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Abstract

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Reference


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Multiparticle Interference, Greenberger-Horne-Zeilinger Entanglement, and Full Counting Statistics

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We investigate the quantum transport in a generalized N-particle Hanbury Brown–Twiss setup enclosing magnetic flux, and demonstrate that the Nth-order cumulant of current cross correlations exhibits Aharonov-Bohm oscillations, while there is no such oscillation in all the lower-order cumulants. The multiparticle interference results from the orbital Greenberger-Horne-Zeilinger entanglement of N indistinguishable particles. For sufficiently strong Aharonov-Bohm oscillations the generalized Bell inequalities may be violated, proving the N-particle quantum nonlocality.

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The Aharonov-Bohm (AB) effect [1], being a most remarkable manifestation of quantum coherence, is at the heart of quantum mechanics. It is essentially a topological effect, because it requires a multiple-connected physical system, e.g., a quantum ring, and consists in a periodic variation of physical observables as a function of the magnetic field threading the loop. It is also a nonlocal effect, since no local physical observable is sensitive to the field. Originally introduced for a single particle [1], it can be generalized as a two-particle AB effect in the average current [2], or in the noise power [3], if only two particles are able to enclose the loop.

The last effect [3], being implemented in the Hanbury Brown–Twiss (HBT) geometry [4], shows the two-particle character in a most dramatic way [5], because single-particle observables, such as the average current, do not contain AB oscillations. In contrast to the single-particle AB effect, which may have a classical analog [6], the two-particle AB effect is essentially quantum, because it originates from quantum indistinguishability of particles. Moreover, it is strongly related to the orbital entanglement in HBT setup [5], and to the quantum nonlocality as expressed via the violation of Bell inequalities [7,8].

In this Letter, we investigate the generalized HBT setup, exemplified in Fig. 1, and introduce the N-particle AB effect for N ≳ 3. We start with analyzing the full counting statistics (FCS) [9,10] and demonstrating that in the N-particle HBT setup the Nth-order cumulant of current cross correlations may exhibit AB oscillations, while there is no such oscillation in all the lower-order cumulants. Next we show that the N-particle AB effect originates from the orbital N-particle entanglement. Namely, we prove that a many-particle state injected into the HBT setup contains the Greenberger-Horne-Zeilinger-type (GHZ) state [6] [11], which via the postselection [12] contributes to the particle transport. We remark here that the concept of the FCS has been used in earlier works in the context of a two- [13] and three-particle [14] entanglement. We further demonstrate that the Svetlichny inequality [15] may be formulated in terms of the joint detection probability and then violated, if the visibility of the AB oscillations exceeds the value 1/√2. This inequality, being most restrictive in the whole family of generalized N-particle Bell inequalities [15–18], discriminates quantum mechanics and all hybrid local-nonlocal theories [19], proving the N-particle quantum nonlocality in the generalized HBT setup. We note that in contrast to the two-particle case, the overall sign of the Nth-order cumulant cannot be uniquely associated with statistics of particles in the cases of N ≥ 3.

FIG. 1. Left: Hanbury Brown–Twiss (HBT) interferometer with N = 3 independent electron sources (reservoirs 2, 6, 10), 2N beam splitters Mᵢ, 2N chiral current paths, each connecting two beam splitters, and 2N detector reservoirs, 3, 4, 7, 8, 11, and 12. Arrows indicate electron trajectories. In the setup, any single electron cannot enclose magnetic flux Φ, while N electrons emitted from N sources can do so. For example, electrons incident from the reservoir 2 move towards the splitters M₆ or M₅ and disappear in the detectors 3, 4, 11, or 12. This HBT setup can be easily generalized to the case of arbitrary N. Right: in the HBT interferometer N-electron states incident from the sources are decomposed into 2ᴺ states, shown for N = 3. The states (a) and (b) result in Aharonov-Bohm (AB) oscillations in the Nth-order cumulant of the current cross correlations at detectors. These two states together form a Greenberger-Horne-Zeilinger-type (GHZ) state of N pseudospins. All the others, (c)–(h), do not contribute to the AB effect.
Generalized HBT interferometer.—We consider the coherent electron transport in the mesoscopic multiterminal conductor consisting of $4N$ electron reservoirs, $i = 1, 2, \ldots, 4N$ (enumerated clockwise), $2N$ beam splitters $M_i$, $i = 1, 2, \ldots, 2N$, and $2N$ chiral current paths enclosing magnetic flux $\Phi$ (see Fig. 1). The $(2i - 1)$th and the $2i$th reservoirs couple to the splitter $M_i$, at which electrons can be transmitted with probability $T_i$ or reflected with probability $R_i = 1 - T_i$. Among $4N$ reservoirs, $N$ reservoirs, $2, 6, \ldots, 4N - 2$, behave as independent electron sources, and $N$ pairs of reservoirs with indexes $\alpha_1 = \{3, 4\}$, $\alpha_2 = \{7, 8\}$, $\alpha_N = \{4N - 1, 4N\}$, act as detectors. All sources are biased with voltage $eV$, while the rest of $3N$ reservoirs are grounded. Neighboring beam splitters $M_i$ and $M_{i+1}$ are connected via a one-dimensional spinless current path, where phase difference $\phi_i$ is accumulated.

The total scattering matrix of the generalized HBT setup consists of two $2N$-dimensional unitary chiral blocks $\hat{U}$ and $\hat{V}$. The block $\hat{U}$ is determined as follows: the electron propagation from source $4i + 2$ to detectors $4i - 1, 4i, 4i + 3$, or $4i + 4$ gives the matrix elements $\exp(-i\phi_{2i}) \sqrt{T_{2i+1}} T_{2i}^{-1/2} \exp(-i\phi_{2i}) \sqrt{R_{2i+1}} R_{2i}^{-1/2}$, respectively. The block $\hat{V}$ describes noiseless outgoing currents from detector reservoirs. It is not relevant for the following discussion; therefore, the requirement of the coherence in this sector may be relaxed. All the other matrix elements are zero, thus no single electron can enclose the flux $\Phi$ in the generalized HBT setup. However, $N$ electrons can do so, and this leads to the $N$-particle interference in the FCS.

$N$-particle Aharonov-Bohm effect.—In the long measurement time limit $t \gg \tau_C$, where $\tau_C = 2\pi/|eV|$ is the correlation time, the electron transport is a Markovian random process. The characteristic property of such a process is that the irreducible correlators (cumulants) of the number of electrons arriving at detectors are proportional to the number of transmission attempts, $t/\tau_C$. Therefore, we normalize the cumulants to $t/\tau_C$, so that the cumulant generating function of the FCS at the detector reservoirs takes the form [9]:

$$S(\chi) = (\tau_C/2\pi) \int dE Tr \ln[1 - \hat{f} T_{\hat{U}} \hat{U} \hat{A} \hat{A}^\dagger].$$ (1)

Here $\hat{f}$ is the diagonal matrix with elements $\delta_{ij} f_j(E)$ being Fermi-Dirac occupations: $f_j(E) = f_n(E - eV)$ at the sources, and $f_j(E) = f_F(E)$ in the rest of reservoirs. The matrix $\hat{A}$ is diagonal with elements $\delta_{ij} \exp(i\chi_i)$, $\chi_i$ is the set of counting variables in the detector reservoirs. The cumulants may be obtained by evaluating the derivatives of $S(\chi)$ and setting $\chi = 0$.

The generating function (1) is simplified by introducing matrices $(\hat{A}_\alpha)_{nm} = f_{\alpha_n} u_{\alpha_n}^* u_{\alpha_m} [20]:$

$$S(\chi) = \frac{\tau_C}{2\pi} \int dE Tr \ln[1 + \sum_{\alpha} (e^{i\chi_{\alpha}} - 1) \hat{A}_{\alpha}].$$ (2)

where the sum runs over all the detectors $\alpha_i$ and $n, m = 1, 2, 5, 6, \ldots, 4N - 3, 4N - 2$. In order to generalize the two-particle interference to the cases of $N \geq 3$, we consider the lowest-order cross-correlation functions with all detectors being different. It turns out that the number of such cross correlators is limited, because between all the possible lowest-order products of the form $\prod_{\alpha} \hat{A}_{\alpha}$, where all $\alpha$’s are different, only few products give nonvanishing traces: $\Tr \hat{A}_{\alpha_1} \hat{A}_{\alpha_3} \hat{A}_{\alpha_4} \hat{A}_{\alpha_5}$ for all $i$, and $\Tr \hat{A}_{\alpha_1} \hat{A}_{\alpha_2} \hat{A}_{\alpha_3} \hat{A}_{\alpha_4}$ and $\Tr \hat{A}_{\alpha_1} \hat{A}_{\alpha_2} \hat{A}_{\alpha_3} \hat{A}_{\alpha_4} \hat{A}_{\alpha_5} \hat{A}_{\alpha_6}$, where $\alpha_i$ are the normalized zero-frequency noise power,

$$Q_{\alpha_i} = P_{\alpha_i} R_{2i-1} + (1 - P_{\alpha_i}) T_{2i+1}$$ (3a)

$$Q_{\alpha_i, \alpha_j} = -P_{\alpha_i} P_{\alpha_j} (1 - P_{\alpha_i} P_{\alpha_j}) T_{2i+1} R_{2j+1},$$ (3b)

where $P_{\alpha_i} = R_{2i} (P_{\alpha_i} = T_{2i})$ for $\alpha_i$ being odd (even).

The next nonzero cumulant is the $N$th-order cross-correlation function

$$Q_{\alpha_1, \alpha_2, \ldots, \alpha_N} = 2 \sum_{\alpha_1, \alpha_2, \ldots, \alpha_N} \Gamma_{BS} \cos(\phi_{\alpha_1}),$$ (4)

where the sign depends on the choice of the detectors, $\Gamma_{BS} = \sum_{\alpha} \phi_{\alpha}$ characterizes the transmission of beam splitters, and the total phase accumulated around the HBT loop is $\phi_{\alpha} = 2\pi \Phi / \Phi_0 + \sum_{i=1}^{2N} \phi_i$, with $\Phi_0 = 2\pi e / h$ being the flux quantum. Below we will use a notation $\{\alpha\} = \{\alpha_1, \alpha_2, \ldots, \alpha_N\}$ for an arbitrary set of $N$ detectors, so that $Q_{\alpha_1, \alpha_2, \ldots, \alpha_N} = Q_{\{\alpha\}}$.

We note several important points. First, the result (4) holds under the usual condition [4,5] of the “cancellation of paths,” which prevents dephasing due to the energy averaging: the difference of the total lengths of the clockwise [see Fig. 1(a)] and counterclockwise [Fig. 1(b)] paths should not exceed $\tau_C \nu_F$, where $\nu_F$ is Fermi velocity. Second, every cumulant in Eqs. (3a), (3b), and (4), has the prefactor $(\tau_C/2\pi) \int dE f_{\alpha}(f - f_0)^k$, where $k = 1, 2, N$. We consider the zero temperature limit and set this prefactor to 1. Next, the sign function in the equation (4) has two contributions: the term $(-1)^{\sum_{i \neq j} \phi_j}$ comes from the detector phases, while the term $(-1)^{N-1}$ originates from the fermionic exchange effect. Thus, in contrast to the case of the second cumulant (3b), the overall sign of high-order cumulants is not universal. Finally, only the $N$th-order cross correlator shows oscillations as a function of the magnetic flux threading the HBT loop. We stress that these oscillations appear in the FCS of electrons injected from $N$ uncorrelated sources, and thus they can be regarded as an $N$-particle AB effect. Below we connect this effect with $N$-particle GHZ entanglement.

GHZ entanglement.—To clarify the origin of the AB oscillations in the cumulant $Q_{\omega_1}$ in Eq. (4), we analyze the multiparticle state injected from the sources,
where \( \gamma_a = -\frac{1}{N} \sum_{i=1}^{N} \langle \psi_N | a_i | 0 \rangle \langle a_i | \psi_N \rangle \) and \( \gamma_b = -\frac{1}{N} \sum_{i=1}^{N} \langle \psi_N | b_i | 0 \rangle \langle b_i | \psi_N \rangle \). The operator \( C_0 = \prod_{i=1}^{N} a_i \) creates the \( N \)-particle state as

\[
\langle \psi_N | = -\frac{1}{N} \sum_{i=1}^{N} \langle \psi_N | a_i | 0 \rangle \langle a_i | \psi_N \rangle \text{,}
\]

where \( p^2 = 1 - q^2 = |\gamma_a|^2/|\gamma_b|^2 \). The expectation value of the \( \theta \)-th order correlation function can now be found as

\[
E_N = g(p, q) \prod_{i=1}^{N} \cos \theta_i + 2pq \cos \theta \prod_{i=1}^{N} \sin \theta_i \text{,}
\]

where \( g(p, q) = p^2 + (-1)^N q^2 \) and \( \theta = \theta_p - \theta_q + \sum_{i=1}^{N} \theta_i \). Turning now to the generalized HBT setup, we note that the detector beam splitters \( M_{2i} \) implement the orbital pseudospin rotation, while the reservoirs \( 4i \) and \( 4i + 1 \) detect pseudospins in \( z \) direction. The pseudospin correlation function can now be found as

\[
E_N = \sum_{\alpha} (-1)^{\sum_{i=1}^{N} \alpha_i} P_{\alpha} \text{,}
\]

where \( P_{\alpha} \) are the current operators in reservoirs \( \alpha_i \). The entangled state then moves towards the detectors. The entanglement of the \( N \)-particle state is given by

\[
(\epsilon/2) \int dE' c_{\alpha}^\dagger(E') c_{\alpha} (E') \text{,}
\]

where \( \epsilon \) is the energy of the state and \( P_{\alpha} \) are the current operators in reservoirs \( \alpha_i \).
inequality \cite{5}, \( \langle M_2 \rangle = (1/2)[E(\bar{n}_1, \bar{n}_2) + E(\bar{n}'_1, \bar{n}_2) + E(\bar{n}_1, \bar{n}'_2) - E(\bar{n}'_1, \bar{n}'_2)] \leq 1 \). The violation of more restrictive Svetlichny inequalities \( \langle S_N \rangle \leq 2^{(N-2)/2} \) \cite{15}, where \( S_N = (1/\sqrt{2})(M_N + M'_N) \) for \( N \) being odd and \( S_N = M_N \), otherwise, rules out all hybrid local-nonlocal models \cite{19}.

Here we focus on the sufficient condition for the violation of generalized Bell inequalities, which can be found as follows: after fixing \( T_{2j} = R_{2j} = 1/2 \) for all detectors the maximum values \( \langle M_N \rangle_{\text{max}} = 2pq2^{(N-1)/2} \) \cite{22} and \( \langle S_N \rangle_{\text{max}} = 2pq2^{(N-1)/2} \) \cite{19} can be reached for a particular choices of detector phases \( \phi_j \). This will violate MABK inequalities if \( 2pq > 1/2^{(N-1)/2} \), and Svetlichny inequalities if \( 2pq > 1/\sqrt{2} \). We note that according to Eqs. (4) and (7) the value \( 2pq \) is nothing but the visibility \( V_{AB} = (\mathcal{P}_{\text{max}} - \mathcal{P}_{\text{min}}) / (\mathcal{P}_{\text{max}} + \mathcal{P}_{\text{min}}) \) of AB oscillations in the JDP for a fixed set \{a\} of detectors. Thus, we come to the important practical conclusion that the observation of sufficiently strong AB oscillations in \( \mathcal{P}_a \),

\[
V_{AB} > 1/\sqrt{2},
\]  

will guarantee the possibility of the violation of Svetlichny inequalities \cite{23}. This is also true in the case of a weak dephasing, since in our HBT setup, where single pseudo-spin flips are not allowed \cite{5}, its only effect is to suppress the second term in the correlator (8). Summarizing this discussion we conclude that the \( N \)-particle AB effect may be viewed as a manifestation of genuine quantum nonlocality in the generalized HBT setup.

Feasibility of experimental realization.—The mesoscopic implementation of two-particle HBT setup proposed in Ref. \cite{5} may be well utilized in the cases of \( N \geq 3 \). It relies on the quantum Hall edge states as chiral channels, and quantum point contacts as beam splitters, and generalizes the electronic Mach-Zehnder interferometer, which has been recently experimentally realized \cite{24}. All limitations not specific to \( N \geq 3 \) can be found in \cite{5}.

Recent experiments \cite{25} revealed a number of specific difficulties in measuring FCS. First of all, the detection of current fluctuations on long time scale \( t \gg \tau_C = 2\pi/|eV| \) reduces the signal-to-noise ratio for the \( N \)th-order cumulants by the factor \( (\tau_C/t)^{N/2-1} \), the consequence of the central limit theorem. This may dramatically increase the total measurement time for high-order cumulants. Second, experimentally measurable high-order cumulants contain nonuniversal low-order corrections from electrical circuit, which makes it difficult to extract intrinsic noise. We believe, however, that all these difficulties should not be that severe in our case, because low-order cumulants do not contain AB oscillations and appear merely as a background contribution.

Finally, in contrast to the two-particle case in Ref. \cite{5}, for the demonstration of the quantum nonlocality in the generalized HBT setup the measurement on the short time scale \( t < \tau_C \) is preferable. This is because the nonlocality condition (10) may require a more accurate determination of the visibility \( V_{AB} \) via measuring the JDP. Such high-frequency “quantum noise” detection techniques have recently become available \cite{26}.

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