Improving on mass flow rate estimates of volcanic eruptions

DEGRUYTER, Wim, BONADONNA, Costanza


DOI: 10.1029/2012GL052566
Improving on mass flow rate estimates of volcanic eruptions

W. Degruyter¹ and C. Bonadonna²

Received 29 May 2012; revised 15 July 2012; accepted 20 July 2012; published 29 August 2012.

[1] We introduce a novel analytical expression that allows for fast assessment of mass flow rate of both vertically-rising and bent-over volcanic plumes as a function of their height, while first order physical insight is maintained. This relationship is compared with a one-dimensional plume model to demonstrate its flexibility and then validated with observations of the 1980 Mount St. Helens and of the 2010 Eyjafjallajökull eruptions. The influence of wind on the dynamics of volcanic plumes is quantified by a new dimensionless parameter (II) and it is shown how even vertically-rising plumes, such as the one associated with the Mount St. Helens 1980 eruption, can be significantly affected by strong wind. Comparison between a one-dimensional model and the analytical equation gives an $R^2$-value of 0.88, while existing expressions give negative $R^2$-values due to their inability to adapt to different source and atmospheric conditions. Therefore, this new expression has important implications both for current strategies of real-time forecasting of ash transport in the atmosphere and for the characterization of explosive eruptions based on the study of tephra deposits. In addition, this work provides a framework for the application of more complete three-dimensional numerical models as it greatly reduces the parameter space that needs to be explored.


1. Introduction

[2] The 2010 Eyjafjallajökull and 2011 Puyehue-Cordón Caulle eruptions demonstrated how even moderate volcanic eruptions can inject hazardous amounts of ash into the atmosphere that disrupt the air-travel infrastructure worldwide. Real-time forecasting of cloud spreading and atmospheric ash concentration based on volcanic ash transport and dispersion models (VATDMs) is, therefore, essential to reduce impact [Guffanti et al., 2010; Bonadonna et al., 2011a; Langmann et al., 2012]. Currently, ash concentration in the atmosphere forecasted by VATDMs can only be constrained within a factor of ten [Mastin et al., 2009]. This is mainly due to the large uncertainty associated with the operationally used relations between plume height and mass flow rate, which omit first order effects related to the variability of the eruption source and the atmosphere. The mass flow rates estimated from the characterization of tephra deposits are based on similar expressions [Wilson et al., 1980; Wilson and Walker, 1987; Sparks et al., 1997] and consequently suffer from similar uncertainties. Here we address this issue by developing an expression that more flexibly incorporates variable atmospheric and source conditions.

[3] Analytical and one-dimensional models of volcanic plumes are based on the general theory developed for turbulent gravitational convection [Morton et al., 1956]. Under the assumptions that (i) the vertical velocity and buoyancy profile are self-similar at all heights, (ii) the rate of entrainment is proportional to the characteristic velocity of the plume at every height, and (iii) the largest local density variations in the plume are small in comparison with the density of the atmosphere at the source, an analytical expression can be derived for the maximum height $H$ reached by a purely buoyant plume (i.e., zero initial mass and momentum flow rate) released from a maintained source into a calm and dry atmosphere [see Morton et al., 1956, equation (10)]:

$$H = 2^{-5/8} \pi^{-1/4} \alpha^{-1/2} \hat{F}^{1/4} N^{-3/4} z_1$$

(1)

with $\hat{F}$ being the buoyancy flow rate at the source, and $\alpha$ the radial entrainment coefficient under the assumption of a top-hat velocity and buoyancy profile, determined through observations and experiments [Morton et al., 1956; Carazzo et al., 2008; Devenish et al., 2010] (auxiliary material).¹ Morton et al. [1956, equation (10)] uses the radial entrainment coefficient of a Gaussian profile, which is that of a top-hat profile divided by $2^{1/2}$. We use $\alpha = 0.1$ unless stated otherwise (see auxiliary material). $N$ is the buoyancy frequency and quantifies the density stratification of the fluid in which the source is released (for a standard atmosphere $N = 1.065 \times 10^{-2}$ s$^{-1}$). Note that Morton et al. [1956] use the parameter $G$ defined in equation (24) in their paper, which is identical too $N^2$. $z_1$ is the maximum non-dimensional height of Morton et al. [1956] and was determined by numerical integration of the non-dimensional governing equations. It has a value of 2.8 [Morton et al., 1956, Table 1]. In the case of a purely buoyant plume rising in a cross flow with a uniform velocity $v$ much larger than the characteristic plume rise velocity, an analogous expression to equation (1) can be obtained [see Hewett et al., 1971, equation (A.35); Briggs, 1972, equation (2)]:

$$H = 6^{1/3} \pi^{-1/3} \beta^{-2/3} \hat{F}^{1/3} N^{-2/3} v^{-1/3}$$

(2)

with $\beta$ being the wind entrainment coefficient, which is determined from observations and experiments [Briggs,

¹Earth and Planetary Science, University of California, Berkeley, California, USA.

²Section of Earth and Environmental Sciences, University of Geneva, Geneva, Switzerland.

Corresponding author: W. Degruyter, Earth and Planetary Science, University of California, 355 McConic Hall, Berkeley, CA 94720-4767, USA. (wim.degruyter@berkeley.edu)

©2012 American Geophysical Union. All Rights Reserved.
0094-8276/12/2012GL052566

1Auxiliary materials are available in the HTML. doi:10.1029/2012GL052566.
1972; Devenish et al., 2010]. We use a value of $\beta = 0.5$ unless stated otherwise (auxiliary material). Furthermore, a moist atmosphere can significantly increase plume height, through the production of latent heat released by the formation of water droplets [Morton, 1957]. The addition of the latent heat term in the conservation of energy, however, does not allow for a general scaling expression [Morton, 1957], and a numerical integration of the governing equations is necessary to obtain a relationship between buoyancy flow rate and maximum height.

These findings have been applied to volcanic plumes [Settle, 1978; Wilson et al., 1978, 1980; Woods, 1988, 1993; Glaze et al., 1997; Sparks et al., 1997; Hort and Gardner, 2000; Bursik, 2001; Mastin, 2007], which differ from purely buoyant plumes in that (i) they consist of a multiphase mixture of particles and gas [Woods, 1988; Hort and Gardner, 2000], (ii) they are characterized by a non-zero mass and momentum flow rate [Morton, 1959; Woods, 1988], and (iii) their initial density is much higher than the surroundings [Wilson et al., 1980; Woods, 1988]. This is highly important for the initial jet phase and leads to plume collapse if mixing with the surrounding air is insufficient [Wilson et al., 1980; Woods, 1988]. If we consider that the particles and the gas are in thermal and mechanical equilibrium, which is reasonable for particles <1 cm and <100 μm respectively [Woods, 1988; Hort and Gardner, 2000; Stroberg et al., 2010], and the plume becomes buoyant beyond the jet phase, equation (1) remains robust in describing plume height of volcanic plumes in a dry and calm atmosphere [Woods, 1988]. However, volcanic plumes are often released in atmospheres with variable wind and humidity conditions, which are shown to have significant effects [Woods, 1993; Glaze et al., 1997; Bursik, 2001; Mastin, 2007].

2. Analytical Expression

We introduce a new analytical expression to estimate source mass flow rate from plume height, which can be adopted for a range of source enthalpies, atmospheric temperature and wind profiles, and radial and wind entrainment coefficients. First, we need a formula relating the buoyancy flow rate in equations (1) and (2) to mass flow rate $M$ [Morton et al., 1956; Wilson et al., 1980; Woods, 1988; Glaze et al., 1997] (see auxiliary material):

$$F = \frac{g'}{\rho_0}M$$

with $g' = g \left( \frac{c_0\theta_0 - c_\infty\theta_0}{c_\infty\theta_0} \right)$ (3)

with $g$ being the gravitational acceleration, $\rho_0$, $c_0$, $\theta_0$, $c_\infty$, and $\theta_\infty$ being the reference density, heat capacity, and temperature of the surrounding atmosphere, respectively, and $c_0$ and $\theta_0$ being the source specific heat capacity and temperature, respectively. $g'$ can be regarded as equivalent to the reduced gravity at the source. Previous analytical expressions for the determination of mass flow rate of volcanic plumes are based on a combination of an equivalent of equation (3) and equation (1) [Wilson et al., 1980; Carazzo et al., 2008; Kaminiski et al., 2011] or on observation fitting rendering similar results [Mastin et al., 2009; Sparks et al., 1997; Dacre et al., 2011]. Second, for a given maximum plume height above the vent $H$, an ambient temperature profile $\theta_\infty(z)$ and wind profile $v(z)$ we define the average buoyancy frequency $N$ and wind velocity $\bar{v}$ across the plume height,

$$N^2 = \frac{1}{H} \int_0^H N^2(z)dz \quad \text{with} \quad N^2(z) = \frac{g^2}{c_\infty \theta_\infty} \left( 1 + \frac{c_\infty d\theta_\infty(z)}{g} \right)$$

$$\bar{v} = \frac{1}{H} \int_0^H v(z)dz$$

with $z$ being the vertical coordinate above the source. Finally, we calculate the mass flow rate from plume height based on equations (1) and (2) as

$$M = \pi \rho_0 \theta_\infty \left( \frac{2^{5/2} \alpha^2 N^3}{z^4} \right) \left( H^4 + \frac{\beta^2 N^2 \bar{v}}{6} H^3 \right),$$

whereby the ratio of the two terms can quantify the influence of wind on plume dynamics

$$\Pi = 6 \frac{2^{5/2} \alpha H \left( \frac{\alpha}{3} \right)^2}{\bar{v}}.$$ (7)

This shows how the wind becomes dominant if the height, buoyancy frequency, and radial entrainment are small and the wind speed and wind entrainment are large. For the wind dominating case ($\Pi \ll 1$), the second term in equation (6) dominates. In the case the buoyancy frequency and wind velocity are constant we obtain equation (2). On the other hand, if $\Pi \gg 1$, the second term becomes negligible and for a constant buoyancy frequency we recover equation (1), demonstrating that the two end members are reproduced.

3. Results

Equation (6) is compared with Monte Carlo simulations of a one-dimensional plume model in order to show the applicability for intermediate values of $\Pi$ (Figure 1; auxiliary material). The one-dimensional plume model is based on the theory of turbulent gravitational convection from a maintained source [Morton et al., 1956] and extensions [Morton, 1957; Hoult et al., 1969; Woods, 1988; Glaze et al., 1997; Bursik, 2001] to adapt it to volcanic plumes and take into account wind and humidity in the atmosphere (see auxiliary material). Such models have been used widely in volcanology [Sparks et al., 1997] but were usually restricted to incorporate either only wind or only humidity. Figure 1a shows results for $10^5$ Monte Carlo simulations run over a large parameter space of source conditions (5 parameters: temperature, exit velocity, exsolved gas mass fraction, vent radius, vent height), temperature and wind profiles (5 parameters: ambient temperature at the vent, temperature gradient in the troposphere, tropopause height, stratosphere height, maximum wind velocity at the tropopause) and radial and wind entrainment coefficients in a dry atmosphere. Previous equations [Mastin et al., 2009] can underestimate the mass flow rate by as much as a factor of 10 (Figure 1a). Fitting the equation of Mastin et al. [2009] to the one-dimensional model gives an $R^2$-value of $-0.65$. Given that equation (6) accounts for changes in the parameter space, it remains accurate within a factor of two compared with a
one-dimensional plume model and gives an $R^2$-value of 0.88 (Figure 1a). The scaling breaks down for plumes characterized by low mass flow rate rising in a humid atmosphere. As an example, within an atmosphere of 100% relative humidity the heights reached by plumes with $M < 10^7 \text{ kg/s}$ are close to those produced by plumes with $M \sim 10^7 \text{ kg/s}$ (Figure 1) in agreement with previous work [Woods, 1993]. Nonetheless, equation (6) still provides an upper bound for the amount of ash injected into the atmosphere.

The May 18, 1980 eruption of Mount St. Helens produced a total tephra mass well constrained between 5 and 7 $\times 10^{11} \text{ kg}$ [Bonadonna and Costa, 2012]. However, the partition of mass between the different phases is not well understood. Previous estimates of mass flow rate attributed 23% of the total erupted mass to Plinian tephra fall (B1, B2, B4), and the remaining 77% to the co-ignimbrite plume deposition (B3) [Carey et al., 1990]. This finding is contradicted by the fact that B3 is the smallest unit while the morning eruption phases (B1 and B2) contributed the most to the final deposit [Lipman and Mullineaux, 1981; Criswell, 1987]. Underestimation of mass flow rate can simply be explained based on the effect of wind, which was not accounted for in the previous estimates [Carey et al., 1990]. Even though the plume could rise nearly vertically, the wind was very strong during the course of this eruption, with a maximum of 30–35 m/s near the tropopause [Carey et al., 1990]. Using the parameterization for atmospheric temperature profile and wind [Carey et al., 1990] in combination with the plume heights observed by radar [Lipman and Mullineaux, 1981] we obtain $i = 0.20 - 0.34$, showing that the wind was important. The mass flow rate derived from both our analytical expression and Monte Carlo simulations of the one-dimensional model is significantly larger than the previously calculated mass flow rate [Carey et al., 1990] (Figure 2). Our new values of mass flow rate result in a more consistent mass partitioning during the eruption, i.e., 60–70% of the mass.

**Figure 1.** Comparison between analytical predictions (blue circles: Mastin et al. [2009]; red circles: equation (6)) and $10^5$ Monte Carlo simulations of the one-dimensional model (auxiliary material), whereby 12 input parameters describing source and atmospheric parameters were stochastically sampled for a (a) dry and (b) wet atmosphere (only $10^3$ comparisons picked randomly are shown for clarity). Predictions of equation (6) mostly fall within a factor of two of the one-dimensional model predictions, whereas the predictions using Mastin et al. [2009] can deviate by a factor of ten (the closer to the points to the one-to-one line (full black line) the better the prediction). Both expressions overestimate the mass flow rate in the case of wet atmosphere for $<10^7 \text{ kg/s}$, in which case equation (6) can be considered as an upper bound. (c) Illustration of the prediction of equation (6) for fixed source temperature, atmosphere temperature profile and dry conditions for a wind profile defined by a maximum velocity at the tropopause (see auxiliary material) varying between 0 m/s (blue) and 60 m/s (red). (d) Same as Figure 1c for fixed source temperature and atmospheric temperature profile and no wind, varying the relative humidity between 0% and 100% showing a large deviation for small mass flow rates.
Evolution of the mass flow rate during the May 18, 1980 Mount St. Helens eruption (see also Table 1). The one-dimensional model predictions for both dry, 0% relative humidity (squares) and wet, 100% relative humidity (triangles) conditions are well matched by equation (6) (circles), which are 2 to 5 times higher than previous estimates [Carey et al., 1990] (solid horizontal lines). Given the large plume height during this eruption, the model estimates for wet and dry atmosphere are very close, which also closely match estimates of equation (6).

4. Concluding Remarks

The International Civil Aviation Organization is currently discussing new aviation-safety strategies that will likely be based on ash-concentration thresholds in which we anticipate the new expression can play a vital role due to (i) the ease of implementation within operational strategies of real-time ash forecasting and (ii) the insight into first order processes. This represents an important step forward towards improved estimates of source parameters of both vertically-rising and bent-over volcanic plumes required by Volcanic Ash Advisory Centers during volcanic crisis [Mastin et al., 2009; Guffanti et al., 2010; Bonadonna et al., 2011a]. Other applications that can benefit from the expression are the characterization of mass flow rate and duration of past eruptions estimated from tephra deposits [Wilson et al., 1980; Wilson and Walker, 1987; Carey et al., 1990], and long-term probabilistic hazard assessment that work with probability density functions of mass [Bonadonna et al., 2005]. A matlab script is provided in the auxiliary material.

Table 1. Mass Partitioning Between the Morning and Afternoon Eruption Phases of the May 18, 1980 Mount St. Helens Eruption

<table>
<thead>
<tr>
<th></th>
<th>Morning (58–63)%</th>
<th>Afternoon (37–42)%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B1–B2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local time</td>
<td>8:32 a.m.–9 a.m.</td>
<td>9 a.m.–12:15 p.m.</td>
</tr>
<tr>
<td>Radar height above vent (km)</td>
<td>21.5</td>
<td>11.1–14.9</td>
</tr>
<tr>
<td>Carey et al. [1990]</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>equation (6)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1D model dry</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1D model wet</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>B3</td>
<td>B4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local time</td>
<td>12:15 p.m.–4:25 p.m.</td>
<td>4:25 p.m.–5:15 p.m.</td>
</tr>
<tr>
<td>Radar height above vent (km)</td>
<td>10.9–14.9</td>
<td>13.3–16.7</td>
</tr>
<tr>
<td>Carey et al. [1990]</td>
<td>-</td>
<td>77 %</td>
</tr>
<tr>
<td>equation (6)</td>
<td>-</td>
<td>2–7 %</td>
</tr>
<tr>
<td>1D model dry</td>
<td>-</td>
<td>69–72 %</td>
</tr>
<tr>
<td>1D model wet</td>
<td>-</td>
<td>61–67 %</td>
</tr>
</tbody>
</table>

aFollowing the directed blast (stratigraphic unit A), the eruption generated a buoyant plume that sedimented stratigraphic units B1 and B2. In the afternoon the activity changed to pyroclastic density currents with co-ignimbrite fallout associated with stratigraphic unit B3. In the final stage of the eruption a buoyant plume was formed that deposited stratigraphic unit B4 [Lipman and Mullineaux, 1981; Criswell, 1987; Carey et al., 1990]. The total mass is estimated between 5 and $7 \times 10^{11}$ kg [Bonadonna and Costa, 2012]. Percentage variations related to the erupted mass derived from the isopach maps of Criswell [1987] are associated with the integrations of various fits (i.e., exponential, power law and Weibull [Bonadonna and Costa, 2012]). Estimates of the A and B3 phases are omitted in our calculation, as they are modeled more appropriately as thermals [Woods and Kienle, 1994], and did not significantly contribute to the total tephra deposition [Lipman and Mullineaux, 1981; Criswell, 1987].

bFrom isopach maps by Criswell [1987].
Figure 3. Evolution of the mass flow rate of the Eyjafjallajökull eruption plume between 4th and 8th May 2010. Black lines indicate the average mass flow rate estimated from field mapping of the deposits [Bonadonna et al., 2011b]. The one-dimensional model predictions (blue) match the best with the Weibull estimate of mass, which is considered as being the most reliable as it is not based on subjective choices of number of exponential segments for the exponential fit and extremes of integration for the power law fit [Bonadonna and Costa, 2012].

material that calculates equation (6) for a given set of source and atmospheric conditions, which can be changed by the user.

[10] Estimates of mass flow rate for small plumes rising in a wet atmosphere (e.g., for an atmosphere with 100% humidity and $M < 10^7$ kg/s) could be further improved and still be computationally efficient if derived based on Monte Carlo simulations of a one-dimensional plume model such as the one presented in the auxiliary material that accounts for both wind and humidity. More accurate predictions of ash loading into the atmosphere can be provided by unsteady three-dimensional models [Devenish et al., 2010; Herzog and Graf, 2010]. However, these are currently too computationally expensive to be yet used operationally. This study significantly reduces the number of computational runs required by such models as the most appropriate region of interest that needs to be explored in a larger parameter space can easily be identified. This offers a stepping stone to bring three-dimensional models closer to becoming operational for future ash forecasting.

[11] Acknowledgments. WD and CB were supported by the Swiss National Science Foundation grant PBGP2-131251 and 200020-125024, respectively. We thank Arnaud Folch for providing detailed meteorological data for the 4–8 May 2010 period of the Eyjafjallajökull eruption, Michael Manga and Joe Dufek for providing comments on an earlier version of this manuscript. We are also grateful for the contributions of two reviewers, which improved the description of our model and results.

[12] The Editor thanks Larry Mastin and Lionel Wilson for assisting in the evaluation of this paper.

References


