = 1.98 \pm 0.20$ for a state with a maximum expected gain $G = 2$. We also demonstrate that there is no need for a stable phase reference between the initial signal state and the local auxiliary photons used by the amplifier. We discuss these results in the context of experimental device-independent quantum key distribution based on heralded qubit amplification, and we highlight several key challenges for its realization.

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Transmission loss is a fundamental limitation in quantum communication. A photon $|1\rangle$ propagating in a channel with a transmission efficiency $\eta_t$ ends up in a state given by $\eta_t |1\rangle (1 - \eta_t) |0\rangle |0\rangle$. Amplifying the single-photon component $|1\rangle$ is not possible in a deterministic, noiseless, and coherent operation. In fact, if such an operation were possible, it could be used for signaling [1]. This restriction makes long-distance quantum communication very challenging, but it also offers a unique opportunity for key distribution with unequalled security [2].

Even when the amplification of a quantum state cannot be performed deterministically with unit fidelity, approximate quantum amplification is possible. Probabilistic noiseless amplifiers have recently attracted a lot of attention. Of particular interest are those for which the success of the amplification process is heralded [3]. This includes techniques based on single-photon addition [4,5] or thermal noise addition followed by heralded photon subtraction [6,7].

Reference [3] presents an interesting protocol for realizing heralded quantum amplification. Inspired by the concept of quantum scissors [8], the authors propose a scheme requiring only single-photon sources and linear optics. This is an attractive proposal from a practical point of view, and it has already triggered a couple of proof-of-principle experiments [9,10]. These experiments focus on the applications of the amplifiers in continuous-variable-based quantum information science, e.g., distilling continuous-variable entanglement [9] or improving continuous-variable quantum key distribution [10,11].

Unlike recent realizations [9,10], we focus on the potential of the amplifier in Ref. [3] for tasks based on discrete variables. Our experiment uses photons at telecom wavelengths, so it is ideally suited for long-distance quantum communication. Furthermore, since it is based on polarization-independent elements, our device could also be used as a qubit amplifier, opening up the way for experimental device-independent quantum key distribution (DI-QKD) [14].

In our system, we have direct access to the behavior of the gain, and we also demonstrate the coherence-preserving nature of the process. Additionally, we show that no specific phase stability is needed between the input photon and the auxiliary photon, as required in a distributed quantum network [15].

This article is divided into three sections: The first describes the theoretical aspects of our single-photon amplifier; the second explains the experimental setup, the measurements performed, and the results obtained; and the final section discusses the additional requirements to implement the amplifier in a real quantum network.

I. THEORY

Figure 1 depicts a heralded single-photon amplifier. We consider an input state that is a statistical mixture between the vacuum and a single photon,

$$|\alpha^2\rangle |0\rangle + |\beta^2\rangle |0\rangle |0\rangle.$$  \hspace{1cm} (1)

An auxiliary photon, sent through a variable beamsplitter (VBS) with transmission $\nu$, produces the entangled state between the modes $\hat{c}$ and $\hat{k}_{\text{out}}$ given by

$$|\sqrt{1 - \nu}\langle c|0\rangle + \sqrt{\nu}\langle k_{\text{out}}|0\rangle.$$  \hspace{1cm} (2)

Then, a 50% beamsplitter combines $\hat{c}$ and $\hat{k}_{\text{in}}$ producing the modes $\hat{d}_+ = \frac{1}{\sqrt{2}} (\hat{c} + \hat{k}_{\text{in}})$ and $\hat{d}_- = \frac{1}{\sqrt{2}} (\hat{c} - \hat{k}_{\text{in}})$. If the auxiliary and the input photon are indistinguishable, the detection of a single photon in one of these modes, say in the mode $\hat{d}_+$, acts as a Bell state measurement and projects the output state $\hat{k}_{\text{out}}$ into

$$\frac{1}{N}(1 - t)|\alpha^2\rangle |0\rangle |0\rangle + t|\beta^2\rangle |0\rangle |0\rangle |k_{\text{out}}|.$$  \hspace{1cm} (3)

where $N = (1 - t)|\alpha^2| + t|\beta^2|$ is a normalization factor. Since the detection of one photon in $\hat{d}_+$ also leads to Eq. (3), the probability to successfully produce that state is given by twice its norm.

The process of transforming the input into the output state changes the weight of the single-photon component. The gain $G$ of the process quantifies the change. It is defined as the ratio of the single-photon component weights before and after the process, and it is given by $G = \frac{t}{1 - t|\alpha^2|}$. Importantly, if there is a well defined phase between the single-photon and the vacuum component on Eq. (1), it is preserved by the process. Therefore, in the case in which $t = \frac{1}{2}$, the input state is mapped to the output, the gain $G = 1$, and the protocol reduces to the teleportation of a vacuum-single-photon qubit [12,13]. When $t > \frac{1}{2}$, the gain $G > 1$ and the desired noiseless amplification is realized.

The amplification, as explained before, is always heralded by a signal coming from either the detection of a single photon
FIG. 1. (Color online) A heralded single-photon amplifier. The auxiliary photon \( |1\rangle \) generates an entangled state between the modes \( \hat{c} \) and \( \hat{k}_{\text{out}} \) after passing through a variable beamsplitter (VBS) with transmission \( t \). The input mode \( \hat{k}_{\text{in}} \) is then combined with \( \hat{c} \) in a 50 : 50 beamsplitter (BS). When \( t = 50\% \), a measurement in \( \hat{d}_+ \) or \( \hat{d}_- \) produces a teleportation transformation between the \( \hat{k}_{\text{in}} \) and \( \hat{k}_{\text{out}} \) modes [12,13]. When \( t > 50\% \), such a detection heralds the desired noiseless amplification.

in \( \hat{d}_+ \) or \( \hat{d}_- \). However, in the case of non-photon-number resolving detectors, it can be shown that the gain reduces to \( G = \frac{t}{1-\alpha^2 t} \). For instance, for \( \alpha^2 = 1/2 \) the amplification only takes place for \( t > \frac{1}{2} \). In practice, the gain depends on myriad parameters that affect the output state given by Eq. (3). To describe our experimental amplifier, we developed a model for the gain, taking into account the coupling, transmission losses, and detector efficiency, which can be quantified independently.

II. EXPERIMENT

Figure 2 shows a schematic of the experimental setup. A 25-mm-long type-II PPLN waveguide (University of Paderborn) generates both the input and the auxiliary photon through spontaneous parametric down conversion. The interaction of the waveguide with a continuous-wave diode laser at 780 nm (Toptica DL100, 3.3 mW) results in the creation of degenerated photon pairs at 1560 nm. After the waveguide, a silicon filter blocks the laser, and an interference filter (IF) with 1.3 nm bandwidth guarantees the spectral indistinguishability of the photons. The pairs are separated by a polarization beamsplitter (PBS) and then coupled into single-mode optical fibers, which ensures their spatial indistinguishability. A delay line in one of the fibers controls the path length difference.

The heralded photon amplifier is embedded in an interferometer formed by two variable fiber beamsplitters (VBSs) and two bulk 50 : 50 beamsplitters (BSs). While VBS1 is part of the amplifier scheme, VBS2 is used to prepare the input state. An input photon passing through VBS2 produces an entangled state between the modes \( \hat{a} \) and \( \hat{k}_{\text{in}} \). When considered independently, the state of each of those modes is given by Eq. (1) (with \( \alpha \) and \( \beta \) set by the splitting ratio of VBS2). We use \( \hat{k}_{\text{in}} \) as an input mode, and, as we will explain later, we use \( \hat{a} \) to test the coherence of the process by combining it with the mode \( \hat{k}_{\text{out}} \) at BS2.

The outputs of the two bulk beamsplitters BS1 and BS2 are coupled into single-mode fibers so that they can be connected to the detectors. The hybrid configuration of bulk and fiber optics, although not necessary for amplification, has been adopted to facilitate the measurements needed to quantify the gain.

The two-photon coincidence events were measured via two InGaAs/InP avalanche photodiodes. We measure single-photon events using a free running detector (IDQ-ID210 with 10\% efficiency, 3 KHz noise, and 20\% dead time). This detection triggers a second detector in gated mode that measures the coincidences (IDQ-ID200 with 15\% efficiency, 2.5 ns detection gate, and a dark count probability of \( 3 \times 10^{-5} \) per detection gate).

The amplification process requires that the input and auxiliary photon are indistinguishable in every degree of freedom. To quantify the indistinguishability, we first measure the Hong-Ou-Mandel (HOM) interference on each beamsplitter.
Heralded Photon Amplification for Quantum ... Physical Review A 86, 023815 (2012)

The interferometer is made such that the path difference between the inputs of both beamsplitters is equal (within the coherence length of the photons). The coincidences after BS1 and BS2 were measured as a function of the delay between the input photons. The characteristic dips had visibilities of 93.4 ± 5.9% and 92.1 ± 5.7% for BS1 and BS2, respectively. The right-hand side of Fig. 2 shows the results for BS1.

Even with indistinguishable photons, double pair emission and detector imperfections decrease the visibility of a HOM dip. Based on the model presented in Ref. [16], we calculated a maximum visibility for our source of 92.7% [17]. The measured visibilities correspond then to highly indistinguishable photons.

To quantify the gain of the heralded amplifier, we compare the probability $P_{\text{in}}$ of detecting a photon at $k_{\text{in}}$ with the probability $P_{\text{out}}$ of detecting a heralded photon in the mode $k_{\text{out}}$. These probabilities are obtained in two independent coincidence measurements. The first is made by blocking the paths $\hat{a}$ and $\hat{c}$ of the interferometer and using the free running detector after BS2 and the second detector after BS1. The coincidences to singles ratio ($C_1 : S_1$) provides our initial reference probability $P_{\text{in}}$ (≈ $15 \times 10^{-4}$ per detection gate) [18]. In the second measurement, only the path $\hat{a}$ of the interferometer is blocked, and the positions of the detectors are reversed. In this case, the coincidences to singles ratio ($C_2 : S_2$) gives the amplified output probability $P_{\text{out}}$. We can then define the gain, in terms of the measured quantities, as

$$G = P_{\text{out}} / P_{\text{in}}.$$ (4)

Figure 3 shows the gain measured for different values of the transmission $t$ of VBS1, where the splitting ratio of VBS2 was fixed at 50 : 50. The input state is then given by Eq. (1) with $\alpha^2 = \beta^2 = 1/2$, and therefore the maximum possible gain is $G = 2$ for $t = 1$. In our system, the amplification $G > 1$ occurs for $t > 75\%$, and the gain reaches a value of $G = 1.98 \pm 0.20$ at $t = 98\%$. The measured values of the gain are in good agreement with those predicted taking into account the coupling, transmission losses, and detector efficiency.

To fully demonstrate amplification, for each gain measurement it is necessary to verify the coherence-preserving nature of the process. To do so, we can take advantage of the fact that for $t = 50\%$, our experiment is analogous to an entanglement swapping experiment [19]. Following Fig. 2, we generate two maximally entangled states after VBS1 and VBS2. Then, a Bell state measurement at BS1 swaps the entanglement onto the two modes $\hat{a}$ and $\hat{k}_{\text{out}}$.

With $\hat{a}$ and $\hat{c}$ unblocked, we measured the coincidences between the heralding detector at $\hat{d}$, and one output of BS2, as the phase between the inputs of BS2 is varied. To control this phase, part of the interferometer’s fiber is wrapped around a piezoelectric tube. Figure 4 shows the interference pattern obtained in the coincidences for $t = 50\%$. The interference clearly indicates that the coherence is preserved during the process.

At higher values of $t$, due to an increasing imbalance in the interferometer, the expected value of the interference visibility $V_{\text{max}}$ decreases following the expression $V_{\text{max}} = 2\sqrt{\theta(1-t)}$ [20]. Figure 3 shows the measured values of the visibility as well as $V_{\text{max}}$ for different values of $t$.

If we are to use the amplifier in a distributed quantum network, it is important for the protocol to be independent of the phase between the different sources. Therefore, in addition to the measurements of gain and coherence, we studied the phase relation between the input modes of the interferometer. During our experiment, this phase fluctuates rapidly (the fibers between the source and the interferometer are not stabilized). The effect of this fluctuation is clear if we turn a $\lambda/2$ wave plate (before the PBS in Fig. 2) in order to have an input state...
of the form

$$|\psi\rangle = \frac{1}{\sqrt{2}} e^{i\phi}|1,1\rangle + \frac{1}{2}|2,0\rangle + \frac{1}{2} e^{2i\phi}|0,2\rangle,$$

(5)

where $n,m$ are the input modes of VBS1 and VBS2, and $\phi$ is a phase between these two modes. Figure 4 compares the interference fringes obtained for the amplifier with those obtained with the input state given by Eq. (5). In the second case, the $|1,1\rangle$ component interferes as before, but the rapid fluctuations of $\phi$ erase the interference of the last two terms. Figure 4 shows how the visibility decreases by a factor of 2, as expected. Therefore, in our scheme the phase between the input state and the auxiliary photon is not stable, which does not affect the amplification process.

III. DISCUSSION

In contrast to our amplifier, where one source generates both photons, in a real quantum network the input state and the auxiliary photon would be independently generated. That is also the case for the implementation of a qubit amplifier [14]. With independently generated photons, the interference requires purity in addition to indistinguishability. While the indistinguishability can be achieved as shown here, to obtain photons in pure states is challenging. Strong filters may remove the correlations, but they also reduce the amount of available photons. A more convenient solution is to obtain pure photons directly from the source. There has been a large effort to design and implement pure photon sources as reported, for example, in Refs. [21–24]. However, it is still a challenge to build a single-photon source satisfying all the requirements on frequency, bandwidth, and maximum transmission losses necessary for an amplifier in a real quantum network.

IV. CONCLUSIONS

We have reported in this article the successful heralded amplification of a single photon in the telecommunication regime. We have shown a maximum gain of $\sim 2$ and demonstrated the coherence of this process. We have also shown that the amplification scheme does not require a stable phase between the input state and the auxiliary photon. All of these results highlight the potential of heralded quantum amplifiers in long-distance quantum communication based on quantum repeaters [15]. The experiment reported here provides experimental results that can be used in the development of device-independent QKD [14].

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2 A complete characterisation of the heralded amplification of photons
A complete characterization of the heralded noiseless amplification of photons

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A complete characterization of the heralded noiseless amplification of photons

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Abstract. Heralded noiseless amplification of photons has recently been shown to provide a means to overcome losses in complex quantum communication tasks. In particular, to overcome transmission losses that could allow for the violation of a Bell inequality free from the detection loophole, for device independent quantum key distribution (DI-QKD). Several implementations of a heralded photon amplifier have been proposed and the first proof of principle experiments realized. Here we present the first full characterization of such a device to test its functional limits and potential for DI-QKD. This device is tested at telecom wavelengths and is shown to be capable of overcoming losses corresponding to a transmission through 20 km of single mode telecom fibre. We demonstrate heralded photon amplifier with a gain >100 and a heralding probability >83%, required by DI-QKD protocols that use the Clauser–Horne–Shimony–Holt inequality. The heralded photon amplifier clearly represents a key technology for the realization of DI-QKD in the real world and over typical network distances.

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1. Introduction

The concept of amplification in communication systems has long been used in the classical regime to overcome transmission loss. However, for quantum systems, amplification of quantum states is generally not possible due to the no (perfect) cloning theory—amplification will normally introduce noise, thus degrading the quality of the quantum state [1]. Heralded photon amplification can allow one to overcome these constraints as it operates in a probabilistic fashion. Importantly, while it is probabilistic in nature, when successful, it provides a heralding signal that allows one to then perform subsequent operations. This heralding signal is what makes this approach interesting for device independent quantum key distribution (DI-QKD) as it can herald the arrival of a photon (or qubit) and hence prepares the system so that the Bell test may be performed [2–4]. More recently, it has also been incorporated into a quantum repeater protocol where it is used to herald the storage of a photon in a quantum memory [5], opening the door to even greater distance for device independent operations. It is clear that such a device could find wide spread use in myriad quantum systems where one needs to overcome inefficiencies associated with loss or multiple probabilistic operations, as well as where feed-forward signals can help in scaling complex quantum systems.

Noiseless photon amplification is related to quantum scissors [6] and relies on quantum teleportation to herald the amplified state. It was first proposed by Ralph and Lund [7] and has found several different implementations [8], either exploiting polarization modes [7, 9, 10], spatial modes in fibre optics [11] or using techniques such as single photon addition and subtraction [12–15]. Also, it has shown potential application in both discrete and continuous systems [4, 16].

In this paper, we first present the principle operation of a heralded photon amplifier, then we introduce how our test device is realized. The purpose of this paper is to completely characterize the performance of the heralded photon amplifier at telecom wavelengths, independently of the source and detector characteristics. We then discuss the operational limits of such devices and give some perspectives on further improvements in the context of DI-QKD.

2. Principle of the heralded photon amplifier

The concept of a heralded photon amplifier is illustrated in figure 1(a). The incoming state that we are interested in is usually a single photon that has been mixed with some vacuum due to transmission loss and has the form \( \rho_{\text{in}} = p|0\rangle\langle 0| + (1 - p)|1\rangle\langle 1| \). An ancilla photon is first used to generate single photon entanglement [17]. The input state is then combined
Figure 1. (a) Standard representation of heralded photon amplifier, composed by a beam splitter with variable transmission $t$ and a balanced beam splitter followed by a photon detector. (b) Experimental setup for our test-device based on bulk optics and using, instead of spatial, polarization modes (see main text for details).

on a beam splitter with one mode of the entangled state and subsequently one photon is detected which corresponds to a Bell state measurement. This requires that the two photons are indistinguishable and that the detector ($D_b$, see figure 1(a)) can resolve the number of photons. If the initial ancilla state is maximally entangled, i.e. for a transmission of $t = 0.5$, then this corresponds directly to the standard teleportation scenario. However, if we now vary this transmission we can bias the output state such that it has the form
\[
\rho_{\text{out}} = \frac{1}{N(t)} [p|0\rangle\langle 0| + g^2(t)(1-p)|1\rangle\langle 1|].
\] (1)

Here $g^2(t) = t/(1-t)$ is the gain factor, while $N(t) = p + g^2(t)(1-p)$ is a normalization factor. The renormalized gain is defined as the ratio between the probability for the single photon component before and after the amplification and is given by
\[
G(t) = \frac{g^2(t)}{p + g^2(t)(1-p)} = \frac{t}{(1-t)p+t(1-p)}.
\] (2)

We see that for $G(t = \frac{1}{2}) = 1$ the protocol reduces to a teleportation of the input state, and the gain is then greater than 1 for $t > \frac{1}{2}$.

One can notice that the gain depends on both $p$ and $t$. In particular, $G$ tends to infinity as $p \to 1$ (high losses) and $t \to 1$ (high transmission). However, a high gain does not imply a high heralding probability, which, on the contrary, is inversely proportional to the losses.

In addition, one should note that equation (2) does not take into account that, in practice, we have non-photon-number resolving detectors and non-zero losses through the components of the amplifier. This can result in a reduction in the actual experimentally achievable gain for a fixed input state and, in general, change the response of the amplifier as a function of $t$. 

3. Experiment

In practice, our test device is of the form represented in figure 1(b), as this provides a more flexible setup for testing. The half wave plate and the polarizing beam splitter (HWP$_2$ + PBS$_1$) are used to simulate losses on the input state, while HWP$_1$ and PBS$_1$ play the role of a beam splitter with variable splitting ratio to define the transmission ($t$).

In this experiment, both the input and the ancillary photons belong to the same pair created in a type II spontaneous parametric down conversion process. For this purpose a 2 cm periodically poled lithium–niobate crystal is pumped by a mode locked Ti:sapphire laser at 780 nm, pulsed in the picosecond regime. The photons at 1560 nm are filtered down to 1 nm by an interference filter to eliminate spectral distinguishability, before being separated by a PBS. Coupling into single mode fibre with $\sim$50% efficiency, ensures a well defined spatial mode. Photon counting is performed by using two gated avalanche photo-diodes (IDQ-210) with 25% detection efficiency, 3 ns gate and a noise probability of $10^{-5}$ per gate that are synchronized with the laser. In all the performed measurements the laser is used to trigger one detector at 80 MHz, which, in turn, triggers the second one.

To ensure indistinguishability between the two photons in all degrees of freedom, a Hong Ou Mandel (HOM) type interference measurement is performed [18], using the two polarization modes at PBS$_3$ [19]. Following the setup reported in figure 1(b), it can be seen that in order for the amplification to take place the two photons are required to arrive at the same time on the PBS$_1$. From this point they travel through the same optical path until they arrive at HWP$_4$ in two orthogonal polarization states: $|H\rangle|V\rangle$. Here, the polarization is rotated by $\pi/4 : (|H\rangle + |V\rangle) (|H\rangle - |V\rangle)$. If there is perfect indistinguishability the terms $|H\rangle|V\rangle$ and $|V\rangle|H\rangle$ interfere and vanish, therefore two detectors at the outputs of PBS$_3$ will not click in coincidence. The measured net HOM visibility is $0.98 \pm 0.03$, with a pair creation probability per pulse $p = 0.01$. The visibility is limited only by double pair emission, and is in good agreement with the theory [20], indicating that all degrees of freedom are well controlled in the experiment.

The gain is evaluated as the ratio between the probability of having a photon in the output state and the probability of having a photon in the input: $G = p_{\text{out}} / p_{\text{in}}$. We estimate the input probability $p_{\text{in}}$ as the ratio between the rate of triggers (counts in $D_a$) and the rate of coincidences with $D_b$. The input losses are varied between 0.5 and 1 by turning the wave plate HWP$_2$. In a second measurement the detector $D_b$ triggers $D_a$, and the output probability $p_{\text{out}}$ is given by the ratio between singles in $D_b$ and coincidences with $D_a$. For each value of input loss we vary the amplifier transmission $t$ between 0.5 and 1.

The gain $G$ is measured as a function of the transmission of the amplifier for eleven loss values. As shown in figure 2, the resulting gain is in agreement with the theoretical prediction taking into account losses, detection efficiency and the use of non-photon-number resolving detection. Figure 2(b) shows the curves for three fixed values of $t$ as a function of $p$. We notice that the gain is measured to be $>100$ for the limit of high losses ($p$) and high transmission (see figure 2(a), blue stars), but in this regime the performance of the heralded photon amplifier tends to be less efficient in terms of success probability, as we will see in the following analysis.

To complete the characterization of the heralded photon amplifier, it is necessary to verify the coherence of the process. As we see in figure 1(b), the input state is separated into two modes after PBS$_1$, the ratio depending on the angle of HWP$_2$. The reflected and the transmitted modes correspond to the state to be ‘amplified’ and the ‘lost’ part, respectively. The latter is obviously
not present in a communication channel, where lost photons are mainly absorbed or reflected, but in our case these two modes remain in a coherent superposition, such that we can use the state in the ‘lost’ mode as a reference and interfere it with the state in the ‘amplified’ mode [11]. The visibility of the interference pattern is a signature of the coherence of the amplification process, i.e. that the teleportation protocol preserves the state. The visibility of the interference pattern is related to the coherence of the amplification process, and it allows one to complete the characterization of the device. The interference fringes are measured for each setting and the corresponding visibilities are represented in figure 3(a), and found to be consistent with the expected behaviour. In particular, it is maximal (∼94%) when \( p \) and \( t \) are complementary, i.e. the amplitudes of the two interfering modes are balanced. Changing the two parameters, the visibility inevitably decreases only because of the imbalance in the amplitudes, thus confirming the coherence of the amplification process. Figure 3(b) shows two examples of the measured interference fringes with maximal visibility.

Summarizing the result in a more intuitive way, as in figure 4, it is convenient to look at the heralding probability as a function of the losses introduced in the input state. With an amount of loss corresponding to the typical network distances, i.e. sending a photon through more than 20 km of network installed fibre, it is still possible to have a heralding probability greater than 83%. The results are renormalized taking into account the probability of pair emission and the losses before and after the device, i.e. they consider the amplifier performance only, and as such, it is limited only by its intrinsic losses.

4. Discussion and conclusion

We have fully characterized a heralded noiseless photon amplifier at telecom wavelengths and obtained a gain \( >100 \) associated with a heralding probability greater than 83% up to a distance in fibre of 20 km. Moreover, by duplicating the amplification stage it is possible to
Figure 3. (a) Single photon interference visibility measured as a function of input losses and transmission of the amplifier, the theoretical prediction is represented in the inset. (b) Interference fringes for two measurement settings such that the visibility is maximal, i.e. losses and transmission are balanced. The visibilities are $0.92 \pm 0.01$ and $0.94 \pm 0.03$ for the red (dots) and the blue (stars) curves, respectively. Visibilities are mainly limited by double pair emission in the down conversion process and polarization dependent losses in optical elements in the setup, which change the weight of the two interfering modes.

Figure 4. Heralding probability as a function of the input losses, for the eleven settings of $t$. The probability is higher than 80% when $t \geq 0.95$ even for losses of around 70%. The coloured region represents the theoretical prediction. The upper bound on the heralding probability is given by the intrinsic losses of the amplifier. On the upper axis, the equivalent transmission distance for an installed single mode fibre at telecommunication wavelength is given (losses are $0.24 \text{ dB km}^{-1}$). In the extreme regime of high losses and high transmission the performance of the amplifier no longer follows the theory, because the trigger and coincidences rates fall to the noise level and the error bars significantly increase, as expected.
have a heralded polarization qubit amplification [21], which could allow the violation of a Bell inequality without the detection loophole by compensating for the losses [4]. The heralded efficiency of such devices could be improved by reducing losses in the setup, in particular, by using anti-reflection coated and optimized optical elements [22]. However, for a more practical implementation of such a device, a fibre-based approach [11] with a fixed gain for a fixed amount of loss would provide a realistically efficient solution with even lower internal losses, i.e. a heralding efficiency >83%. One of the biggest challenges, though, is the generation of pure photons and coupling them into the heralded photon amplifier [22–26]. To resolve the coupling problem, one could also think of a device completely realized with integrated optics on a chip, which could include, sources of pure photons—either from single photon emitters, or heralded single photons realized via engineering the phase-matching of photon pair sources [27, 28] as well as wavelength division multiplexers [29], variable couplers [30] and potentially even detectors [31].

At the moment, all of these devices have been shown to work independently and it remains a grand challenge to bring these together on one chip, however, the heralded photon amplifier would provide an excellent motivation.

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References

3 Displacement of entanglement back and forth between the micro and macro domains
Displacement of entanglement back and forth between the micro and macro domains

N. Bruno, A. Martin, P. Sekatski, N. Sangouard, R. T. Thew* and N. Gisin

Quantum theory is often presented as the theory describing the microscopic world, and admittedly, it has done this extremely well for decades. Nonetheless, the question of whether it applies to macroscopic scales remains open, despite many efforts\(^1^3\). Here, we report on entanglement exhibiting strong analogies with the Schrödinger cat state as it involves two macroscopically distinct states—two states that can be efficiently distinguished using detectors with no microscopic resolution\(^4\). Specifically, we start by generating entanglement between two spatial optical modes at the single-photon level and subsequently displace one of these modes up to almost a thousand photons\(^2\). To reliably check whether entanglement is preserved, the state is redispersed back to the single-photon level and the field correlation is confirmed through a Hong–Ou–Mandel (HOM) type interference\(^6\). This path-entangled state, known as single-photon entanglement, can be seen as the signature of the non-classical feature of the heralded signal photon\(^7\,8\).

The second step consists of amplifying the mode \(A\) by applying a unitary operation \(D_\alpha(\lambda)\) corresponding to a displacement in the phase space. The displacement is obtained by combining the mode \(A\) and an intense local oscillator on a highly unbalanced beam splitter\(^9\,10\). The physics behind the displacement is based on an interference process. Hence, the field \(A\) and the local oscillator need to be indistinguishable. This is ensured in practice by producing the local oscillator by means of a difference frequency generation (DFG), using an identical nonlinear crystal to the one used for the photon pair creation but stimulated by a continuous-wave telecom laser. The indistinguishability between the resulting local oscillator and the field \(A\) is confirmed through a Hong–Ou–Mandel (HOM) type interference\(^11\) whose dip, reported in Fig. 2, has a visibility limited only by the reflectivity of the beam splitter and the photon statistics. As such, after the displacement, the detection of an idler photon heralds the creation of a single signal photon. By further placing a narrowband filter before the heralding detector, the signal photons are heralded in spectrally pure states with a bandwidth limited by the pump spectrum. To ensure their spatial purity, they are coupled into a monomode fibre with an efficiency of 50\%. The measurement of the second-order autocorrelation \(g^{(2)}(0) = 1.9(1)\) unambiguously demonstrates the purity of the signal field\(^12\). By sending the heralded photon into a balanced beam splitter, one obtains, leaving aside the loss, a maximally entangled state that describes the two output modes \(A\) and \(B\) sharing a single photon \((1/\sqrt{2})(|1\rangle_0 + |0\rangle_1)_A\). The path-entangled state, as such, becomes the reference for non-classical tests.

The experiment starts with the heralded creation of a path-entangled state lying in the macro domain (see Fig. 1). A nonlinear optical crystal is pumped by a pulsed laser in the picosecond regime to produce telecom photon pairs by means of spontaneous parametric down-conversion. We set the pump intensity such that the probability of creating a single pair per run is very small. Thus, the detection of one idler photon heralds the creation of a single signal photon. By further placing a narrowband filter before the heralding detector, the signal photons are heralded in spectrally pure states with a bandwidth limited by the pump spectrum. To ensure their spatial purity, they are coupled into a monomode fibre with an efficiency of 50\%. The measurement of the second-order autocorrelation \(g^{(2)}(0) = 1.9(1)\) unambiguously demonstrates the purity of the signal field\(^12\). By sending the heralded photon into a balanced beam splitter, one obtains, leaving aside the loss, a maximally entangled state that describes the two output modes \(A\) and \(B\) sharing a single photon \((1/\sqrt{2})(|1\rangle_0 + |0\rangle_1)_A\). The path-entangled state, known as single-photon entanglement, can be seen as the signature of the non-classical feature of the heralded signal photon\(^7\,8\).

The second step consists of amplifying the mode \(A\) by applying a unitary operation \(D_\alpha(\lambda)\) corresponding to a displacement in the phase space. The displacement is obtained by combining the mode \(A\) and an intense local oscillator on a highly unbalanced beam splitter\(^9\,10\). The physics behind the displacement is based on an interference process. Hence, the field \(A\) and the local oscillator need to be indistinguishable. This is ensured in practice by producing the local oscillator by means of a difference frequency generation (DFG), using an identical nonlinear crystal to the one used for the photon pair creation but stimulated by a continuous-wave telecom laser. The indistinguishability between the resulting local oscillator and the field \(A\) is confirmed through a Hong–Ou–Mandel (HOM) type interference\(^11\) whose dip, reported in Fig. 2, has a visibility limited only by the reflectivity of the beam splitter and the photon statistics. As such, after the displacement, the detection of an idler photon heralds the generation of an entangled state of the form

\[
\frac{1}{\sqrt{2}} (D_\alpha(\lambda)|1\rangle_0 + |\alpha\rangle_1)|1\rangle_B)
\]

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whether entanglement survives as its size increases, the displaced microscopic resolution
where the dead and alive components can be distinguished without
these states
micro domain. On the contrary, the probability to distinguish the
detected (the larger the photon number is, the further the pointer
pictured as a pointer on a scale that is shifted when photons are
read) represents the displacement of the vacuum, which follows a
Poissonian photon number distribution with mean photon number $|\alpha|^2$ equal to the variance. $D_\lambda(\alpha)|1\rangle_\lambda$ represents the displacement of a single photon that is a non-Gaussian state characterized by a photon number distribution with a mean photon number $|\alpha|^2 + 1$ and a variance $3|\alpha|^2$. Although $|\alpha\rangle_\lambda$ and $D_\lambda(\alpha)|1\rangle_\lambda$ differ by only one photon on average, the distance between their photon number distributions is about the square root of their size. Checking whether the photon number falls in the interval $[|\alpha|^2 - |\alpha|,|\alpha|^2 + |\alpha|]$ or not, allows one to distinguish their photon number distributions with a probability that increases with the size and approaches 74% in the limit of large $|\alpha|$. This probability goes up to 90% if the state (1) is seen in another basis so that it reads $(1/\sqrt{2})D_{\lambda}(\alpha)(|0+\rangle + |0-\rangle))$ by dividing the photon number into two intervals $[0,|\alpha|^2]$ and $[|\alpha|^2,\infty)$ (for real positive $\alpha$). This allows one to distinguish efficiently the two components of the displaced mode with classical detectors, which we now describe. Any measurement of the photon number can be pictured as a pointer on a scale that is shifted when photons are detected (the larger the photon number is, the further the pointer shifts). If the initial state of the pointer is not a delta-function, but has a large spread, for example, due to thermal noise, the probability to distinguish a single photon from the vacuum state in a single shot approximation that of a simple guess. We call such a measurement classical, because it does not resolve the quantum features in the micro domain. On the contrary, the probability to distinguish the states $D_{\lambda}(\alpha)(|0+\rangle + |0-\rangle))$ and $D_{\lambda}(\alpha)(|0-\rangle - |0+\rangle))$ in a single shot with such a classical measurement approaches 90% (for large enough $\alpha$). This makes the state (1) analogous to the famous Schrödinger’s cat state where the dead and alive components can be distinguished without microscopic resolution.

In the last step of the experiment where we wish to probe whether entanglement survives as its size increases, the displaced state in mode $A$ undergoes another displacement $D_{\lambda}(\alpha)^{-1}$, which ideally returns it back to the single-photon level. In practice, the de-amplification is realized in a similar manner as the amplification stage but inverting the phase of the local oscillator. The resulting fields can then be probed using single-photon detectors to reveal heralded entanglement between the modes $A$ and $B$. We emphasize that as the re-displacement is performed locally, it cannot increase the entanglement. A measure of the entanglement after the displacement thus provides a lower bound for the entanglement before the displacement, thus between the macro and micro states.

The state to be measured is described by a density matrix that includes noise and loss. To reveal entanglement, we use the tomographic approach based on single-photon detections presented in ref. 6 and successfully applied in many experiments.22-24 Specifically, from the measured values of the heralded probabilities $p_{nm}$ for detecting $n$ photons in mode $A$ and $m$ in mode $B$ ($m,n \in \{0,1\}$) and of the visibility $V$ of the interference obtained by combining the modes $A$ and $B$ on a balanced beam splitter, a lower bound on the concurrence at the level of the detection is obtained through $C \geq V(p_{01} + p_{10}) - 2\sqrt{p_{01}p_{10}}$. Let us mention that to maximize the observed concurrence, the interference visibility $V$ needs to be maximized and the probability for having one detection in each mode, $p_{ij}$, has to be minimized. This can be done with ease at the single-photon level; however, as the system size increases, this becomes increasingly difficult, as we shall see in the following.

The typical size that can be achieved primarily depends on the precision with which the relative phase of the local oscillators is controlled. This can be intuitively understood in two complementary ways depending on whether the phase noise is regarded as a decoherence mechanism operating on the macro state or as an imprecision of the measurement apparatus. In the first picture, a phase noise acts as if the environment performs a weak measurement of the photon number. (A phase noise, for example, the thermal fluctuations of the fibre length, can be described by a random unitary $U(\varphi) = e^{i\varphi a^+ a}$, where $a^+$ is the creation
We performed a series of measurements without displacement to verify that the modes $A$ and $B$ are indeed entangled when sharing a single photon. The interference fringe obtained by combining the two modes on a 50:50 beam splitter exhibits a visibility of $V = 0.966(6)$. The conditional probabilities are measured to be $p_0 = 0.9719(1)$, $p_1 = 1.313(7) \times 10^{-4}$, $p_0 = 1.492(8) \times 10^{-2}$ and $p_1 = 1.0(2) \times 10^{-2}$ leading to $C = 0.0208(7)$. These measurements are then repeated for various displacements. Figure 2 shows the resulting concurrence. The obtained values are in excellent agreement with the theoretical model described before that uses independent measurements of the transmission loss, detection efficiencies and pair creation probability. They show that the concurrence decreases as the state describing the mode $A$ becomes more macroscopic, and remains non-zero up to more than 500 photons. This strengthens the idea that one has to pay the price of increasing phase resolution to maintain and observe the quantum nature of a physical system as its size increases. The observed values are lower bounds on the amount of entanglement at the detection level and are mainly limited by optical loss. Factoring out the detector inefficiency and transmission loss yields an increase in the concurrence by more than one order of magnitude, for example, a lower bound of almost 20% for the concurrence of the micro–macro state with $|\alpha|^2 = 100$. Putting aside the loss before the displacement (the coupling inefficiency of the idler photon into the fibre), the concurrence would further increase to around 40% for $|\alpha|^2 = 100$.

We have reported an experimental observation of heralded entanglement involving two components that could be distinguished with detectors resolving only large photon number differences. Although there is no consensus on the definition of macroscopic entanglement, we believe that this property captures the essence of what is a genuinely macroscopic quantum state. Similarly to NOON states of or squeezed states of, the phase sensitivity of the macro state under consideration may be useful for precision measurements, especially in the presence of loss, where alternative states have a limited usefulness. Returning to a more fundamental perspective, one could imagine mapping these superpositions into the motion of a massive object by momentum transfer. This could open up the possibility of testing unconventional decoherence models.

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Author contributions

A.M., N.S. and R.T.T. conceived and designed the research. N.B. and A.M. carried out the experiment. P.S. and N.S. contributed with theoretical analysis. All authors participated in writing the manuscript. N.S., R.T.T. and N.G. supervised the project.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to R.T.T.

Competing financial interests

The authors declare no competing financial interests.
Displacement of entanglement back and forth between the micro and macro domains

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A. Local oscillator source

The coherent state source is composed of a 2 cm PPLN crystal pumped by the same 780 nm laser used in the heralded single photon (HSP) source (see main text) and seeded by a CW laser at 1563 nm (idler wavelength in the HSP source). Emission of pulsed light is stimulated by difference frequency generation (DFG). This process is governed by the energy conservation, which fixes the wavelength at 1557.5 nm and limits the bandwidth to that of the pump laser. Having a common pump laser allows to prepare a HSP and a coherent state with same spectral properties. A measurement of the second order autocorrelation function $\chi^2(0) = 1$ is used to confirm the Poissonian statistics of the state.

B. Indistinguishability study

To prove the indistinguishability between the photons emitted by the two sources, HSP source and local oscillator, we realise a two photon interference experiment. The HOM dip maxi-
mal visibility is given by:

$$V_{\text{max}} = \frac{P_{1,1}}{P_{2,0} + P_{0,2} + \frac{r^2 + t^2}{2rt} P_{1,1}}$$

where $P_{i,j}$ represents the probability to have $i$ and $j$ photons at the input ports of the beamsplitter characterised by a transmission $t$ and a reflection $r$. To optimise this visibility, it is necessary to minimise the contribution $P_{2,0}$ and $P_{0,2}$ by adjusting the average number of photons in the two sources. In our experiment, the HSP source is characterised by a coupling efficiency close to 50% and a mean number of photons of 0.01 per pulse. By setting the mean number of photon in the local oscillator at 0.05 photons per pulse, the maximal visibility is 80% and 22% for a 50:50 and a 90:10 couplers, respectively. As shown in Figure 1, and in Figure 2 (see main text), we obtain experimentally visibilities of 82(5)% and 23(4)%, which proves the perfect indistinguishability between the photons from our two sources.

C. Stabilisation

In the last part of the setup (see "analysis" in Figure 1 in main text) the state analysis stage is depicted. This part starts with a projection of the state into the single photon subspace, which is done in practice by applying an inverse displacement operator $D_u(-\alpha)$ on the mode $A$. In order to do this it is necessary to combine the mode $A$ with a local oscillator with the same properties as the first one and with opposite phase. For this purpose an interferometer with high visibility and phase stability is used. Due to the 4.5 ps pulse duration of our local oscillator, the maximum visibility can be achieved with a perfectly balanced interferometer. Moreover, for the phase stability, it is necessary to lock the interferometer on a reference laser, but this cannot be done with 0 path length.
difference. A good solution is the introduction of a different dispersion in one of the two arms. This is achieved by introducing a free space path (\(\sim 1\) cm) in one arm, compensated (at the experimental wavelength) by fibre path in the other arm. In this configuration, scanning the wavelength of the reference laser, we observe a full fringe every 400 nm, from which we estimate a path length difference of 20 \(\mu m\). Furthermore, we achieve visibilities of 99.98810(8)\% and 99.985(2)\% with a monochromatic laser and our local oscillator, respectively. The visibility in the pulse regime is mainly limited by the path length difference. To set the phase of \(\alpha\) in the re-displacement operation the interferometer is locked on the side of an interference fringe of the reference laser at 1460 nm.

D. Experimental results

In Figure 2 the raw data for the interference visibility and the probabilities \(p_{m,n}\) (see main text) are shown.
**Figure 1** Hong, Ou, and Mandel dip between HSP and LO on a balanced beam splitter. The visibility (82(5)% ) is in good agreement with what is expected taking in account the beam splitting ratio and the photon number statistics.(The error bars are the standard deviation of 100 acquisitions per points.)

**Figure 2** a) Single photon interference visibility as a function of the mean number of photons $|\alpha|^2$ involved in the displacement. The drop in visibility comes from the additional noise introduced by the imperfect re-displacement operation. b) Conditional probabilities $p_{m,n}$ as a function of the mean number of photons $|\alpha|^2$ involved in the displacement. As expected, since the displacement is applied only on arm A (see Figure 1), $p_{01}$ is constant. On the other hand, $p_{10}$ and $p_{11}$ grow linearly with $|\alpha|^2$, leading to a decreasing value for the concurrence (see Figure 2 in main text). (The error bars are the standard deviation of 100 acquisitions per points.)
4 Generation of tunable wavelength coherent states and heralded single photons for quantum optics applications
Generation of tunable wavelength coherent states and heralded single photons for quantum optics applications

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Quantum optics experiments frequently involve interfering single photons and coherent states. In the case of multi-photon experiments this requires all photons to be frequency degenerate. We report a simple and practical approach to generate coherent states that can be readily tuned to any wavelength required, for example by non-degenerate photon pair creation. We demonstrate this by performing a two-photon (Hong–Ou–Mandel) interference experiment between a coherent state and a pure heralded single photon source. Neither interfering photon passes through a spectral filter, the coherent state is constrained by the pump and seed lasers and the heralded photon exploits non-local filtering. We expect that such an approach can find a wide range of applications in photonic based quantum information science.

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1. Introduction

The interplay between coherent states and single photons has a rich history. Some of the first interference experiments [1,2] and Bell tests [3] between independent sources and the characterization of single photon states using homodyne detection [4] exploited these different photonic resources. Recently, the combination of single photons and homodyne detection [5], or displacement operations [6], has opened up new avenues for testing fundamental questions about entanglement and macroscopic systems, as well as witnessing single photon entanglement [7]. Coherent states have also been shown to be a useful resource for characterizing the purity of photon states [8] or generating entangled coherent states and quantum metrology [9]. The combination of single photon systems and coherent states also opens up possibilities for hybrid systems involving continuous variable and discrete detection schemes [10].

Typically, experiments of this type involve using a pump laser to generate the coherent states that are then interfered with photon pairs, for example from spontaneous parametric downconversion (SPDC) that is pumped by the frequency doubled laser. This approach constrains all the photons to be degenerate in frequency. In the case of multiple independent sources it is also important that the photons are in a pure single mode [11,12]. In the context of an increasing range of applications, an interesting task is to develop schemes to efficiently generate perfectly indistinguishable coherent states and single photons with low loss, low noise and, importantly, with wavelengths tunable over a wide range.

In the following we describe how we can efficiently generate pure heralded single photon (HSP) states and interfere this with a wavelength-tunable coherent state, without spectral filtering. We show the indistinguishability of these two sources by performing a two photon interference experiment [13] between the two independent sources.

2. Heralded single photon source

There has been enormous progress in developing heralded single photon sources and engineering their characteristics to suit myriad applications [11,14–17]. Typically we are interested in photonic sources for quantum communication, therefore we need to ensure that the photons are pure, indistinguishable, narrowband and at telecom wavelengths, which has proven to be a more demanding challenge. The most straightforward way to herald pure single photons is using nonlocal filtering to prepare the heralded photon in a pure state [18].

To realize the heralded single photon source we use pairs of photons produced via SPDC. The Hamiltonian for the process has the form

\[ H = \xi \int d\omega_1 d\omega_2 S(\omega_1, \omega_2) a_1^\dagger(\omega_1) a_2^\dagger(\omega_2) + h.c \]  \hspace{1cm} (1)

where \( S(\omega_1, \omega_2) \) is the Joint Spectral Amplitude (JSA) of the photons, as a function of the signal (s) and idler (i) frequencies.
The process is governed by the conservation of energy and momentum between final and initial configuration

$$\omega_f = \omega_i + \omega_s;$$

(2)

$$k_i(\omega_i) + k_i(\omega_i) - k_p(\omega_i + \omega_s) - \frac{2\pi}{\Lambda} = 0$$

(3)

where $\Lambda$ is the poling period of the crystal. Periodical poling is a technique based on the inversion of the crystal polarization with a period $\Lambda$ that can be tuned and optimized in order to achieve the desired phase matching condition (quasi-phase matching), with a greater efficiency than typical phase-matched materials.

In our experiment, SPDC is achieved in a periodically poled Lithium Niobate (PPLN) bulk crystal, with type-II quasi-phase matching. To generate photon pairs at telecom wavelengths, the crystal is pumped at 780 nm by a pulsed ($\Delta t = 2$ ps) laser with a repetition rate of 76 MHz. The two photons are generated with orthogonal polarization, thus they can be deterministically separated.

To observe two photon interference effects [13] with a good visibility we need to maximize the spectral and temporal overlap of the two photons. If the two photons are coming from different sources, this condition imposes the need for a single spectral and temporal mode, i.e. to produce separable photons [14]. Narrow spectral filtering is the most common solution, but is not suitable for configuration where low loss is required [19]. We take advantage of the fact that selecting a single spectral mode on the heralding (idler) photon with a sufficiently narrowband filter heralds the presence of a signal photon in the correlated mode. Hence, the signal photon is heralded in a pure state [18], without any need for additional filtering, thus minimizing loss. The final efficiency of this heralding then only depends on the coupling efficiency [20].

The single- or multi-mode nature of the photon’s state can be determined by measuring the second order autocorrelation function ($g^{(2)}(\tau)$) in a Hanbury Brown and Twiss like experiment [21]. The setup consists in sending light on a balanced beam splitter and measuring the histogram of coincidence rate as a function of the time with two APDs placed at the output of the beam-splitter. This measurement allows one to estimate the photon number statistics. For a single mode source the photon number distribution is thermal, and for a large number of independent modes with thermal statistics the photon number becomes Poissonian. The value of $g^{(2)}(0)$ is directly related to the purity (and, consequently, to the number of modes) from the relation $g^{(2)}(0) = 1 + P = 1 + 1/K$, where $P$ and $K$ represent the purity and the number of Schmidt modes, respectively. For a pure photon we expect a $g^{(2)}$ close to 2, which corresponds to thermal statistics. For a large number of modes, $K \rightarrow \infty$, we find that $g^{(2)}(0) \rightarrow 1$, which is the signature of a Poissonian distribution [12,23].

Fig. 2 shows a measurement of the purity $P$ of the signal photon as a function of the filter bandwidth. The spectral correlation can be quantified evaluating the average number of Schmidt modes, which is found to be $K = 5 \pm 1$ without filtering. The spectral correlation is due to a combination between the properties of the material and the bandwidth of the pump laser. Choosing a 0.2 nm filter for the idler photon allows one to achieve a purity of $0.9 \pm 0.1$ for both photons of the pair. The bandwidth of such heralded single photon is then limited by the pump bandwidth, due to energy conservation, and it is found to be equal to $79 \pm 1$ GHz.

3. Tunable coherent state source

The purpose of this paper is to demonstrate a tunable coherent state source that can be used in conjunction with heralded single photons. This kind of interference has historically been realized doubling the frequency of a laser to subsequently down-convert it and using the same laser as a local oscillator [23,24,5]. Femtosecond pulsed lasers allow one to have a broad spectrum, such that filtering both heralded single photon and coherent state with similar band-pass filters is possible, although lossy. Indeed, to efficiently generate coherent states with the same temporal and spectral characteristics as the heralded photon is difficult. The solution we propose here consists in the use of a coherent state produced by means of difference frequency generation (DFG) in a nonlinear crystal, as in [25].

DFG takes place in materials with a $\chi^{(2)}$ nonlinearity and it is a stimulated process that happens in the presence of two strong coherent states, namely a pump laser and a seed laser. This is in contrast to SPDC, which is a spontaneous process that requires only one strong pump and is stimulated by the vacuum field. In

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1. The Schmidt number is defined $K = 1/\sum |c_i|^2$ where $c_i$ is the coefficient of the Schmidt decomposition of the joint spectral amplitude of the pairs: $f(\omega_s, \omega_i) = \sum \Psi(\omega_s)\Psi(\omega_i)$ (see Ref. [22] for more details).
DFG, energy and momentum conservation (Eq. (3)) also need to be satisfied
\[ \hbar \omega_p + \hbar \omega_i = \hbar \omega_s + 2\hbar \omega_i; \quad (4) \]
where the frequency of the idler photon is given by the difference \( \omega_p - \omega_s = \omega_i \).

The setup is shown in Fig. 3: the crystal we use is similar to the one used for the HSP, and the pump light is coming from the same laser. The seed for the nonlinear process is a laser at telecom wavelengths (1563.5 nm). Since the latter is a monochromatic laser, the bandwidth of the coherent state resulting from the DFG process is fixed by the bandwidth of the pulsed pump.

The coherent state is characterized in the same way as the HSP source, by measuring the second order autocorrelation function \( g^2(0) \). For a coherent state \( g^2(0) = 1 \). In the case of DFG, where the process is stimulated, this function becomes
\[ g^2(0) = 1 + \frac{N_{SP}}{N_{SP} + N_{ST}} \quad (5) \]
where \( N_{SP} \) and \( N_{ST} \) represent the number of photon per mode emitted by spontaneous emission and the number of photons emitted by stimulated emission, respectively. Moreover, in this configuration, where just one mode is stimulated \( N_{ST} \) is given by \( N_{ST} = N_i N_{SP} \) (see Refs. [26,27] for more details), with \( N_i \) corresponding to the number of photons in the seed field. So, we obtain
\[ g^2(0) = 1 + \frac{1}{N_i + K} \quad (6) \]
where \( K \) is the number of photons per mode in the pump.

In Fig. 4, we see that as the input intensity of the seed field increases, the output idler state is coherent and the spontaneous emission is negligible. \( N_i \) is chosen to be greater than 30 and an attenuator is placed at the output of the crystal to produce a coherent state \( |\alpha\rangle \) with \( |\alpha|^2 < 1 \).

4. Hong–Ou–Mandel interference

To prove the indistinguishability between the HSP and the coherent state generated by DFG, a two photon interference experiment is performed [13,3,2,24]. As presented in Fig. 5, the two states are mixed on a 50/50 fiber coupler. To avoid polarization and temporal discernibility that could reduce the interference visibility, two Lefèvre fiber polarization controllers and an adjustable optical delay line are employed, respectively. Note that the two interfering states never pass through a filter. The spectral overlap is only ensured by the filter placed on the correlated heralding photons, and the energy conservation for the coherent
state. The absence of filters allows one to achieve a heralding coupling efficiency of 50%. Losses are mainly due to fiber coupling at the output of the PPLN crystal.

Furthermore, the visibility is also limited by the statistics of the sources. Two photons coming at the same time from one source can give a coincidence which reduces the dip visibility. Assuming that the probability of having three or four photons is negligible, one can give a coincidence which reduces the dip visibility. Assuming that this technique will be beneficial in a wide range of experiments where one needs to combine single photons and coherent states.

Fig. 6 shows the experimental results and the theoretical prediction for a mean number of photons $|\alpha|^2 = 0.05$ and $P_1 = 0.05$ and 0.01. The theoretical prediction fits perfectly with the experimental measurement for $R$ close to 1 which proves the indistinguishability between the coherent state and the HSP.

5. Conclusion

We have demonstrated how difference-frequency generation and spontaneous parametric down conversion in twin crystals can be exploited to generate states of light with common spectral and temporal properties. This was confirmed by performing a Hong-Ou-Mandel like experiment that showed that the two states overlap perfectly. We utilized sufficiently nonlocal narrow-band filtering (26 GHz in our case) to produce heralded single photons with the same bandwidth of the pump (80 GHz) in a pure spectral mode. As the CW laser, which seeds the DFG, is monochromatic, the coherent state is constrained to have the same spectral bandwidth as the HSPs but with the advantage that one can work in the non-degenerate wavelength regime. We believe that this technique will be beneficial in a wide range of experiments where one needs to combine single photons and coherent states.

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References


Fig. 6. HOM dip obtained for two different values of the probability of photon pairs emission for the HSPS. The blue and black data points correspond to $P_1 = 0.05$ and 0.01, respectively. The mean number of photons per pulse in the coherent state $|\alpha|^2 = 0.05$. The curves represent the theoretical prediction for two photons with a bandwidth of 80 GHz and a maximum visibility given by Eq. (7) for $R = 1$. The error bars are given by the photon counting statistics, with acquisition times of 120 s per point for $P_1 = 0.01$ and 20 s for $P_1 = 0.05$, resulting in smaller error bars for the former. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)


5 Pulsed source of spectrally uncorrelated and indistinguishable photons at telecom wavelengths
Pulsed source of spectrally uncorrelated and indistinguishable photons at telecom wavelengths

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Abstract: We report on the generation of indistinguishable photon pairs at telecom wavelengths based on a type-II parametric down conversion process in a periodically poled potassium titanyl phosphate (PPKTP) crystal. The phase matching, pump laser characteristics and coupling geometry are optimised to obtain spectrally uncorrelated photons with high coupling efficiencies. Four photons are generated by a counter-propagating pump in the same crystal and analysed via two photon interference experiments between photons from each pair source as well as joint spectral and g(2) measurements. We obtain a spectral purity of 0.91 and coupling efficiencies around 90% for all four photons without any filtering. These pure indistinguishable photon sources at telecom wavelengths are perfectly adapted for quantum network demonstrations and other multi-photon protocols.

OCIS codes: (190.4410) Nonlinear optics, parametric processes; (270.5565) Quantum communications.

References and links


1. Introduction

Engineering multi-photon quantum states of light is becoming increasingly important as we move beyond point-to-point, or two-party systems, entangled systems and towards entanglement based protocols such as quantum teleportation, swapping and relays, useful for QKD [1]. Already entanglement swapping protocols rely on the interference between quantum states carried by independent photons [2, 3]. As we look towards more complex quantum networks [4] and simulation experiments [5], efficient coupling and spectrally pure photon generation is of critical importance.

Currently, spontaneous parametric down conversion (SPDC) is the simplest and most versatile way to generate photon pairs and historically PPLN waveguides have been exploited in the telecom regime due to their high brightness [6]. However, most of the time the twin photons generated in this way have spectral correlations, which decrease the purity of their quantum state [7]. Spectral filtering is usually employed to erase these correlations, consequently introducing additional losses [8–11]. Another solution is to adapt the pump laser spectrum and the dispersion properties of the non-linear material. This approach has been implemented in various media such as bulk crystals [12–14], waveguides [15, 16], or fibers [17]. Quantum communication requires narrowband sources and recent focus has shifted to using periodically poled potassium titanyl phosphate (PPKTP) nonlinear crystals with ps pulsed lasers [18–23] to address these demands of quantum communication. We have combined this approach with some
of our recent work on optimising source-to-fibre coupling [24] to realise multi-photon experiments with pure, indistinguishable narrowband telecom photons with high levels of purity and coupling efficiencies that are ideally suited to testing quantum network protocols.

After a presentation of the theoretical model employed to simulate the non-linear interaction in the crystal, we describe the experimental setup. In the third part, we present the characterization of the purity of the photons emitted by our source. In the last part, we report the brightness of the source.

2. Theoretical model for phase matching and photon coupling

In the following we outline the theory used for modeling the phase matching as well as the focusing characteristics for the pump laser and photon pairs that are needed to optimise both the purity and the fibre coupling. SPDC processes are governed by energy and momentum conservation laws:

$$\omega_p = \omega_s + \omega_i; \quad \vec{k}_p = \vec{k}_s + \frac{2\pi}{\Lambda} \vec{z}$$  \hspace{1cm} (1)

where $\omega$ and $\vec{k}$ represent the frequencies and the wavevectors of the pump (p), signal (s), and idler (i) photons, respectively. In a periodically poled crystal, a poling period $\Lambda$ is inscribed in the crystal to compensate the dispersion and achieve quasi-phase matching [25]. The non-linear process that takes place in this kind of crystal is described by a Hamiltonian function of the creation and annihilation operators of the signal and idler fields, of the following form:

$$H = c \int d\omega_p d\omega_s d\vec{k}_p d\omega_i d\vec{k}_i S(\omega_s, \omega_i, \vec{k}_i, \vec{k}_s, \vec{k}_p) a^{\dagger}(\omega_s, \vec{k}_s) a^\dagger(\omega_i, \vec{k}_i) + \text{h.c.}. \hspace{1cm} (2)$$

We define the joint spectral amplitude as $S(\omega_s, \omega_i, \vec{k}_s, \vec{k}_i, \vec{k}_p) = \epsilon(\omega_s, \omega_i) \phi(\omega_s, \omega_i, \vec{k}_s, \vec{k}_i, \vec{k}_p)$, where $\epsilon(\omega_s, \omega_i)$ and $\phi(\omega_s, \omega_i, \vec{k}_s, \vec{k}_i, \vec{k}_p)$ are the pump energy envelope and the phase matching function, respectively. These two functions depend on the energy conservation and the phase matching conditions. For a sech²-shaped pump pulse, such as a pulse generated in a mode-locked laser, the first factor is given by:

$$\epsilon(\omega_s, \omega_i) \approx \text{sech} \left[ \frac{(\omega_s + \omega_i - \omega_p)}{3\pi} \frac{2\log(2 + \sqrt{3})}{\Delta\omega_p} \right] \hspace{1cm} (3)$$

with $\Delta\omega_p$ the full width at half maximum of the pump pulse in frequency. In the Gaussian approximation, this becomes:

$$\epsilon(\omega_s, \omega_i) \approx \exp \left( \frac{\omega_s + \omega_i - \omega_p}{4\sigma_p^2} \right) \hspace{1cm} (4)$$

with $\sigma_p = \Delta\omega_p/2\sqrt{2\ln2}$. The phase matching function for a crystal of a length $L$ depends, in general, on the direction of $\vec{k}_{s,i,p}$ and on the poling period [26]. However, in the approximation of collinear propagation we can consider only the $z$ direction, and the function is given by:

$$\phi(\omega_s, \omega_i, k_s, k_i, k_p) \approx \text{sinc} \left[ \frac{L}{2} \left( k_z(\omega_s) + k_z(\omega_i) + \frac{2\pi}{\Lambda} - k_z(\omega_p) \right) \right] \hspace{1cm} (5)$$

In the Gaussian approximation this function can be written as:

$$\phi(\omega_s, \omega_i) \approx \exp \left( -\alpha^2 \frac{\Delta\kappa^2 L^2}{4} \right) \hspace{1cm} (6)$$

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with $\alpha = 0.439$ and $\Delta k = k_i(\omega_i) + k_s(\omega_s) + \frac{2\pi}{\Lambda} - k_p(\omega_p)$. Furthermore, for very small deviations from the central wavelengths of the pump $\omega_{p,0}$ and of the emitted photons $\omega_{i,0}$ and $\omega_{s,0}$, we have $\Delta k = (\omega_i - \omega_{i,0})k_i' + (\omega_s - \omega_{s,0})k_s' - (\omega_p - \omega_{p,0})k_p'$, with $k'_j = \frac{\partial k_j(\omega_j)}{\partial \omega_j}|_{\omega_j,0}$. By using these approximations, the joint spectral amplitude can be rewritten as a Gaussian function \[7, 27\]:

$$S(\Delta \omega_i, \Delta \omega_s) = \epsilon(\omega_i, \omega_s) \varphi(\omega_i, \omega_s)$$

$$\propto \exp \left( -\frac{1}{\sigma_p^2} + \alpha^2 L^2(k_p' - k_i')^2 \right) \frac{\Delta \omega_p^2}{4}$$

$$- \left( \frac{1}{\sigma_p^2} + \alpha^2 L^2(k_p' - k_i')^2 \right) \frac{\Delta \omega_i^2}{4}$$

$$- \left( \frac{1}{\sigma_p^2} + \alpha^2 L^2(k_p' - k_i')(k_p' - k_s') \right) \frac{\Delta \omega_p \Delta \omega_s}{2}, $$

(7)

with $\Delta \omega_j = \omega_j - \omega_{j,0}$. The purity of the two-photons state is defined as the inverse of the number of Schmidt modes $K$, more precisely $P = 1/K = Tr[\rho Tr[\rho]]$, where $\rho$ is the two photon state, with the joint spectral amplitude defined in Eq. 7. To generate pairs of photons in a separable state, i.e. $K = 1$, it must be:

$$\alpha^2 L^2 \sigma_p^2 (k_p' - k_i')(k_p' - k_s') = -1. $$

(8)

To satisfy this equation, the condition $k_i' < k_p' < k_s'$ must be fulfilled in the non-linear medium.

Following Eq. (8), we find that a PPKTP crystal can satisfy these constraints. To satisfy the phase matching condition, a poling period of $47.8 \mu$m inscribed in the crystal enables the type-II non-linear interaction between pump, signal and idler photons polarised along the X, Z, and Y axes of the crystal, respectively. In this configuration, we obtain $(k_p' - k_i')(k_p' - k_s') = 2.63 \times 10^{-2}$ m$^{-1}$ GHz$^{-1}$. Thus, for a 3 cm long crystal a purity close to one should be achievable with a pump bandwidth of 467 rad$^{-1}$ GHz, which corresponds to a pulse duration with FWHM $= 2.6 \text{ps}$, assuming a Gaussian approximation of the pump as in Eq. (2). In this way we are able to generate degenerate photon pairs at 1544 nm with a pump laser at 772 nm.

If we consider these experimental characteristics, we can simulate the non-linear interaction defined in Eq. (2) without any approximations, i.e. taking into account the shape of the pump, Eq. (3), the crystal geometry and the wavevector orientations of Eq. (5). This simulation gives us access to all of the correlations for the signal and idler photon in wavelength, wavevector, as well as joint wavelength - wavevector basis [24]. The first is useful in order to estimate the purity in frequency, and the other two are used to determine the coupling efficiencies of the generated twin photons into single mode fibres. To obtain pure photons, we need to reduce the correlation between the signal and idler wavelengths. As mentioned above, the wavelength correlation depends on the type and length of the crystal, on the pulse duration of the pump laser, and also on the spatial profile of the three fields. To have an efficient coupling, one needs to reduce the wavelength wavevector correlations for the signal and idler photons and increase the signal-idler wavevector correlation (see [24] for more detail). Thus, from this simulation we can study the influence of the spatial modes of the fields on the spectral purity and on the coupling efficiencies.

The wavevector correlations only depend on the wavevector of the pump, and to be maximised, the pump field should be close to a plane wave. Consequently, the focusing parameter denoted $\xi$, defined as the ratio between the half length of the crystal and the Rayleigh range of the beam $z_R (\xi = L/2z_R)$, needs to be close to zero. For this reason we choose a waist of $296 \mu$m for the pump laser in the center of the crystal, which corresponds to a focal parameter
of 0.0425. Figure 1 illustrates the dependence on purity and coupling. To define the optimal focal parameter for the photon pairs generated in the crystal, we studied, as shown in Fig. 1, the photon pair spectral purity and the photon’s spatial-spectral purity, as a function of the photon collection waist. We chose a collection waist of 187 µm (ξ = 0.212), which is a good compromise between spectral purity and coupling efficiency. To obtain this waist, lenses with a 400 mm focal length need to be placed at the output of the crystal combined with 11 mm focal length lenses to then couple into the single mode fibres.

3. Experimental realisation

A schematic of the experimental setup is given in Fig. 2. The light from a Ti-Sapphire picosecond mode-locked laser at 772 nm is injected into a 3 cm long PPKTP bulk crystal and we exploit a double-pass configuration. The source is built in a double-pass configuration via a set of two dichroic mirrors employed to inject the laser and separate it from the generated photons. This enables the production of two photon pairs emitted independently in opposite directions, with an adjustable delay. The photon pairs are deterministically separated and all four photons are then coupled into single mode telecom fibres. All four photons are generated around 1544 nm and we measure a fibre coupling efficiency of (90 ± 4)% for all four photons.

4. Characterisation of the spectral purity

There are several different techniques for characterising the spectral purity of the photons. We test three of these, which are represented in the lower half of Fig. 2: the second order autocorrelation function (g^{(2)}(τ)) for each photon, the joint spectral intensity for the pair of photons from each source, and finally the two-photon (HOM) interference between photons from independent sources.

The second order autocorrelation function (g^{(2)}(τ)) in a Hanbury Brown and Twiss like experiment [28], involves sending the signal photons to a balanced beam splitter and placing two single photon detectors based on InGaAs avalanche photodiodes (APD) at the output. A time-to-digital convertor (TDC) is used to record the coincidence histogram as a function of the time difference between clicks on the two detectors. This measurement allows one to estimate the photon number statistics. For a pure photon we expect a g^{(2)}(0) close to 2, which corresponds to thermal statistics typical of a single mode, and when the number of modes increases g^{(2)}(0) goes to 1 which is the signature of a Poissonian distribution, characteristic of a multimode emission from the nonlinear medium [29]. For a SPDC source we can define the
Fig. 2. a) Experimental setup. A 772nm ps-pulsed laser pumps a periodically poled potassium titanylphosphate (PPKTP) crystal in a double-pass configuration. Dichroic mirrors (D) separate the telecom and visible wavelengths. A variable delay is placed before the second passage of the pump laser through the crystal. The generated photon pairs are first collimated by a set of lenses (L) and then separated with polarising beam-splitters (PBS) and coupled into single mode fibres. Three characterisation techniques are shown: b) the joint spectral intensity; c) the second order autocorrelation function \(g^2(\tau)\), and d) Hong-Ou-Mandel (HOM) interference.

\[ g^2(0) = 1 + P = 1 + \frac{1}{K}, \]

where \(P\) and \(K\) represent the purity and the number of Schmidt modes, respectively [11,30,31]. The measurement of \(g^2(0)\) as a function of the pump laser bandwidth is reported in Fig. 3. The maximum of purity is obtained for a pump laser with a FWHM equal to 0.33 nm as predicted by the numerical simulation. For the following measurements, we use this value. This results in an experimentally measured value of \(g^2(0) = 1.91 \pm 0.04\), corresponding to a purity of \(P = 91\% \pm 4\%\).

Fig. 3. \(g^2(0)\) as a function of the laser pump FWHM. The point and the line represent the experimental data and the simulation predictions, respectively.

Another quantity that is useful in order to evaluate the purity of the two photon state is the
Joint spectral intensity (JSI). To measure it we use two tunable filters with a bandwidth of 0.2 nm, one for each photon of the independent pairs. Two InGaAs APDs are placed at the output of the filters and connected to a coincidence measurement apparatus. By recording the coincidence rate as a function of the position of the two filters, the JSI can be reconstructed.

Fig. 4(a) and 4(b) represent the simulated and measured JSIs, respectively, and it shows the good agreement between the theoretical prediction and the experimental results. The simulation, calculated from the joint spectral amplitude defined in Eq. (7) before any Gaussian approximation, predicts a purity of 91.8%. By fitting the experimental data with a two dimensional Gaussian function derived in Eq. (7) we obtain a purity of 91% ± 3%. N. b. that the sidepeaks typical of a squared sinc function are attenuated as an effect of the spatial filtering with single mode fibres, given that the collection waist is optimised to reduce wavelength-wavevector correlations and at the same time improve the spectral purity.

\[
V = \frac{\text{Tr}[\rho_a \rho_b] + \text{Tr}[\rho_a^2] + \text{Tr}[\rho_b^2] - ||\rho_a - \rho_b||}{2}, \tag{9}
\]

where \(\rho_a\) and \(\rho_b\) represent the density matrices of the two photons at the input of the beamsplitter. Accordingly, the visibility measured for the signal-signal photons interference directly gives the spectral purity, while for the dip between a signal and an idler photon the visibility...
Fig. 5. HOM dips obtained between signal-signal (a) and signal-idler photons from independent sources as a function of the delay between the photons. The blue and black points represent the four-fold coincidences and accidental coincidences, respectively. For both configurations a visibility of $91 \pm 2\%$ is obtained.

gives also information about the distinguishability. The obtained visibilities prove that less than 1.1 spectral modes are emitted by the crystal and that the four photons are perfectly spectrally indistinguishable, as predicted by the numerical simulation.

5. Source brightness

As well as the photon purity, one of the key features of this source is its brightness, since it provides rates that allow one to concretely implement quantum communication tasks. We define the brightness as the number of photon pairs available at the output of the source per mode and pump power. The brightness depends on the pair creation efficiency and on the transmission losses of the source. For a pulsed laser with pump power of 200 mW, the crystal generates 0.01 photons per pulse at a repetition rate of 80 MHz, which correspond to 4.0 pairs/µJ. At the fibred output of the source we obtain a brightness of 2.6 pairs/µJ, the losses per photon being of 0.96 dB, 0.50 dB due to the optical elements which build the source and 0.46 dB introduced by the coupling into single mode fibre. It can be noted that for all the presented results the laser pump power is set to 600 pJ per pulse to generate just 0.0025 photon pairs in order to avoid errors due to double pair contributions.

6. Conclusion

We have presented an experimental set-up for generating pure indistinguishable narrowband telecom photons in a double-pass configuration using a PPKTP crystal. By combining concepts for pure photon generation based on phase matching non-linear crystals and pump laser bandwidths along with the constraints of coupling photons into single mode fibres we obtain high coupling efficiencies and purities of around 90% for all four photons. The type II phase matching condition allows us to deterministically separate all four photons providing us with a flexible multi-photon source of photons for diverse applications in quantum communication and networking.

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6 High efficiency coupling of photon pairs in practice
High efficiency coupling of photon pairs in practice

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Abstract: Multi-photon and quantum communication experiments such as loophole-free Bell tests and device independent quantum key distribution require entangled photon sources which display high coupling efficiency. In this paper we put forward a simple quantum theoretical model which allows the experimenter to design a source with high pair coupling efficiency. In particular we apply this approach to a situation where high coupling has not been previously obtained: we demonstrate a symmetric coupling efficiency of more than 80% in a highly frequency non-degenerate configuration. Furthermore, we demonstrate this technique in a broad range of configurations, i.e. in continuous wave and pulsed pump regimes, and for different nonlinear crystals.

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References and links


1. Introduction

The efficient generation of photon pairs is fundamental to multi-photon experiments and the scaling of quantum networks. In particular, distributing quantum correlations through the use of photons is a promising resource for quantum cryptography technologies such as device independent quantum key distribution (DI-QKD) [1]. Thus, efficiently producing, distributing and detecting photon pairs from spontaneous parametric down conversion (SPDC) [2–4] is a major objective for current research, as an inefficient transmission or detection process opens loopholes in the verification of quantum correlations [5].

The problem of detection has recently undergone significant progress, and single photon detectors with near unit efficiency have been demonstrated both for visible and telecommunication wavelengths [6–8]. The problem is therefore shifted towards the transmission of the photons. For long distance communication, single mode telecommunication fibers and wavelengths can be used [9]. The single mode fibers have the additional advantage of guaranteeing the produced photon pairs to be in a single (spatial) mode, thus increasing the correlation in their detection. Residual loss can be compensated by heralding the presence of a photon with protocols such as qubit amplification [10] or sum frequency generation [11]. This leaves the problem of efficiently coupling the photons into single mode optical fibers as the last problem to be solved.

To efficiently couple SPDC photon pairs into single mode optical fibers, they must be spatially correlated in such a way that if either photon is coupled in one fiber, the other is coupled in the other fiber. This is equivalent to maximizing the number of coincidences $C$ with respect to the number of single counts $R_s$ and $R_i$ for the signal and idler photons, respectively. This is usually called the symmetric heralding efficiency $\mu_{si} = C/\sqrt{R_sR_i}$ [3], and is the geometric average of the single heralding efficiencies defined as $\mu_{si}(i) = C/R_{si}(i)$.

Recently, general theoretical analyses have been put forward to solve this problem [2–4]. In these proposals it has been shown that there exists pump (p), signal (s) and idler (i) focusing parameters for which both the heralding efficiency into single mode optical fibers and the generation rates are high. In addition, high heralding efficiency has been demonstrated experimentally [12–17]. These articles, however, do not provide a practical methodology to obtain high heralding efficiencies and the general theoretical treatments presented in [2, 3] still require considerable adaptation on an experimental level. In this article we show how $\mu_{si}$ can be
maximized. The method which we describe should give the reader a good intuition and understanding of how to achieve high heralding efficiency in practice, with a variety of crystals and in different experimental configurations. As will become clear later on, this method relies on the entanglement of the generated photons, and is thus quantum mechanical.

In what follows, we explain in simple terms the intuition behind obtaining high coupling through this method, and show how phasematching calculations are used to determine the pump, signal and idler focusing parameters that maximize heralding efficiency. We then describe the experimental implementation of these ideas in a highly non degenerate SPDC setup (for which high $\mu_s$ has not been demonstrated in the literature) based on potassium niobate (KNbO$_3$, type-I angular phasematching) and periodically poled lithium niobate (PPLN, type-0 quasi-phasematching). We do this in two different pump regimes, continuous wave (CW) and pulsed, and demonstrate $\mu_s(i) > 80\%$ and $\mu_s \sim 80\%$.

2. Intuition

In this section we explain, in terms of a simple one-dimensional model, the intuition behind the technique used here to achieve high heralding efficiency.

Let’s consider the SPDC process, where a pump photon ($p$) interacts with a non-linear medium to decay into a pair of signal ($s$) and idler ($i$) photons. If the pump has a well defined wavevector (i.e. it can be approximated as a plane wave), and the energy of the photons is uncorrelated with their wavevector directions, the resulting state can be written as a correlated two-photon wavefunction in the wavevector basis [18]:

$$|\Psi_{s,i}\rangle = \int dk |k_s\rangle |k_p - k_i\rangle.$$  \hspace{1cm} (1)

Now we want to “project” the idler photon in the state corresponding to the fundamental mode of an optical fiber, denoted by $|\phi\rangle_i$. The representation of this state in the position basis is

$$\phi(x) = \langle x|\phi\rangle_i = \frac{1}{(2\pi\sigma^2)^{1/4}} e^{-x^2/4\sigma^2},$$  \hspace{1cm} (2)

where $\sigma$ is the width of the Gaussian packet. Writing Eq. (2) in the momentum basis

$$\phi(k) = \langle k|\phi\rangle_i = \left(\frac{2}{\pi}\right)^{1/4} \sigma^{1/2} e^{-k^2\sigma^2},$$  \hspace{1cm} (3)

we can calculate the resulting state of the signal photon $\Psi_s(x) = \langle x|\langle \phi|\Psi_{s,i}\rangle_i $ after the idler photon has been projected onto the state $|\phi\rangle_i$:

$$\Psi_s(x) = \int dk \langle x|k_s\rangle \langle \phi|k_p - k_i\rangle_i$$  \hspace{1cm} (4)

$$= \frac{e^{ik_px}}{(2\pi\sigma^2)^{1/4}} e^{-x^2/4\sigma^2}$$  \hspace{1cm} (5)

It is possible to see from Eq. (5) that every time photon $i$ is measured to be in the Gaussian state Eq. (2), photon $s$ must also be found in a Gaussian state of the same form up to a global phase. This means that once photon $i$ is detected to be in the fundamental mode of an optical fiber, photon $s$ will be heralded in the fundamental mode of a fiber, leading to a high signal-idler coincidence and a high heralding efficiency. Of course, the effect is symmetric under exchange $s \leftrightarrow i$, so it does not matter if the idler photon is used to herald the signal photon or the opposite.

It is the correlations in the state shown in Eq. (1), generated by a plane wave pump which leads to this high probability of heralding a photon in the mode of an optical fiber. If such a
state cannot be generated, the above method will not work. Of particular interest is the situation in which the wavevectors of the signal and idler photons are correlated to their respective wavelengths. If this is the case, the detection of the idler photon in a state $|\phi\rangle_i$ will project the signal photon onto an entangled state between wavevector and wavelength; once the information about the wavelength is traced out, the signal photon will be left in a statistical mixture of different wavevectors, which will not couple well into the fundamental mode of an optical fiber.

With this we see that two points are crucial to efficiently herald a photon in the mode of an optical fiber through the method described above: first, it is necessary to pump the non-linear medium with a plane-wave (collimated) pump. Second, it is fundamentally important to have photons in a wavelength-wavevector separable state, thus working in a collection regime where these spectral-spatial correlations are reduced as much as possible. In the following sections we address these two points.

3. Choosing pump focusing and collection parameters

To address the above-mentioned issues we considered the phasematching of our crystal (we used a custom-made program but there are programs which can be found online, for example [19]). In the first instance we use a specific nonlinear crystal and pump regime, though the approach here is general and can be applied to a vast number of configurations. The success of the methods described depends only on the crystal length, refractive index, pump focusing and collection optics of the setup.

As an example, we consider the phase-matching for the interaction:

$$532\text{nm} \rightarrow 810\text{nm} + 1550\text{nm}$$

in a 10 mm long KNbO$_3$ crystal pumped in pulsed regime (8 ps). In what follows we determine the pump focusing and the collection waists for this example, but we note that the same results for pump focusing and collection are obtained if we considered as an example the case of CW pump regime.

3.1. Pump focusing

To maximise the correlations between $s$ and $i$, the pump field should be close to a plane wave. In this section we examine to what extent the pump wavefront should be plane for the photon coupling to be good.

First, we define the focusing parameter, which tells how much a beam is focused inside a non-linear crystal with respect to the crystal length. This definition follows the one presented in [3]. The focusing parameter denoted $\xi$ is simply the half length of the crystal $L/2$ divided by the Rayleigh range $z_R$ of the beam

$$\xi = \frac{L}{2z_R} = \frac{\lambda L}{2\pi w_0^2}$$

where $\lambda$ and $w_0$ correspond to the wavelength and waist at the center of the crystal, respectively. The closer $\xi$ is to zero, the more the beam can be approximated by a plane wave. On the other hand, if $\xi \gg 0$ the beam is tightly focused and localized at a point in space.

To achieve a high wavevector signal-idler correlation, and consequently a high heralding efficiency, it is necessary to operate in a regime where the pump focusing parameter is close to zero. Fig. 1 shows the degree of separability, or purity (analogous to the one defined in [20]), of the two photon joint angular correlation function, as a function of the pump waist. We can see that the larger the pump waist (the lower $\xi$), the lower the purity and consequently the...
higher the correlation in wavevectors of signal and idler. In the limit that the waist of the pump
is infinite (plane wave) we have zero purity and approach a perfectly entangled state like the
one in Eq. (1).

Fig. 1. Purity of the joint signal-idler angular correlation function as a function of the pump
waist. Correlation is higher with large pump waists: we wish to operate as close to this
regime as possible.

There are, however, limits on the pump focusing parameter that one should keep in mind.
First, the lower the focusing of the pump, the smaller the generation rates will be. Second,
special attention must be taken about the cross section of the crystal: if the pump focusing is
too low, the beam might hit the edges of the crystal. These two observations set a practical
upper limit on the pump focusing. A third point, which sets a lower limit on the pump focusing,
is the uncertainty in the wavevector: the pump wavevector uncertainty cannot be higher than the
wavevector uncertainty in the collection modes of the signal and idler photons. This wavevector
uncertainty corresponds to the angular spread in the case of a Gaussian beam.

In our setup we would like to optimise only the heralding efficiency. For this reason, it suf-
fices to set the pump angular spread much smaller that the collection modes angular spread,
keeping in mind the restrictions given by the cross section of the nonlinear crystal. Taking
these points into consideration, a good value for pump focusing in our setup (for both PPLN
and KNbO3) was $\xi_p = 0.02$.

3.2. Signal and Idler collection

As mentioned above, the second point is to operate in a collection regime where the wavelength
of the signal (idler) photon is uncorrelated from its wavevector direction. To verify this we
considered the phase-matching of our crystal and calculated the correlations between angle
of emission and the photon’s wavelengths. As we will show below, these calculations will
determine the size of the image of the mode of the optical fiber at the center of the nonlinear
crystal and consequently all the lenses of the setup. Since we chose $\xi_p = 0.02$ this implies a
waist for the pump of 200 µm, which yields an angular spread of $\Delta \Theta_p \sim 0.0017$ rad.

In figure 2 it is possible to see the complete characterization of the KNbO$_3$ crystal pumped in pulsed regime. Fig. 2(a) shows the joint spectral function while 2(b) shows the angular correlation intensity function. Fig. 2(c) and 2(d) present the spectral-spatial correlations for signal and idler photons, respectively. These functions are all we need to determine the collection waists of the setup. All the angles involved are external to the nonlinear crystal.

To choose a collection waist for the signal photon, we look at Fig. 2(c), where it is possible to see that for a certain range of emission angles, approximately from $-0.1^\circ$ to $0.1^\circ$ ($\Delta \Theta_s \sim 0.2^\circ \sim 0.0035$ rad) the angle of emission is independent of the wavelength, i.e. the photon is pure in this basis. This determines the collection waist for the signal photon to be approximately

$$w_{0,s} \simeq \frac{2\lambda_s}{\pi \cdot \Delta \Theta_s} \simeq 145 \mu m \quad (7)$$

This choice can be readily justified if we look at Fig. 3, where the spectral-spatial “purity” of the state is shown as a function of $\Delta \Theta_s$. As $\Delta \Theta_s$ increases, the spectral-spatial purity (and consequently the heralding efficiency) decreases and stabilizes at a value around 50%. To determine the corresponding collection waist for the idler we look at the joint signal idler angle correlation function in Fig. 2 (b), where we can see then that the corresponding $\Delta \Theta_i$ for the idler is approximately twice the one for the signal $\Delta \Theta_i \sim 2 \cdot \Delta \Theta_s \simeq 0.4^\circ (= 0.007$ rad),

![Figure 2](image-url)
giving an idler collection waist of

\[ w_{0,i} \approx \frac{2\lambda_i}{\pi \cdot \Delta \Theta_i} \approx 140 \, \mu m \]  

(8)

Figure 3. Purity of the spectral-spatial wavefunction as a function of the signal angular collection. The purity starts to drop after the signal angular collection becomes \( \sim 0.3^\circ \), which justifies the choice \( \Delta \Theta_i = 0.2^\circ \).

Figure 4 shows the various phasematching functions after imaging these collection waists in single mode optical fibers. Notice that the joint spectral function, depicted in Fig. 4(a) remains unchanged after coupling into the fiber. On the other hand, the emission angle of signal and idler photons become uncorrelated from the spectrum, as can be seen in Fig. 4(c) and 4(d).

If we consider the focusing parameters for the pump (\( w_{0,p} \approx 200 \, \mu m \)), signal (\( w_{0,s} \approx 145 \, \mu m \)) and idler (\( w_{0,i} \approx 140 \, \mu m \)) with a crystal length of 10 mm, we find \( \xi_{532} = 0.02 \), \( \xi_{810} = 0.06 \) and \( \xi_{1550} = 0.13 \), respectively. This shows that our considerations are outside any of the optimal values predicted by [2, 3], since here we only optimize the heralding efficiency and not the generation rates.

4. Experiment

A schematic of the experimental setup is shown in Fig. 5. A laser at 532 nm passes through a single mode optical fiber to ensure a perfect Gaussian profile. The output of the fiber is directed to an aspheric lens of focal \( f = 15.29 \, \text{mm} \) (L1), to produce a 200 \( \mu m \) waist at the center of a 10 mm long nonlinear crystal, either a PPLN with cross section \( 0.5 \, \text{mm} \times 0.5 \, \text{mm} \) or a KNbO3, with cross section \( 5 \, \text{mm} \times 5 \, \text{mm} \). The waist of the pump at the position of the center of the crystal is verified with a CCD camera.

The crystal produces pairs of photons at 1550 nm and 810 nm, which are collected from a waist of \( \sim 140 \, \mu m \) and \( \sim 145 \, \mu m \) respectively, as calculated above. To image these collection waists on the optical fibers, we use two lenses on each path. On the 1550 nm path, an
Fig. 4. Phasematching functions after coupling into single mode optical fibers, collection waists $w_{0,s} \simeq 145\,\mu m$ for signal and $w_{0,i} \simeq 140\,\mu m$ for idler.

Fig. 5. Experimental setup. A 532 nm laser passes through a single mode optical fiber at that wavelength, and is used to pump a nonlinear crystal. The remaining pump light is removed by a high pass filter, and the signal and idler photons are split by dichroic mirror (DM). The transmitted photons (1550 nm) are collimated by the lens L2 and focused on the fiber by L3. The reflected photons (810 nm) are collimated by L4 and focused inside the fiber by L5. The zoom shows the output of the nonlinear crystal, where emission happens through a wide range of angles. Only a small fraction of the emitted light, as determined above, is imaged (collected) into the optical fibers.

$f = 150\,\text{mm}$ lens (L2) is used to collimate the beam, and an $f = 7.5\,\text{mm}$ lens (L3) is used to focus the beam on the optical fiber. These lenses were selected taking into account the mode
field diameter of the telecom single mode fiber (5.1 µm) and were verified to produce the desired waist at the crystal position during the alignment, using a CCD camera. The 810 nm path has a similar setup, with an \( f = 150 \) mm lens (L4) to collimate and an \( f = 3.1 \) mm lens (L5) to focus. Again, lenses were chosen according to single mode fiber diameter (2.8 µm) and the produced waist was verified with a CCD. After the crystal, the remaining pump light is filtered by a high-pass filter and a prism (P) on the 810 nm path.

When coupled into fibers, the photons are then directed to detectors via a time-to-digital converter (TDC). For the 810 nm photon we use a free running Si-APD (D1), with efficiency of \( (48.0 \pm 2.5)\% \) at 810 nm. On the 1550 nm side we use an InGaAs detector (D2), with efficiency of \( (24 \pm 2)\% \). The heralding efficiency \( \mu_{ij} \) for the signal (idler) is defined to be the ratio between coincidences \( C \) to the number of singles (detector noise subtracted) in the idler (signal) detector \( S_{ij} \) corrected by the detection efficiency \( \eta_{ij} \). We are interested in characterizing the coupling into the optical fibers. For this reason we also correct for the transmissions in the signal (idler) path \( t_{ij} \):

\[
\mu_{ij} = \frac{C}{R_{ij}} = \frac{C}{S_{ij} \cdot \eta_{ij} \cdot t_{ij}}
\]

The measured values for overall transmissions in the setup are \( t_{1550} = (87.0 \pm 0.2)\% \) and \( t_{810} = (78.0 \pm 0.2)\% \). These values are limited mainly by the transmission of the high pass filter used to remove the pump, which was measured to be \( (88.0 \pm 0.2)\% \). The value \( t_{810} \) is smaller than \( t_{1550} \) due to the prism in that path. In principle these values can be made closer to 100% by using higher quality optics and appropriate coatings [16].

If we consider the KnbO₃ crystal pumped by a CW laser at 5 mW the rate of singles at 810 nm was 39.0 KHz, while the coincidence rate was around 7.0 KHz. Integrating for 30 s a heralding efficiency of \( \mu_{1550} = (86 \pm 7)\% \) was measured. The error on this value is dominated by the systematic error in the measurement of the detector efficiency. If we reverse the detection roles, heralding the 810 nm photons with those at 1550 nm, the rate of singles was 3.2 KHz, while the coincidence rate was around 0.9 KHz. Integrating also for 30 s a heralding efficiency of \( \mu_{810} = (75 \pm 5)\% \) was measured. This yields a symmetric heralding efficiency of \( \mu_s = \sqrt{\mu_{810} \cdot \mu_{1550}} \sim 80\% \). The same heralding efficiencies are obtained in both CW and pulsed regimes.

5. Summary of the method

We can summarize the technique to achieve high heralding efficiency in the following simple steps.

1) The wavevector uncertainty for the pump must be smaller than the collection modes’ wavevector uncertainty. This means the angular spread for the pump must be smaller than the angular spread of the signal and idler collection modes, a condition which can be achieved by setting the pump focusing parameter close to zero. In our setup we have about \( \xi_p \sim 0.02 \).

2) Calculate the signal spectral-spatial correlation function for the crystal used (shown here in Fig. 2(c)). This will determine the signal angular collection and consequently the signal collection waist.

3) Calculate the signal-idler joint angle correlation function. Given the signal angular collection, this function will determine the idler angular collection and consequently the idler collection waist.
4) Purity is highest towards the centre of the beam. For this reason, in the presence of wave-length dependent optical elements, it is very important to align with the exact wave-lengths which will be collected into the fiber.

This method was described using a 10 mm KNbO$_3$ crystal (type-I, frequency non-degenerate angular phasematching) as an example, pumped in CW and pulsed (8 ps) regimes. To test the universality of this approach we also verified the technique using a 10 mm PPLN (type - 0, frequency non-degenerate quasi-phasematching) pumped in pulsed (8 ps) regime. In addition to that, we have also employed the same method in a completely different setup, using a pulsed (2 ps) laser at 780 nm to pump a 30 mm PPKTP crystal with type-II degenerate quasi-phasematching, generating photons at 1560 nm. The design of these sources using this technique all yielded experimental results for $\mu_{si} \sim 80\%$. We summarize the different tested configurations in Table 1. In all of these experiments we have neglected effects of spatial walk-off.

Table 1. Different setups where the proposed method was employed. The wide range of configurations proves the universality of the technique.

<table>
<thead>
<tr>
<th>NL crystal</th>
<th>Length</th>
<th>Pump</th>
<th>Phasematching</th>
<th>Type</th>
<th>$\mu_{si}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KNbO$_3$</td>
<td>10 mm</td>
<td>CW</td>
<td>532nm $\rightarrow$ 810nm + 1550nm</td>
<td>I</td>
<td>(80 $\pm$ 4)$%$</td>
</tr>
<tr>
<td>KNbO$_3$</td>
<td>10 mm</td>
<td>Pulsed, 8 ps</td>
<td>532nm $\rightarrow$ 810nm + 1550nm</td>
<td>I</td>
<td>(80 $\pm$ 4)$%$</td>
</tr>
<tr>
<td>PPLN</td>
<td>10 mm</td>
<td>Pulsed, 8 ps</td>
<td>532nm $\rightarrow$ 810nm + 1550nm</td>
<td>0</td>
<td>(77 $\pm$ 4)$%$</td>
</tr>
<tr>
<td>PPKTP</td>
<td>30 mm</td>
<td>Pulsed, 2 ps</td>
<td>780nm $\rightarrow$ 1560nm + 1560nm</td>
<td>II</td>
<td>(88 $\pm$ 4)$%$</td>
</tr>
</tbody>
</table>

For the first three configurations in the Table 1 we considered the transmissions and detection efficiencies mentioned in the section **Experiment**. For the PPKTP, the corresponding values of transmission and detection efficiencies are $t_{1560} = (81.0 \pm 0.2)\%$ and $\eta_{1560} = (25 \pm 3)\%$. Notice that for the PPKTP, the length of 30 mm imposes a different pump waist from the one used in the examples described in the last sections, and the simulations shown in figure 2 yield different results, thus implying other collection conditions. This provides strong experimental evidence for the universality of this method, showing that the above four steps can be employed in a wide range of configurations to obtain high heralding efficiencies.

6. Conclusion

In this paper we report a method to design a high heralding efficiency photon pair source for bulk crystals. This technique exploits the purity of photons in the wavelength-wavevector basis and the signal-idler correlations in wavevector.

A theoretical and experimental study of different crystals, for both degenerate and non-degenerate photon pairs as well as pulsed and CW regimes shows that the method works in a broad range of configurations. The main parameters are the pump focusing, the length and the refractive index of the crystal. Given these parameters we show how to determine the signal and idler collection waists and consequently all the lenses of the setup.

In our method, there is a trade-off between the maximum coupling achieved and the generation rates. The bigger the angular spread of the pump with respect to the angular spread of the collection modes, higher the coupling will be; on the other hand, the pump power will be spread over a large area thus reducing the generation rates in the collected modes. If the rates can be compensated by an increase in the pump power, the ultimate limitation for the method is given only by the quality of the optics, and in principle the heralding efficiency can be made close to unity. This work has important implications for anyone designing a photon pair source based on SPDC bulk crystals, and can be applied to a wide range of experiments.

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7 Simple, pulsed, polarization entangled photon pair source
Simple, pulsed, polarization entangled photon pair source

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1. Introduction

Entanglement is a fundamental resource in quantum information science and sources of photonic entanglement are key enabling technologies for quantum communication [1]. Entangled photons can be generated from a wide range of single- or heralded-photon sources [2], or directly via interactions in materials with nonlinear optical susceptibility, such as spontaneous parametric down conversion (SPDC). A simple way to generate polarization entangled photon pairs consists in employing a nonlinear crystal with type-II phase-matching conditions [3], which allows one to generate pairs of photons with orthogonal polarization and to exploit the conservation of energy and momentum to produce entanglement.

However, a constraint arises when we want to use a collinear configuration or waveguide crystals [4], for example, to increase the source brightness. The simplest approach is to use a balanced beam splitter, in which case the two photons take different paths half of the times. Hence, one immediately has only 50% efficiency and the scheme only works in post-selection. We can no longer produce polarization entanglement without post compensation or manipulation in fiber based setups [9] or on chip [10,11].

In this work, we report on an extension of a scheme proposed in Ref. [12], which exploits energy conservation and degenerate photon pairs in SPDC along with readily available fiber components such as dense wavelength division multiplexer (DWDM) filters. However, in our case, the picosecond pulsed regime allows one to generate narrowband photons and overcome synchronization problems associated with distributed systems. We can also adapt the filtering bandwidth to obtain spectrally separable [13] photons that are a requirement for more complex quantum communication tasks.

2. Principle

In the present work, the realization of the entangled photon pair is based on a type II spontaneous parametric down conversion (SPDC) process in a nonlinear crystal. As proposed in Ref. [12], in order to generate entangled photon pairs with minimal losses (i.e. the collinear photons are deterministically separated), it is necessary to be in a degenerated configuration. A filtering stage with wavelengths slightly detuned from the central one, like a dense wavelength division multiplexer (DWDM) [12], has the role of separating the photons. At the two outputs, labeled a (Alice) and b (Bob), if all the distinguishabilities between the H and V polarized photons are erased, we obtain an entangled state of the form:

$$|Ψ⟩ = \frac{1}{√2} [ (|H_{δ\omega_a}⟩ + |V_{δ\omega_b}⟩) |V_{δ\omega_b}⟩ + |V_{δ\omega_a}⟩ |H_{δ\omega_a}⟩ ]$$

(1)

where $δ\omega_f$ represents the detuning between the central filter frequency and the degeneracy frequency of the photon pairs.
Parametric down conversion processes in nonlinear crystals are governed by energy and momentum conservation laws:

\[ \omega_p = \omega_s + \omega_i; \quad k_p = k_s + k_i + \frac{2\pi}{\Lambda} z \]

where \( \omega \) and \( k \) represent respectively the frequency and the wavenumber for the pump (p), signal (s), and idler (i) photons [14]. \( \Lambda \) is the crystal poling period employed to compensate the crystal dispersion. It was chosen to produce degenerate photon pairs at 1560 nm. This nonlinear process is governed by the Hamiltonian:

\[ H = c \int d\omega_s d\omega_i \varepsilon(\omega_s, \omega_i) \phi(\omega_s, \omega_i) \phi^\dagger(\omega_i) \phi^\dagger(\omega_s) + h.c. \]  

(3)

\( \varepsilon(\omega_s, \omega_i) \) and \( \phi(\omega_s, \omega_i) \) are the pump pulse envelope and the phase matching function, respectively, which fix the energy conservation and the phase matching conditions. If the pump pulse is Gaussian, the first factor is given by \( \varepsilon(\omega_s, \omega_i) = \exp[-(\omega_s + \omega_i - \omega_p)^2/4\Delta\omega_p^2] \), with \( \Delta\omega_p \) the pump frequency bandwidth. The second factor, for a crystal of a length \( L \), is given by: \( \phi(\omega_s, \omega_i) = \sin(\Delta k p + k s - k i) / C_0 \). 

Fig. 1 represents the joint spectral intensity (JSI) \( J(\omega, \omega) = |\varepsilon(\omega_s, \omega_i) \phi(\omega_s, \omega_i)|^2 \) corresponding to our experimental configuration. From Fig. 1 it is possible to observe a correlation in wavelength: indeed in this case the state of the two photons at the output of the crystal is not separable in frequency. This state can be made spectrally separable by filtering one photon down to 200 pm [13].

If we include the action of such a filter in the JSI function, we can define the wavelength distribution of Bob’s photons that are correlated with the photons sent to Alice. Fig. 2a gives an example of this distribution. H and V polarized photons are spectrally distinguishable, which will clearly reduce the entanglement visibility [15]. The visibility in the diagonal basis is given directly by the overlap in frequency of the two polarization modes. Fig. 2b shows this overlap as a function of the relative position of the filter compared to the degeneracy wavelength.

This distinguishability is not observed in the CW regime [12], due to perfect spectral correlation between the photons. In the pulsed regime the JSI contour is no longer an ellipse at 45° (see Fig. 1). The “tilt” angle \( \theta \) of the phase matching function is given by

\[ \tan \theta = \frac{k_p - k_i}{k_s - k_i} \]  

(4)

where \( k' = dk/d\omega \) is the first derivative of the wavenumber, which in this case corresponds to a \( \theta = 59.96° \). To avoid this distinguishability it is necessary to add a second filter on Bob’s arm, with a bandwidth adapted to select the part of the spectrum where the two photons, H and V, overlap.

3. Experimental realization

A scheme of the experimental setup is depicted in Fig. 3. The two photons are generated via SPDC in a 2 cm-long PPLN bulk crystal (Covesion), pumped by a 2 ps pulsed laser at 780 nm with a repetition rate of 76 MHz (Coherent Mira 900). Type II quasi-phase matching generates two orthogonally polarized photons. A birefringent medium, e.g. a polarization maintaining single mode fiber (PMF) is used to compensate the temporal walk-off introduced by the LN birefringence between the two orthogonally polarized photons. The photon pairs are directly coupled into the PMF with an efficiency of 50%, the slow and fast axis as well as the length of the fibre (1.44 m) are adapted to compensate the temporal walk-off introduced by the crystal. To separate the two photons, a circulator is placed at the output of the PMF and a fiber

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**Fig. 1.** (a) Numerical simulation of the joint spectral intensity for a 2 cm-long type II periodically poled Lithium Niobate (PPLN) crystal pumped by a 2 ps pulsed laser and (b) corresponding measurement realized scanning two tunable filters with a bandwidth of 200 pm and counting the coincidences between signal and idler photons. Note that signal and idler are associated to H and V polarization, respectively.

**Fig. 2.** (a) Coincidence spectrum heralded by the photon passing the 0.2 nm filter. The lines and the points represent respectively the theoretical prediction and the experimental results and (b) overlap in frequency between the two polarization modes as a function of \( \omega \), the difference between the degeneracy wavelength and the central wavelength of the 0.2 nm filter.
Bragg grating (FBG) with a 200 pm bandwidth is connected at the second port of the circulator, thus reflecting the filtered light towards the third port. The central wavelength of the filter is detuned by $\sim 0.5$ nm compared to the photons degeneracy wavelength. This detuning is such that one and only one photon per pair is selected by the filter. A second narrow filter (FWHM=200 pm), used to eliminate residual distinguishabilities, can be added in Bob's arm.

4. Result

The photons reflected by the FBG are sent to Alice and the transmitted photons are sent to Bob, producing the state defined in Eq. (1). The polarization analysers are composed of a half-wave plate, a polarizing beam splitter and a gated InGaAs avalanche photodiode (APD, ID Quantique id210). A time-to-digital converter (TDC) records the coincidence counts between Alice and Bob for different polarization settings. To quantify the entanglement quality of the source, we perform a standard Bell inequality measurement. Alice’s analyser is fixed to horizontal (H), vertical (V), diagonal (D) and anti-diagonal (A) positions, and the coincidences are recorded as a function of Bob’s analyser angle ($\theta$). Fig. 4(a) presents the experimental results. A high visibility is obtained in the (H, V) basis, as expected. However, in the (D, A) basis, the visibility is limited to 78%, due to the fact that on Bob’s side. Fig. 4(b) shows the experimental data.

5. Conclusion

Currently, the filtering is the major disadvantage with such an approach. The need for a second filter on Bob’s arm does not arise in the CW regime as the correlations in frequency between the two photons are perfect, i.e. the tilt angle of the JSI is 45°. To overcome the double filtering in the pulsed regime, one could look at engineering the phase matching conditions in such a way that the JSI orientation is diagonal, which would lead to a perfect overlap between the entangled photons’ spectra. In either case, the filtering can be easily replaced by ultra-DWDMs, which have a bandwidth of 25 GHz, which would be well suited to such a system.

We have demonstrated a simple way to generate polarization entangled photon pairs based on collinear type-II PPLN phase matching in the pulsed regime. The high quality of the entanglement is demonstrated by visibilities above 98% in both bases. The pulsed regime also allows one to have temporally well-defined photons for synchronization with other sources and the filters allow one to deterministically separate the pairs and erase the frequency correlations. These characteristics, together with the simplicity and the intrinsic stability of the setup, make this approach a good candidate for future applications in entanglement-based quantum communication networks.

Table 1

Summary of the interference pattern visibility for the different orientations of the Alice’s polarization analyser and relative Bell parameter ($S$) for the configuration without and with the filter on Bob’s arm.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>H</th>
<th>V</th>
<th>D</th>
<th>A</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>No filter</td>
<td>98.9 (3.3)%</td>
<td>96.5 (2.7)%</td>
<td>78.0 (2.2)%</td>
<td>74.5 (3.1)%</td>
<td>2.46 (12)</td>
</tr>
<tr>
<td>Filter</td>
<td>98.9 (5.3)%</td>
<td>99.1 (4.8)%</td>
<td>98.0 (3.6)%</td>
<td>98.3 (3.3)%</td>
<td>2.79 (17)</td>
</tr>
</tbody>
</table>

Fig. 3. Experimental setup. From left to right: pairs of orthogonally polarized photons degenerate at telecom wavelengths are generated via SPDC in a PPLN crystal, pumped by a 780 nm pulsed laser. The two photons are coupled into a PM fiber, then a filter based on a circulator followed by a FBG splits $\lambda_a$ and $\lambda_b$ in order to generate the polarization entangled pair. Note that the additional filter described in the main text is placed directly at the output of the FGB. Half wave plates (HWP) and polarizing beam splitters (PBS) followed by single photon avalanche photodiodes constitute the measurement apparatuses.

Fig. 4. Interference pattern for the standard entanglement measurement, i.e. horizontal (H), vertical (V), diagonal (D), and anti-diagonal (A), for one filter and two filters configurations, (a) and (b), respectively. To reduce the probability of having double pairs, the pair creation probability was fixed at $P=0.01$. 

Previously explained. This is related to the spectral properties of the photon pair, and it is a peculiar effect of the nonlinear material combined with the pump pulse and filter shapes. Placing a second narrowband filter on Bob’s arm allows one to improve the visibility up to 98% for both the natural creation basis (HV) and the diagonal (DA) basis.
Acknowledgments

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References

8 Herald amplification of photonics qubits
Heralded amplification of photonic qubits

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Quantum communication relies on the efficient transmission of photonic states through fibre optic networks [1]. Photon loss represents one of the most significant sources of decoherence for photonic quantum technologies, not only for applications in quantum communication, but for metrology [2] and fundamental tests of quantum physics [3]. While the amplification of quantum states is fundamentally limited by the no-cloning theorem [4], heralded qubit amplifiers provide a promising solution that teleports the state in a way that can compensate for this loss [5, 6]. Here we demonstrate heralded qubit amplification for Time-Bin and Fock-state qubits in an all-fibre, telecom-wavelength, scheme that highlights the simplicity and potential for a fully integrated photonic solution. The efficiency of the amplifier is studied for several scenarios and an amplification gain of 9 is realised. We also demonstrate the first heralded qubit amplifier without post-selection, which is critical for Device Independent communication scenarios [7] as well as fundamental tests such as Bell inequalities that close the detection loophole over extended distances.

Heralded qubit amplification (HQA) provides a teleportation-based solution to overcome loss in photonic quantum technologies [5, 6]. HQA are of particular interest for quantum communication, where, protocols are typically based on qubits encoded on photons at telecommunication wavelengths, due to the high transmission efficiency in single mode optical fibres. Nonetheless, loss remains a limiting factor to the maximal communication distances. This is even more critical for recently proposed protocols such as Device Independent Quantum Key Distribution (DIQKD) [6, 8, 9], which need to overcome transmission loss in order to observe the violation of a Bell inequality free from the detection loophole.

Heralded amplification has previously been demonstrated in proof-of-principle experiments in the visible [10] and telecom regimes [11, 12]. There has also been interest in exploiting this in the continuous variable regime to reduce noise [13, 14] and extend transmission distances [15, 16]. Fundamental tests of single photon addition and subtraction have also been explored in the same context [17, 18]. Heralded qubit amplification was recently realised in the visible regime in a polarisation-based experiment [10]. Here, we extend this to the telecom regime and develop simple, all-fibre, linear optic circuits for schemes adapted for quantum communication: exploiting Time-bin [19] and Fock-state [20] encodings. In the case of the Fock-state scheme we demonstrate the first HQA without post-selection.

The principle for heralded qubit amplification follows directly from a teleportation protocol. Consider FIG. 1 (Left), an unknown input qubit state is combined with one half of an entangled pair, which then undergoes a Bell state measurement (BSM), teleporting the input state into the output channel. The use of an unbalanced beam splitter (with transmission t) allows for the teleportation to be biased so as to reduce the vacuum contribution.

Consider an initial qubit state $|\psi_{in}\rangle$, transmitted, for example through a fibre optic channel. It will undergo loss, such that the resulting state is in a statistical mixture of the initial state and vacuum:

$$\rho_{in} = (1 - P_{in})|00\rangle\langle00| + P_{in}|\psi_{in}\rangle\langle\psi_{in}|,$$

where $P_{in}$ is the channel transmission. This will be the input state for the HQA, which also requires orthogonal input states that span the relevant qubit space, $|\psi_{in}\rangle \equiv |\phi, \phi^{+}\rangle$. In the case of Fock-state qubits, the ancilla state is simply $|1\rangle$, as the vacuum comes for free. This is easily extended to polarisation qubits, or similarly time-bin qubits, by coherently combining two amplifiers - one for each mode, see FIG. 1 (Right).

The resulting output state is still a mixture between the vacuum and the initial qubit, up to a unitary transfor-
The amplification is heralded by a 3-fold coincidence given by a click in the detector D1, which heralds the input photon, followed by D2 and D3, heralding its teleportation. 4-fold coincidence with the output photon in detector D4 are recorded. We use MoSi SNSPDs with 70% detection efficiency at 1550 nm, at a temperature $T < 2.5$ K. Two polarisation modes which accompany the two temporal modes, short and long, allow one to separate the $\ell$ and $s$ components in the analysis without further loss. A temporal delay introduced in one of the outputs of the BSM allows one to project the input modes onto two Bell states ($\Psi^\pm$) using two detectors instead of four. The analysis is performed in the same interferometer used for the preparation, with the aid of a fibred circulator. In this way, active stabilisation of the phase is not needed. The relative phase in the input qubit is controlled after the interferometer, taking again advantage of the two polarisation modes, using a variable liquid crystal retarder.

The gain $G(t)$ is, in the ideal case, equal to $t/(1-t)$. In practice, considering realistic experimental conditions, it is also a function of the coupling of the ancilla $P_a$ and the detection efficiency $\eta$ (see Methods). Having non-photon-number-resolving (PNR) detectors increases the probability of heralding the vacuum, and therefore it reduces the gain with respect to the ideal case. A proposed modification of the original scheme allows one to improve the gain by eliminating the case in which two auxiliary photons are reflected [21]. However, this scheme is more demanding from an experimental point of view as it requires a phase-stabilised interferometer after the unbalanced beam splitter.

In the case of time-bin qubits, e.g. $|\psi_{in}\rangle = \frac{1}{\sqrt{3}} (|s_H\rangle + e^{i\phi_{in}} |t_V\rangle)$, the polarisation can be used to label and switch the short and long paths to improve efficiency [22, 23], requiring the ancilla state to be of the form $|\psi_a\rangle = |s_H, t_V\rangle$. In this way we can exploit the two polarisation modes in a single fibre. A BSM is now heralded after a coincidence detection for any pair of H-V photons. In FIG. 2 we see a further simplification of the HQA, which combines paths to the detectors to reduce complexity and the number of detectors required for the four possible outcomes. This simple linear optical circuit lends itself to an all-fibre implementation.

FIG. 2 shows the experimental setup where four indistinguishable photons are generated by two SPDC sources (see Methods). Detector $D_1$ heralds a single photon that is sent into an interferometer to prepare the time-bin qubit. The other source provides the two ancillae photons, where one is delayed with respect to the other by $\Delta_2$, corresponding to the path-length difference in the time-bin interferometer. The delay between the short photon from each source is determined by $\Delta_1$. It is critical that both the short photons’, and similarly, both long photons’, arrival times, and the photons themselves, are indistinguishable for the BSM to function with high fidelity [24]. The phase of the initial qubit can be varied and loss can be added before the HQA. The output state is then sent back to the same interferometer to verify the fidelity with respect to the initial state.

Ideally, we would like that the BSM heralds the output of the desired state, however, this would typically require three photons, either from deterministic single photon sources or heralded single photon sources (HSPS) based on spontaneous parametric downconversion (SPDC) - one for the initial qubit and two ancillae. The former are not yet available and the latter implies a six photon experiment, which is extremely challenging. By using two SPDC sources we can herald the initial qubit and use the second SPDC source for both ancillae, which means the final output state has to be detected to verify the HQA functioned correctly, i.e. we post-select the results,
as in [10], based on the four-fold coincidences between the heralded single photon $D_1$, a successful BSM $D_2$ and $D_3$, and the output state $D_4$. We use MoSi superconducting nanowire single photon detectors (SNWPD) with high (70%) efficiency (see Methods). By exploiting the fast recovery time (80 ns) of these detectors, each one is used twice in each run of the experiment, i.e. with two different and well distinguished arrival times.

Experimentally the probability of having a qubit $P_{in}$ is measured by dividing the number of coincidences between detectors $D_1$-$D_3$ by the number of counts in detector $D_1$. The output probability $P_{out} = G(t)P_{in}$ is given by dividing the four-fold coincidence $D_1$-$D_2$-$D_3$-$D_4$, by $D_1$-$D_2$-$D_3$. This allows us to determine the gain of the amplifier as a function of $P_{in}$. The measurement is done for two different values of $t$: 0.7 and 0.9. The coincidence rate is around 10 counts/minute for each of the two curves, and it drops linearly with the input probability $P_{in}$. For this reason, when measuring the gain, we increase the time of measurement by a factor $1/P_{in}$ in order to have the same statistics. FIG. 3 shows that the experimental results are in agreement with the theoretical prediction. Due to the need for a variable delay line to prepare the ancillae photons when changing the variable beam splitter, we introduce extra loss, which reduces the gain. Nonetheless, in the case of $t = 0.9$, the gain reaches the maximum value of 9, fixed by $t/(1-t)$. The dashed lines in FIG. 3 show the gain when the excess loss for the ancillae are factored out – we keep experimentally feasible values for $P_{in} = 90\%$.

To measure the Fidelity of the output state, defined as $\mathcal{F} = \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle$, we project the output state on the input. Varying the phase $e^{i\phi}$ of the input qubit, we can observe oscillations in the fourfold coincidence rates, as shown in FIG. 4. The two curves correspond to the two possible outputs of the BSM. at $\Delta \phi_{in} = 0$, the output obtained when projecting onto $\Psi^+$ gives the maximum Fidelity, while the output related to $\Psi^-$ is orthogonal to the qubit, since a phase shift of $\pi$ (the unitary operation from equation 2) is needed to recover the input state.

The main figure of merit is, however, the output probability $P_{out}$, plotted in FIG. 3 as a function of $P_{in}$. To be more specific, $P_{out}$ is the probability of having a photon in output once a successful BSM is announced. From FIG. 3 one can see that the main limitation on the performance of the amplifier is given by the probability of having an ancilla $P_a$, which multiplied by $t$ gives the upper bound on the output probability. Again, the theory and experiment are in good agreement and when the excess (ancillae preparation) loss is factored out we are easily in a regime of net gain. n.b. if a beam splitting ration of $t = 0.99$ is used, an output probability $P_{out} > 82.3\%$ is achievable for distances up to 40 km, thus meeting the requirements for DI-QKD.

Finally, using the same setup, we demonstrate heralded single photon amplification (HPA) with no post-selection. Indeed, this experiment requires two independent heralded single photons, therefore it is a four photon experiment and it can be done with our four-photon source. In previous works [11, 12], we have shown the heralded amplification of photons using two photons coming
from the same SPDC process as input and ancilla. In the present work, referring to the experimental scheme depicted in FIG.2, the time-bin qubit preparation (and analysis) stage can be removed and only one (heralded) ancilla photon is sent to the amplifier. This enables us to perform non-postselected HPA, i.e. the amplification is conditioned only upon the detection of the two heralding photons plus the BSM. FIG.5 shows the gain and the output probability as a function of the input probability measured in our setup. As before the dashed lines indicate the performance without excess loss.

We have demonstrated an all-fibre heralded qubit amplifier functioning in the telecom regime for both time-bin and Fock-state qubits, showing the first post-selection free operation in the case of the latter. The implementation highlights the suitability for a fully integrated photonics solution, where the addition of photonic sources and even detectors could further assist the efficiency of these devices. While a gain of 9 is realised with the current system, the key parameter is the efficiency of the HQA - the output probability for the qubit after heralding. In the cases presented here we sacrificed this parameter for experimental control - the interferometer that defines the two modes for the ancillae photons needs to be adjusted for each different fibre coupler, but also introduces significant loss - however, this is only limited by the initial ancilla photon coupling and the excess loss in the fibre coupler. Ancilla efficiencies $P_a \sim 90\%$ are feasible [25]. Coupled with recent demonstrations for characterising entanglement for Fock-state systems [26], this approach may provide a new path towards loop-hole-free Bell tests and DIQKD exploiting heralded qubit amplification.

**Methods**

**SPDC Source.** The HSUS used in this work is based on a SPDC process in a 3 cm bulk Periodically Poled Potassium Titanyl Phosphate (PP-KTP) crystal, with type II phase matching conditions. With this material, adjusting the poling period, the crystal length and the pump pulse duration, it is possible to achieve quasi phase matching conditions that allow the generation of spectrally separable pairs of photons [27–29]. Moreover, purity is optimised when the photons are selected in a single spatial mode with a high heralding efficiency [25].

A picosecond pulsed laser at 772 nm, 76 MHz, pumps the crystal in a double-pass configuration, generating pairs of photons in each direction, for a total of four photons, degenerate at 1544 nm. The setup is symmetrical: for each of the two passes of the pump through the crystal, the two daughter photons are separated by a polarisation beam splitter (PBS) and then coupled into single mode fibres. The achieved heralding efficiency after coupling into single mode fibers (SMF) is measured to be $0.86 \pm 0.04$ for the qubit (pair generated in the first passage through the crystal), $0.80 \pm 0.04$ on average for the two ancillae (generated in the second passage through the crystal), not considering the detection efficiency. A Hong-Ou-Mandel (HOM) [24] interference experiment between independent single mode fibres allows verification of the genuine entanglement. This purity without any filtering. To increase the single mode character of the generated photons, two 100 GHz filters can be placed before the heralding, BSM, detectors, allowing one to herald the output state in a well defined spectral mode.

The initial transmission of the qubit to the amplifier is $0.55 \pm 0.04$, this includes the preparation interferometer, a circulator and a free space path where a liquid crystal retarder is placed, allowing one to vary the relative phase between the two components of the qubit ($s$ and $t$, which have orthogonal polarisations). The transmission of the ancillae is $0.37 \pm 0.02$, the losses are mainly due to an adjustable temporal delay setup to synchronize the ancillary photons temporally with the input qubit in the HQA.

Two HOM interference measurements between the qubit in the input mode and each of the two auxiliary photons in the modes short and long are performed in order to adjust the arrival times onto the 50/50 beam splitter (BS) needed for the BSM. After the 50/50 BS a system of four polarisation beam splitters (PBS) and a fixed delay of 50ns on one of the BS outputs allow one to project onto the two Bell states $\Psi^\pm$, using two detectors (D2 and D3 in FIG. 2) instead of four as shown in the right of FIG. 1.

**Detectors.** The MoSi detectors were fabricated to obtain maximum efficiency at a wavelength of 1550 nm and for operation at 2.5 K in a two-stage closed-cycle cryocooler. Their fabrication and characterization is detailed in (in preparation). A gold mirror was fabricated on top of Ti on a 3 inch Silicon wafer using electron-beam evaporation, and was photolithographically patterned using a lift-off process. A SiO$_2$ space layer between the mirror and the MoSi was then deposited by plasma-enhanced chemical vapour deposition (PECVD). A 6.6 nm-thick Mo$_x$Si$_{1-x}$ layer ($x \approx 0.8$) was sputtered at room temperature. Electron-beam lithography and etching in an SF$_6$ plasma were used to define nanowire meanders. An antireflection coating was deposited on the top surface. A key-hole shape was etched through the Si wafer around each SNPSP, which could then be removed from the wafer and self-aligned to a single mode optical fibre [30]. The size of the SNPSP is 16 x 16 µm$^2$, larger than the 10 µm mode field diameter of a standard single mode fibre, to a large extent. The optimal system detection efficiency reaches 70% with a dark count rate of the order of a few hundred counts per second.

**Definition of Gain.** If one takes into account non PNR detectors with efficiency $\eta$ on the BSM and a probability $P_a$ of having an ancilla (for simplicity, we consider $P_a$ to be the same for both auxiliary photons), we can define the gain as:

$$G(t) = \frac{P_{out}}{P_{in}} = \frac{P_a t}{P_a (1 - t) (1 - P_a \eta) + P_a},$$

(3)
i.e. the ratio between the probability of having a photon in the output given a successful BSM, $P_{\text{out}}$, and the probability of having a qubit at the input, $P_{\text{in}}$. In this way we clearly see the importance not only of the ancillae input probability $P_{\text{in}}$, but also the detection efficiency $\eta$.

References


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