Macroscopic Optomechanics from Displaced Single-Photon Entanglement

SEKATSKI, Pavel, ASPELMEYER, Markus, SANGOUARD, Nicolas Bruno

Abstract
Displaced single-photon entanglement is a simple form of optical entanglement, obtained by sending a photon on a beam splitter and subsequently applying a displacement operation. We show that it can generate, through a momentum transfer in the pulsed regime, an optomechanical entangled state involving macroscopically distinct mechanical components, even if the optomechanical system operates in the single-photon weak coupling regime. We discuss the experimental feasibility of this approach and show that it might open up a way for testing unconventional decoherence models.

Reference

DOI: 10.1103/PhysRevLett.112.080502
Macroscopic Optomechanics from Displaced Single-Photon Entanglement

Pavel Sekatski,$^{1,2}$ Markus Aspelmeyer,$^3$ and Nicolas Sangouard$^1$

$^1$Group of Applied Physics, University of Geneva, CH-1211 Geneva 4, Switzerland
$^2$Institut for Theorettische Physik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
$^3$Vienna Center for Quantum Science and Technology, Faculty of Physics, University of Vienna, Boltzmannaggasse 5, I-1090 Vienna, Austria

(Received 30 August 2013; published 27 February 2014)

Displaced single-photon entanglement is a simple form of optical entanglement, obtained by sending a photon on a beam splitter and subsequently applying a displacement operation. We show that it can generate, through a momentum transfer in the pulsed regime, an optomechanical entangled state involving macroscopically distinct mechanical components, even if the optomechanical system operates in the single-photon weak coupling regime. We discuss the experimental feasibility of this approach and show that it might open up a way for testing unconventional decoherence models.

Introduction.—Can a macroscopic massive object be in a superposition of two well distinguishable positions? It has been argued that such superpositions undergo intrinsic decoherence, e.g., due to a nonlinear stochastic classical field [1–3] or caused by the superposition’s perturbation of spacetime [4,5]. These decoherence mechanisms are different from conventional decoherence that occurs through entanglement with the environment [6] and that has been nicely demonstrated in Refs. [7–11]. In contrast, testing for unconventional decoherence models requires a combination of large masses and superpositions of states corresponding to well separated positions. Matter-wave interferometry with large clusters [12] or with submicron particles [13] is one possible route. Another approach is to manipulate states of motion of massive mechanical resonators, a fast moving field of research that has now succeeded in entering the quantum regime [14–17]. In the framework of optically controlled mechanical devices [18], the proposals [19,20] have the potential to create a superposition of mechanical states with a distance of the order of the mechanical zero-point fluctuation where the effects of unconventional decoherences might be observable [21,22]. However, this requires (i) one to work in the single-photon strong coupling regime, (ii) a coupling rate at least of the order of the mechanical frequency so that the displacement induced by a single photon is larger than the mechanical zero-point spread, and (iii) one to work in the resolved sideband regime where the mechanical frequency is larger than the cavity decay rate to allow ground state cooling. While (i) and (ii) can be relaxed, e.g., using nested interferometry [23] and (iii) can be circumvented by cooling, e.g., via pulsed optomechanical interactions [24], the distance between the superposed states remains small, of the order of the mechanical ground state extension.

Here, we show how to create macroscopic optomechanical entanglement with relatively simple ingredients. Our proposal starts with an optical entangled state of the type $|\mp\rangle_A|\pm\rangle_B - |\plus\rangle_A|\mp\rangle_B$, involving two spatial modes $A$ and $B$. Concretely, this state is obtained by sending a single photon into a beam splitter (with output modes $A$ and $B$) and by subsequently applying a phase-space displacement on $A$. The displaced photons in $A$ then interact with a mechanical system $M$ through radiation pressure. If the interaction between $A$ and $M$ falls within the pulsed regime [24–26] where the pulse duration is much smaller than the mechanical period, the optical and mechanical modes entangle, $|\mp\rangle_{AM}|\pm\rangle_B - |\pm\rangle_{AM}|\mp\rangle_B$. Because $|\mp\rangle_A$ and $|\pm\rangle_A$ are well distinguishable in photon number, the mechanical components $\rho_M^{(2)} = \text{tr}_A[|\mp\rangle_{AM}\langle\mp|\langle\pm|_AM]$ are well distinct in the phase space even in the weak coupling regime and if the coupling rate is smaller than the mechanical frequency. This relaxes the constraints on the initial cooling of the mechanical oscillator and makes our proposal well suited to test unconventional decoherence processes, as we show below.

Optomechanical entanglement.—Consider an optomechanical cavity described by $H = \hbar \omega_m m^\dagger m - \hbar g_0 a^\dagger a (\gamma^m + m)$, where $\omega_m$ is the angular frequency of the center of mass motion of the mechanical system, $m$, $m^\dagger$ $(a, a^\dagger)$ are the bosonic operators for the phononic (photonic) modes, and $g_0 = (\omega_c/L)/\sqrt{\hbar/2\omega_m}$ for a Fabry-Perot cavity with a mechanically moving end mirror $(\omega_c$ is the optical angular frequency, $L$ is the cavity length, and $M$ is the effective mass of the mechanical mode). The form of the optomechanical interaction, proportional to $a^\dagger a \tilde{x}_m$ [where $\tilde{x}_m = x_0 (m + m^\dagger)$ is the position operator, $x_0 = \sqrt{\hbar/2\omega_m}$ being the mechanical zero-point fluctuation amplitude] tells us that starting with a superposition of photonic components which are well distinguishable in photon number space, we can create a superposition of mechanical states corresponding to well distinct momenta. Displaced single-photon entanglement exhibits such a...
property [27,28] and has the advantage of being easily prepared, see Fig. 1. It can be written as

$$\frac{1}{\sqrt{2}} (D(\beta)|+\rangle_A|\rangle_B - D(\beta)|-\rangle_A|\rangle_B),$$

(1)

where $|\pm\rangle = 2^{-1/2}(|0\rangle \pm |1\rangle)$, $|0\rangle$ being the vacuum, and $|1\rangle$ the single photon Fock state [29]. $D(\beta) = e^{i|\beta|^2/2}$ stands for the displacement operator and can be implemented using an unbalanced beam splitter and a coherent state [30]. Although the photon number distributions for $D(\beta)|+\rangle_A$ and $D(\beta)|-\rangle_A$ partially overlap (their variance is given by $\beta^2 + (1/4)$), their mean photon numbers $\beta^2 \pm \beta + (1/2)$ are separated by $2\beta$ [27]. (Here $\beta$ is considered real, as throughout Letter). In other words, their distance in the photon number space is of the order of the square root of their size. This makes the state (1) macroscopic in the sense that its components can be distinguished without a microscopic resolution [28].

Consider first the case where $|n\rangle_A$ photons interact with the mechanical mode initially prepared in its motional ground state $|0\rangle_M$. According to Ref. [19], they induce a coherent displacement of the mechanical state whose amplitude varies periodically in time $e^{i(\gamma n^2/\omega_m - \sin(\omega_m t))}|(g_0n/\omega_m)(1-e^{-i\omega_m t})\rangle_M|n\rangle_A$. The first exponential term corresponds to the variation of the cavity length and is quadratic in the photon number because the mean position of the mechanical oscillator depends on the photon number. To avoid this nonlinear behavior, we consider the pulsed regime where the interaction time $\tau$ is much smaller than the mechanical period $|\sin(\omega_m \tau) \sim \omega_m \tau$, cf. below for the detailed conditions. Right after this interaction, the propagator has the simple form $e^{ig_0|\alpha|^{2n+m'}\tau}$ and after a free evolution of duration $t$, the overall propagator can be written as $U(t) = e^{ig_0\omega_m t + e^{i\omega_m^{\prime}m'}\tau} e^{-i\omega_m^{\prime}m\tau}$. An initial state $|0\rangle_M |n\rangle_A$ now evolves towards $|na(t)\rangle_M |n\rangle_A$ where $|na(t)\rangle_M$ is a coherent state with a fixed amplitude and a periodic phase $na(t) = -i g_0 n t e^{-i\omega_m t}$. In other words, the $n$ photons kick the mechanical mode that gets an additional momentum $2g_0n\hbar\pi p_0$ at time $t = 0$ ($p_0 = \sqrt{\hbar M\omega_m}/2$ is the initial mechanical momentum spread). The mechanical state then starts to rotate in phase space. It reaches a minimal position $-2g_0n\hbar\pi x_0$ after $\pi/2\omega_m$, then gets a momentum $-2g_0n\hbar\pi p_0$ after $\pi/\omega_m$ and so on.

Let us now come back to the initial state (1). The pulse in $A$ enters the optomechanical cavity, the mechanical mode being in $|0\rangle_M$, as before. A time $t$ after the interaction, the state of the system is

$$\frac{1}{\sqrt{2}} \left( \sum_k a_{\beta}^{(\pm)}(k)|k\rangle_A|\alpha(t)\rangle_M|\mp\rangle_B \right)$$

(2)

where $a_{\beta}^{(\pm)}(k) = (1/\sqrt{2})e^{-i|\beta|^2/2} (\beta^k / \sqrt{k!})(1 \pm ((k/\beta) - \beta))$ are the probability amplitudes for having $k$ photons in $D(\beta)|\pm\rangle_A$. Since $\sum_k a_{\beta}^{(\pm)}(k)^* a_{\beta}^{(\pm)}(k) = 0$, the mechanical mode entangles with the optical modes. Specifically, after $\pi/2\omega_m$, the state (2) involves two mechanical states $|\rho_{\pm}^{(\pm)}\rangle = \sum_k |a_{\beta}^{(\pm)}(k)|^2 |-g_0\pi k\rangle_M |-g_0\pi k\rangle_A$, each having a variance $(1 + g_0^2\hbar^2 (1 + 4\beta^2))/x_0^2$ in space and for which the mean position is separated by $4g_0\pi\beta x_0$ (see Fig. 2). These two mechanical states can thus be distinguished with a detector having a resolution $\delta x \sim 2g_0\pi\beta x_0$, see below. For $g_0\pi\beta \geq 1$, such a detector cannot resolve two phononic Fock states with $n$ and $n + 1$ excitations (no microscopic resolution) and the entangled state (2) can fairly be defined as being macroscopic.

**Macroscopic correlations.**—We now show how to demonstrate that the mechanical mode involves macroscopically distinct states $\rho_{\pm}^{(\pm)}$. More precisely, we show that $B$ and $M$ are correlated, i.e., when the state of $B$ is projected into $|-\rangle \langle +|$, the mechanical mode is found in $\rho_{\pm}^{(\pm)} [\rho_{\pm}^{(-)}]$ a quarter of a mechanical period after the interaction, (cf. Fig. 3) and that these correlations can be revealed without the need for a microscopic resolution. This is done by tracing out $A$, and by measuring the $\tilde{X} = 2^{-1/2}(b^\dagger + b)$ quadrature of $B$ and the mirror position. The latter can be realized following Ref. [24], by observing through a

![FIG. 2](color online). Trajectory of the mechanical state in the phase space. (I) The mirror first gets a momentum proportional to the mean photon number. The superposition of two mechanical states (corresponding to the two ovals) result from the interaction with a superposition of $D(\beta)|-\rangle_A$ and $D(\beta)|+\rangle_A$. (II) After a quarter of a period, the positions of the two superposed states are maximally distinct and are correlated with the $\tilde{X}$ quadrature of the mode B. (III) By measuring the position after a multiple of a half period, the information about the number of photons in $A$ (contained in the mirror) is erased, which enables us to observe the entanglement between $A$ and $B$. 

080502-2
quadrature measurement the phase acquired by a strong, short light pulse reflected by the mechanical oscillator. We attribute the value $+1 (-1)$ to a positive (negative) result of the quadrature measurement on $B$ and $+1 (-1)$ if the mirror is found to be shifted more to the left (right) with respect to its mean position $-g_0 \epsilon x_0 (1 + 2\beta^2)$. For an uncertainty $\delta x$ on the measurement of the mirror position, the probability $P_{\pm E_y}$ for having the same results $\{\pm 1, \pm 1\}$ is given by $(1/4) + (g_0 \epsilon \beta )/\sqrt{1 + g_0^2 \epsilon^2 \beta^2 + \delta x^2/(4x_0^2)}$ (for $\beta \gg 1$) while the probability for having different results $P_{\pm E_y} = (1/2) - P_{\pm E_y}$. Therefore, the correlations between the outcomes (the probability for having correlated results minus the probability for having anticorrelated results) are given by $(2/\pi)(g_0 \epsilon \beta)/\sqrt{1 + g_0^2 \epsilon^2 \beta^2 + \delta x^2/(4x_0^2)}$. In the regime of interest $g_0 \epsilon \beta \gg 1$, even a coarse grained measurement with the resolution $\delta x = 2g_0 \epsilon \beta x_0$ leads to substantial correlations $\sim 0.45$. This is a consequence of the macroscopic characteristic of the optomechanical state (2).

Testing unconventional decoherence models.—Figure 4 shows how to probe the effect of mirror decoherence. First, the mechanical position is measured at any time that is a multiple of half a mechanical period where no information is obtained about the state of $A$. Finding the mirror at the position $y$ projects the overall state into

$$I(2) \left( \sum_k c^{(\pm)}(k)e^{i\sqrt{2g_0 \epsilon}ky}|k\rangle_A|\pm \rangle_B \right. \left. - \sum_k c^{(-)}(k)e^{i\sqrt{2g_0 \epsilon}ky}|k\rangle_A|\pm \rangle_B \right) \left| y \right\rangle_M. \quad (3)$$

Actively controlling the relative length of paths $A$ and $B$ to get rid of the undesired phase term $e^{i\sqrt{2g_0 \epsilon}ky}$ and subsequently applying $\mathcal{D}(-\beta)$ leaves the optomechanical state in $(1/\sqrt{2})(|1\rangle_A|0\rangle_B - |0\rangle_A|1\rangle_B)|y\rangle_M$. The modes $A$ and $B$ can then be combined on a beam splitter and varying their relative phase leads to interference fringes, ideally with a unit visibility ($V$). Note here that from the values of the probabilities $p_{mn}$ of detecting $m \in \{0, 1\}$ photons in $A$ and $n \in \{0, 1\}$ in $B$, a lower bound on the negativity between $A$ and $B$ can be obtained $N_{AB} \geq (1/2)(p_{00} - p_{11})^2 + (V(p_{01} + p_{10})^2 - (p_{00} + p_{11}))$ through the approach presented in Ref. [31]. Decoherence of the mirror operates as a weak measurement of the photon number on $A$ (see the Supplemental Material [32]). Therefore, if the measurement of the mechanical position is delayed, more and more “which path” information is revealed, which decreases the visibility as the delay time increases. In particular, we compare conventional (environmentally induced) decoherence with unconventional decoherence proposed by gravitationally induced collapse [4,5] and by quantum gravity [33] (see the Supplemental Material [32]). For sufficiently large $\beta$, i.e., macroscopic entanglement, and small thermal dissipation we find an experimentally feasible parameter regime, in which the unconventional decoherence rates surpass the conventional ones, hence opening up the possibility for experimental tests (see below). Finally, note that the observed visibility is degraded if the mirror position is not accurately measured. A small imprecision $\delta x$ would indeed introduce an additional phase on $A$ that prevents its redisplacement to the single photon level and degrades the quality of the interference between $A$ and $B$ [27,34]. Quantitatively,

$$V \approx 1 - 3\delta \phi A^4 + o(\delta \phi A^4), \quad (4)$$

where $\delta \phi = (\delta x/x_0)(2(\delta x/x_0)^2 + 1)^{-1/2}\sqrt{2g_0 \epsilon}$. A high accuracy $\delta x \lesssim (x_0/(g_0 \epsilon \beta x_0)^2)$ is thus required to observe high visibility and to see the effect of mirror decoherence.

Witnessing optomechanical entanglement.—We can prove that the mirror is entangled with the optical modes from an entanglement witness that uses the values of $\{P_{\pm E_y}, P_{\mp E_y}\}$ and $N_{AB}$ only (see the Supplemental Material [32]). The witness is based on the following intuitive argument: since $B$ is a qubit, the only way for $M$ to be correlated to $B$ and for $B$ to be entangled with the joint system $AM$ is that $M$ is entangled with $AB$. Concretely, we can conclude about optomechanical entanglement if $N_{AB} > \sqrt{P_{+E}P_{-E} + P_{-E}P_{+E}}$. We emphasize that in contrast to the correlation measurement, the detection of entanglement $N_{AB}$ requires a measurement of the mirror position with a very high accuracy (through $V$). We are retrieving what seems to be the essence of macroentangled states: although they involve components that can easily be distinguished without microscopic resolution, one needs detectors with a very high precision to reveal their quantum nature [28,35].

Experimental feasibility.—We now address the question of the experimental feasibility in detail. First, we require
To further guarantee a high visibility of the interference between $A$ and $B$, the system needs to operate in the linear regime. For a pulsed optomechanical interaction ($\tau \ll \omega_A^{-1}$), the nonlinear response of the optomechanical system degrades the visibility of the interference pattern according to $V \to V(1-e)$ where $e = (g_0\beta)^2/(\omega_A^2/\delta v^2)^2$ [36]. This undesired effect is thus negligible if $g_0\beta \gg \omega_m$. The requirement of observing a high interference visibility also imposes the mirror position to be accurately estimated, cf. Eq. (4). It has been established in Ref. [24] that the maximum accuracy is obtained by choosing an input drive with a duration $\sim (\ln 2/\kappa)$. The achievable precision then depends on its number of photons $N_p$ via $\langle \delta x/x_0 \rangle = (\kappa/\sqrt{5g_0\sqrt{N_p}})$ and is thus high if $g_0\sqrt{N_p} > \kappa$. The primary limitation for $N_p$ is the power that can be homodyned before photodetection begins to saturate. Assuming a saturation power of 10 mW results in $N_p \sim 5 \times 10^{16}/\kappa$. To build up a proposal as simple as possible, we consider the case where a single local oscillator with a controllable amplitude is used both for implementing the displacement and for measuring the mirror position ($\tau = (\ln 2/\kappa)$). Using Eq. (4), this results in the reduced visibility $V \to V(1-e)$ where $e \sim 2 \times 10^{-35} \kappa^2 \beta^2$.

The mechanical device also needs to be prepared in its ground state. More precisely, if the mechanical oscillator is initially in a thermal state with a mean occupation $n_0$, the interference visibility is unchanged but the observed correlations decrease according to $(2/\pi)(g_0\tau\beta/\sqrt{n_{th} + 1 + g_0^2\tau^2\beta^2 + \delta x^2/(4\kappa^2)})$. High correlations can thus be observed if $\sqrt{n_{th}} \ll g_0\tau\beta$; i.e., the constraint on the initial cooling is relaxed for macroscopically distinct mechanical states. Cooling in the pulsed regime can be obtained through various schemes [24,25]. For example, Refs. [24,37] show that two subsequent pulses (identical to the pulses used for the measurement of the mechanical position) that are separated by $\pi/2\omega_m$ allow one to cool the mechanical mode to an effective thermal occupation of $n_{eff} = (1/2)(\sqrt{1 + (\kappa^4/g_0^4N_p^2} - 1)$. For $g_0\sqrt{N_p} > \kappa$, this results in $n_{eff} \ll 1$, i.e., ground state cooling.

For concreteness, we consider a mechanical mirror with resonance frequency $\omega_m = 2\pi \times 20 \times 10^3$ s$^{-1}$ ($\omega_m = 50$ s) and an effective mass $M = 60$ mg in a 0.5 cm long cavity ($g_0/\omega_m = 5 \times 10^{-3}$). We require correlations larger than 0.5 ($4g_0\tau\beta = 6$) and an error on the overall visibility of $\sim 1\%$. This imposes a cavity finesse of $\sim 80000$ ($\beta \sim 40000$, $N_p \sim 4 \times 10^8$, $\kappa \sim 2 \times 2 \times 10^8$ s$^{-1}$, $\tau \sim 60$ ns). For comparison, the highest reported finesse in an optical Perot-Fabry cavity with micromirrors is $1.5 \times 10^5$ [38].

The photons in $A$ need to be stored on the time scale of the decoherence being probed. A simple fiber loop allows one to reach delay times up to 100 $\mu$s without significant loss at telecom wavelength. Much longer delays can be obtained with such a technique if one is willing to use postselections [39].

The surrounding temperature $T$ must also be low enough so that the effect of conventional (environmentally induced) decoherence [6] is negligible on the time scale of the decoherence being probed. This requires $T \ll \hbar\omega_m Q_m/k_B (1/2(g_0\tau^2\beta^2)(1/2\pi n)$ for $n$ mechanical periods. In other words, for a base temperature of $T = 800$ mK and a mechanical quality factor of $Q_m = 10^8$, conventional decoherence operates on a time scale of $1$ $\mu$s, which is long enough to observe optomechanical entanglement. Lower temperatures and/or higher $Q_m$ are required for testing unconventional decoherence models. For example, for quantum gravity induced collapse [33], we find a time scale $\sim 415$ $\mu$s following Ref. [40], which would be testable with the proposed device with $Q_m \sim 1.5 \times 10^7$ and $T \sim 20$ mK where conventional decoherence operates on $\sim 630$ $\mu$s. Gravitationally induced decoherence [4,5] provides another example, despite the known ambiguity with respect to the mass distributions. Under the assumption where the mass is distributed over spheres corresponding to the size of atomic nuclei, we find a time scale $\sim 10$ $\mu$s following Ref. [22]. This is testable with the proposed device for $T \sim 300$ mK and $Q_m \sim 10^7$ where conventional decoherence operates on $\sim 30$ $\mu$s. Note that in addition to absolute decoherence rates, the scaling behavior with respect to mechanical parameters, e.g., the mass, provides an independent assessment of the nature of the observed decoherence (see the Supplemental Material [32]).

**Conclusion.**—We have proposed a way for creating and detecting macroscopic optomechanical entanglement that combines displaced single-photon entanglement and pulsed optomechanical interaction. Our proposal can be implemented in a wide variety of systems. The optomechanical photonic crystal cavity device introduced in Ref. [41] could exhibit correlations of 0.6 and an interference visibility of 0.95 at a temperature of a few kelvins, while more massive systems, like the one proposed before, open up a way to measure unconventional decoherence models.

We thank N. Gisin, K. Hammerer, S. Hofer, N. Timoney, and P. Treutlein for discussions. This work was supported by the Swiss NCCR QSIT, the Austrian Science Fund FWF (SFB FoQuS, P24273-N16, SFB F40-FoQuS F4012-N16), the Vienna Science and Technology Fund WWTF, the European Research Council ERC (StG QOM), and the European Commission (IP SIQS, ITN cQOM).

[29] The singlet state can equivalently be written as $(1/\sqrt{2})((01) - (10))$ and as $(1/\sqrt{2})((+ +) - (- -))$.
[34] Note that the same constraint also applies to the phase of the local oscillator used to displace A back to the single-photon level (see Ref. [27]).
[36] To obtain this result, the unitary $\rho^{\dagger}(\hat{a}^{\dagger}\hat{b}^{\dagger}\omega_{m}/3)\rho^{a^{\dagger}}a$ needs to be applied on A between the interaction with the mechanics and the redispacement.
[39] Considering that the detectors need to be opened for approximately $2\pi c^{-1}$ and assuming a detector dark count rate of about 1 Hz, delays up to 500 $\mu$s could be obtained by degrading the observed visibility by less than 1% only.