Quantum-to-Classical Crossover in Full Counting Statistics

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Abstract

The reduction of quantum scattering leads to the suppression of shot noise. In this Letter, we analyze the crossover from the quantum transport regime with universal shot noise to the classical regime where noise vanishes. By making use of the stochastic path integral approach, we find the statistics of transport and the transmission properties of a chaotic cavity as a function of a system parameter controlling the crossover. We identify three different scenarios of the crossover.

Reference


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Quantum-to-Classical Crossover in Full Counting Statistics

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The reduction of quantum scattering leads to the suppression of shot noise. In this Letter, we analyze the crossover from the quantum transport regime with universal shot noise to the classical regime where noise vanishes. By making use of the stochastic path integral approach, we find the statistics of transport and the transmission properties of a chaotic cavity as a function of a system parameter controlling the crossover. We identify three different scenarios of the crossover.

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Random transfer of charge in electrical conductors leads to time-dependent fluctuations of the current, a phenomenon called shot noise. In recent years, the shot noise has been extensively studied in mesoscopic conductors [1], small degenerate electron systems of a size comparable to the coherence length of electrons. In contrast to the classical shot noise in vacuum tubes, which was explained by Schottky already in 1918 [2], the shot noise in mesoscopic conductors originates from the quantum-mechanical scattering of electrons. Consequently, in a non-interacting mesoscopic conductor biased by the chemical potential difference \( \Delta \mu \), the average current \( \langle I \rangle = \Delta \mu \sum_n T_n \) [3] (setting electron charge and the Planck constant \( e = h = 1 \) ), the noise power \( S = \langle \langle I^2 \rangle \rangle = \Delta \mu \sum_n T_n (1 - T_n) \) at zero temperature [1], and, in general, the higher cumulants of current \( \langle I^n \rangle \) [4] are determined by the transmission matrix \( T_n \), namely, by the eigenvalues \( T_n, n = 1, \ldots, N \), of \( \hat{T} \). The current cumulants \( \langle I^n \rangle \) can be expressed via the cumulant generating function (CGF) \( C_m = \partial^m \mathcal{H}(\lambda) / \partial \lambda^m |_{\lambda = 0} \). In the semiclassical limit, \( N \gg 1 \), the CGF is given by [5]

\[
\mathcal{H}(\lambda) = \int_0^1 dT \rho(T) \ln[1 + T(e^\lambda - 1)],
\]

where \( \rho(T) = N^{-1} \sum_n \delta(T - T_n) \) is the transmission eigenvalue distribution. Equation (1) generalizes the binomial statistics, and together with the inverse formula (12), provides a connection between the full counting statistics (FCS) and the scattering properties of a mesoscopic system to leading order in \( 1/N \).

The quantum origin of shot noise in mesoscopic conductors implies that, regardless of the character of disorder, current can flow without noise if quantum scattering is suppressed [6]. Therefore, in the classical limit the noise should vanish even in a chaotic system, such as a mesoscopic cavity, where the transport in the quantum regime is universally described by random matrix theory (RMT) [7]. It has been predicted [8] that in a mesoscopic cavity with a long-range disorder the noise power shows an exponential crossover \( S = S_{\text{RMT}} \exp(-\tau_e/\tau_D) \) as a function of the ratio of the Ehrenfest (diffraction) time \( \tau_e \) to the average dwell time of electrons \( \tau_D \). Reference [9] suggested that this crossover results from a sharp cutoff introduced by the Ehrenfest time in the exponential distribution \( \mathcal{P}(t) = \tau_D^{-1} \exp(-t/\tau_D) \) of the dwell times of classical trajectories. Recent numerical analysis [10] has demonstrated that the cutoff leads to a complete separation of the cavity’s phase space into the quantum universal part of relative volume \( v = \exp(-\tau_e/\tau_D) \) and the classical noiseless part of the volume \( 1 - v \). As a result, the eigenvalue distribution splits into two terms, \( \rho = v \rho_{\text{RMT}} + (1 - v) \rho_\infty \), where

\[
\rho_{\text{RMT}}(T) = \frac{1}{\pi \sqrt{T(1 - T)}}
\]

is the universal RMT result, and \( \rho_\infty(T) = [\delta(T) + \delta(1 - T)]/2 \) is the classical distribution. The onset of the quantum-to-classical crossover has been observed in the experiment on the shot noise of a mesoscopic cavity [11]. Since then interest in the physics of the crossover has grown dramatically and brought new results in the context of the shot noise suppression [9,12], the proximity effect in Andreev billiards [13], mesoscopic conductance fluctuations [10,14], and many other phenomena.

In this Letter, we demonstrate that the presence of the homogeneous short-range disorder in a chaotic cavity dramatically changes the quantum-to-classical crossover. It leads to the large-angle quantum scattering of electrons and results in the relaxation of the deterministic occupation function \( f_B \), which takes values 0 and 1, to its fully quantum isotropic value \( f_C < 1 \). The relaxation with the constant rate \( \tau_Q^{-1} \), where \( \tau_Q \) is the quantum scattering time, does not introduce a sharp cut-off in the dwell time distribution. As a result, all cumulants have a power-law dependence on the crossover parameter \( \gamma = \tau_Q/\tau_D \). In particular, in contrast to the case of the long-range disorder discussed above, the noise power shows the power-law crossover [15]

\[
S = S_{\text{RMT}}/(1 + \gamma), \quad \gamma = \tau_Q/\tau_D.
\]

The distribution \( \rho(T) \) gradually evolves as a function of the
considered to be noise sources which are correlated solely
modes where the classical chaotic dynamics of electrons is taken
into account by the “gradient” term and the quantum
scattering is described by the second, the collision integral
in the scattering time approximation. In the classical limit
\( \gamma = \gamma_0 / \tau_D \gg 1 \) the second term can be neglected, and
the solution of Eq. (6) takes one of the boundary values
\( f_{L,R} = 0, 1 \) giving 
\( \mathcal{H}_i = (f_C - f_i) \lambda_i \). Then the minimization of
the function (5) leads to \( \mathcal{H} = \lambda / 2 \) giving the average
current \( \langle I \rangle = \Delta \mu N / 2 \) and no noise. In the quantum limit
\( \tau_0 / \tau_D \ll 1 \) the second term in Eq. (6) dominates; there-
fore, in Eq. (4) \( f_p \) may be replaced with \( f_c \). Minimizing \( \mathcal{H} \)
given by (5), we obtain the result [17]

\[
\mathcal{H} = 2 \ln(1 + e^{\lambda/2}) - 2 \ln 2, \quad \gamma = 0, \tag{7}
\]

which agrees with the RMT result [20].

In order to obtain the coarse-grained value of the loga-
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a consequence of the zero temperature limit and of the fact that the cavity is symmetric. The first three nonvanishing cumulants are $C_1 = 1/2$, which determines the mean current, $C_2 = 1/[8(\gamma + 1)]$, which determines the noise power and agrees with the result (3), and $C_4 = (\gamma - 1)/[32(\gamma + 1)(\gamma + 2)]$.

The logarithm of the distribution of transmitted charge in the stationary phase approximation is given by
\[
\ln P(Q) = Q_0 \min_x (\mathcal{H}(\lambda - Q\lambda) - \frac{1}{2} Q^2)
\]
where $Q = Q/Q_0$ is the transmitted charge normalized to its maximum value $Q_0 = \Delta \mu N t$. The result of the evaluation using Eq. (11) is shown in Fig. 1 for different values of $\gamma$. In the quantum limit, we use Eq. (7) to obtain
\[
\ln P(Q) = -2 \ln 2 - 2 [\ln(Q + (1 - Q)(1 - Q))] - 1
\]
which vanishes at the average value of charge $Q = 1/2$, giving the correct normalization of $P(Q)$. In the classical limit $\gamma \gg 1$ the noise is Gaussian, $\ln P(Q)/Q_0 = -2 \ln 2 - 1$, for $|Q - 1/2| \ll 1$. Surprisingly, the extreme value distribution in the range $\gamma^{-1} \leq |Q - 1/2| \leq 1/2$ shows a weak $\gamma$ dependence: $\ln P(Q)/Q_0 = -2 |Q - 1/2| \ln(\gamma Q - 1/2)$; see Fig. 1. This remarkable behavior may be attributed to the formation of almost open (closed) quantum channels, the situation specific to the short-range disorder considered here. The number of such channels is nearly independent of $\gamma$ and close to the total number of modes $N$ (see the discussion below).

**Distribution of transmission eigenvalues.**—Having found the CGF, we now invert Eq. (1) in order to obtain the distribution of transmission eigenvalues $\rho(T)$. We note that $\mathcal{H}$ as a function of the variable $\Lambda \equiv e^\lambda - 1$ has a branch cut in the complex plane at $-\infty < \Lambda < -1$. Analytically continuing from $\Lambda > 1$ to the branch cut [22], we obtain
\[
\rho(T) = \frac{1}{\pi T^2} \text{Im}(\partial \mathcal{H}/\partial \lambda)|_{\lambda \to -1/T - i 0}).
\]

Using this relation together with Eq. (11), we arrive at the following result for $\rho(T)$ in the crossover regime:
\[
\rho(T) = \frac{\gamma}{\pi \sqrt{T(1 - T)}} \int_{-1}^{1} du (1 - u^2)|u|^{2\gamma - 1} (1 + u^2)^{-2 - 4\gamma T u}.
\]

The distribution is symmetric with respect to $T \to 1 - T$ and properly normalized: one can verify that $\int_0^1 dT \rho(T) = 1$. In the quantum limit $\gamma \to 0$, Eq. (13) leads to the RMT result (2). In the classical limit we obtain the asymptotic formula
\[
\rho(T)|_{\gamma \to \infty} = \frac{1}{8 \pi \gamma [T(1 - T)]^{3/2}} + O(\gamma^{-2})
\]
which is valid away from the points $T = 0$ and $T = 1$ where $\rho(T)$ in (14) is divergent. This divergence, being cut at $T, 1 - T \sim \gamma^{-2}$ in Eq. (13), gives the main contribution to the normalization of $\rho(T)$, as well as to the average current, and determines the extreme value statistics discussed above. However, it is integrable for high cumulants of current.

The distribution $\rho(T)$ for several values of $\gamma$ is illustrated in Fig. 2. The quantum-to-classical crossover appears as a gradual transition from the RMT distribution at $\gamma = 0$ to two $\delta$ functions at $T = 0$ and $T = 1$. Following Ref. [10] we plot the integrated distribution $I(T) = \int_0^T \rho(T')dT'$, which turns out to be a smooth function of $T$. This is in contrast to the case of a long-range disorder, where $I(T)$ shows an offset at $T = 0$ [10], indicating the separation of phase space into a classical and a quantum part. Our result implies that such a separation does not occur in the case of a homogeneous short-range disorder.

**Inhomogeneous disorder.**—So far we have considered a relatively weak homogeneous disorder with the strength characterized by the scattering time $\tau_D$. Another experimentally relevant situation is the case of a strong inhomogeneous short-range disorder. For instance, a few strong impurities, sharp openings to the leads or irregularities at

FIG. 1 (color online). The logarithm of the distribution of transmitted charge $Q$ plotted versus charge normalized to its maximum value $Q_0 = \Delta \mu N t$. It is symmetric around the average value $Q/Q_0 = 0.5$. Note a relatively weak dependence of the extreme value statistics on the crossover parameter $\gamma = \tau_D/\tau_Q$. The dashed line is the Gaussian distribution shown for a comparison.

FIG. 2 (color online). The crossover of the distribution of transmission eigenvalues $\rho(T)$ between the quantum ($\gamma = 0$) and classical ($\gamma \to \infty$) regimes. $\rho(T)$ is symmetric around $T = 0.5$. Inset: Integrated probability distribution $I(T) = \int_0^T \rho(T')dT'$ for the same set of parameters.

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the boundary of the cavity belong to this class of disorder. The inhomogeneity implies that some trajectories do not enter the disordered region and remain classical with f_p = 0, 1. The fact that the disorder is strong means that all trajectories entering the disordered region acquire the isotropic occupation f_C. For the coarse-grained occupation function \langle f_p \rangle, this leads to the relaxation with the collision rate \tau_{imp}. This process is described by Eq. (6) with \tau_Q replaced by \tau_{imp}. In the present case the solution of this equation determines the relative volume of the quantum phase space v = 1/(1 + \gamma), where \gamma = \tau_{imp}/\tau_Q is the new crossover parameter. Therefore we conclude that inhomogeneous strong disorder leads to the complete separation of the phase space on the classical and quantum parts with the consequence that \rho = v\rho_Q + (1 - v)\rho_\gamma, where \rho_Q is a quantum (nonuniversal) distribution. However, in contrast to the case of long-range disorder, the FCS is a power-law function of the crossover parameter: C_\gamma \sim 1/\gamma.

Asymmetric cavity.—From the above analysis it follows that the noise power has the same dependence (3) on the crossover parameter \gamma for both types of a short-range disorder. Moreover, since the odd cumulants of current determine the extreme value statistics. In the case of an asymmetric cavity vanish at zero temperature, the disorder. Moreover, since the odd cumulants of current determine the extreme value statistics. In the case of an asymmetric cavity vanish at zero temperature, the disorder.

In conclusion, we have analyzed the FCS and transmission properties of a mesoscopic cavity at the crossover from the universal quantum to the classical transport regimes. We have found new different scenarios of the crossover in a cavity with short-range disorder. In case of homogeneous disorder, the crossover occurs via the formation of almost open (closed) quantum channels, which determine the extreme value statistics. In the case of an inhomogeneous strong disorder, the phase space of the cavity splits into two parts: classical noiseless channels and quantum channels. In both cases the FCS has a power-law dependence on the crossover parameter.

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[16] Here we assume \tau_e \gg \tau_\gamma. The combined effect of short-range and long-range disorders will be addressed elsewhere.