A Wave-guide Model for Packetized Media Streaming in Lossless Networks

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***Abstract*-Optimal operation of network based multimedia applications requires a precise specification of the underlying network parameters. Different models have been used in the past in calculating the behavior of the network and defining parameters like throughput and delays of packets, using among others fluid analogy. In this paper we extend the bundled packet level perspective towards the macroscopic level of propagation dynamics focusing on the impedance that observed packets experienced during propagation using the analogy of particles propagating in a wave-guide. By defining notions like *packet pressure*, *packet velocity* and *stream impedance*, we define the required frame that allows us to directly use existing methods and equations for the study of streaming networks.**

Keywords: multimedia streaming, traffic shaping, performance estimation, quality of service.

I. INTRODUCTION

In multimedia applications like audio-visual tele-conferencing systems or streaming media applications, multimedia packets have to be delivered regularly at a receiving peer for a timely presentation. In the underlying network, however, a stream of packetized multimedia undergoes impeding disturbances due to the sharing of network resources by traffic of other applications. An estimate of the throughput bottleneck in a stream channel can be used to prevent throttle down of the stream, for example by timely re-reservation of resources. The bottleneck estimate may also be used to control the settings of the multimedia encoder such that the generated packet stream better matches to the bottleneck of the channel.

In this paper, we propose a traffic-shaping model of an end-to-end stream channel based on continuous, one dimensional and lossless wave equations. The model is defined by the dynamics of the propagation of packets at a macroscopic level, which is an abstraction of packet forwarding behaviour at packet level. From the packet propagation dynamics we derive an iterative algorithm that contains internal recursions to compute the propagation of the packets in terms of waves in the stream channel. These wave computations require as boundary values the observed (e.g. measured) streams of packets at the inlet and the outlet of the channel. Using these computed waves, we can estimate the course of the throughput of links of the stream channel. On the other hand, the iterative part of the algorithm computes refined estimates of the channel characterizing function that represents a measure of the amount of backpressure that propagating packets experienced. These estimates can also be used to identify the bottlenecks of the stream channel.

Methods for the estimation of throughput bottlenecks in networks that apply the packet pair principle, which uses a fluid analogy but at packet level, are well known [1, 2, 3]. In [4], bottleneck estimation uses a 'bunch' of packets to improve the stability of the estimation, among others to overcome multi transport channel problems when using only two consecutive packets. For the same reason, trains of packets have been used for these estimations in [5]. The proposed stream channel model extends the bundled packet level perspective towards the macroscopic level of propagation dynamics. Despite the discrete property of today's networks due to the discrete number of switches and routers along the propagation path of packets, we apply an analogy of a continuous wave guide. Therefore, we do not distinguish between the network performance concepts "transmission (delay)" and "propagation (delay)" because they are intertwined in continuous models. Continuous models are only able to approximate the discrete stream channel characteristics, but this is sufficient for most cases (e.g. throughput and bottlenecks estimations). The advantage of a continuous model is that the corresponding computational algorithms are less sensitive to the (precise) locations of these switches or routers.

We address packetized media streams from a user-oriented end-to-end perspective and consider networks that internally apply backpressure mechanisms (e.g. sliding window protocols) and mechanisms that enable fluent propagation of packets, such as token-buffer fluid mechanisms or other fair queue based mechanisms that ensure fluent, possibly dripping, packet forwarding. In this perspective, we do not directly address utilization of network resources, for example dynamics of fluid buffers in networks. Instead, we focus on the impedance that observed packets experienced during propagation. Packets of a stream in this context will therefore propagate as if they propagate in a virtual channel from an inlet to an outlet of the network and experience the same forwarding dynamics as other packets of other streams that may share the virtual channel link-wisely.

Since the proposed channel model is an analogy of a wave-guide in which particles propagate, we reuse earlier results in signal processing, in particular, results from the area of inverse scattering in acoustic or seismic media and signal estimation [6, 7]. In wave-guides, backpressure can be easily measured. This will be different for streaming packets, but packets also experience backpressure caused by the sliding window mechanisms, for example.

We also peek into one-lane highway traffic models that have similar Lagrangian's propagation dynamics, but particles are intelligent in these models [8]. That is, the drivers of cars have a more autonomous behaviour but they typically keep a larger safe-gap to ahead cars. In these models, traffic jams of dense and slowly moving cars often occur, even in the absence of externally impeding road conditions. However, both highway traffic and stream channel models may posses the so-called instantaneous adaptation property, i.e. particles adapt instantaneously to their surrounding.

The structure of this paper is as follows. In the next section, we heuristically associate required packet propagation behaviour in networks to the proposed particle dynamics and derive the wave equations induced by the packet propagation. That section also describes the stream channel model that embodies the wave equations and it shows the lossless property of the model. Section III describes the numerical algorithm derived from the wave equations to compute the reflection function, which is a stream channel characteristic that defines the backpressure ratio along the channel. Finally, Section IV presents our conclusions.

II. STREAM SHAPING MODEL

In this section, we elaborate our key idea to view a stream of data packets that are flowing in a network as particles propagating in a wave-guide. As described earlier, the proposed stream channel model is an analogy of a continuous one-dimensional lossless wave-guide in which particles propagate. Accordingly, the stream channel does not loss packets and is continuous. This means that its characteristics, like bandwidth capacity, are real or complex valued functions with some continuity constraint along the real valued length coordinate of the channel. In this paper, we focus on stream channels which characteristics are real valued and time-invariant. The model is therefore suitable for circuit switched networks, networks that apply some kind of reservation of resources or networks that use some kind of rate control (e.g. sliding windows) and fair queue mechanisms. If, however, variation of channel characteristics is infrequent, it is expected that iterative algorithms, like the one derived in Section III, can cope with these network load variations.

In the next section, we explore the required behaviour of streaming packets at microscopic packet level. In Section II-B, we discuss the dynamics of packetized media that abstract the microscopic behaviour of propagating packets to a macroscopic level. In that section, we therefore introduce macro-level terms like packet pressure, packet velocity and stream impedance. Section II-C and Section II-D describe the differential and the corresponding integral equations of the waves induced by the packet propagation. In Section II-E, we discuss the stream channel that embodies the wave dynamics.

A. *Requirements on the stream channel model*

The packet propagation behaviour that we need at microscopic level will be the following:

- *Absence of background traffic*: In the absence of external influences on an end-to-end stream, packets in the stream should propagate in accordance with the packet pair principle ([1, 2, 3]).

  Fig. 1 shows a 1-dimensional stream channel conveying packets of the same size (the gray boxes in the figure). The channel has capacity C_0 at the left and C_1 (> C_0) at the right part, respectively. In this microscopic perspective, the packet pair distance d_0 of packets propagating from left to right is *invariant* in absence of external influences.

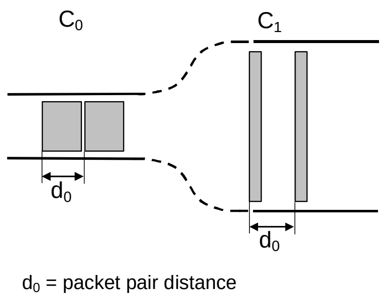

Fig. 1 Packet pair principle in a 1-D stream channel

- *Influence of background traffic*: The streaming of packetized media in networks will usually be influenced by external elements, such as control on links of the observed end-to-end stream by sliding window protocols or server utilization consumed by joining packets from other streams in links of the observed stream. These influences may impede the propagation of the packets of the observed stream. To cope with these influences, we therefore require the following additional behaviour of packet propagation:
  - Since the right side of the channel in Fig. 1 can convey more packets, we introduce the notion of slots in the microscopic model (Fig. 2). Slot distances depend on the channel capacity, i.e. the notion introduces packet pair distances relative to the channel capacity along the channel length coordinate. Slots may be empty, carry packets from the observed stream, or carry packets from other streams that join the observed stream at a particular channel link. Since the proposed stream channel model will

be based on wave propagation, the joining strategy (i.e. the servers' scheduling strategy in networked queue models) should be fair, in the sense that the packets propagate sufficiently fluent, when viewed from a macroscopic point of view. After joining, these packets will be treated similarly to the packets of the observed stream, i.e. they will undergo the same packet propagation dynamics. This will yield a per hop based behaviour, because the joining packets will eventually leave the observed end-to-end stream;

- In cruise control cases of highway traffic models [8], cars drive at cruise speed unless the free slots ahead drops below a safe car-gap threshold. Similarly, packets simultaneously move forward to next free slots at previous value of packet pair distances (which are not necessarily equal to the slot distances at the particular spatial coordinates) if, while moving, ahead slots are also free. In this way, packet pair distance remains invariant in the absence of external influences.

  On the other hand, if some of the slots ahead are not free, e.g. due to newly joining packets, packets may only move to available slots at closer distance. More specifically, the proposed model needs to cope immediately with changes in the density of ahead packets. That is, the model will adopt the so-called instantaneous adaptation property (see also [8]);

- Buffers in queuing network models that may contain packets awaiting to be served will be viewed as dense channel links with slowly moving packets in the proposed model. As mentioned earlier, the relativity of the packet pair distance to the channel capacity introduces empty slots. These slots enable packets in front of buffers to be moved to a next channel link and to be scheduled immediately in the empty slots. Furthermore, the stream channel model has to be able to accelerate these packets in case they leave the buffer and experience a decrease in packet density while moving.

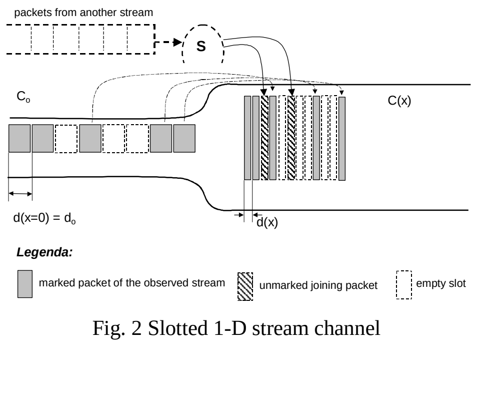

Fig. 2 Slotted 1-D stream channel

B. Dynamics of packetized media in the stream channel

This section addresses packet propagation at macroscopic level that abstracts from individual packet forwarding behaviour at microscopic level, discussed in the previous section. First, we introduce two wave-guide variables, i.e. packet pressure and packet velocity.

Definition:

We define the *packet pressure* p(x,t) and *packet velocity* v(x,t) as follows:

p(x,t) = the number of packets per unit channel length;

v(x,t) = displacement of a packet per unit time,

with $x, t \in \Re$, x corresponds with the length coordinate of the 1-D channel and t the time dimension.

Let model the dynamics of packetized media that streams in a network by the following set of coupled first order partial differential equations:

$$\frac{\partial p(x,t)}{\partial x} = -\rho(x)\frac{\partial v(x,t)}{\partial t} \qquad (1a)$$

$$\frac{\partial v(x,t)}{\partial x} = -\mu^{-1}(x)\frac{\partial p(x,t)}{\partial t} \qquad (1b)$$

with $\rho(x)$ denoting the channel density characteristic and $\mu(x)$ the channel elasticity characteristic. Let further $\rho(x)$ and $\mu(x)$ be positive and differentiable.

In the field of acoustics, these equations describe the dynamics of pressure waves propagating in an isotropic heterogeneous acoustic medium [9]. They are also analogous to the Maxwell equations for cylindrical wave-guides with perfectly conducting surface and filled with passive dielectrical material [10].

The first Newtonian equation prescribes invariant packet velocity in absence of pressure changes along the path of propagation; therefore satisfying the packet pair distance principle described earlier. On the other hand, if packet pressure increases along the stream channel, packet propagation will be impeded. The second equation is the continuum Hooke's equation.

Next, we define a new variable that combines the channel characteristics $\rho(x)$ and $\mu(x)$ into a single characteristic. This will suit our case better, because we are only interested in a single channel characteristic that can be used to indicate transmission bottlenecks along the channel. Furthermore, we transform the spatial coordinate x onto a channel length coordinate that has the dimensionality of (propagation) time. This better suits performance studies, which usually are more interested in delays rather than distances.

Definitions:

We define the *stream impedance* Z(x) as

$$Z(x)=\sqrt{\rho(x)\mu(x)} \tag{2}$$

and we transform the spatial channel length coordinate onto a length coordinate of time dimensionality,

$$T(x)=\int_0^x\sqrt{\rho(\xi)\mu^{-1}(\xi)}d\xi \tag{3}$$

Proposition 1

The coupled first order differential equations (1a) and (1b) are equivalent to the following system of coupled differential equations in the Laplacian domain:

$$\frac{\partial}{\partial T}\begin{bmatrix}P(T,s)\\V(T,s)\end{bmatrix}=\begin{bmatrix}0 & -sZ(T)\\ -sZ^{-1}(T) & 0\end{bmatrix}\begin{bmatrix}P(T,s)\\V(T,s)\end{bmatrix} \tag{4}$$

with P(T,s) and V(T,s) the Laplace transforms of p(T,t) and v(T,t), respectively, and s the Laplace variable.

Proof:

Straightforward, by calculating the uncoupled second order equations derived from (1a) and (1b) and substituting the new variables defined in (2) and (3).

□

C. Wave dynamics in the stream channel

Instead of exploring the dynamics of packet propagation in terms of pressure and velocity, it is more convenient to describe these dynamics from the point of view of the waves that are induced by the packet propagation. This approach is also used in areas like signal estimation and geophysics [11, 12].

Definition:

We define the *incident* and *reflected waves*, A(T,s) and B(T,s) respectively, as follows:

$$A(T,s)=\frac{P(T,s)+Z(T)\,V(T,s)}{2\sqrt{Z(T)}} \tag{5a}$$

$$B(T,s)=\frac{P(T,s)-Z(T)\,V(T,s)}{2\sqrt{Z(T)}} \tag{5b}$$

The time domain representation of A(T,s), i.e. A(T,t), propagates along the positive channel length ‘T’ direction, while B(T,t) propagates in the opposite direction (see also (8a) and (8b)). A comprehensive representation of (5a) and (5b) is:

$$\begin{bmatrix}A(T,s)\\B(T,s)\end{bmatrix}=\frac{1}{2}\begin{bmatrix}Z^{-\frac{1}{2}}(T) & Z^{\frac{1}{2}}(T)\\ Z^{-\frac{1}{2}}(T) & -Z^{\frac{1}{2}}(T)\end{bmatrix}\begin{bmatrix}P(T,s)\\V(T,s)\end{bmatrix} \tag{5c}$$

Proposition 2:

The waves A(T,s) and B(T,s) satisfy the following system of equations:

$$\frac{\partial}{\partial T}\begin{bmatrix}A(T,s)\\B(T,s)\end{bmatrix}=\begin{bmatrix}-s & -\kappa(T)\\ -\kappa(T) & s\end{bmatrix}\begin{bmatrix}A(T,s)\\B(T,s)\end{bmatrix} \tag{6}$$

with *reflection function* $\kappa(T)=\frac{1}{2}\frac{\partial}{\partial T}\ln Z(T)$, (7)

which represents the channel backpressure ratio[1] at location T.

Proof:

By straightforward calculations.

□

In [14], the equations (6) have been called the Krein-Schur equations, because Schur [15] has inspired the work and Krein [16] has extended the work of Schur from the discrete domain to the continuous domain.

D. Incident and reflected waves in the stream channel

This section discusses the incident and reflected waves A(T,s) and B(T,s), respectively. These waves will be expressed by integral equations derived from the differential wave equations (6). Although the reflected wave B(T,s) propagates in the opposite direction, both incident and reflected waves will be expressed in terms of the waves at the channel inlet, i.e. A(0,s) and B(0,s).

Proposition 3:

The waves A(T,s) and B(T,s) satisfy the following integral equations:

$$A(T,s)=e^{-sT}A(0,s)-\int_0^T\kappa(\tau)\,B(\tau,s)\,e^{-s(T-\tau)}\,d\tau \tag{8a}$$

$$B(T,s)=e^{sT}B(0,s)-\int_0^T\kappa(\tau)\,A(\tau,s)\,e^{s(T-\tau)}\,d\tau \tag{8b}$$

Proof:

An alternative presentation of the Krein Schur equations (6) is

[1] In a discrete case analogy, say at a router at location T_1, where the impedance Z shows a jump, we deal with a reflection coefficient of the form $\rho(T_1)=(Z(T_1^+)-Z(T_1^-))/(Z(T_1^+)+Z(T_1^-))$ instead of a reflection function value. Moreover, we may relaxed our differentiability constraint on the propagation characteristics in equations (1a) and (1b) to differentiability almost everywhere [13] to cope with jumps in Z(T).

$$\frac{\partial}{\partial T}\begin{bmatrix} A(T,s) \\ B(T,s) \end{bmatrix} = \begin{bmatrix} -s & 0 \\ 0 & s \end{bmatrix}\begin{bmatrix} A(T,s) \\ B(T,s) \end{bmatrix} + \begin{bmatrix} 0 & -\kappa(T) \\ -\kappa(T) & 0 \end{bmatrix}\begin{bmatrix} A(T,s) \\ B(T,s) \end{bmatrix}$$

The first parts of the expressions on the right hand side of the equations characterize a homogeneous channel and the second parts represent the perturbation. Therefore, the left parts of the solutions ((8a) and (8b)) represent the solutions of the homogeneous channel for which the inlet waves travel undisturbed. The second parts are the sums of the perturbation contributions along the channel ($\tau \in [0,T]$) that for the case of the incident wave A(T,s) are delayed until T. The case for the reflected wave B(T,s) is similar but in the opposite direction.

□

Corollary 4:

The time domain representations of (8a) and (8b) are the following:

$$A(T,t) = A(0, t - T) - \int_0^T \kappa(\tau)\, B(\tau, t - T + \tau)\, d\tau \qquad (9a)$$

$$B(T,t) = B(0, t + T) - \int_0^T \kappa(\tau)\, A(\tau, t + T - \tau)\, d\tau \qquad (9b)$$

E. Stream channel characteristics and representations

The solution of the earlier described Krein-Schur equations models the stream channel that shapes the A(T,s) and B(T,s) waves along the channel. The solution is presented in the next proposition and is characterized by the reflection function κ(T), which represents the channel backpressure at T. The rest of this section addresses the properties of the stream channel model with a focus on the lossless property, meaning that the model excludes cases with packet losses.

Proposition 5:

Let Θ(T,s) be the solution of the Krein-Schur equations, i.e.

$$\begin{bmatrix} A(T,s) \\ B(T,s) \end{bmatrix} = \Theta(T,s)\begin{bmatrix} A(0,s) \\ B(0,s) \end{bmatrix} \qquad (10)$$

then by (6)

$$\frac{\partial}{\partial T}\Theta(T,s) = \begin{bmatrix} -s & -\kappa(T) \\ -\kappa(T) & s \end{bmatrix}\Theta(T,s) \qquad (11)$$

with initial condition Θ(0,s) = I. □

The matrix Θ(T,s), also shown in Fig. 3, is a so-called *chain scattering matrix* [11]. It can be expressed as a multiplicative integral [17] as follows:

$$\Theta(T,s) = \overset{\curvearrowleft}{\int_0^T} e^{X(\xi,s)d\xi}$$, with the generator

$$X(T,s) = \begin{bmatrix} -s & -\kappa(T) \\ -\kappa(T) & s \end{bmatrix} \qquad (12)$$

The following proposition and its corollary show that the stream channel model does neither generate packets nor loss packets.

Proposition 6:

The matrix Θ(T,s) is J-lossless, i.e.

- in the open right half plane ($\Re e(s) > 0$)
  $J - \tilde{\Theta}(T,s)\, J\, \Theta(T,s) \geq 0$, (13a)
  the passive property for any T along the channel.
- and on the imaginary axis ($s = e^{j\omega t}$)
  $J - \tilde{\Theta}(T,s)\, J\, \Theta(T,s) = 0$ (lossless property) (13b)

with ~ the conjugate transposition operator and

$$J = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad (13c)$$

Proof:
A heuristic proof can be found in [14], proof in its details can be found in [18].

□

Corollary 7:

The matrix Θ(T,s) is a model for a stream channel that does not generate nor loss packets.

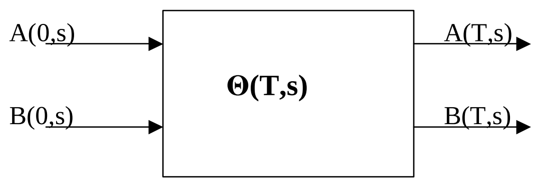

Fig. 3 the chain scattering matrix Θ(T,s)

Proof:
On the imaginary axis, we have for any T along the channel

$$\begin{bmatrix} \tilde{A}(0,s) & \tilde{B}(0,s) \end{bmatrix}\left(J - \tilde{\Theta}(T,s)\, J\, \Theta(T,s)\right)\begin{bmatrix} A(0,s) \\ B(0,s) \end{bmatrix} = 0$$

$$A^{\sim}(0,s)A(0,s) - B^{\sim}(0,s)B(0,s) - \left[A^{\sim}(T,s) \quad B^{\sim}(T,s)\right] J \begin{bmatrix} A(T,s) \\ B(T,s) \end{bmatrix} = 0$$

by (13b). This leads to the energy preservation equation:

$$\|A(0,s)\|_2^2 - \|B(0,s)\|_2^2 = \|A(T,s)\|_2^2 - \|B(T,s)\|_2^2 \qquad (14a)$$

for any T within the channel.

By (5c), we also get on the imaginary axis

$$\begin{aligned} &A^{\sim}(T,s)A(T,s) - B^{\sim}(T,s)B(T,s) = \\ &\frac{1}{2}(P^{\sim}(T,s)V(T,s) + V^{\sim}(T,s)P(T,s)) \end{aligned} \qquad (14b)$$

By applying Parseval's isometry on the energy equations (14a) and (14b) and because the packet pressure-velocity product represents the throughput, we get the lossless property of the stream channel as given below:

$$\begin{aligned} &\int_{-\infty}^{\infty} p(T,\tau)\, v(T,\tau)\, d\tau = \int_{-\infty}^{\infty} P^{\sim}(T,\omega)\, V(T,\omega)\, d\omega = \\ &\int_{-\infty}^{\infty} p(0,\tau)\, v(0,\tau)\, d\tau \end{aligned} \qquad (14c)$$

That is, the total number of packets passing at any T along the channel is eventually equal to the total number of packets entering at the inlet.

□

Corollary 8:

Associated to the chain scattering matrix Θ(T,s), we have a lossless *scattering matrix* Σ(T,s) satisfying:

$$\begin{bmatrix} A(T,s) \\ B(0,s) \end{bmatrix} = \Sigma(T,s) \begin{bmatrix} A(0,s) \\ B(T,s) \end{bmatrix} \qquad (15a)$$

with the property:

- in the open right half plane ($\Re e(s) > 0$)

  $I - \Sigma^{\sim}(T,s)\,\Sigma(T,s) \geq 0$ (15b)
- and on the imaginary axis ($s = e^{j\omega t}$) it is unitair (or orthogonal, in our real valued case), i.e.

  $I - \Sigma^{\sim}(T,s)\,\Sigma(T,s) = 0$ (15c)

□

In a scatterer like Σ(T,s), reflected waves flow in its natural direction as indicated by (15a) and shown in Fig. 4.

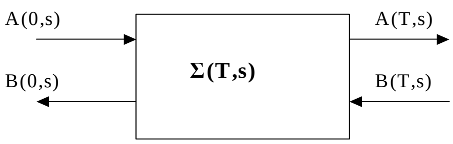

Fig. 4 the scattering matrix Σ(T,s).

The unitair property of the matrix Σ(T,s) makes this scatterer an excellent starting point for algorithms to compute the A(T,t) and B(T,t) waves, because of its numerical stability. In Section III, we develop a computational schema in accordance with wave propagation in Σ(T,s).

III. NUMERICAL ALGORITHMS

In this section, we derive an algorithm to compute the reflection function κ in an iterative way, given (e.g. measured) packet pressures at both the inlet and the outlet of an observed end-to-end stream channel. The RTCP [19] protocol may for example be used to transfer these inlet or outlet values to the computing peer. On the other hand, the initial guess of the reflection function may for example be calculated from RSVP [19] conveyed resource reservation parameters during channel establishment.

Nested within an iteration that refines the reflection function κ, recursive procedures compute the waves A(T,t) and B(T,t) in accordance with a scatterer Σ(T,t) setting.

Section III-B explains these wave computations that use (a refined) estimate of the reflection function. In Section III-C, we discuss the iteration procedure that refines the estimates of the reflection function. First, we discuss the boundary conditions of the computations.

A. Boundary conditions of the computations

The packets injected into the stream channel at the inlet obey the applied rate control or sliding window mechanism, these packets therefore undergo backpressure at the left boundary (Fig. 5). The next proposition and corollary specify the boundary conditions at the channel outlet and at the incident wave-front before it reaches the outlet.

Proposition 9:

At the wave-front of the incident wave, the reflected wave B(T,T) ≡ 0, if all packets are absorbed immediately at the channel outlet (i.e. B(L,t) ≡ 0 with L representing the channel length).

Proof:

This comes from the passivity of the scatterer Σ(T,t) given in (15b), see also the wave propagation in (8a) and (8b).

□

Remark_1: in the previous proposition we have assumed that the incident wave A(T,t) (resp. p(T,t)) is not peaked in the sense that A(T,T) (resp. p(T,T)) is not some order of magnitude higher than other local maximum values of A(T,t). However, if A(T,t) is peaked, for example containing a Dirac function in its extreme, the value $B(T,T^{+}) = \kappa(T)$, see e.g. [14]. In peaked cases, the algorithms to compute A(T,T) along the channel may need perturbation contributions of $B(T,T^{+})$. An interpretation of peaked cases is that a packet pressure peak will overflow network buffers instantaneously causing immediate backpressure (i.e. $B(T,T) \neq 0$).

Corollary 10:

If $B(L,t) \equiv 0$ with L denoting the stream channel length, $A(L,t) = p'(L,t)$, the normalized (by the square-root of the channel impedance at L) packet pressure at the outlet of the stream channel.

□

B. *Σ-algorithm to compute incident and reflected waves*

The next proposition describes the elementary computational element of a Σ–scatterer to compute numerically and recursively the incident and reflected waves in the stream channel. In this work, we apply the trapezoidal integration rule, similar algorithms may however be derived for other numerical integration rules.

Proposition 11:

Let $h > 0$ be the numerical integration step-size. By applying the trapezoidal rule on pieces of length h of the integral equations (9a) and (9b), we have

$$A(T,t) = A(T-h, t-h) - \frac{h}{2}\kappa(T)B(T,t) - \frac{h}{2}\kappa(T-h)B(T-h, t-h) + \mathcal{O}(h^3) \quad (16a)$$

$$B(T-h,t) = B(T, t-h) + \frac{h}{2}\kappa(T)A(T, t-h) + \frac{h}{2}\kappa(T-h)A(T-h, t) + \mathcal{O}(h^3) \quad (16b)$$

Proof:

Compute the difference between $A(T,t)$ and $A(T-h, t-h)$ using (9a) and apply the trapezoidal rule, which has $\mathcal{O}(h^3)$ accuracy. Equation (16b) can be derived in a similar way.

□

In the following, we discuss the length first Σ-algorithm to compute the incident and reflected waves in the stream channel. This algorithm contains two recursions; a secondary recursion is nested in a primary (Fig. 5). An alternative algorithm is indicated in the Remark_3. We start the discussion with the boundary conditions of this Σ-algorithm.

Left boundary waves at T = 0:

Associated to this algorithm, the incident and reflected waves at the boundaries satisfy the following expressions:

$$A(0,2n) = \frac{1}{1+\frac{h}{2}\kappa(0)}\left[p'(0,2n) - B(1,2n-1) - \frac{h}{2}\kappa(1)A(1,2n-1)\right] \quad (17a)$$

which comes from (17b) and the reflected wave expression (16b) but at T = 0:

$$B(0,2n) = B(1, 2n-1) + \frac{h}{2}\kappa(1)\, A(1, 2n-1) + \frac{h}{2}\kappa(0)\, A(0, 2n)\,.$$

Moreover, from (5c) we also have the reflected wave expression at T = 0:

$$B(0,2n) = p'(0, 2n) - A(0, 2n) \quad (17b)$$

with $p'(0,2n)$ the (measured) packet pressure normalized by the square root of the channel impedance at T = 0.

Remark_2: for the odd values of the discretized time t values, we may interpolate some right boundary values to compute the waves at all points in Fig. 5.

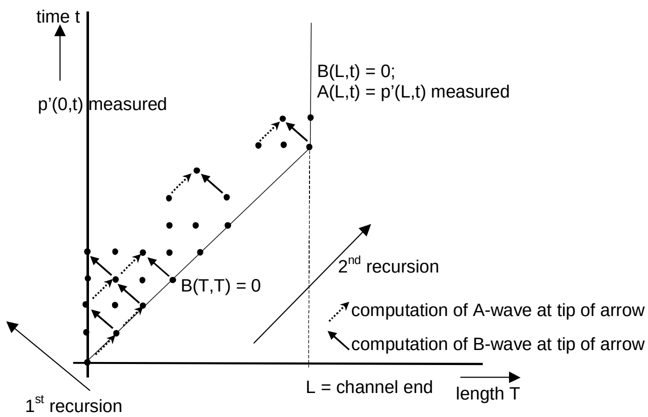

Fig. 5 Σ-based length-first recursive procedure & the boundary conditions

Remark_3:

Another alternative is to base the left boundary incident and reflected waves on both packet pressure and packet velocity measured at T = 0. For example, we may measure $p(0,t)$ from the incoming packets into the output buffer of the encoder that await for transmission (but experienced backpressure from the stream channel rate control) and measure the throughput $p(0,t).v(0,t)$ at the inlet to derive $v(0,t)$. Given these two measurements, the incident and reflected wave can be computed using (5c) and a normalized value for the impedance Z(0) value.

Instead of the proposed Σ-algorithm, we may also use another algorithm to compute the waves inside the channel, for example, an algorithm that is based on the chain scatterer Θ, which computes both incident and reflected waves from T = 0 towards T = L, with L denoting the channel length.

Right boundary waves at channel length L for $t \geq L$ or at the wave front coordinate (T,T):

For T < L, A(T,T) and B(T,T) can be determined using Proposition 9 or Remark_1 given immediately after this proposition.

At the right outlet of the stream channel, for $t \geq L$, B(L,t) = 0 if all packets are well absorbed by the receiver. Accordingly, A(L,t) = p'(L,t), the normalized packet pressure measured at the outlet of the stream channel.

Wave computations inside the stream channel:

The incident and reflected waves in the channel can be computed recursively by the following Σ-based length-first procedure:

for i = 1, 2, .. , M/2, with M denoting the end of the observation time (*primary recursions*);

for j = 1, 2, . . , L-1, with L denoting the channel length (*secondary recursions*);

- *at boundaries*: apply the left boundary equations (17a), (17b) and the right boundary conditions discussed earlier; and
- *inside the channel*: apply the expressions (18a) and (18b), which are results from some elimination process applied on (16a) and (16b):

$$A(j,2i+j) = \frac{1}{1+\frac{h^2}{4}\kappa^2(j)} \begin{bmatrix} A(j-1, 2i+j-1) \\ -\frac{h^2}{4}\kappa(j)\kappa(j+1)A(j+1, 2i+j-1) \\ -\frac{h}{2}\kappa(j)B(j+1, 2i+j-1) \\ -\frac{h}{2}\kappa(j-1)B(j-1,2i+j-1) \end{bmatrix} \quad (18a)$$

$$B(j,2i+j) = \frac{1}{1+\frac{h^2}{4}\kappa^2(j)} \begin{bmatrix} B(j+1, 2i+j-1) \\ -\frac{h^2}{4}\kappa(j)\kappa(j-1)B(j-1, 2i+j-1) \\ +\frac{h}{2}\kappa(j+1)A(j+1, 2i+j-1) \\ +\frac{h}{2}\kappa(j)A(j-1, 2i+j-1) \end{bmatrix} \quad (18b)$$

C. *Iterative algorithm to refine the reflection function:*

In this section, we propose an iterative algorithm to refine the reflection function κ given a first guess. As mentioned earlier, this guess can be based on the reservation of resources during the stream channel set-up. In each iteration, the innovation information needed as a seed to generate an update of κ(T), with $T \in [0, L]$ and L the channel length, comes from the discrepancy between the given (e.g. measured) packet pressure at the outlet and the waves along the stream channel that have been computed using a preceding iteration of the reflection function.

In the earlier described recursive procedures, the incident wave A(L,t) was not included in the computation (therefore, not used). The discrepancy between this computed wave and the measured packet pressure at the outlet may therefore be used to refine κ(T), for example, using a minimum least squares errors method together with an extended observation period (L,M], with M sufficiently large. Essentially, we may enforce the normalized packet pressure at the outlet to satisfy the following integral equation for the computation of the $(i+1)^{th}$ iteration of κ(T), given the $(i)^{th}$ calculations of the incident and reflective waves:

$$A(L,t) \cong p'(L,t) = A^{(i)}(0, t-L) - \int_0^L \kappa^{(i+1)}(\tau)\, B^{(i)}(\tau, t-L+\tau)\, d\tau$$

for $L \leq t \leq L+M$ (19)

If we use the trapezoidal rule with step-size h, we may represent (19) by the following system of (over-determined) equations:

$$P' = A_0 - \mathbf{B}\,\kappa^{(i+1)} \quad (20)$$

with the (measured) normalized packet pressure vector at the outlet:

$P' = [p'(L,L)\ p'(L,L+h)\ p'(L,L+2h) \ldots p'(L,L+M)]^T$,

the incident wave vector at the inlet of the $(i)^{th}$ iteration:

$A_0 = [A(0,0)\ A(0,h)\ A(0,2h) \ldots A(0,M)]^T$,

the rectangular (M+1) x (L+1) matrix of reflected waves of the $(i)^{th}$ iteration:

$$\mathbf{B} = h \begin{bmatrix} \frac{1}{2}B(0,0) & B(h,h) & B(2h,2h) & .. & \frac{1}{2}B(L,L) \\ \frac{1}{2}B(0,h) & B(h,2h) & B(2h,3h) & .. & \frac{1}{2}B(L,L+h) \\ \frac{1}{2}B(0,2h) & B(h,3h) & B(2h,4h) & .. & \frac{1}{2}B(L,L+2h) \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \end{bmatrix},$$

and the $(i+1)^{th}$ iteration of κ(T), i.e. the vector:

$\kappa^{(i+1)} = [\kappa(0)\ \kappa(h) \ldots \kappa(L)]^T$

We may solve this set of equations using SVD (Singular Value Decomposition) or a Householderbased method [20] and we may iterate the reflection function κ(T) until the iterated estimates do not converge further or their difference, e.g. in terms of their L_2 energy, becomes smaller than a specified threshold.

Since κ(T) represents the backpressure ratio in the stream channel, we may use the computed estimate of κ to identify channel bottlenecks. Moreover, we may also compute the throughput (i.e. the product p(T,t).v(T,t)) at any T and any time t, using (5c) and the computed incident and reflected waves at T.

IV. CONCLUSIONS

Multimedia applications using data streaming, like audio and video, require a timely and consistent delivery of the streamed packets. However, the ever-increasing complexity of the underlying networks makes it difficult to estimate in advance the actual network parameters allowing a correct calculation of the different quality of service parameters, like available bandwidth, delays, etc. that are required for an optimal operation of the applications. In this paper, we have presented a traffic-shaping model of an end-to-end stream channel based on continuous, one dimensional and lossless wave equations. We have used these equations to derive an algorithm that can be used to track the throughput of links of the stream channel by feeding a packetized media stream at the inlet (e.g. instead of probes) and measuring the shaped stream at the outlet of the channel. We therefore have developed a kind of passive probing mechanism based on packet pressure that is able to track throughput of channel links over the whole period of streaming the users data. In this perspective, we have extended the packet pair based bottleneck throughput estimation methods.

Further research is needed to validate our results. We will verify the results with simulations and on a longer term, we will validate the results using streaming media over networks. For the computations, we first need to investigate some practical issues like the influence of interval length in the calculation of packet pressure, the influence of network load changes and the observation length of the iterative algorithm to the accuracy of the estimations. Other relevant research topics are the comparison of the different alternative algorithms (the scatterer-based and the chain-scatterer-based algorithms mentioned in Remark_3 of the previous section), the relaxations of the constraints of the model to cope with packet losses in the stream channel, packet reordering or interleaving multi-paths in the underlying networks, and large delays on routers or gateways.

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