How Difficult Is It to Prove the Quantumness of Macroscopic States?

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Abstract

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How Difficult Is It to Prove the Quantumness of Macroscopic States?

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General wisdom tells us that if two quantum states are “macroscopically distinguishable” then their superposition should be hard to observe. We make this intuition precise and general by quantifying the difficulty to observe the quantum nature of a superposition of two states that can be distinguished without microscopic accuracy. First, we quantify the distinguishability of any given pair of quantum states with measurement devices lacking microscopic accuracy, i.e., measurements suffering from limited resolution or limited sensitivity. Next, we quantify the required stability that has to be fulfilled by any measurement setup able to distinguish their superposition from a mere mixture. Finally, by establishing a relationship between the stability requirement and the “distinguishability with inaccurate measurements” of the two superposed states, we demonstrate that, indeed, the more distinguishable the states are, the more demanding the stability requirements.

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Introduction.—The predictions of quantum physics have been extremely well reproduced in experiments all over the world for almost a century. Even the most counterintuitive effects, such as entanglement and nonlocality, have been repeatedly confirmed. Those effects forbid a local interpretation of physical reality that is at the core of classical physics. Once one accepts a radical departure from classical realism, it becomes quite puzzling that quantum physics is unnecessary when describing macroscopic phenomena, because there is no explicit quantum to classical transition mechanism within quantum theory itself. There are two ways to tackle this problem.

From one side, there are attempts to derive a new theory that reproduces quantum and classical physics as two asymptotic cases. Phenomenologically, those new theories can be seen as quantum theory supplemented with an explicit collapse mechanism and might have a rich underlying physics; see Ref. [1] for a recent review.

On the other hand, there are attempts to mimic the arousal of classical reality from within quantum theory itself. Decoherence [2] is usually given as a solution: a system unavoidably interacts with its environment, which measures its state and destroys the quantum correlations. It is argued that decoherence is more and more important when the “size of the system” increases, in such a way that in practice no quantum property can be observed in “large systems.” Another possibility is to blame the measurement [3–6]. Here, the central intuition is that revealing entanglement in a large system requires measurements with an extreme precision. The two approaches are closely related, since a decoherence channel can be seen as acting on the observable, spoiling the accuracy of the measurement.

A lot of examples have been presented in the literature confirming the general intuition that the quantum features of “macroscopic” states are very fragile with respect to experimental imperfections (uncontrolled interactions with the environment and imperfect measurements) and, therefore, extremely hard to observe.

In this Letter, we go beyond specific examples of states and put this intuition in a quantitative form. Naturally, the first step is to elucidate the meaning of “macroscopic quantum state” that is not obvious. Indeed, measuring the size of a quantum state (either a superposition or an entangled state) is at the heart of many recent papers [7–14], which result in various definitions, but they are not easily connected to decoherence or measurement inaccuracy. Here, we analyze the distinctness of the superposed components $|A\rangle_M$ and $|D\rangle_M$ contained in the system $M$ from the observer’s point of view by asking how easy is it for the observer to distinguish them.

Concretely, we consider states of the form inspired by Schrödinger’s cat

$$\left|\zeta\right\rangle_M = \left|A\right\rangle_M\uparrow + \left|D\right\rangle_M\downarrow_Q,$$

where the system $M$ is entangled with a microscopic two-level system $Q$. We characterize such a state by quantifying how well the components $|A\rangle_M$ and $|D\rangle_M$ can be distinguished with inaccurate measurements, the inaccuracy being either coarse graining (limited resolution) or inefficiency (limited sensitivity); see below. In both cases, entanglement is reduced by experimental defects, namely, by noise that we identify with a weak measurement [15] of the observable that is coarse grained (for limited resolution) or by loss (for limited sensitivity). We prove that the
amount of entanglement surviving the defects is upper bounded by an expression solely involving the probability to distinguish the two components with inaccurate measurements; see Eqs. (10) and (11). This shows quantitatively that the observation of entanglement becomes progressively harder as the distinctness of the components increases. We also demonstrate that this holds for quantum superpositions instead of entangled states; i.e., the task to distinguish a superposition of two states from a statistical mixture becomes more and more difficult as the distinctness of the superposed states increases. We conclude that observing the quantum nature of macrostates, either described by quantum superpositions or by entangled states, requires an extreme control. Our results also provide a quantitative framework for unifying and comparing the plethora of specific studies on the loss of coherence in various scenarios.

Setting the problem.—Consider a bipartite entangled state (1), where the system of interest $M$ is in a superposition of states $|A\rangle$ and $|D\rangle$ that are potentially macroscopically distinct. The test qubit $Q$ entangled with $M$ contains two orthogonal components $|\uparrow\rangle$ and $|\downarrow\rangle$. It fixes what are the superposed components $|A\rangle$ and $|D\rangle$ [16].

Let us take two orthogonal components $<A|D> = 0$. Then, the states $|A\rangle$ and $|D\rangle$ can be distinguished with certainty in a single shot. However, the measurement that allows one to do so might be very complicated, and more importantly this operational distinctness might be very fragile with respect to technical imperfections within the measurement device. We call a pair of states $|A\rangle$ and $|D\rangle$ macroscopically (or easily) distinguishable if this is not the case, i.e., if an imperfect measurement still allows one to distinguish them.

To put it more formally, let $\sigma$ be the parameter describing the imperfection of our measurement device (with $\sigma = 0$ giving an ideal measurement). The probability to guess between the two states $|A\rangle$ and $|D\rangle$ in a single shot, labeled $P_\sigma$, is then a function of $\sigma$. It generally drops when $\sigma$ increases. The pair $(\sigma, P_\sigma)$ characterizes the distinctness of states $|A\rangle$ and $|D\rangle$.

Easily distinguishable states are such that they can be distinguished with high probability ($P_\sigma$) with an inaccurate measurement device (high $\sigma$) in a single shot. Below, we show that a superposition of easily distinguishable states is necessarily very hard to observe. We consider two aspects in which a measurement device can be inaccurate, namely, a limited resolution (coarse graining) or a limited sensitivity (probability to interact with the measured system). The intuition is that, in each case, the which-path information is easily extracted from their superposition by either noise or loss, which turns the superposition state into a statistical mixture.

Distinguishability with inaccurate measurements.— Measurement with limited resolution: Consider an ideal measurement of an arbitrary operator $\hat{X}$. The probability (or probability density in the continuous case) to observe an outcome $x$ within the spectra of $\hat{X}$ is obtained from the operator $\delta(\hat{X} - x)$; i.e., for a state $|S\rangle$ it is $p^S_\lambda(x) = \langle S|\delta(\hat{X} - x)|S\rangle$. The effect of coarse graining on $\hat{X}$ is to smear out the outcome distribution

$$p^\sigma_\lambda(x) = \int g_\sigma(\lambda) p^S_\lambda(x + \lambda) d\lambda = \langle S|g_\sigma(\hat{X} - x)|S\rangle,$$

with a “noise function” $g_\sigma(\lambda)$ with zero mean and standard deviation $\sigma$. For concreteness, we assume Gaussian noise in such a way that $g_\sigma$ is solely characterized by $\sigma$.

The probability to correctly guess between two states $|A\rangle$ and $|D\rangle$ in a single shot with a measurement device specified by $g_\sigma$ is given by

$$P^\sigma_\sigma(A,D) = \frac{1}{2}(1 + D^\sigma_\sigma(A,D)),$$

with $D^\sigma_\sigma(A,D) = \frac{1}{2} \int |p^\sigma_\lambda(x) - p^\lambda_\sigma(x)| dx$, the trace distance between the outcome probability distributions corresponding to the two input states. Note also that to any fixed value of the guessing probability $P_\sigma$ corresponds a value $R^\sigma_\sigma$ that gives the worst possible measurement that allows one to distinguish the states with at least the required probability $P_\sigma$ (inspired by probabilistic algorithmic one could, for example, set the threshold $P_\sigma = 2/3$). Both $P^\sigma_\sigma$ and $R^\sigma_\sigma$ characterize the distinctness of $|A\rangle$ and $|D\rangle$ with respect to $\hat{X}$.

Measurement with limited sensitivity: For a measurement device with limited sensitivity $\eta < 1$, there is a chance that the system or its part goes through without being detected. In other words, the interaction between the measured system and the measurement device is a probabilistic process. Such a process can be modeled by splitting the measured system into two parts and then sending only one part to an ideal measurement device (the other part is given to the environment and traced out). In the case of photons, such a splitting is produced by a beam splitter with transmission $\eta$, and in the case of qubit ensembles, it is produced by uncorrelated depolarization channels. But such a probabilistic loss channel $L_\eta$ can be defined in full generality, and it has to satisfy several conditions. Let $\rho^M_\eta = L_\eta(|S\rangle\langle S|)$ be the partial state of the system after the interaction ($\rho^F_\eta$ is the partial state of the environment), cf. Fig. 1. Then (i) for a unit efficiency $\rho^M_\eta = |S\rangle\langle S|$ equals to the input state $|S\rangle$, (ii) for a zero efficiency $\rho^M_{\eta=0} = |0\rangle\langle 0|$ contains no information about the input state $|S\rangle$, and (iii) the outputs are symmetric $\rho^M_\eta = \rho^{1-\eta}_\eta$ [17]. The probability to distinguish $|A\rangle$ and $|D\rangle$ with a measurement device of efficiency $\eta$ is at most given by
random unitary identify the noise channel that consists in applying a.
 Specifically, for the states containing components that are
 M of entanglement contained in the state of Eq. (1) when the
 global unitary

 \[ P^E_{\eta}(A, D) = \frac{1}{2} (1 + D^E_{\eta}(A, D)), \quad (5) \]

 with the trace distance \( D^E_{\eta}(A, D) \equiv \frac{1}{2} \text{tr}[\rho_{\eta}(|A\rangle) - \rho_{\eta}(|D\rangle)] \).
 As before, by fixing the required guessing probability
 \( P'_{\eta} \) (e.g., \( P_y = 2/3 \)), one can extract the minimal efficiency
 \( S_{P_y} \) allowing us to achieve

 \[ S^E_{P_y}(A, D) \equiv \min\{\eta : P^E_{\eta}(A, D) \geq P_y\}. \quad (6) \]

 **Fragility of entanglement.**—We now analyze the fragility of
 entanglement contained in the state of Eq. (1) when the
 system \( M \) is sent through various decoherence channels \( \mathcal{E} \).
 Specifically, for the states containing components that are
 easily distinguishable with limited resolution on \( \hat{X} \), we first
 identify the noise channel that consists in applying a
 random unitary \( e^{i\hat{X}\lambda} \) on the state (with the random variable
 \( \lambda \)) as a weak measurement of \( \hat{X} \) by the environment. It
 is then clear that this channel rapidly extracts the which-path
 information if the two components under consideration are
 easily distinguishable with \( \hat{X} \) measurements. For the states
 containing components that are easily distinguishable with
 limited sensitivity, we use the symmetry of the outputs of the
 loss channel. In particular, we show that when such a
 superposition is sent through a loss channel, the environment
 can extract the which-path information rapidly, i.e., from a
 small fraction of the state, because the observer can do so.

 Quantitatively, consider the system-environment represent-
 ation of some decoherence channel described by a
 global unitary \( U \), as depicted in Fig. 2. It can be shown that
 the negativity \([18]\) of the state \( \rho_f = \mathcal{E}(|\zeta\rangle_M) \) is upper
 bounded by the which-path information available to the
 environment after the interaction with the system (see the
 Supplemental Material \([19]\])

 \[ 2N(\rho_f) \leq \sqrt{1 - D(\rho^A_{E}; \rho^D_{E})^2} \leq \sqrt{1 - D_{\{E_m\}}(\rho^A_{m}, \rho^D_{m})^2}, \quad (7) \]

 where \( D(\rho^A_{E}; \rho^D_{E}) \) is the trace distance between the partial
 states of the environment \( \rho^E_{E} = \text{tr}_M U|S, 0_E\rangle\langle S, 0_E|U^\dagger \), and
 \( D_{\{E_m\}}(\rho^A_{m}, \rho^D_{m}) \) is the trace distance between the distributions
 \( \{\rho^A_{m}\} \) and \( \{\rho^D_{m}\} \) that are the probabilities of outcomes
 for the positive-operator valued measure \( \{E_m\} \) and for the
 states \( \rho^A_{E} \) and \( \rho^D_{E} \). We remark that

 \[ D(\rho^A_{E}; \rho^D_{E}) = \max_{\{E_m\}} D_{\{E_m\}}(\rho^A_{m}, \rho^D_{m}) \]

 and that a strictly zero rhs in Eq. (7) implies that the state
 \( \rho_f \) is separable (no positive partial transpose entanglement; see
 the Supplemental Material \([19]\)). Importantly, upper bounds similar to
 Eq. (7) can be derived for any convex entanglement measure. In
 particular, we give an explicit bound in the Supplemental Material \([19]\) for any
 entanglement measure that is equal to the entanglement of formation
 for pure states. We now apply this result to the super-
 positions of easily distinguishable states.

 Noise and superpositions of states distinct with coarse-
 grained detectors: Consider the state (1) with the functions
 \( P^X_{\eta}(A, D) \) and \( R^X_{\eta}(A, D) \) characterizing the distinctness of
 the components \( |A\rangle \) and \( |D\rangle \) under noisy measurements of
 \( \hat{X} \). Further consider the following noise channel

 \[ \mathcal{E}^X_{\Delta} = \int e^{i\hat{X}\lambda} \rho e^{-i\hat{X}\lambda} \hat{f}_{\Delta}(\lambda)d\lambda. \quad (8) \]

 It corresponds to diffusion of state \( \rho \) in the directions
 complementary to \( \hat{X} \) in the Hilbert space. This channel is
 characterized by the standard deviation \( \Delta \) of the distribu-
 tion \( \hat{f}_{\Delta}(\lambda) \) that we assume is Gaussian. So \( \Delta \) describes
 the instability of the setup with respect to transformations
 generated by \( \hat{X} \). We give three examples of such a channel
 below, but let us first show that it corresponds to a weak
 measurement of \( \hat{X} \) (without postselection). For this, we
 rewrite Eq. (8) in the system-environment representation
 depicted in Fig. 2. Let the environment be a particle in
 one dimension with the initial state \( |0_E\rangle = \int \psi(q)|q\rangle dq =
 \int \tilde{\psi}(p)|p\rangle dp \). Then the channel of Eq. (8) is equivalently
 given by

 \[ \mathcal{E}^X_{\Delta} \rho = \text{tr}_E e^{i\hat{X}\hat{\psi}} \rho \otimes |0_E\rangle\langle 0_E| e^{-i\hat{X}\hat{\psi}} \quad (9) \]

 for \( |\tilde{\psi}(p)|^2 = \Delta(p) \). Equation (9) represents a weak
 measurement of the \( \hat{X} \) observable performed by the
 environment: \( e^{i\hat{X}\hat{\psi}} \) shifts the initial state of the environment
 \( \psi(q) \) in the \( q \) line by exactly \( \hat{X} \). So when reading its
 position, the probability of having the outcome \( q \)
 (corresponding to the projector \( |q\rangle\langle q| \)) equals
\[ \text{tr}_M \otimes |q\rangle \langle q| e^{i\hat{\chi}p} \otimes |0\rangle_E \langle 0| e^{-i\hat{\chi}p} = \text{tr}_M [\psi(q - \hat{\chi})]^2 \rho. \]

Identifying \(|\psi(x)\rangle|^2 = g_\sigma(x)\) with \(\sigma = 1/\Delta\) the measurement of the position of the final state of environment corresponds to the coarse-grained measurement of \(\hat{\chi}\) defined in Eq. (2). Combining Eq. (7) for \(\hat{\chi}\) since in this case the channel (8) corresponds to a coarse graining of the momenta observable \(\hat{p}\) and the spatial position \(\hat{x}\) of the clock precision (or fluctuations of the length of optical paths for photons).

\[ N(\mathcal{E}_\Delta^\chi(|\xi\rangle_M\rangle)) = \sqrt{P_{1/\Delta}^\chi(A,D)(1 - P_{1/\Delta}^\chi(A,D))}. \]

The entanglement remaining in the state \(|A\rangle_M\rangle\rangle_Q + |D\rangle_M\rangle\rangle_Q\rangle after the channel \(\mathcal{E}_\Delta^\chi\) is upper bounded by an expression involving \(P_{1/\Delta}^\chi(A,D)\) — the probability to guess between the two components \(|A\rangle\) and \(|D\rangle\) with an \(\hat{x}\) measurement coarse grained with \(1/\Delta\) noise in a single shot. As an example, for the guessing probability \(P_g = u^2/(u^2 + 4),\) the decoherence channel (8) with strength \(\Delta = 1/R_{\xi}^{\chi}\) reduces the entanglement by at least \(u\). In other words, when the two superposed components are well distinguishable with coarse-grained measurements \((R_{\xi}^{\chi} \gg 1),\) their superposition reduces to a statistical mixture even for a tiny imperfection of the experimental setup \((\Delta = 1/R_{\xi}^{\chi} \ll 1).\) Note that the channel \(\mathcal{E}_\Delta^\chi\) can be equivalently seen as a decoherence affecting the state or as a lack of control on the measurement setup, i.e., a degradation or a finite resolution of an experimentalist’s reference frame, which occurs to some extent in any realistic scenario. Of course, there are measurements that are not affected by this process, and one might wonder if such measurements allow one to reveal more entanglement than that prescribed by Eq. (10). The answer is no, since such measurements cannot tell the difference between the state of Eq. (1) and the same state spoiled by the entanglement breaking channel (8) with \(\Delta \rightarrow \infty\) (corresponding to a projective measurement of \(\hat{X}\) by the environment [21]). In particular, this yields the following concrete results.

(I) A superposition of two states \(|A\rangle\) and \(|D\rangle\) distinct in energy \(\hat{X} = \hat{H}_M\) is necessarily fragile with respect to the phase noise channel of Eq. (8), which stands for a lack of the clock precision (or fluctuations of the length of optical paths for photons).

(II) A superposition of states distinct with respect to the spatial position \(\hat{X} = \hat{x}\) is necessarily very demanding on the precision of momenta measurement \(\hat{p}\) to be revealed, since in this case the channel (8) corresponds to a coarse graining of the momenta observable \(\delta(\hat{p}) \rightarrow f_\Delta(\hat{p}).\) (The same applies for any quadrature measurements.)

(III) A superposition of states distinct with respect to spin \(\hat{S}_z\) is increasingly demanding on the control of the polar angle on the Bloch sphere, since the channel of Eq. (8) for \(\hat{X} = \hat{S}_z\) stands for random rotations around the \(z\) axis.

Loss and superpositions of states distinct with insensitive detectors: Consider the state of Eq. (1) with the functions \(P_\eta^\chi(A,D)\) and \(S_\eta^\chi(A,D)\) characterizing the distinctness of the components \(|A\rangle\) and \(|D\rangle\) with insensitive detectors. The entanglement in this state after the loss channel \(\mathcal{L}_\eta(|\xi\rangle_M\rangle\rangle)\) is upper bounded by the trace distance between environmental partial states \(\rho_\eta^\chi(|A\rangle\rangle)\) and \(\rho_\eta^\chi(|D\rangle\rangle)\); see Eq. (7). Using the symmetry property of the loss interaction outputs \(\rho_\eta = \rho_{M\eta}^{\chi}\), we find

\[ N(\mathcal{L}_\eta(|\xi\rangle_M\rangle)) \leq \sqrt{P_1(\eta)P_\eta^\chi(A,D) - P_1(\eta)P_\eta^\chi(A,D)}. \]

For example, for \(P_g = u^2/(u^2 + 4),\) any loss channel with transmission \(\eta \leq 1 - S_\eta^\chi\) reduces the entanglement in the state \(|A\rangle_M\rangle\rangle + |D\rangle_M\rangle\rangle\rangle by at least \(u\).

Concluding discussion.—We have shown in Eqs. (10) and (11) that for any state of the form of Eq. (1), the amount of entanglement that can be revealed in the presence of experimental defects is limited by the distinctness of components \(|A\rangle\) and \(|D\rangle\) that can be achieved with inaccurate measurements. Several remarks naturally arise from this result.

Measure of macroscopicity: The idea of looking at the distinctness between the components of a superposition was at the core of measures of macroscopicity presented in Ref. [9] for insensitive measurements and in Ref. [14] for measurements with limited resolution. This definition seems very intuitive. Indeed, to distinguish between a dead and an alive cat, one does not need measurements with precision at the level of a single atom contrary to the precision required for the observation of typical microscopic properties. Along the same lines, we note that the maximal tolerable inaccuracy \(R_{\xi}^\chi\) or \(S_\eta^\chi\) itself (or any monotonic function of these parameters) can be used as a measure of the size of a superposition state [22].

Certifiability: Instead of talking about entanglement, one could drop the qubit from the state (1) and formulate the problem differently: Is it possible after the decoherence channel \(\mathcal{E}\) (or \(\mathcal{L}\)) to certify that the state was prepared in a superposition

\[ |\xi_S\rangle = |A\rangle + |D\rangle \]

and not a mixture \(|\xi_M\rangle = |A\rangle\langle A| + |D\rangle\langle D|\)? This problem is closely related to the notion of certifiability introduced in Ref. [23], where the authors showed for the case of qubit ensembles and uncorrelated depolarizing noise that the superpositions of macroscopically distinct states are uncer-tifiable. Our findings allow us to draw similar conclusions. By replacing the negativity \(N\) by the trace distance \(D(\mathcal{E}(|\xi_S\rangle\rangle), \mathcal{E}(|\xi_M\rangle\rangle))\) in the inequalities of Eqs. (10) and (11) [as follows from Eqs. (12)–(15) of the Supplemental Material [19]], we conclude that to certify the superposition
of easily distinguishable states becomes more and more difficult as the distinguishability increases.

Observing entanglement with coarse-grained measurements: In Ref. [24], it was conjectured that in order to “detect quantum effects such that superposition or entanglement in macroscopic systems either the outcome precision or the control precision of the measurements has to increase with the system size.” The conjecture was illustrated with an example involving two coherent states $|\alpha\rangle = |\alpha\rangle$ and $|\beta\rangle = |\alpha\rangle$. By increasing $\alpha$ one can make these states distinguishable even with low outcome precision in the computational basis. However, to reveal a quantum feature, it is necessary to also perform a measurement in the computational basis. However, to reveal a quantum feature, it is necessary to also perform a measurement in the computational basis. Therefore, in this context, the channel $\hat{E}_X$ acting on the state can be interpreted as a lack of the control precision (on $U$). Reciprocally, any lack of control precision generated by $\hat{X}$ can be seen as a noise channel operating on the state and equivalently as a weak measurement of $\hat{X}$ by the environment. This shows in full generality that the demand on the control precision increases with the distinctness of the components $|\alpha\rangle$ and $|\beta\rangle$ as suggested by Eq. (10).

Effective classical-to-quantum transition: Finally, let us emphasise that our results can be seen as an effective bound between the classical and the quantum domains. If the macroscopic quantum states are defined as those containing a superposition of components that can be distinguished with imperfect measurements, then their quantum nature is very difficult to observe.

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[16] Since the work of Schrödinger, it is customary to refer to the state (1) as a macroscopic superposition, as if there was only the system $M$. We believe that the presence of $Q$ is necessary. It fixes the superposed components $|\alpha\rangle$ and $|\beta\rangle$ (up to a rotation), and we remark that $|\alpha\rangle + |\beta\rangle = |\alpha\rangle$ admits an infinite number of decompositions. Furthermore, it allows one to talk about quantum correlations; in the absence of $Q$, it is unclear how to certify that $|\alpha\rangle$ and $|\beta\rangle$ are genuinely superposed (and not mixed) without knowing the Hilbert space they belong to.
[17] In fact, one only requires symmetry up to a local unitary $U_M\rho^M U_M^\dagger = U_M\rho^M U_M^\dagger$. Note that it is also reasonable to assume that (iv) for a varying efficiency $\eta \in [0,1]$ the trajectory in the space of states $\rho^M_\eta$ is continuous.
[21] A projective measurement of $\hat{X}$ by the environment reduces the negativity to at most $\sqrt{P^X_\eta(A,D)[1-P^X_\eta(A,D)]}$, which is more restrictive than Eq. (10). Note that it does not necessarily breaks the entanglement completely, as $\hat{X}$ might have invariant eigenspaces, within which some entanglement survives.
[22] For example, it is appropriate to calibrate the size $N$ with a family of superposition states $|\Psi(N)\rangle$ with a naturally defined size, as was done with superpositions of Fock states in Ref. [14].