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Abstract

We present a novel Bell-state analyzer (BSA) for time-bin qubits allowing the detection of three out of four Bell states with linear optics, two detectors, and no auxiliary photons. The theoretical success rate of this scheme is 50%. Our new BSA demonstrates the power of generalized quantum measurements, known as positive operator valued measurements. A teleportation experiment was performed to demonstrate its functionality. We also present a teleportation experiment with a fidelity larger than the cloning limit.

Reference


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Quantum Teleportation with a Three-Bell-State Analyzer

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We present a novel Bell-state analyzer (BSA) for time-bin qubits allowing the detection of three out of four Bell states with linear optics, two detectors, and no auxiliary photons. The theoretical success rate of this scheme is 50%. Our new BSA demonstrates the power of generalized quantum measurements, known as positive operator valued measurements. A teleportation experiment was performed to demonstrate its functionality. We also present a teleportation experiment with a fidelity larger than the cloning limit.

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A Bell-state analyzer (BSA) is an essential part of quantum communications protocols such as a quantum relay based on quantum teleportation [1–7], entanglement swapping [8,9], or quantum dense coding [10,11]. It has been shown that, using only linear optics, a BSA for qubits has a maximal success rate of 50% when no auxiliary photons are used [12,13]. This, however, does not limit the number of Bell states one can measure, but only the overall success rate. A complete BSA could be achieved using either nonlinear optics [14] or using continuous variable encoding [15,16]. However, each of these two alternatives carry some significant drawback. The nonlinear optics approach has exceedingly low efficiency; while continuous variable encoding has the disadvantage that postselection is not possible. Note that postselection is a very useful technique that allows one to use only “good” measurement results and straightforward eliminate all others without the need for a lot of computing power [17].

Today’s optimal BSA schemes based on linear optics for qubits are only able to detect two out of four Bell states [2,6,8,9], or are able to detect more than two states but not optimally [18]. Here we present a novel scheme for a BSA which achieves the 50% upper bound of success rate, but can distinguish three out of the four Bell states. We demonstrate this scheme in a quantum teleportation experiment at telecom wavelengths. The new BSA is inspired by, although not limited to, the time-bin implementation of qubits, and is thus fully compatible with the field of quantum communications [19].

At first, one may think that detecting 3 out of 4 Bell states provides a full BSA. Indeed, if the BSA would consist of a standard von Neumann projective measurement, then the fourth Bell state would merely correspond to the nondetection of the 3 others. But our BSA is a new example of the power of generalized quantum measurements, it uses a positive operator valued measurement (POVM) with 21 possible outcomes. Some outcomes of this POVM (see Fig. 1) correspond to one of the 3 Bell states that can be distinguished unambiguously and thus detect this state. The others correspond to inconclusive results. More specifically, the Bell state \(|\phi^+\rangle\) is always detected, \(|\phi^-\rangle\) is never detected, while \(|\psi^-\rangle\) and \(|\phi^+\rangle\) are detected with a 50% success rate.

In previous BSAs, the main method was to use a beam splitter followed by detectors to determine the input Bell state. We replace this standard approach by a time-bin interferometer equivalent to the ones used to encode and decode time-bin qubits (Fig. 1) [19]. The two photons of the Bell state enter in port \(a\) and \(b\), respectively. In a BSA using only a beam splitter, one is able to distinguish between \(\psi^\pm\) but not \(\phi^\pm\), since in the later case the two photons will experience photon bunching but this interference does not contain the phase information that distinguishes \(\phi^-\) from \(\phi^+\). In our case, the first beam splitter acts like above, but we introduce a second interference possibility between the two photons on the second beam splitter. This second interference allows one to distinguish more than just two Bell states. The input modes \(a\) and \(b\) evolve in the interferometer as follows (see Fig. 1):

\[
\hat{a}_i^+ \equiv \frac{1}{\sqrt{4}} (-\hat{e}_i^+ + e^{i\delta} \hat{e}_{i+\Delta t}^+ + i\hat{j}_i^+ + e^{i\delta} \hat{j}_{i+\Delta t}^+),
\]

\[
\hat{b}_i^+ \equiv \frac{1}{\sqrt{4}} (\hat{j}_i^+ - e^{i\delta} \hat{j}_{i+\Delta t}^+ + i\hat{e}_i^+ + i e^{i\delta} \hat{e}_{i+\Delta t}^+),
\]

where \(\hat{j}_i^j\) is a photon at time \(j\) in mode \(i\). Using these

![FIG. 1. A schematic representation of the new type of Bell-state measurement. When two qubit states are sent into a time-bin interferometer the output state is a mixture of photons in two directional modes and three temporal modes. By looking at certain combinations of these photons a Bell-state measurement can be performed for three different Bell states.](image)
TABLE I. The table shows the probability to find any of the 21 possible coincidences as a function of the input Bell-State. A 0 in row D1 means that a photon was found at detector “D1” and at a time corresponding to the photon having taken the short path in the interferometer and it was originally a photon in time-bin \( t_0 \), a 1 corresponds to \( t_0 + 1 \times \tau \) with \( \tau \) corresponding to a the difference between the time-bins, etc., Note that several combinations of detection are possible for only one Bell-state (the bold entries), therefor when such a combination is found a Bell-state measurement was performed. The theoretical probability of a successful measurement is 0.5 which is the optimal value using only linear optics [12].

| \( \phi_+ \) | 1/16 1/16 1/16 1/16 | 1/8 1/8 1/2 |
| \( \phi_- \) | 1/16 1/16 1/4 1/4 1/16 1/16 | 1/8 1/8 |
| \( \psi_+ \) | 1/8 1/8 1/8 1/8 1/8 1/8 | 1/8 1/8 |
| \( \psi_- \) | 1/4 1/4 | 1/8 1/8 |

formulas it is possible to calculate all possible outputs of the interferometer as a function of any input state.

These output coincidences in ports \( e \) and \( f \), i.e., on detectors D1 and D2, are summarized in Table I. By convention, a photon detected at time “0” means that the photon did not accumulate any delay with regards to a fixed reference. This is only possible if the photon took the short path in the BSA and it was originally a photon in time-bin \( t_0 \) (Fig. 1). A photon detected at time “1” signifies that the photon was originally in \( t_1 \) and took the short path of the BSA interferometer or it was in \( t_0 \) and took the long path. A detection at time “2” then means the photon was in \( t_1 \) and took the long path.

In Table I one can distinguish two cases. Either the result unambiguously distinguishes one Bell state. Or the result could have been caused by two specific Bell states, i.e., the result is ambiguous and hence inconclusive.

The above described approach is correct in the case were the phase \( \delta \) of the BSA interferometer is set to 0. Let us thus briefly analyze in this paragraph the situation of an arbitrary phase \( \delta \). In such a case, our BSA still distinguishes 3 Bell states, but these are no longer the Bell states of the computational basis. For a teleportation experiment this means the basis for the measured Bell states is not the same as the basis for the entangled states shared between Bob and Charlie. Still, perfect teleportation is possible, but with the difference that the unitary transformations that Bob has to apply after receiving the classical information about the result of the BSA have changed and no longer include the identity: all unitary transformations are non-trivial, but they remain experimentally feasible. More specifically, the analyzed Bell states are:

\[
\phi'_\pm = |00\rangle \pm e^{2i\delta}|11\rangle, \tag{3}
\]

\[
\psi'_\pm = e^{i\delta}(|01\rangle \pm |10\rangle). \tag{4}
\]

These Bell states are equivalent to the standard states except that the \(|1\rangle\) is replaced by \( e^{i\delta}|1\rangle \) for each of the input modes. Therefore the unitary transformations that have to be applied to retrieve the original state of the teleported photon also have to be modified from \([I, \sigma_x, \sigma_y, \sigma_z, \sigma^0_x] \) to \([\sigma_{2\delta}, \sigma_z \sigma_{2\delta}, \sigma_x, \sigma_z \sigma_{2\delta}] \). Here \( \sigma_{2\delta} = e^{-2i\delta} P_{|1\rangle} + P_{|0\rangle} \) is a phase shift of \( 2\delta \) to be applied to the time-bin \(|1\rangle\).

In a realistic experimental environment the success probabilities of the BSA will be affected by detector limitations, because existing photon detectors are not fast enough to distinguish photons which follow each other closely (in our case two photons separated by \( \tau = 1.2 \) ns) in a single measurement cycle. Hence a coincidence, for example, “02” on D2, cannot be detected with our detectors. This limitation arises from the dead time of the photodetectors. When including this limitation we find that the maximal attainable probabilities of success in our experimental setup are reduced to 1/2, 1/4, and 1/2 for \( \phi'_+, \psi'_-, \) and \( \phi'_+ \), respectively. This leads to an overall probability of success of \( 5/16 \), which is greater by 25% than the success rate of 1/4 with a BSA consisting only of one beam splitter and two identical detectors.

In order to demonstrate successful Bell-state analysis we performed a teleportation experiment. A schematic of the experimental setup is shown in Fig. 2. Alice prepares a photon in the state \(|\xi\rangle = |0\rangle + e^{i\alpha}|1\rangle\). Bob analyzes the teleported photon and measures interference fringes for each successful BSA announced by Charlie. The setup consist of a mode-locked Ti:sapphire laser creating 150 fs pulses with a spectral width of 4 nm, a central
wavelength of 711 nm and a mean power of 400 mW. This beam is split in two beams using a variable coupler (λ/2 and a PBS). The reflected light (Alice) is sent to a Lithium tri-Borate crystal (LBO, Crystal Laser) were by parametric down-conversion a pair of photons is created at 1.31 and 1.55 μm. Pump light is suppressed with a Si filter, and the created photons are collected by a single mode optical fiber and separated with a wavelength-division multiplexer (WDM). The 1.55 μm photon is ignored whereas the 1.31 μm is send to a fiber interferometer which encodes the qubit on the equator of the Bloch sphere. In the same way, the transmitted beam (Bob) is send onto another LBO crystal after having passed through an unbalanced Michelson bulk optics interferometer, the phase of this interferometer is considered as the reference phase. The nondegenerate entangled photons produced in this way corresponds to the φ⁺ state. The photons at 1.31 μm are send to Charlie in order to perform the Bell-state measurement. In order to assure temporal indistinguishability, Charlie filters the received photons down to a spectral width of 5 nm. In this way the coherence time of the generated photons is greater than that of the photons in the pump beam, and as such we can consider the photons to be emitted at the same time. Bob filters his 1.55 μm photon to 15 nm in order to avoid multiphoton events [20]. A liquid-nitrogen-cooled Ge avalanche photon detector (APD) D1 with passive quenching detects one of the two photons in the BSA and triggers the commercial infrared APDs (id Quantique) D2 and D3. Events are analyzed with a time to digital converter (TDC, Acam) conditioned on a successful BSA. When Bob scans the phase of his interferometer we obtained a raw visibility of \( V = 57\% (F = 79\%) \) and a net visibility of \( V = 83\% \pm 4 (F = 91\% \pm 2) \) clearly higher than the cloning limit of \( F = 5/6 \) (Fig. 3). We then switched to the new BSA. This new setup introduces about 3 dB of excess loss, due to added optical elements including the interferometer and its stabilization optics. These losses result in a lower count rate. For experimental reasons we now scan the interferometer of Alice instead of Bob. The experiments were performed for approximately 4.4 hours per point in order to accumulate enough data to have low statistical noise. The expected interference fringes after Bob’s interferometer are of the form \( 1 \pm \cos(\alpha + \beta) \) for a projection on \( \psi_\pm \) and \( 1 + \cos(\alpha - \beta - 2\delta) \) for a projection on \( \phi_\pm \). Hence one would

![FIG. 3 (color online). Left: The result of the 1-Bell-state teleportation experiment (a beam splitter instead of the interferometer) with \( F_{\text{raw}} = 79\% \) and \( F_{\text{net}} = 91\% \). Right: A typical result of the scan in delay of the coincidences “00” and “22”. The predicted antidip is clearly visible in both curves with a visibility after noise subtraction of 32% for “0” on D1 and D2 and 26% for “2” on D1 and D2 (theoretical maximum = 1/3 [22]).](https://example.com/fig3.png)

![FIG. 4 (color online). These graphs show the interference fringes found when scanning the interferometer at Bob. we found visibilities of 0.22, 0.43 and 0.38 for \( |\psi_\pm \rangle \), \( |\phi_\pm \rangle \) and \( |\psi_- \rangle \). The average visibility of the BSA is \( V_{\text{avg}} = 0.34 (F = 0.67) \).](https://example.com/fig4.png)
interference possibility for H and V polarization. This would require 4 detectors, but an overall efficiency of 50% can be achieved with current day detectors. The limit of 50% can also be achieved with time-bin encoded qubits but would require the use of ultra fast optical switches and two more detectors. We did not implement this due to the losses associated with introducing current day high speed integrated modulators. Second, even though three out of four Bell states can be distinguished, one cannot use this scheme in order to increase the limit of log_2 3 bits per symbol for quantum dense coding.

In conclusion, we have shown experimentally that it is possible to perform a three-state Bell analysis while using only linear optics and without any actively controlled local operations on a single qubit. In principle this measurement can obtain a success rate of 50%. We have used this BSA to perform a teleportation experiment, and obtained a non-corrected overall fidelity of 67%, after noise subtraction we find $F = 76\%$. Also, we performed a teleportation experiment with a one state BSA which exceeded the cloning limit.

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**References**

[17] For experiments on the nature of physics, such as Bell tests, postselection can be a drawback. For applications in quantum communication, however, it is advantageous since it selects useful measurement results.