Implementation In Class of a Theory Stemming From a Research: A Question of Didactical Transposition

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This article aims to point out some results of the implementation of a theory stemming from a research in mathematics education and we are going to discuss its contributions and limitations. A future primary teacher in his masters year at the University of Geneva decided to try to implement the notion of sets of tasks (« jeux de tâches »), developed in our PhD (Del Notaro, 2010). That concept describes the experimenter as an element of what Brousseau (1998) calls the « milieu1 » who involves his own knowledge to interact with pupils. After describing what we mean by the notion of sets of tasks and its origins, we are going to give a definition and to state the problem by exposing our main idea: the fact that the exploration of the milieu by the experimenter will interact with the exploration done by the pupils. This will demonstrate that this interaction is an interaction of knowledge. Our research methodology is also going to be presented. Finally, we are going to reveal certain effects of the didactical transposition and to analyze the interpretation the student has made. We are going to show how the transposition [...]
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Implementation in Class of a Theory Stemming from a Research: a Question of Didactical Transposition

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Abstract
This article aims to point out some results of the implementation of a theory stemming from a research in mathematics education and we are going to discuss its contributions and limitations. A future primary teacher in his masters year at the University of Geneva decided to try to implement the notion of sets of tasks (« jeux de tâches »), developed in our PhD (Del Notaro, 2010). That concept describes the experimenter as an element of what Brousseau (1998) calls the « milieu » who involves his own knowledge to interact with pupils. After describing what we mean by the notion of sets of tasks and its origins, we are going to give a definition and to state the problem by exposing our main idea: the fact that the exploration of the milieu by the experimenter will interact with the exploration done by the pupils. This will demonstrate that this interaction is an interaction of knowledge. Our research methodology is also going to be presented. Finally, we are going to reveal certain effects of the didactical transposition and to analyze the interpretation the student has made. We are going to show how the transposition of a theory in class transforms and makes the knowledge evolve. As a conclusion, we are going to mention that although the exercise was quite successful in some aspects, there should be a discussion about the effects of the transposition to understand the evolution of the knowledge. It is certainly not easy to define this point, but we are going to propose some elements of thinking.

Keywords
Teacher education, didactics of mathematics, sets of tasks (jeux de tâches), interaction of knowledge, 11-12-year-old pupils, experience.

1. INTRODUCTION
1.1. Presentation of the Context
Our researches are the continuity of the Theory of Didactical Situations in Mathematics (Brousseau, 1998). After starting with empirical observations of teaching situations in several classrooms and the lesson plan contexts of future teachers, we got interested in the concept of situation, and especially

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1 The milieu concerns everything which surrounds the pupil’s mathematical activity. It can be the mathematical content, the material, the teacher’s intervention.
in the concept of **fundamental situation** that we have questioned through the notion of **set of tasks** developed in this article. We are going to use the concept of **milieu** to question the relationship between the experimenter and the pupils in a mathematical situation. The **milieu** of the pupil is defined by Brousseau like the “**antagonistic system of the taught system**” (our translation) (1998). Indeed, to learn, a pupil must act against the **milieu**. There is a double interaction: the “pupil ↔ milieu” system interacts with the mathematical knowledge of the situation. “The **milieu** is a set or part of set which behaves like a non-finalized system” (our translation) (ibid, 1990). The professor, on the side of knowledge, plays with the system “pupil ↔ milieu”.

**Figure 1. Modeling of the interactions between Teacher-Pupil-Milieu**

We have particularly highlighted (Del Notaro, 2010) the causal link between the way pupils build their experience and the investigation of the **milieu** by the experimenter. We have thus developed the idea that the experience is especially built in that kind of interaction, stimulated by a set of tasks.

### 1.2. Development of the Question

Using our sets of tasks, we have tried to understand how a mathematical content appears in the pupil’s **milieu** as well as in the experimenter’s, and how both of them interact. Our hypothesis is that the **milieu** and its exploration by the experimenter interact with the exploration done by the pupils themselves.

Why a set of tasks? As researchers, when we question pupils, it is to understand how they build their knowledge. In doing so, we are part of the **milieu**, as our words have an impact on what pupils will do and how they will react. In a current of dynamic scientific thought, we cannot reasonably consider the experimenter as a neutral element, without any influence on the thought of a subject. There is an impact of our own mathematical knowledge on the one of the pupils, and vice versa. We are currently looking at how the pupil’s knowledge is being built within the interaction with the experimenter. Our observations have shown that the mathematical
knowledge is built during an active process, while leaving a little part of improvisation of both parts: pupils and teachers. In fact, the experimenter “piloting” the situation can put into the milieu a new task he had not foreseen in its first analysis. That task is created spontaneously and influenced by the interaction with the pupils. The experimenter can then decide to propose it or not. These unplanned tasks are probably related to the personal exploration of the experimenter, but we have decided not to try to demonstrate this fact. Therefore, the study of didactical milieu and situations will be in the center of our work.

2. ORIGIN, DEFINITION, AND USE OF THE SET OF TASKS

2.1. Origin

The set of tasks was a concept first developed by the group DDMES\(^2\) (2003), which brought together teachers and researchers. Their work focused specifically on the field of geometry in primary grades, in special education. In its 2003 text, the Group outlined a very decisive idea for our own research: the idea of an extension of the milieu. This notion means exploring and investigating deeply the milieu with pupils. The notion of extending (or stretching) contains the idea to explore the limits of the task which is going to grow and extend in a mathematical way. The milieu is not a static entity, but a dynamic one. Only a few papers have been written on this topic and we are going to try to contribute to the analysis of this matter. We have included this idea in our research (Del Notaro, 2010) on the numerical field in standard primary classes; we have especially explored the criteria of divisibility of numbers and their connections. To the image of stretching or extension just described, we have associated a certain autonomy in the pupil’s questioning. Thus, we have questioned them according to our own representation of the task and sometimes even beside our preliminary analysis. The goal here, is not to see if pupils succeed or not, but to understand in an epistemological perspective how their knowledge is built and then fixed. We are trying to identify how they link their prior knowledge to the mathematical task assigned and find the elements. As the success of the pupils is not our main priority, we may interrupt a task abruptly to understand a fact or to ask another question or even introduce counterexamples.

2.2. Definition

We define as *set of tasks* as a group of, generally but not necessarily, interdependent tasks of equal importance. The knowledge of both the experimenter and the pupils will interact with each other in the milieu. The challenge is therefore to use the pupil’s answers to ask further questions, depending on our own interpretation of the mathematic knowledge we suppose the pupils have used.

First of all, you have to define one or more tasks while leaving the possibility of changing the way of questioning, according to the pupils’ contributions on the spot. This presupposes a good knowledge of what is needed for the tasks. Therefore, they are determined prior to, and during the experiment. This kind of questioning is not neutral for the researcher; it allows intrusion in the pupil’s reasoning. To dissect the milieu, it is necessary to be somehow intrusive because you cannot simply observe as a bystander. We believe that to find answers you must somehow provoke them. This double meaning word contains both the idea of encouraging them and challenging them in order to test the strength of the knowledge. So we questioned the pupils’ mathematical knowledge through their own experience of the number structure. This allowed us to open the milieu and to explore it by using various experiments. In other words, the researcher and the pupils will proceed together in the set of tasks, stimulated by the knowledge of both parts. This is typical of what we have called interaction of knowledge. In our PhD thesis, we have shown the link between an exploration of the milieu by the pupils and the construction of experiences, often missing in education, whose consequence is that the pupils don’t have a lot of opportunities to construct their mathematical knowledge. We have also highlighted (Del Notaro, 2011) the set of tasks’ particularity, the interactions it causes, and the experiences done by the pupils. We noticed that the knowledge and the experience of the pupils interacted with the ones of the researcher, and we then tried to establish how this experience is shown in the actions of the pupils.

2.3. Example of a set of tasks
The tasks proposed below are the result of a real set of tasks. We will make a difference between «the original set of tasks » originally planned by the experimenter, and the «actual set of tasks » which was actually done in class. The actual set of tasks can either be identical to the original one, a little bit different or even totally different from the original set of tasks. In the following example there is only one task (in bold) and the tasks 2 to 5 were created during the interaction.

<table>
<thead>
<tr>
<th>My tasks</th>
<th>The pupil’s answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Give a number that can be divided by 3. Give others: one bigger, one smaller, one with 3 digits, and one with 4…</td>
<td>→ One pupil said: 108 but is not sure if it can be divided by 3, 6, or 9</td>
</tr>
<tr>
<td></td>
<td><strong>I decided to start from 108</strong></td>
</tr>
<tr>
<td></td>
<td>Here is what I said:</td>
</tr>
<tr>
<td>2. Look for all the multiplication tables 108 belongs to.</td>
<td>→ One pupil said : « We have written them all, but there might be others, 2 and 4»</td>
</tr>
<tr>
<td></td>
<td><strong>That means they don’t see in what they have written that 108 can also be divided by 2 (1 x 108, 2 x 54, 3 x 36, etc.)</strong></td>
</tr>
<tr>
<td></td>
<td>After that, I asked:</td>
</tr>
<tr>
<td>3. Are the numbers dividing 108, the same as the multiplication tables belongs to?</td>
<td>→ They answered: « Absolutely not! It’s totally different! »</td>
</tr>
</tbody>
</table>
I then said:

4. Say if 1532 can be divided by 3.  → Confusion between the two criteria used in 3 and 4

I answered:

5. Transform this number in order for it to be divisible by 3. Only change one digit.
   *I supposed that the difference of divisibility criterion between 3 (adding the digits) and 4 (only looking at the last 2 digits forming a number divisible by 4 or not) is going to “appear” to them.*

...And so on

This way of interacting with the milieu of the pupils brings an opening in the exchanges. Indeed, the set of tasks permits a larger development of logic insofar as it manages an experiment, even recommends it.

2.4. **Methodology of Research for an Exploitation of the Set of Tasks**

We proceeded by clinical interviews which can be qualified as interventionist, i.e. we are authorized to intervene in what pupils say, in order to understand an answer, or to ask for details (for the most banal interventions), or to put ourselves in the interaction by introducing, for example, new elements in the milieu. This way of carrying out an interview creates surprises not only for the experimenter, but also for the pupil. Thus, we do not feel the effects related to a routine because the surprises are productive and we never really know what will come out of the interaction, even if we propose the same original task several times. Our tasks are related to the field of numbers in standard primary education. The criteria of divisibility and the relations between the numbers remain our ground of predilection to explore relations in the number suites and the connection digit/number the pupils can establish.

3. **AN APPLICATION OF THE THEORY TO BE QUESTIONED**

3.1. **Why a Set of Tasks?**

Before using this concept in the context of teacher training, we have to specify that the challenge lies in avoiding considering the set of tasks as a teaching technique. Despite that fear, we decided to supervise a student in Master’s Degree to have the opportunity to involve her in this research. After attending our doctoral thesis, this student expressed the desire to work in this context to get her Master’s degree in educational sciences, with a concentration in education. It was a very interesting experience that we are going to discuss now. Before proceeding further, it seems important to point out the interest shown by this student before even knowing what the experiment was really going to be. What seemed important to us, in a way, was the fact that only the presentation of the set of tasks had given the desire to a future teacher to test it. So we decided to take this opportunity to discuss the concept and test its strength and its transfer in teaching. Thévenaz (2010) drew up a list of not hierarchical tasks, as a support for the
resolution of a game. She then followed the approach of the pupils, proposing tasks like playing cards and creating new tasks if the proposal of the pupils justified it. It showed that behind this idea, something attracted the experts. Let us specify however that our first interest is to understand these phenomena and not to propose a method. The set of tasks supposes an interaction of knowledge between the experimenter and a pupil forcing the experimenter to involve his own knowledge to interact with what the pupil proposes. It is a difficult exercise insofar as the knowledge of the experimenter can fail at some point, because, as one knows it, the mathematical reasoning does not take marked out path. That supposes to be aware and to take into account its own knowledge and the one of the pupils, what cannot be done without a personal and thorough exploration of the mathematical milieu. It is where the difficulty for a teacher lies, but it is not impossible. The student demonstrated in her master's paper how she had created sets of tasks around the concept of powers and what these games brought to her and to the pupils. She allowed herself to improvise from what the pupils had said or done and she left herself being dragged rather far into the relations that could be established between the knowledge of the powers, the one of the pupils, and her own. She showed two things: firstly, although the basic framework was established beforehand by her, it is the interaction between the teacher and the pupils that created and enriched the full content of the lesson. From a trainer point of view, what seems interesting for the continuation of our work is to note the way in which an object can fill a future teacher with enthusiasm and how, by the means of the set of tasks, the latter let herself go into her own exploration. We also noted that by the use of the set of tasks, the student allowed herself to go out of a typical book exercise like this one: «This week, I am writing to 4 friends. Next week, each of them is going to write a post card to 4 other friends. These friends will then do the same the following week... and so on. How many people will get a card on the fourth week? and at the end of the tenth week? ». While she was working on the powers, she “suddenly” decided to switch for Pascal’s Triangle which allowed us to observe the following interaction of knowledge between her and the pupils: what the pupils were doing made her think about her own knowledge what encouraged her to propose a task on the Pascal’s Triangle. We could wonder if the student would have allowed herself this change if she had conducted a lesson in a more usual way. However, we noticed in her paper a too obvious intent of teaching using the set of tasks as a teaching method and not only as an experiment method, which made us question ourselves on possible slips, which will be discussed in the rest of this article.

3.2. Effects of the Didactical Transposition
The question of interpretation of the set of tasks notion by the protagonist has to be examined and assessed to understand the transposition process. The application to the contingency of the class causes some slips. It is indeed very difficult to make the difference between teaching and experimenting, especially if you are an unexperienced young teacher. The set of tasks is not a technique and shouldn’t be understood as a right/false dichotomy. Research must continue to understand under what conditions it
could be used in a classroom, and what it would really imply for the teacher. What is new is the fact that we don’t operate any selection or filtering of the experience. Errors or irregularities may occur and will be considered as a result of the exploration of the pupils. The student, in her set of tasks, attempted to switch from one task to another, according to what pupils were saying, but in order to avoid errors to help her pupils to solve the tasks (even if she wouldn’t probably agree with this). Let’s take two exchanges in 4th session about Pascal’s Triangle to try to illustrate this (our translation):

- (task) Compare discoveries in the triangle with the results of the starting task → (obs) This is not enough to ensure that pupils make the connection between the exponentiations and the initial task → (hyp) Construction of a tree diagram can help make this link.

The construction of a tree diagram is not a pupil’s idea, but the hypothesis made by the student. Shen then proposes the following tasks:

- (task) To build a tree diagram that represents the initial task → (obs) Tree diagram allows us to make the link between the exponentiations of initial task → (task) Continue research in Pascal’s triangle.

**Figure 2. The student’s set of tasks (our translation)**
We think that the tasks were all suggested by the student, and that none were really proposed by the pupils. This is a first step toward the set of tasks, yet imperfect, but however positive, if we consider the curiosity of the student and her perseverance in such an uncertain experimental field. Nevertheless, we have to point out some intent “to ensure that students learn”. This is the kind of slips we would like to avoid as researchers, not to turn the set of tasks into a technique. The following extracts, in bold, highlights her own exploration of the *milieu*. She rediscovers some mathematical relations that make her say at the end of the extract that she finally got her link. Here is an overview (*our translation*):

"I then asked myself what had taken them to that direction and I finally made the hypothesis that the pupils had not understood the initial situation. I then searched for a way to help them get a representation which corresponded to the situation we were working on. When they told me they needed to visualize what was happening, I decided to use the drawing. Then I proposed to the group of pupils to represent the initial situation using a drawing, a diagram or a tree. (…). After that, I was forced to admit that my idea of drawing was not useful for the pupils, and therefore I needed to find another way to make them understand the exercise 15. I then read several books and articles, until I found the “Demon of math” (1998). It was then that I saw the solution I had been looking for! Indeed, by adding all the numbers of each row, you get powers of 2. Therefore, by taking one line out of two, you obtain the powers of 4: I finally got my link! »

By quickly analyzing these few elements, we realized that, in spite of the efforts made to open the debate about the situation – and this simple fact is remarkable – the teacher’s position became more important. The fact that the student said the pupils had not understood the initial situation is a value judgment while the interaction in a set of tasks should only be to understand what knowledge was required by the situation and used by the pupils. In the set of tasks, what encourages learning is the interaction through knowledge: both parts learn. In the test of the student, learning is promoted by an action of a part on the other: the student “knows”; she finds a way to help the pupils, to make things comprehensible to them, and to lead them to the expected solution. She managed it thanks to the link she had made and not to the one the pupils could have made alone. Consequently, by establishing the link herself instead of letting the pupils make it, she made a generalization, but the pupils did not. This situation arose as if it contained the discoveries of the pupils. The nuance is subtle because, sometimes very skillfully, one can manage to guide the pupils and to make them formulate the expected discoveries, while maintaining the illusion that they come from them. In our experiments, we did the exact opposite in order to open the *milieu*. By not filtering the experience or the links which were revealed spontaneously (that means, we did not intervene), we inexorably caused a destabilization of the *milieu*. What came out of this experiment and was useful to our research, is the fact that we are now well documented and
extremely informed on the knowledge the pupils implement when they carry out this kind of exploration and we can thus attest that their experiments are undoubtedly more than simple tests/trials. When a pupil makes a problem his own, and decides to solve it, the interest for the task is guaranteed and the desire to discover more is assured.

4. CONCLUSION
4.1. Contributions and Limits
Although the exercise was rather positive, we must discuss the possible slips when transposing a theory in a classroom. It is certainly not easy to control this issue, but we are going to point out some elements. First of all, one of the main contributions is that this way of interacting with the milieu of the pupil allows more open interactions. The adjustment of the milieu in the sets of tasks allows the pupils not only to explore it but also to constitute their own experience around the concept of powers. This emphasized that prior knowledge has to be adapted in new knowledge. The student, just as the pupils, uses her own knowledge. Let us talk again about the example of the first lesson, where she said that: “the pupils make multiplications and not powers”. You can obviously quickly explain to the pupils that they are wrong and show them the right way to solve the problem, but in that case, you prevent them from thinking alone. This matter guided the student’s thinking and maintained her enthusiasm to continue the experiment. Thus, she was convinced that if you let the pupils experiment the multiplications by themselves as long as they need to, you allow them to constitute their own knowledge. That idea was perfectly understood by the student and also strengthened her convictions to use the set of tasks as she had seen for herself how helpful it could be for both teacher and pupils. Another contribution lies in the fact that the set of tasks is an answer to the institutional regulations which require to put the pupils “in situation”, but whose practical application remains without any explanation for the teachers. They don’t find any help in the handbooks’ theory, which does not take into account the cases where “it does not work” for certain pupils. Therefore, it seems much more comfortable to use practical exercises. The set of tasks is a way to put the pupils in situation, making sure that they learn, even if you cannot always control this fact.

4.2. How to avoid a Switch to a Simple Technique?
The challenge is to avoid considering the set of tasks as a new technique. As already mentioned, we could imagine a possible slip of the use of the set of tasks to a confirmed technique, which might result in reducing it to a simple method: the set of tasks is, first of all, an interaction of knowledge which supposes the investment of both experimenter/teacher and pupil. We used a theoretical framework (Bloch, 2002) to control our experiments in order to avoid possible slips during the implementation but we decided not to explain it in this paper. Moreover, this model also allowed us to build experimental situations to study and to analyze them during the implementation. This modeling helps to guarantee a scientific validity and to avoid the slips of the empiricism, as Bloch (2002) declares (our translation): “The goal of the use of an experimental model is to make sure
that the observation is neither controlled nor limited to the declared or not declared choices of the teacher”. This type of epistemological vigilance is missing in the student’s paper – which is perfectly normal as she is not a researcher. Thus, the consequence is that her desire of teaching took over the experiment. She then forgot to leave the pupils think by themselves. However, we noticed that the student’s intention really was to let the pupils search, even if she didn’t always succeed in doing so. The epistemological posture consists in the idea that pupils learn in the interaction. The experimenter interacts with them in order to help them build their own knowledge.

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Figure 2. *The student’s set of tasks (our translation)*

1st session

- **Launch of exercise 15**
- **Observation:** pupils make multiplication and not the powers.
- **Hypothesis:** pupils did not fully understand the initial situation.

2nd session

- **Task:** To represent the initial situation using drawings, diagrams or trees.
- **Observation:** pupils draw the result that they had found and the situation.
- **Hypothesis:** Has not agreed them the passage by drawing: need to try something else.

3rd session

- **Task:** Build and explore the Pascal's triangle.
- **Observation:** pupils make various discoveries, including that of powers of 2.
- **Hypothesis:** It is possible to use this discovery to return to exercise 15.

4th session

- **Task:** Compare the discovery in the triangle with the results of the exercise 15.
- **Observation:** This is not enough to ensure that pupils make the connection between the powers and exercise 15.
- **Hypothesis:** The construction of a tree can help make this link.

5th session

- **Task:** Build a tree that represents the initial situation of exercise 15.
- **Observation:** The tree allows us to make the link between the powers and the exercise 15.
- **Task:** Continue research in Pascal's triangle.

6th session

- **Task:** Make the results found in order to communicate to the rest of the class.
- **Task:** Communicate the results to the rest of the class.