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Abstract

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Reference


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A gravitational wave background from the decay of the standard model Higgs after inflation

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ABSTRACT: The stability of the Standard Model (SM) at high energies implies that the SM Higgs forms a condensate during inflation, which starts oscillating soon after the inflationary stage ends. This causes the Higgs to decay very fast, via non-perturbative effects, into all the SM fields coupled directly to it. The excited species act as a source of gravitational waves (GWs), and as a result, all Yukawa and SU(2)_L gauge couplings of the SM are imprinted as features in the GW spectrum. In practice, the signal is dominated by the most strongly interacting species, rendering the information on the other species inaccessible. To detect this background new high frequency GW detection technology is required, beyond that of currently planned detectors. If detected, this signal could be used for measuring properties of high-energy particle physics, including beyond the SM scenarios.

KEYWORDS: Higgs Physics, Cosmology of Theories beyond the SM, Standard Model

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1 Introduction

Compelling evidence strongly supports the idea of inflation, i.e. a phase of accelerated expansion in the early Universe. The precise measurements of the cosmic microwave background (CMB) temperature anisotropies by the Planck satellite [1], show an impressive agreement with the basic inflationary predictions: a spatially flat Universe, with gaussian, adiabatic and slightly red-tilded scalar perturbations. The BICEP2 collaboration[2] has recently announced the first detection of B-modes at large angular scales in the CMB. If these turn out to be due to inflationary tensor perturbations, this detection will represent a milestone in cosmology, and will become most likely the ultimate tool to validate the inflationary paradigm. However, much work is needed yet on the observational side, in order to discard other possible — more mundane — astrophysical sources.

The interpretation of the BICEP2 B-mode detection in light of inflation, implies that the inflationary energy scale is of the order of $\sim 10^{16}$ GeV, which remarkably coincides with the typical scale of grand unified theories (GUT). The determination of such scale constitutes an invaluable piece of information by itself, disfavouring all inflationary models with a lower energy scale. However, it tell us little about the specific model realisation of inflation, which still remains uncertain.

Besides, the end inflation must be followed by a period of reheating, during which all the energy available is converted into different particle species, which eventually thermalize and dominate energy budget of the Universe. However, the details of reheating and of the first stages of the thermal era, are also unknown. In general, they are expected to be high energy phenomena, which cannot be probed by the Large Hadron Collider (LHC), nor by future planned particle colliders. Most likely, only cosmic relics, remnants of these primeval instants, can be used to probe the physics of these early stages of the Universe.

One of the most promising relics are gravitational waves (GWs). Once produced, GWs decouple and propagate at the speed of light, carrying information about the source that originated them. The leading primordial candidate to explain the BICEP2 B-mode signal
are indeed GWs generated during inflation, due to quantum fluctuations of the metric tensor perturbations \[ 3 \]. It is precisely the interpretation\[ 1 \] of the BICEP2 B-signal as due to the inflationary GW background, which allow us to infer the aforementioned energy scale of inflation.

Other backgrounds of GWs are also expected from the early Universe, like those from preheating \[ 8–15 \], phase transitions \[ 16–22 \], or cosmic defects \[ 23–28 \], all corresponding to phenomena in the post-inflationary period. Each of these backgrounds has a characteristic spectrum depending on the high energy process that generated them. If detected, they will provide direct information about the physics of that epoch.

In this paper we want to describe a GW background generated soon after the end of inflation, due to the decay of the Standard Model (SM) Higgs. The ATLAS and CMS collaborations have firmly established \[ 29, 30 \] the existence of the Higgs, with a mass of 125–126 GeV. We ignore however the role of the Higgs in the early Universe or, more precisely, during inflation. Generically, one expects that the Higgs played no dynamical role during inflation, though in principle it could also be responsible for it if a non-minimal coupling to gravity is present \[ 31 \]. The two situations share in common that at the end of inflation, the Higgs is in the form of a condensate with a high amplitude, oscillating around the minimum of its effective potential \[ 32–35 \]. This gives rise to particle creation through non-perturbative parametric effects \[ 36–43 \]. All particle species coupled directly to the Higgs are then created out-of-equilibrium \[ 33, 44 \]. The transverse-traceless (TT) part of the energy-momentum tensor of the Higgs decay products represents a source of GWs. As a result, each of the produced species contributes to generate a GW background.

In this work we compute the spectral shape of such background of GWs. We find that each of the species coupled to the Higgs leaves an imprint in the GW spectrum. However, in practice, the signal from the most strongly interacting species dominates over the rest, rendering inaccessible the information on the other species. We discuss the implications of this result as a probe for particle couplings in high-energy physics. We focus on the situation when the Higgs plays no dynamical role during inflation. We consider also, albeit more briefly, the case when the Higgs is responsible for inflation. All through the text \( a(t) \) is the scale factor, \( t \) conformal time, \( h = c = 1 \), and \( M_p = 1/8\pi G \simeq 2.44 \times 10^{18} \) GeV is the reduced Planck mass, with \( G \) the gravitational constant.

2 Higgs oscillations after inflation

Let us characterize inflation as a de Sitter period with Hubble rate \( H_e \), simply demanding that \( H_e \gg M_{EW} \), where \( M_{EW} \sim O(10^2) \) GeV is the electroweak (EW) scale. The inflationary interpretation of the BICEP2 results indicate that \( H_e \simeq 10^{14} \) GeV, so our demand, in principle, is very much fulfilled. In the unitary gauge, the Standard Model Higgs doublet can be written as \( \Phi = \varphi/\sqrt{2} \), with the large field effective potential of \( \varphi \) given

\[ 1 \] Alternative primordial sources to the inflationary background of GWs have been formulated, like e.g. the presence of magnetic fields \[ 4 \] or cosmic defects \[ 5–7 \], both of which also create B-modes in the CMB. However, in their simplest formulation, these alternatives do not predict the correct shape of the observed B-mode angular power spectrum, unless very ‘contrived’ parameters are chosen.
by $V = \lambda(\mu)\varphi^4/4$, where $\lambda(\mu)$ is the Higgs self-coupling at the renormalization scale $\mu = \varphi$ [45, 46]. If the Higgs is decoupled (or weakly coupled) from (to) the inflationary sector, it plays no dynamical role during inflation, behaving as a light spectator field, independently of its initial amplitude [32, 33]. The Higgs then performs a random walk at superhorizon scales, reaching quickly an equilibrium distribution $P_{eq} \propto \exp\left(-2\pi^2\lambda/3)(\varphi/H_e)^4\right)$ [33, 47], with variance $\langle \varphi^2 \rangle \simeq 0.13\lambda^{-1/2}H_e^2$. A typical Higgs amplitude at the end of inflation is $\varphi_e \sim O(0.1)H_e/\lambda_e^{1/4}$, with $\lambda_e = \lambda(\varphi_e)$. More concretely, $\varphi_e$ ranges between $0.01H_e/\lambda_e^{1/4}$ and $H_e/\lambda_e^{1/4}$ with $\sim 98\%$ probability.

Note that the running of the Higgs self-coupling shows that $\lambda(\mu_c) = 0$ at some scale $\mu_c$, above which $\lambda(\mu)$ becomes negative [45, 46, 48, 49]. For the best fit SM parameters one finds $\mu_c \sim 10^{11}$ GeV. This scale, however, can be pushed up even to $\sim 0.1H_e/\lambda_e^{1/4}$, where $\lambda_1 \equiv \lambda(\varphi_1) > \lambda_e$, with $\varphi_1$ the Hubble rate when the Higgs is decoupled (or weakly coupled) from (to) the inflationary sector, behaving as a light spectator field, independently of its initial amplitude [32, 33]. The Higgs then performs a random walk at superhorizon scales, reaching quickly an equilibrium distribution $P_{eq} \propto \exp\left(-2\pi^2\lambda/3)(\varphi/H_e)^4\right)$ [33, 47], with variance $\langle \varphi^2 \rangle \simeq 0.13\lambda^{-1/2}H_e^2$. A typical Higgs amplitude at the end of inflation is $\varphi_e \sim O(0.1)H_e/\lambda_e^{1/4}$, with $\lambda_e = \lambda(\varphi_e)$. More concretely, $\varphi_e$ ranges between $0.01H_e/\lambda_e^{1/4}$ and $H_e/\lambda_e^{1/4}$ with $\sim 98\%$ probability.

The Higgs slowly starts rolling down its potential as soon as inflation ends. Depending on the inflationary sector (which we do not specify here), the universe can be, just after inflation, matter-dominated (MD), radiation-dominated (RD), or in-between. The Hubble rate $H$ decreases in any case faster than $\varphi$, eventually becoming sufficiently small, verifying $H^2 < d^2V/d\varphi^2$. From then on, the Higgs starts oscillating around $\varphi = 0$, with an initial amplitude $\varphi_1 = H_1/\lambda_1^{1/2}(\varphi_e)$, where $\lambda_1 \equiv \lambda(\varphi_1) > \lambda_e$, and $H_1$ is the Hubble rate when $H = d^2V/d\varphi^2$ is satisfied. The initial velocity can be read from the slow-roll condition, $d\varphi_1/d\tau = -V'/2H_1$. Ignoring the logarithmic running of $\lambda$ and rescaling the Higgs amplitude as $h \equiv a\varphi/\varphi_1$, the Higgs condensate oscillates according to

$$\dot{h}(\tau) + h^3(\tau) = (\ddot{a}/a)h(\tau),$$

where the dots denote derivatives with respect the time variable $d\tau \equiv \sqrt{\lambda_1}\varphi_1 dt$, and the initial conditions are given by $h_1 = 1, \dot{h}_1 = 1/2$. If the Universe after inflation is RD, $a(\tau) = (1 + \tau)$, whereas if it is MD, $a(\tau) = (1 + 0.5\tau)^2$. Either way, the term $\ddot{a}/a$ is irrelevant, since $\ddot{a} = 0$ for RD, or $\ddot{a}/a \propto 1/\tau^2$ for MD, which becomes negligible very quickly. Therefore, independently of the expansion rate of the Universe just after the end of inflation (which, let us insist, depends on the unspecified inflationary sector), the Higgs condensate oscillates an-harmonically according to eq. (2.1), with a decaying amplitude as $\varphi \propto 1/a$. As we will see next, the oscillations of the Higgs condensate have a striking consequence.

3 Gravitational waves from the Higgs decay products

It is a well known phenomenon in quantum field theory that whenever a scalar homogeneous scalar field oscillates around the minimum of its potential, if there are quantum fields coupled to it, then the quanta of such fields are created out-of-equilibrium through non-perturbative effects, see for instance [37]. In our case of study, it is expected that every time the Higgs $\varphi$ passes through zero, all particle species coupled to it are non-perturbatively created out-of-equilibrium, as indeed has been recently studied in [33]. This occurs much faster than particle production from the perturbative decay of the Higgs [33]. In particular,
the SU(2)\(_L\) gauge bosons and the charged fermions of the SM are all created at the first and successive Higgs zero crossings. The energy momentum tensor \(T_{\mu\nu}\) of the created species will represent an anisotropic stress over the background and, consequently, its TT part will act as a source of GWs. Thus, all species excited due to the Higgs oscillations, are expected to generate GWs. In this paper we focus on the GW production from the SM charged fermions. Nonetheless we note that gauge bosons are also expected to produce GWs, see section 5. For later convenience, we define now the times \(t_e, t_1, t_F\) and \(t_{RD}\), as the end of inflation, the start of the Higgs oscillations, the end of GW production, and the first moment when the Universe becomes RD.

As it has been shown recently, parametrically excited fermions can indeed generate very efficiently GWs [14, 15]. To see this, let us consider a given fermion species \(\psi_{\alpha}\), coupled to the Higgs with a Yukawa interaction \(\frac{1}{\sqrt{2}}y_\alpha \varphi \bar{\psi} \psi\), with \(y_\alpha\) is the Yukawa coupling strength. We can decompose the fermionic field as

\[
\psi_{\alpha}(x,t) = \frac{1}{a^{3/2}(t)} \int \frac{dk}{(2\pi)^3} e^{-i k x} \left\{ a_{k,\alpha} u_{k,\alpha}(t) + b_{k,\alpha}^\dagger (i\gamma_2) u_{k,\alpha}^\dagger (t) \right\} , \quad u_{k,\alpha} = \left( \begin{array}{c} u_{k,+} \cr u_{k,-} \end{array} \right) ,
\]

(3.1)

with \(S_{1,2}\) eigenvectors of the helicity operator, and \(a_{\gamma}, b_{\gamma}\) the standard creation/annihilation operators, obeying the usual anti-commutation relations \(\{ a_{\gamma}(k), a_{\lambda}^\dagger(q) \} = \{ b_{\gamma}(k), b_{\lambda}^\dagger(q) \} = (2\pi)^3 \delta_{\gamma\lambda} \delta_D(k - q)\). \(\{ a_{\gamma}(k), b_{\lambda}^\dagger(q) \} = 0\). Introducing eq. (3.1) into the Dirac equation, and after some algebraic manipulations, one obtains that the fermion mode functions follow the second order differential equation (see [15] for more details)

\[
\ddot{u}_{k,\pm} \pm \left( \frac{k}{H_1} \right)^2 + q_a \hbar^2 \pm i \sqrt{q_a \hbar} \right) u_{k,\pm} = 0 ,
\]

(3.2)

where \(q_a\) is a ‘resonance’ parameter given by

\[
q_a \equiv \frac{q_a^2}{\lambda_I} ,
\]

(3.3)

and \(u_{k,\pm}(t_1) = [1 \pm (q_a/[q_a+(k/H_1)^2])^{1/2} \right]^{1/2}\) and \(\dot{u}_{k,\pm}(t_1) \equiv i[(k/H_1)u_{k,\mp}(t_1) \mp q_a^{1/2}u_{k,\pm}(t_1)]\) guarantee an initially vanishing fermion number density [15]. From solving eq. (2.1), we then find \(h(t)\), which we plug into eq. (3.2) in order to solve for the mode functions \(u_{k,\pm}(t)\). This scheme is consistent as long as the backreaction from fermions into the Higgs is not relevant.

The energy density spectrum of GWs generated by a fermionic field with mode functions \(u_{k,\pm}(t)\), and normalized to the critical energy density \(\rho_c = \frac{3H^2}{8\pi G}\), is given by [15]

\[
\Omega_{GW}(k, t) \equiv \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\log k} = \frac{4}{3\pi^3} \frac{G^2 k^3}{H^2 a^4(t)} \int d\vec{p} p^2 \sin^2 \theta \left( |I_c|^2 + |I_s|^2 \right) ,
\]

(3.4)

where

\[
I_c(\vec{k}, \vec{p}, t) = \int T_{\alpha\beta} \cos(k t') \delta_{\alpha\beta}(k, p) \left[ u_{|k-p|+,t'} u_{p,+}(t') - u_{|k-p|-,t'} u_{p,-}(t') \right] ,
\]

(3.5)
with $K_{\text{reg}}(k,p) \equiv 2(n_k - n_p)^{1/2}$, $n_p, n_k - p$ the fermion occupation numbers, and $I_{(s)}$ analogously defined as $I_{(c)}$ but with $\sin(kt)$. Note that parametric creation of fermions excites modes up to a given cut-off scale $k_\ast$ $\simeq q_a^{1/4}H_1$, i.e. only infrared (IR) modes ($k \lesssim k_\ast$) are excited, whilst ultraviolet (UV) modes ($k \gtrsim k_\ast$) remain in vacuum. The contribution from the UV modes therefore diverges and must be subtracted ("regularized"). The kernel $K_{\text{reg}}(p,k)$ appears precisely due to the regularization of the anisotropic-stress [15], acting as a IR filter which suppresses the UV contribution, i.e. $K_{\text{reg}}(p,k)$ $\rightarrow$ 0 when $p,k \gg k_\ast$.

Since the fermionic spectrum has a hard cut-off at $k_\ast$, the GW spectrum must be peaked at a scale $k_p \sim k_\ast$, with a $k^3$ slope for $k \ll k_\ast$ (simply because eq. (3.5) becomes independent of $k$ in such regime), and a decaying UV tail at $k \gg k_\ast$ (due to the suppression of the fermion occupation number). Figure 1 shows several GW spectra computed for different resonant parameters $q_a = 10^2, 10^3, 10^4$ in RD. All spectra depict the expected behavior, i.e. the $k^3$ IR tail, a peak at $k_p \sim q_a^{1/4}H_1$, and a decaying amplitude at $k \gg k_p$. The UV tails are well fitted to a power-law $\propto k^{-1.5}$, but this should be taken with care given the limited momenta range probed.\footnote{Note as well that the $k^{-1.5}$ behavior depends on $K_{\text{reg}}(k,p)$, which is based on an ansatz presented in [15]. Current work in progress [50] will eventually review this result using the more rigorous procedure of adiabatic regularization for fermions [51, 52].} From the given shape of the fermion spectrum, the amplitude of the GW peak is expected to scale as $\Omega_{\text{GW}}^{(p)} \propto q_a^{3+\delta}/H_1$, with $\delta < 1$ a small correction depending on the fermion number suppression details at $k \gtrsim k_\ast$. Numerically we find $\Omega_{\text{GW}}^{(p)} \propto q_a^{1.55}$ for either RD or MD, so $\delta \simeq 0.1$ for both cases.

Denoting as $w$ the effective equation of state parameter characterizing the expansion history between $t_1$ and $t_{\text{RD}}$, the GW spectrum for a given resonance parameter $q_a \geq 1$, can be parametrized as

$$\Omega_{\text{GW}}(k,t_1; q_a) = q_a^{1.55} U(k/k_p) \times (H_1/M_p)^4 (a_1/a_F)^{1-3w},$$

(3.6)

with $U(x)$ a ‘universal’ function

$$U(x) \equiv U_1 \cdot \frac{x^3}{(\alpha + \beta x^{4.5})},$$

(3.7)

capturing the essence of the spectral features (peak amplitude and IR/UV slopes), with $U_1 \equiv U(1)$ and $\alpha + \beta = 1$. We find $U_1 \simeq 10^{-5}$ for RD, $U_1 \simeq 10^{-6}$ for MD, and $\alpha = 0.25, \beta = 0.75$ for both RD and MD. Note that $U(x)$ characterizes the shape of the spectrum of GWs independently of the resonance parameter $q_a$. Therefore, this function can be obtained by inverting eq. (3.6) from any GW spectrum calculated for an arbitrary resonance parameter $q_a$, $U(k/k_p) = q_a^{-1.55} \Omega_{\text{GW}}(k,t_1; q_a) (M_p/H_1)^4 (a_1/a_F)^{1-3w}, \forall q_a$. In figure 2 we plot $U(x)$ as extracted from the spectra shown in figure 1 calculated for $q_a = 10^2, 10^3$ and $10^4$ in RD. The overlapping of the extracted $U(x)$ functions from different resonance parameters $q_a$, demonstrate very nicely the universality of the shape of the GW spectra irrespectively of $q_a$. Similar results are obtained as well in MD. In summary, the value of $q_a$ simply determines the height ($\propto q_a^{1.55}$) and peak position ($\propto q_a^{1/4}$) of the GW spectrum, whereas the shape of the spectra, characterized by $U(x)$, is dictated by the form of the Higgs effective potential $V(\varphi) \propto \varphi^4$ as well as by the Higgs-Fermion Yukawa-type interaction $\bar{\psi}_a \varphi \psi_a$. 

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\cite{15}
Redshifting the amplitude and wavenumbers, we can easily find the GW energy density spectrum today from the spectrum computed at the time of production as (see section 2.2 of [15] for details)

\[ f = \epsilon_1^{1/4} \times (k/\rho_1^{1/4}) \times 5 \cdot 10^{10} \text{ Hz}, \]  
\[ h^2 \Omega_{GW}^{(0)}(f) = h^2 \Omega_{\text{rad}}^{(0)}(g_0/g_F)^{1/3} \times (H_1/M_p)^4 \times \epsilon_1 q_a^{1/55} U(k/k_p), \]

where

\[ \epsilon_1 \equiv (\alpha_1/\alpha_{RD})^{(1-\gamma_v)} \leq 1, \]

\[ h^2 \Omega_{\text{rad}}^{(0)} \simeq 4 \cdot 10^{-5} \] is the fractional energy in radiation today, and \((g_0/g_F)^{1/3} \simeq 0.1\) is the (third root of the) ratio of relativistic species today to those at \(t_F\). Using \(\rho_1 = 3\alpha_1 f_1^2 M_p^2\), today’s frequency \(f_p\) and amplitude of the GW background peak \(h^2 \Omega_{GW}^{(p)} = h^2 \Omega_{GW}(f_p)\), are given by

\[ f_p \simeq \epsilon_1^{1/4} y_a^{1/2} (\varphi_1/M_p)^{1/2} \times 5 \cdot 10^{10} \text{ Hz}, \]

\[ h^2 \Omega_{GW}^{(p)} \simeq \epsilon_1 U_1 q_a^{1/55} (H_1/M_p)^4 \times 10^{-6}, \]

where we used \(h^2 \Omega_{\text{rad}}^{(0)}(g_0/g_F)^{1/3} \simeq 10^{-6}\). Eqs. (3.11), (3.12) describe the peak of the GWs from a single fermion species with Yukawa coupling strength \(y_a\). The position and height of the GW peak from a given fermion species is univocally determined by \(y_a\) and the Higgs self-coupling \(\lambda_1\), from which we build the resonance parameter \(q_a \equiv y_a^2 / \lambda_1\).

In the SM every charged fermion couples directly to the Higgs, each with a different Yukawa coupling strength, \(y_e > y_b > y_T > y_c > y_\mu > y_d > y_\tau > y_s > y_u > y_e\), the labels standing...
for the quarks, top (t), bottom (b), strange (s), charm (c), down (d) and up (u), and the charged leptons, electron (e), muon (μ) and tau (τ). The derivation of eqs. (3.4) actually relies on computing an unequal-time-correlator of the type \( \langle T_{ij} T_{ij} \rangle \) [15], assuming that only one fermion species contributes to the energy momentum tensor \( T_{ij} \). However, in our case, there is a sum over all the fermion species \( T_{ij} = \sum_a T_{ij,a} \), so that \( \langle T_{ij} T_{ij} \rangle = \sum_a \langle T_{ij,a} T_{ij,a} \rangle + \sum_{a \neq b} \langle T_{ij,a} T_{ij,b} \rangle \). Since the creation/annihilation operators of different species anticommute, the cross-terms \( \langle T_{ij,a} T_{ij,b} \rangle \) vanish. Eqs. (3.4) and, consequently, eq. (3.6) and eqs. (3.11), (3.12) are valid for each species individually. This implies that the total spectrum of GWs is a superposition of each individual species’ spectra

\[
 h^2 \Omega_{GW}^{(0)}(f) \simeq \epsilon_1 \times 10^{-6} \times \frac{H_1}{M_p} \times 4 \sum_a q_a^{1.55} \mathcal{U}(q_a^{-1/4} \kappa),
\]

with the \( a \)-index running over all SM charged fermions \( \{t, b, s, c, d, u\} \) and \( \{e, \mu, \tau\} \).

Let us note that if the amplitude of the GW peaks had scaled as \( \Omega_{GW}^{(p)} \propto q_a^r \) with \( r < 1 \), a series of peaks would have emerged in the final spectrum, one peak per fermion. The presence of these peaks could have represented a method for probing particle couplings, i.e. a ‘spectroscopy’ of particle physics. However, the real scaling of the peaks amplitude with a much greater exponent, as \( \propto q_a^{1.55} \), implies that the IR tail of the highest peak completely dominates over the amplitude of the lower peaks, see figure 1. Given the Yukawa coupling strengths of the SM, the amplitudes of each species peak are in proportion \( \Omega_{GW}^{(p)} \big|_{t} : \Omega_{GW}^{(p)} \big|_{b} : \Omega_{GW}^{(p)} \big|_{s} : \ldots = y_t^{3.1} : y_b^{3.1} : y_r^{3.1} : \ldots \) located at frequencies \( f_p^{(t)} : f_p^{(b)} : f_p^{(r)} : \ldots = y_t^{1/2} : y_b^{1/2} : y_r^{1/2} : \ldots \). The IR tail of the top quark dominates over the lower peaks, thus making the information on the other species’ couplings inaccessible. In other words,
despite the fact that each of the fermion species coupled to the Higgs produce a GW peak with a characteristic amplitude and position, only the peak from the top quark will remain in the final spectrum. The rest of the peaks contribute only to a tiny distortion of the top quark’s IR tail. Hence, from the final GW spectrum, in principle only the information about $\lambda_I$ and $y_t$ can be extracted. The information about the couplings of the rest of species $y_a < y_t$, unfortunately remain ‘buried’ under the long wavelength tail of the GW signal from the top quark.

To compute the frequency $f_p^{(t)}$ and amplitude $\left|h^2\Omega_{GW}^{(p)}\right|$ of the top quark peak today, we need to fix first the resonance parameter $q_t = y_t^2/\lambda_I$ at the energy scale $E_I$. The Yukawa coupling $y_t$ runs very mildly from $\sim 0.9$ to $\sim 0.4$, between $\sim 10^2$ GeV and $\sim 10^{19}$ GeV, so we can set $y_t(E_t) \sim 0.5$ as a representative value. The resonant parameter is then $q_t \sim \mathcal{O}(0.1)/\lambda_I \gg 1$, for instance $q_t \sim 10^6$ if $\lambda_I \sim 10^{-7}$. The smaller $\lambda_I$ the bigger $q_t$, and hence the higher the GW peak amplitude. Using the fact that $\varphi_l = (a_e/a_I)\varphi_e \simeq 0.1(a_e/a_I)H_e/\lambda_I^{1/4}$, we find

$$f_p^{(t)} \sim e_1^{1/4}(H_e/H_e^{B_2})^{1/2} \times 10^7 \text{ Hz},$$

$$h^2\Omega_{GW}^{(p)}|_{t} \sim e_1 U_1 10^{-24}(H_e/H_e^{B_2})^4 \lambda_I^{-1.55}$$

with $H_e^{B_2} = 10^{14}$ GeV the inflationary Hubble scale inferred from the B-mode BICEP2 results [2]. Since the BICEP2 results yet need an independent confirmation, let us consider the possibility for the time being that the inflationary Hubble rate could be smaller than the BICEP2 value $H_e^{B_2}$. In such case, the lower $H_e$, the smaller $f_p^{(t)}$, shifting the GW peak towards the observable low-frequency window of currently planned detectors. However, lowering $H_e$ also suppresses significantly the amplitude of the signal, which scales as $\propto (H_e/H_e^{B_2})^4 \ll 1$. Therefore, $H_e \lesssim H_e^{B_2}$ is the only situation at which the peak amplitude might not be strongly suppressed. If the BICEP2 results are finally confirmed, the unexpectedly high inflationary energy scale that they indicate, provides indeed the optimal scenario for the enhancement of the signal that we are studying. Yet, in that case, very small values of $\lambda_I$ are needed to reach a sufficiently high peak amplitude. For instance, assuming a RD scenario immediately after inflation (i.e. $\epsilon_1 = 1$), $\lambda_I \lesssim 10^{-7}, 10^{-10}, 10^{-13}$ are needed to achieve $h^2\Omega_{GW}^{(p)}|_{t} \gtrsim 10^{-20}, 10^{-15}, 10^{-10}$, respectively. Such small values of $\lambda_I$ represent indeed a fine-tuning. The GW background predicted here exists independently of how small $\lambda_I$ is (as long as $\lambda_I > 0$), but only for such fine-tuned values of $\lambda_I$ does the signal not become extremely tiny.

In summary, we see that only if the SM is stable but is extremely close to the instability region, i.e. $0 < \lambda_I \ll 1$, does the peak signal of the GWs from the quark top have a significant amplitude.

A subtle aspect that we have not commented about so far, is the following. Since the Higgs at the end of Inflation is a condensate which fluctuates over superhorizon scales, one might think that this could invalidate the notion of parametric excitation of fields. The excited wavelength should not be larger than the coherence length scale of the Higgs.

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4 Hopefully the Planck collaboration will be able to confirm the signal when their new data analysis is released.
$\sim 1/H_e$, so that the notion of a homogeneous condensate applies. A rapid look into this condition indicates that, independently of the expansion rate after Inflation, a necessary condition is that $q_a^{1/4} > (a_e/a_I)(H_e/H_I)$. With the help of $\lambda_I^{1/2} \varphi_I = H_I = H_e(a_e/a_I)^2$ and $\varphi_I \sim (a_e/a_I)H_e/\lambda_e^{1/4}$, the condition translates into a lower bound for the Yukawa coupling, $y_a > (\lambda_e/\lambda_I)^{1/2}$. Since for the enhancement of GWs we need $\lambda_I \ll 1$, it is expected that $\lambda_e/\lambda_I < 1$. The smallness of this ratio is unfortunately uncertain, since it depends on the fine dependence of the running of the Higgs self-coupling with respect the top quark mass, the strong coupling, etc. In principle it is even possible that $\lambda_e/\lambda_I \ll 1$, but it seems more natural that $\lambda_e$ is perhaps only one or two — few at most — orders of magnitude smaller than $\lambda_I$. Therefore, what this condition tells us, is that indeed we can only trust the parametric excitation analysis for the most heavy fermions, possibly only for the top quark, for which $y_t \sim 0.5$ at high energy scales. Fortunately, we just concluded that only the GW peak from the top quark remains as a feature in the GW spectrum, so in principle the whole analysis is consistent at least for the dominant signal by the top quark. Interestingly, let us note that, at the same time, the spatial superhorizon modulation of the Higgs is expected to lead to anisotropies in the amplitude of the GW background, similarly as it has been recently studied [53, 54] in the case of GW production from light scalar fields at preheating.

4 What if the Higgs was responsible for inflation?

In the Higgs-inflation scenario a non-minimal coupling to the Ricci scalar $\xi R \varphi^2$, allows the Higgs to play the role of the inflaton [31]. An intense debate is currently ongoing about the viability of this scenario. As a matter of fact, the BICEP2 results point towards the inability of this scenario to predict the detected B-mode signal, clearly disfavouring it. We will nevertheless compute the GW production after inflation in this scenario, simply assuming the validity of the model. Whereas this should be considered simply as a pure academic exercise, or a viable physical possibility, only time will tell. For the time being, it will serve as an illuminating exercise just for the sake of comparison with the previous Higgs spectator scenario.

Thus, considering that Higgs-inflation describes correctly the inflationary period, the Higgs oscillates as well after the end of inflation, around the minimum of its effective potential. In the Einstein frame, redefining the Higgs amplitude as $h = a^{3/2}(3\xi/4)(\varphi/M_p)^2$, it is found [34, 35] that the Higgs oscillates as $h \simeq \sin(\tau)/\tau$, with $d\tau \equiv a(t)M dt$ and $M \equiv \lambda/\sqrt{3}\xi M_p$ the effective mass of the Higgs. The Higgs pressure averages to zero over the oscillations, so the universe expands effectively as in MD.

A background of GWs is generated after the end of inflation, again due to the non-perturbative decay of the Higgs, which corresponds to preheating in this scenario. From the Yukawa interactions, fermions acquire an effective mass in the Einstein frame given by $m_{\psi}(\tau) = q_a^{1/2}h^{1/2}(\tau) M$, with $q_a \equiv 2r^2(y_a^2/\xi)$ a resonance parameter, $y_a$ the Yukawa coupling of a given species $\psi_a$, and $r \equiv \xi/\lambda^{1/2}$. Using this effective mass, we can solve the corresponding fermion mode equations, choosing again initial conditions corresponding to

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4If the BICEP2 results held, a version of known as ‘critical point Higgs-Inflation’ would still be viable [55], though the universality of the predictions characteristic of the original formulation would be lost.
vanishing fermion number density. To compute the GW spectrum $\Omega_{GW}(k, t; q_a)$, we simply need to insert the new mode functions $u_{k,\pm}(t)$ into eq. (3.4). Following a similar analysis as in section 3, we find that fermions are excited up to a cut-off scale, this time given by $k_* \sim j^{1/3} q_a^{1/3} M$, with $j$ the number of Higgs zero-crossings since the end of inflation. Considering that fermion production ends after $j_F$ zero-crossings, we find the amplitude and frequency of the GW peak today, for a given fermion species, given by

$$f_p \simeq \epsilon_1^{1/4} J_F^{1/3} q_a^{1/3} r^{-1/2} \times 2 \cdot 10^{10} \text{Hz},$$

$$h^2 \Omega_{GW}^{(p)} \simeq \epsilon_1 U_1 q_a^{1.7} r^{-4} \times 10^{-7},$$

where $\epsilon_1 \equiv (a_1/a_{RD}) < 1$, whilst the scaling $\propto q_a^{1.7}$ and the amplitude $U_1 \simeq 10^{2}$ are found from a numerical fit. A 2-loop analysis of the running of the parameters in this model [56] shows that, for the allowed 125 – 126 GeV Higgs mass range, $\xi \sim \mathcal{O}(10^3)$ and $r \sim 5 \cdot 10^4$ at the energy scale of inflation. Besides, in [34, 35, 56, 57] it has been shown that the Higgs transfers efficiently its energy into the decay products after $\mathcal{O}(100)$ zero-crossings. Finally, note that we can estimate $\epsilon_1 \sim j_{RD}^{-2/3}$, with $j_{RD} \gtrsim j_F$ the number of Higgs zero-crossings until RD. Putting everything together, the frequency of each peak today is estimated as $f_p \simeq 2 y_a^{2/3} \times 10^{10}$ Hz, where we used as fiducial values $\xi = 1000$, $r = 5 \cdot 10^4$, $j_{RD} \sim j_F = 100$, and $\epsilon_1^{1/4} J_F^{1/3} \sim j_{RD}^{1/6} \sim 2$. The GW peaks are in a proportion $f_p^{(a)} : f_p^{(b)} = y_a^{2/3} : y_b^{2/3}$, with $y_a, y_b$ the Yukawa couplings of different species. As in the Higgs spectator scenario, the GW peak from the most strongly coupled species — the quark top — dominates over the rest of peaks. Therefore, only the peak associated to the top quark remains in the final spectrum of GWs, this time located at $f_p^{(t)} \sim 10^{10}$ Hz. Choosing the previous fiducial values for $\xi$, $r$ and $\epsilon_1$, the amplitude of the peak today is estimated as $h^2 \Omega_{GW}^{(p)} \simeq U_1 y_t^{3.4} \times 10^{-15} \sim 10^{-14}$. Note that contrary to the Higgs spectator case, there is no freedom to tune the value of the Higgs self-coupling $\lambda$ for modulating the final amplitude. The amplitude of the GW peak is actually fixed, and its position is also at higher frequencies than in the previous case.

As a final comment, let us note that the last remark made in section 3, about the possible limitations of parametric excitation due to the finite correlation length of the Higgs condensate, does not apply here, since the Higgs is the inflaton and therefore is homogeneous over cosmological scales.

5 Discussion and conclusions

A number of aspects not considered in our derivations, might have an impact on the results. The most relevant aspect is the parametric excitation of the SU(2)$_L$ gauge vectors $Z, W^\pm$, from which new peaks are expected to appear in the GW spectrum. On general grounds, these peaks should be higher than the fermionic ones, since bosons can grow in amplitude arbitrarily, but fermions cannot. However, in the absence of lattice simulations considering the non-linearities and charge currents in the bosonic sector, we will not attempt to estimate their peak amplitude. Let us observe, nonetheless, that given the fact that the SU(2)$_L$ gauge coupling is $g_2 \sim y_t$, the GW peaks from the gauge bosons will be located at similar frequencies as that of the top quark, most likely not being possible to resolve them
separately. The SU(2)$_L$ gauge bosons might therefore enhance the amplitude of the final single peak in the GW spectrum, but we leave the study of this for future research.$^5$

Another relevant aspect is the fermion decay width, which for the top quark is $\Gamma_t \sim \mathcal{O}(10^{-3}) g_2^2 (m_t/m_W)^2 m_t$, $m_t = y_t \varphi/\sqrt{2}$, $m_W = g_2 \varphi/2$. The GWs are created in a step manner only, during the brief periods of fermion non-perturbative excitation $\Delta t \ll T_\varphi$, when the Higgs crosses around zero (twice per oscillating period $T_\varphi$). The GW production will not be affected by the top decay unless $\Gamma_t \Delta t > 1$. In the Higgs spectator scenario, $\sqrt{\lambda_I} \varphi_1 \Delta t \sim q_t^{-1/4}$, and the Higgs amplitude during that time is $|\varphi| \lesssim \varphi_* = q_t^{-1/4} \varphi_1$, so $\Gamma_t \Delta t \lesssim \mathcal{O}(10^{-3}) \times (y_t^2/q_t^{1/4}) (y_t/\sqrt{\lambda_I}) (|\varphi|/\varphi_1) \lesssim \mathcal{O}(10^{-3}) y_t^2 \ll 1$. Therefore, the top decay does not affect the GW production. Similar conclusions follow in the Higgs-Inflation case.

Other aspects that could impact on the final details are the fermions’ backreaction onto the Higgs and the possible thermal coupling of the Higgs. Note, for instance, that in previous studies of non-perturbative fermion production in the early Universe $^4$, it was concluded that backreaction from the created fermions into the scalar condensate creating them, becomes only relevant for resonant parameters $q \gtrsim 10^{10}$, which in our case translates to $\lambda_I \lesssim 10^{-11}$. On the other hand, the possible coupling of the Higgs to the expected thermal bath from the decay of the inflaton, represents a ‘model dependent’ question depending on the assumptions about the inflationary sector. The different possibilities need to be studied therefore, separately, case by case $^5$. Finally, let us note that the study of quantum corrections in the fermion dynamics beyond the Dirac equation $^6$ is an interesting aspect that remains to be investigated further in detail, at least in what concerns its impact on the GW production from fermions.

To conclude, let us stress the fact that the generation of GWs from non-perturbatively excited fields can also be expected in beyond the SM scenarios. For instance if the Higgs couples to non-SM fields, say to species heavier than the top quark, right-handed neutrinos, etc. Alternatively, we can also conceive an oscillatory scalar field $\phi$ other than the SM Higgs, coupled to either SM or non-SM fields. The single peak in the final GW spectrum will then probe the coupling of the most strongly interacting particle with the oscillatory field. The rest of the GW peaks from any species more weakly coupled to $\phi$, are expected to be completely ‘buried’ under the long wavelength tail of the signal from the most strongly interacting species. The corresponding GW background, if detected, would provide a methodology for probing couplings at energies much higher than what any particle accelerator will ever reach.

Summarizing, in this paper we predict a background of GWs created due to the non-perturbative decay of the SM Higgs after inflation, with the simple requisite that the SM is stable during inflation. The existence of this background and the location of its spectral features should be considered as a robust prediction, though the final details might be affected by the inclusion of some of the effects mentioned above, to be investigate elsewhere $^5$. The GW spectral features could be used for spectroscopy of elementary particles in/beyond the SM, probing at least the coupling of the most strongly interacting

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$^5$After completion of this work, the preprint $^5$ appeared in the ArXiv, studying analytically the dynamics of the non-abelian gauge bosons after inflation. Their results reinforce the idea that the GW from the bosonic sector might possibly enhance the final GW peak.
species. For this, new high frequency GW detection technology must be developed, beyond that currently planned [60–62].

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References


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