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Reference


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Witnessing Trustworthy Single-Photon Entanglement with Local Homodyne Measurements

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Abstract

Single-photon entangled states, i.e., states describing two optical paths sharing a single photon, constitute the simplest form of entanglement. Yet they provide a valuable resource in quantum information science. Specifically, they lie at the heart of quantum networks, as they can be used for quantum teleportation, swapped, and purified with linear optics. The main drawback of such entanglement is the difficulty in measuring it. Here, we present and experimentally test an entanglement witness allowing one to say whether a given state is path entangled and also that entanglement lies in the subspace, where the optical paths are each filled with one photon at most, i.e., refers to single-photon entanglement. It uses local homodyning only and relies on no assumption about the Hilbert space dimension of the measured system. Our work provides a simple and trustworthy method for verifying the proper functioning of future quantum networks.

Motivations.—Quantum networks [1] provide broad capabilities, ranging from long distance quantum communication at large scales [2,3], to the simulation of quantum many-body systems [4] in tabletop implementations. Remarkable progresses have been made in practice [5–7] and experimental capabilities are now advancing into a domain of rudimentary functionality for quantum nodes connected by quantum channels [8–11]. Surprisingly, the task of checking that a newly implemented quantum network performs well remains nontrivial.

In the past decade, a great number of architectures based on atomic ensembles and linear optics have been proposed [12]. We now know that quantum networks based on single-photon entanglement [13], i.e., entangled states of the form

$$\frac{1}{\sqrt{2}}(|1_A\rangle|0_B\rangle + |0_A\rangle|1_B\rangle),$$

where $A$ and $B$ are two spatial modes sharing a delocalized photon, are very attractive: They require significantly fewer resources than the other architectures and are less sensitive to memory and photon detector inefficiencies [12]. Furthermore, they are efficient when combined with temporal multiplexing [14]. However, such networks have a major drawback: The detection of single-photon entangled states is very challenging. One cannot resort, for example, to violating a Bell inequality given solely photon-counting techniques.

Hitherto, there are three prescribed methods to detect single-photon entanglement. The first one converts two copies of a single-photon entangled state into one copy of two-particle entanglement. Starting from entanglement $|1_A\rangle|0_B\rangle + |0_A\rangle|1_B\rangle$, it basically consists of a postselective projection onto the subspace with one excitation in each location, yielding $|1_A\rangle|1_B\rangle + |1_A\rangle|1_B\rangle$. The latter is analogous to conventional polarization or time-bin entanglement, and any witness suited for such entanglement can thus be used to postselectively detect single-photon entanglement. Nevertheless, this approach is not fully satisfying conceptually because it relies on postselection. Furthermore, for practical implementation, the need to create two copies requires twice the number of resources at each node.

The second method is based on partial quantum state tomography. Specifically, one reconstructs a reduced density matrix that corresponds to a projection of the full density matrix into a subspace with at most one photon locally. The presence of entanglement is then inferred from an entanglement measure computed from the reduced density matrix [15]. Specifically, this tomographic approach requires the knowledge of probabilities $p_{mn}$ of having $m$ photons in mode $A$ and $n$ in mode $B$, where $m, n \in \{0, 1\}$, and the visibility $V$ of the single-photon interference pattern obtained by combining the modes $A$ and $B$ into a beam splitter. Although it has triggered highly successful experiments [15–18], the approach presented in Ref. [15] cannot be directly used in large-scale networks when one needs to check the entanglement between far away locations, since the knowledge of $V$ relies on a joint measurement of $A$ and $B$.
The last method uses local homodyne detections and provides, a priori, a full tomography of the state that can subsequently be used to measure the entanglement [19,20]. However, the tomographic approach requires a number of measurements, which increase with the dimension of the state being measured [21]. In practice, one could be tempted to make an assumption on the regularity of the measured Wigner function to reduce the number of measurements or, equivalently, on the dimension of the system’s Hilbert space, especially when focusing on single-photon entanglement. But this would amount to making an assumption about the system that we want to characterize. One can also estimate the dimension of the state from measurements, but it is not clearly established how errors on this estimation can affect the conclusion about the presence of entanglement. More generally, the exponential increase of required measurements with the number of measured subsystems makes the tomography not suited to increase of required measurements with the number of components and does not reduce to a two-qubit state. We show below how the entanglement witness can be extended to the case of arbitrary dimensional bipartite states. First, we show how Alice and Bob can accurately estimate the probability that their state lies out of a two-qubit space \( \{|0\}, \{|1\}\}^2 \). We then demonstrate that this probability can be used to upper bound the maximal CHSH value that can be obtained with separable states.

Principle. — Here, we propose a simple approach to witness single-photon entanglement which relies on local measurements only and needs neither postselection nor assumption on the tested state. The basic principle is drawn in Fig. 1. Two distant observers, Alice and Bob, share a quantum state. To check whether it is entangled, each of them randomly chooses a measurement among two quadratures, \( \{X, P\} \) for Alice and \( \{X + P, X - P\} \) for Bob. At each run, they obtain a real number. They then process the results to get binary outcomes using a sign binning; i.e., they attribute the result \(-1\) if the result is negative and \(+1\) otherwise. By repeating the experiment several times, Alice and Bob can compute the conditional probabilities \( p(a, b|x, y) \), where \( a, b \in \{-1, +1\} \), \( x \in \{X, P\} \), and \( y \in \{X + P, X - P\} \). Substituting these probabilities by their real numbers into the Clauser-Horne-Shimony-Holt (CHSH) [24] polynomial

\[
S = E_{XX + P} + E_{XX - P} + E_{P X + P} - E_{P X - P},
\]

(2)

where \( E_{a,y} = \sum_{a,b\in\{-1, +1\}} p(a = b|x, y) - p(a \neq b|x, y) \), they obtain a real number \( S \).

The value of \( S \) can easily be obtained under the assumption that Alice and Bob each have a qubit. Indeed, in the Fock basis \( \{|0\}, \{|1\}\} \), the measurement of the \( X \) quadrature with sign binning is equivalent to a noisy \( \sigma_x \) measurement [25,26], 

\[
\int_0^\infty dx|x(x) - \int_0^\infty dx|x|x = \sqrt{2} \sigma_x
\]

and, similarly, the \( P \) quadrature corresponds to \( \sigma_y \) with the same noise. For the setting choice \( \{X, P\} \) and \( \{X + P, X - P\} \), the state (1) thus yields to \( S = 2\sqrt{2} \times \frac{2}{\pi} = 1.8 \). Furthermore, the maximum value that can be obtained with a separable state belonging to the subspace \( \{|0\}, \{|1\}\}^2 \) is \( S_{\text{sep}} = \sqrt{2} \times \frac{2}{\pi} = 0.9 \). Since \( S \) is smaller than 2, the proposed CHSH-like test does not highlight the nonlocal characteristic of a single photon delocalized among two modes, but it does provide an attractive entanglement witness: If the measured CHSH value is larger than \( S_{\text{sep}} \), Alice and Bob can conclude that they share an entangled state.

This holds for qubits only. In practice, however, the state describing the modes A and B includes multiphoton components and does not reduce to a two-qubit state. We show below how the entanglement witness can be extended to the case of arbitrary dimensional bipartite states. First, we show how Alice and Bob can accurately estimate the probability that their state lies out of a two-qubit space \( \{|0\}, \{|1\}\}^2 \). We then demonstrate that this probability can be used to upper bound the maximal CHSH value that can be obtained with separable states.

Bounding the Hilbert space dimension. — Let us consider the case where Alice and Bob do not have qubits but quantum states of arbitrary dimension. First, they need to bound the probability that at least one of their modes is populated with more than one photon \( p(n_A \geq 2 \cup n_B \geq 2) \). This can be realized without assumption on the Hilbert space dimension by first determining the probabilities \( p(n_A = j) (p(n_B = j)) \) of having \( j \) photons in Alice’s (Bob’s) mode, using local homodyning with phase-averaged local oscillators through a direct integration of the obtained data with a pattern function [28]. The joint probability \( p(n_A = 2 \cup n_B = 2) = p(n_A \geq 2) + p(n_B \geq 2) - p(n_A \geq 2 \cap n_B \geq 2) \) can then be bounded by the parameter \( p^* \), defined as follows

\[
p^* = 2 - \left( \sum_{j=0}^1 p(n_A = j) + p(n_B = j) \right).
\]

(3)

We now show how the knowledge of \( p^* \) can be used to construct an operational witness for single-photon entanglement.

Evaluating the maximal CHSH value with separable states. — Consider the general case, where \( p^* \neq 0 \), i.e. \( p(n_A \geq 2 \cup n_B \geq 2) \neq 0 \) a priori. The state of Alice and Bob can be described by the density matrix

\[
\rho = \begin{pmatrix}
\rho_{n_A \leq 1 \cap n_B \leq 1} & \rho_{n_A \leq 1 \cap n_B \geq 2} \\
\rho_{n_A \geq 2 \cap n_B \leq 1} & \rho_{n_A \geq 2 \cap n_B \geq 2}
\end{pmatrix}.
\]

(4)

where \( \rho_{n_A \leq 1 \cap n_B \leq 1} \) denotes the \( 4 \times 4 \) block with, at most, one photon per mode, \( \rho_{n_A \geq 2 \cap n_B \geq 2} \) refers to the block where at least one of the two modes contains at least two photons,
and $\rho_{\text{coh}}$ is associated with the coherence between these two blocks. Since $\rho_{n_A=2\cup n_B=2}$ possibly spans a Hilbert space of infinite dimension, there could be an infinite number of coherence terms. However, a few of them give a nonzero contribution to the CHSH polynomial if a phase-averaged homodyne detection is used at each location. Specifically, consider the case where Alice and Bob perform the measurements $X_{\phi_A} = \cos \phi_A X + \sin \phi_A P$ and $X_{\phi_B} = \cos \phi_B X + \sin \phi_B P$, respectively, where $\phi_A$ and $\phi_B$ are random variables such that $\langle e^{i\phi_{A,B}} \rangle = 0$, $k \in \mathbb{N}^*$ but the phase difference $\phi_A - \phi_B = \Delta \phi$ is fixed. This requires classical but not quantum communication and, hence, can only decrease the entanglement that Alice and Bob potentially share. In particular, if Alice can choose a measurement among the two quadratures $\{X_{\phi_A}^1, X_{\phi_A}^2\}$ and if Bob’s choice reduces to one of the quadratures $\{X_{\phi_B}^i, X_{\phi_B}^j\}$ such that $\phi_A^1 - \phi_B^1 = -\frac{\pi}{4}$, $\phi_A^2 - \phi_B^2 = \frac{\pi}{4}$, $\phi_A^2 - \phi_B^1 = \frac{\pi}{4}$ and $\phi_A^1 - \phi_B^2 = \frac{3\pi}{4}$, we show in the Supplemental Material [29] that the CHSH polynomial corresponding to the state ($4$) is bounded by

$$S_{\text{max}} = \frac{16}{\sqrt{2\pi}} \Im\{01\vert \rho_{n_A=1\cup n_B=1}\vert 10\}$$

$$+ \frac{8}{\pi} \Im\{20\vert \rho_{\text{coh}}\vert 11\} + \Im\{02\vert \rho_{\text{coh}}\vert 11\}$$

$$+ 2\sqrt{2} p(n_A \geq 2 \cup n_B \geq 2),$$

where $\Im$ denotes the real part. For a given value of $p(n_A \geq 2 \cup n_B \geq 2)$, $S_{\text{max}}$ can be directly maximized over the set of physical states $(\rho \in \{0, 1, 2\})^2 \otimes \mathbb{C}^2$, $\text{tr}(\rho) \leq 1$, $\rho \geq 0$ that satisfy the observed photon number distributions, i.e., $p(00) + p(10) + p(01) + p(11) = 1 - p(n_A \geq 2 \cup n_B \geq 2)$ and that are separable in the $\{00, 11\}^2 \otimes \mathbb{C}^2$ subspace, i.e., for which the projection into this subspace remains positive under partial transposition (PPT) [30,31]. Figure 2 shows the result of this optimization $S_{\text{sep}}^{\text{max}}$.

**Witnessing single-photon entanglement.**—This provides a truly state-independent witness [32] of entanglement. First, the protagonists determine the local photon-number distributions from which they deduce an upper bound $p^*$ on the joint probability $p(n_A \geq 2 \cup n_B \geq 2)$. Second, they deduce $S_{\text{sep}}^{\text{max}}(p^*)$ from Fig. 2 (see the Supplemental Material [29]). Third, they measure the CHSH value $S_{\text{obs}}$ by randomly choosing measurements among $\{X_{\phi_A}^i, X_{\phi_A}^j\}$ and $\{X_{\phi_B}^i, X_{\phi_B}^j\}$, respectively, and by subsequently computing the CHSH polynomial through Eq. (2). If $S_{\text{obs}} > S_{\text{sep}}^{\text{max}}(p^*)$, Alice and Bob know that the projection of their state into the subspace $\{00, 11\}^2$ has a negative partial transpose; i.e., they can safely conclude that the state is entangled and that the entanglement resides in the subspace with at most one photon locally.

Importantly, a tighter bound can be obtained if $S_{\text{max}}$ is maximized over the set of states with a positive partial transpose not only satisfying $p(00) + p(10) + p(01) + p(11) = 1 - p(n_A \geq 2 \cup n_B \geq 2)$ but also reproducing the locally measured probabilities $p(n_A = j) (p(n_B = j))$ for having $j = 0$, 1 photon in Alice’s (Bob’s) mode. These additional constraints have been taken into account for the computation of the separable bounds related to the experiment presented below (see the Supplemental Material [29]).

**Proof-of-principle experiment.**—We start off with a heralded single photon generated by a conditional preparation technique operated on a two-mode squeezed vacuum emitted by a type-II optical parametric oscillator [33]. Without correction for detection loss, the overall fidelity reaches 70%. Single-photon entanglement is obtained by sending

![FIG. 2](color online). Separable bound $S_{\text{obs}}^{\text{max}}$ as a function of the probability that at least one of the two protagonists gets more than one photon $p(n_A \geq 2 \cup n_B \geq 2)$. The blue curve allows one to know the maximum value of the CHSH polynomial $S_{\text{sep}}^{\text{max}}$ that a state separable in the $\{00, 11\}^2 \otimes \mathbb{C}^2$ subspace can reach. If $S_{\text{obs}} > S_{\text{sep}}^{\text{max}}$, one can conclude that the projection of the state in the subspace with zero and one photon locally is entangled.

![FIG. 3](color online). Experimental setup. A tunable single-photon entangled state is created by sending a heralded single photon on a tunable beam splitter based on a polarizing beam splitter (PBS) and a half-wave plate ($\lambda/2$). The proposed witness is then tested with two independent homodyne detections (Alice and Bob). The local oscillator is superposed to each mode via the first PBS. Its global phase is swept with a piezoelectric actuator. The relative phase $\Delta \phi$ is set with a combination of birefringent plates.
the created photon into a beam splitter [34]. Specifically, by controlling the angle $\theta$ of a half-wave plate relative to the axis of a polarizing beam splitter (PBS), we create a tunable single-photon entangled state $\cos(2\theta)|0\rangle_A|1\rangle_B + \sin(2\theta)|1\rangle_A|0\rangle_B$ between the two output modes of the PBS, as sketched in Fig. 3.

The local oscillators that Alice and Bob need to reveal entanglement are obtained by impinging a bright beam on the second input of the PBS: one polarization mode is used as the quantum channel, and the orthogonal one conveys the local oscillator. It is worth mentioning that this technique can be implemented over long distances as realized in field implementation of quantum key distribution [35]. The relative phase between Alice’s and Bob’s local oscillators $\Delta \varphi$ is fixed by choosing an appropriate elliptical polarization of the bright beam just before the PBS [36]. In practice, the setting difference is calibrated by observing the dephasing of interference fringes (the quantum state is replaced here by a coherent state). A global phase averaging is obtained by sweeping a piezoelectric transducer located on the path of the bright beam before the PBS.

For each heralding event, Alice and Bob each obtain a real valued outcome, which is extracted from homodyne photocurrents. Accumulating 200,000 events for each quadrature relative measurements, they deduce the value of the CHSH polynomial $S_{\text{obs}}$. The same homodyne measurements also provide the local photon number distributions which are used to compute the separable bound $S_{\text{sep}}$. We emphasize that the separable bound is here obtained by maximizing the CHSH value over the set of separable states that fulfill the locally measured photon-number occupation probabilities $p(n_A = j)$ and $p(n_B = j)$ for having $j = 0, 1$ photon in Alice and Bob’s mode, respectively. Furthermore, it takes several errors into account, for example, errors related to the quadrature measurement imperfections were considered (see the Supplemental Material [29]). The procedure is repeated for various angles $\theta$ ranging from 0 to 45°. Figure 4 shows the main result, i.e., the observed CHSH values and the separable bounds as a function of $\theta$. One sees that they both reach maximal values around $\theta = 22.5°$, where Alice and Bob ideally share a maximally entangled state. The small deviation between the observed value $S_{\text{obs}}(\theta = 22.5°) \approx 1.33$ and the CHSH value that would be obtained with a maximally entangled state (1.8) demonstrates that the overall source and detection efficiencies are very high. Furthermore, the observed CHSH values are almost all larger than the separable bounds when dealing with entangled states ($\theta \neq 0°, 45°$). This shows the great robustness of the proposed witness.

**Conclusion.**—We have presented and experimentally tested a witness for single-photon entanglement that does not need postselection, uses local measurements only, and does not rely on assumptions about the dimension of the measured state. Note that our witness can be easily adapted to detect few-photon entanglement without additional complications. We believe that it will naturally find applications in long-distance quantum communication, allowing users to check whether two remote nodes of a given quantum network are entangled. One important challenge in this context is to reveal the entanglement shared by a large number of parties. Finding Bell inequalities that could be used as witnesses for multipartite single-photon entanglement is work for the future.

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Entanglement between atomic ensembles has been reported in B. Julsgaard, A. Kozhekin, and E.S. Polsik, Nature (London) 413, 400 (2001); J. Simon, H. Tanji, S. Ghosh, and V. Vuletić, Nat. Phys. 3, 765 (2007), and in Refs. [15–18].


Note that in Ref. [19], in addition to a tomographic approach, a Bell test is presented which can be used for witnessing single-photon entanglement. The latter relies on the rejection of some of the acquired data. In comparison, our witness does not need postselection.

For a state with dimension $d$, $d + 1$ measurements are needed; see for example U. Leonhardt and M. Munroe, Phys. Rev. A 54, 3682 (1996).

The number of measurements that are needed to reconstruct a state of dimension $d$ shared by $n$ parties (nodes) is given by $(d + 1)^n = e^{n \log(d + 1)}$.


The witness does not require one to know the way in which the state was created nor its dimensionality. However, it requires that Alice and Bob’s devices perform the desired measurements. In particular, for the proposed witness, measurements are assumed to be homodyne measurements defined in the single optical mode of a local oscillator.


Note that it is sufficient for the input state to be non-classical with respect to the $P$ representation to create entanglement at a beam splitter, as shown, e.g., in J.K Asboth, J. Calsamiglia, and H. Ritsch, Phys. Rev. Lett. 94, 173602 (2005).
