Abstract
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Reference

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Shot-Noise Thermometry of the Quantum Hall Edge States

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We use the nonequilibrium bosonization technique to investigate the effects of the Coulomb interaction on quantum Hall edge states at a filling factor ν = 2, partitioned by a quantum point contact (QPC). We find that, due to the integrability of charge dynamics, edge states evolve to a nonequilibrium stationary state with a number of specific features. In particular, the noise temperature Θ of a weak backscattering current between edge channels is linear in voltage bias applied at the QPC, independently of the interaction strength. In addition, it is a nonanalytical function of the QPC transparency T and scales as \( \Theta \approx T \ln(1/T) \) at \( T \ll 1 \). Our predictions are confirmed by exact numerical calculations.

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Rapid experimental progress in the field of the electron transport in one-dimensional systems has unveiled new exciting phenomena inherent in strong, nonperturbative interactions characteristic of such systems. The notable examples are the recent experiments on the energy relaxation [1] and on the nonequilibrium dephasing of quantum Hall (QH) edge states [2,3]. These chiral electron states may be viewed as quantum analogs of classical skipping orbits arising at the edge of a two-dimensional electron system exposed to a perpendicular magnetic field. The aforementioned experiments utilize quantum point contacts (QPCs) to bring edge states of opposite chirality close to each other in order to mix them, thereby inducing electron backscattering. By applying a voltage bias between these edge states, one may create a nonequilibrium state with the electron distribution function in the form of a “double step” [1] (see the upper panel of Fig. 1).

The double-step distribution is characteristic of the effectively free-fermion behavior of electrons in metals [4]. Weak interactions lead to the equilibration of electrons in the long-time limit. At the QH edge, however, this distribution may evolve in a nontrivial way [5] and, in the weak injection regime, through several intermediate asymptotics [6], before reaching the equilibrium state. At the origin of this behavior are the nonperturbative interaction effects: For the Landau level filling factor ν > 1, when several copropagating channels coexist at the edge, the strong Coulomb interaction leads to the formation of collective excitations called edge magnetoplasmons [7] (see the lower panel of Fig. 1). Propagating with different velocities, these excitations strongly redistribute electrons. We have shown earlier [8] that this process is also responsible for the nonmonotonic dephasing observed in the resent experiments [3].

Instead of determining directly the electron distribution function, as in Ref. [1], one may investigate the effects of interactions in a nonequilibrium state by measuring the effective noise temperature of a system [9]. One way of doing this in a QH system [10] is by attaching a cold Ohmic contact to the copropagating edge channel, via the second QPC, as shown in Fig. 2, and measuring the zero-frequency noise power \( S_{bs} \) of the backscattering current \( j_{bs} \):

\[
S_{bs} = \int dt \langle j_{bs}(t) j_{bs}(0) \rangle. \tag{1}
\]

The important property of this measurement scheme is that, in the absence of the interaction between the channels, one should not expect any influence of the electron injection at the first, source, QPC on the noise at the second, detector, QPC. Therefore, by measuring \( S_{bs} \) as a function of the voltage bias \( \Delta \mu \), of the transparency \( T \) of the source QPC, and of the distance \( D \) between the QPCs, one may investigate interaction effects and the evolution of a nonequilibrium state initially prepared at the first QPC. In this Letter, we demonstrate that the strong interaction and the integrability of the charge dynamics at the QH edge lead to the formation of a nonequilibrium stationary state, which manifests itself in the singular, nonanalytical behavior of the effective noise temperature.

**Effective noise temperature.**—In the regime of weak tunneling at the second detector QPC, one can write [9]

\[
S_{bs} = G_D \int d\epsilon [f(\epsilon)[1 - f_D(\epsilon)] + f_D(\epsilon)[1 - f(\epsilon)]], \tag{2}
\]

where \( G_D \) is the conductance of the QPC, \( f(\epsilon) \) is the electron distribution function in the inner channel, and \( f_D(\epsilon) \) is the equilibrium distribution in the detector’s Ohmic contact. Assuming the Fermi distribution \( f(\epsilon) = f_F(\epsilon - \epsilon_F) \) with the temperature \( \Theta_{eq} \) in the inner edge channel, and with zero temperature at the detector’s Ohmic contact, \( f_D(\epsilon) = \Theta(\epsilon_F - \epsilon) \), one immediately finds that \( S_{bs} = (2 \ln 2) G_D \Theta_{eq} \). Therefore, away from equilibrium, it is natural to define the effective noise temperature \( \Theta \) via the expression
then the distribution function of the outgoing electrons is the transparency \( T \). If the transparency \( T \) of the QPC is independent of the energy, then the distribution function of the outgoing electrons is \( f(e) = (1 - T) f_1(e) + T f_2(e) \). Bottom: Schematic illustration of the strong Coulomb interaction effect at the QH edge at a filling factor of \( \nu = 2 \). The electron wave packet of the charge \( e \), created in the outer edge channel (lower black line), decays into two eigenmodes of the edge Hamiltonian, the charged and dipole modes, which propagate with different speeds and carry the charge \( e/2 \) in the outer channel [19]. A similar situation arises when an electron is injected in the inner channel (upper blue line).

\[
S_{bs} = (2 \ln 2) G_B \Theta. \tag{3}
\]

On the other hand, since the inner and the outer edge channels are electrically isolated from each other, there is no average current contribution from the first QPC to the inner channel, which may be expressed as \( \int d\epsilon f(e) - \theta(\epsilon_F - \epsilon) \) = 0. Combining this identity with the expression (2), one obtains the following simple expression for the effective noise temperature:

\[
\Theta = \frac{1}{(\ln 2)} \int_{\epsilon_F}^{\infty} d\epsilon f(e). \tag{4}
\]

Facing strong interactions that cannot be accounted for perturbatively, one may choose to treat tunneling at the first QPC perturbatively with respect to its small transparency \( T \). Recently, using this method, Ref. [11] has found that the noise temperature \( \Theta \) is linear in \( T \), while nonperturbative interactions manifest themselves in the nontrivial power-law dependence of \( \Theta \) on the voltage bias \( \Delta \mu \). However, it turns out that, far from the injecting QPC, where a non-equilibrium stationary state arises, the perturbation theory fails to correctly describe the behavior of \( \Theta \) at small \( T \). Very roughly, this happens because the weak partitioning noise at the first QPC generates a correction to the distribution function of the form \( f(e) \propto T \Delta \mu / (\epsilon - \epsilon_F) \) [6]; therefore, the integral in Eq. (4) has a logarithmic divergence. At the upper limit, this integral is cut at \( \epsilon - \epsilon_F \sim \Delta \mu \), since this is the maximum energy provided by the source. At the lower limit, the integral has to be cut at \( \epsilon - \epsilon_F \sim T \Delta \mu \), due to broadening of the distribution function induced by the noise. This leads to the behavior \( \Theta \propto T \ln(1/T) \Delta \mu \) at \( T \ll 1 \); i.e., the noise temperature is singular in \( T \) and linear in \( \Delta \mu \), contrary to the prediction of Ref. [11]. In the rest of the Letter, we demonstrate this fact rigorously by resumming weak tunneling using the non-equilibrium bosonization technique [12] and investigate various physical regimes in detail.

Model and theoretical method.---In an experiment, the applied voltage bias \( \Delta \mu \) is typically much smaller than the Fermi energy \( \epsilon_F \). Thus, it is appropriate to use the low-energy effective theory [13] describing the edge states at a filling factor of \( \nu = 2 \) as collective fluctuations of the charge density \( p_{\alpha}(x) \), where \( \alpha = 1, 2 \) enumerates channels at the QH edge and \( x = L, M, U \) denotes the lower, middle, and upper edges (see Fig. 2). The charge density fields are expressed in terms of chiral boson fields, \( \phi_{\alpha}(x) \), satisfying the commutation relations

\[
[\phi_{\alpha}(x), \phi_{\beta}(y)] = i \pi \delta_{\alpha \beta} \delta(x-y), \tag{5}
\]

namely, \( \rho_{\alpha\beta}(x) = (1/2\pi) \partial_x \phi_{\alpha\beta}(x) \). The total Hamiltonian of the system, \( \mathcal{H} = \mathcal{H}_0 + (A + A^I + \text{H.c.}) \), contains the term describing the edge states

\[
\mathcal{H}_0 = \frac{1}{8 \pi^2} \sum_{s, \alpha, \beta} \int dx dy V_{\alpha\beta}(x-y) \partial_x \phi_{\alpha}(x) \partial_y \phi_{\beta}(y), \tag{6}
\]

where the kernel, \( V_{\alpha\beta}(x-y) = 2 \pi \nu_F \delta_{\alpha \beta} \delta(x-y) + U_{\alpha\beta}(x-y) \), includes the free-fermion contribution with the Fermi velocity \( \nu_F \) and the Coulomb interaction potential \( U_{\alpha\beta} \). Vertex operators

\[
A = te^{i \phi_{L1}(0)} e^{-i \phi_{L1}(0)}, \quad A^I = t' e^{i \phi_{M1}(D)} e^{-i \phi_{M1}(D)} \tag{7}
\]

describe electron tunneling between the edge channels at the QPCs. The right QPC, serving as a noninvasive detector, is in the weak tunneling regime. Therefore, we treat the corresponding operator \( A^I \) perturbatively [14].

The backscattering current at the second QPC may be written as \( j_{bs} = i (A^I - A^I) \), and, to the leading order in the tunneling operator \( A^I \), the noise power (1) of this current reads \( S_{bs} = \int dt \langle [A^I(t), A^I(0)] \rangle \) if the relatively straightforward steps lead to the standard result (2) and to the effective noise temperature (4), with


\[ f(\epsilon) \propto \int dt e^{-i(\epsilon - \epsilon_F)t} K(t), \]  
\[ K(t) = \langle e^{-i\phi_{M2}(D,t)} e^{i\phi_{M2}(D,0)} \rangle, \]

(8a) (8b)

where the normalization prefactor in (8a) is determined by the condition \( f(\epsilon) = 1 \) at \( \epsilon \to -\infty \). The average in the definition of \( K(t) \) has to be taken with respect to the nonequilibrium state created by the source QPC. Therefore, we apply the nonequilibrium bosonization technique proposed in our earlier work [12].

The Hamiltonian (6), together with the commutation relations (5), generates equations of motion for the fields \( \phi_{M\alpha} \) that have to be accompanied with boundary conditions:

\[ \partial_t \phi_{M\alpha}(x, t) = -\frac{1}{2\pi} \sum_\beta \int dy V_{a\beta}(x - y) \partial_y \phi_{M\beta}(y, t), \]  
\[ \partial_t \phi_{M\alpha}(0, t) = -2\pi j_{\alpha}(t). \]

(9a) (9b)

We place the boundary at the point \( x = 0 \), right after the source QPC. At low energies of interest, the characteristic length scales are much longer than the screening length \( d \) of the Coulomb potential \( U_{a\beta}(x-y) \). Therefore, we can neglect its logarithmic dispersion and approximate \( U_{a\beta}(x-y) = U_{a\beta}(\delta(x-y)) \) and, consequently, \( V_{a\beta}(x-y) = V_{a\beta}(\delta(x-y)) \). Then, Eq. (9) acquire a form of first-order differential equations. We solve these equations by diagonalizing the matrix \( \tilde{V} = V_{a\beta} \) with the rotation \( \tilde{V} = \hat{S}(\theta) \hat{S}^\dagger(\theta) \) by the angle \( \theta \) defined as \( \tan 2\theta = 2V_{12}/(V_{11} - V_{22}) \). Then, the spectrum of the collective charge excitations splits into two modes, \( \hat{\Lambda} = \text{diag}(u, v) \), with the speeds \( u, v = (V_{11} + V_{22})/2 \pm \sqrt{(V_{11} - V_{22})^2/4 + V_{12}^2} \). Imposing the boundary condition (9b), we arrive at the solution

\[ \phi_{M2}(x, t) = \lambda_1 Q_1(t_u) + \lambda_2 Q_2(t_v) - \lambda_1 Q_1(t_u) + \lambda_2' Q_2(t_v), \]

(10a)
\[ \lambda_1 = \pi \sin 2\theta, \quad \lambda_2 = \pi (1 + \cos 2\theta), \quad \lambda_2' = 2\pi - \lambda_2, \]

(10b)

where we have introduced the injected charges \( Q_\alpha(t) = \int_{-\infty}^t dt' j_\alpha(t') \) and notations \( t_u = t - x/u \) and \( t_v = t - x/v \).

Since the edge state dynamics is chiral and the screened Coulomb interaction is effectively short range, the fields \( \phi_{M\alpha} \) do not influence fluctuations of the currents \( j_\alpha \) at the QPC [8,15]. As a consequence, the electron transport through a single QPC is not affected by the interaction, which seems to be an experimental fact [16]. Therefore, when finding the correlator (8b), one may utilize the free-fermion scattering theory for the statistics of injected charges \( Q_\alpha \) [9,17].

Gaussian noise regime.—It has been shown in Ref. [6] that a weak dispersion of plasmon modes suppresses higher-order cumulants at large distances. Therefore, we first focus on the situation, where the fluctuations of the boson fields may be considered Gaussian. Then, the logarithm of the correlation function (8b) can be written as \( \ln K(t) = -\langle \phi_{M2}(D,t)^2 \rangle \phi_{M2}(D,0) \rangle/2 \), where the linear terms in the field \( \phi_{M2} \) vanish, since there is no contribution to the average current in the inner channel. Using Eqs. (10), we obtain

\[ \ln K(t) = -2\pi \int_0^\infty \frac{d\omega}{\omega^2} \left( 1 - e^{i\omega t} \right) \left( \frac{\lambda_1^2}{\pi} \sin^2 \left( \frac{\omega t}{2} \right) \right) S_1(\omega) \]
\[ + \left[ 1 - \frac{\lambda_2^2}{\pi^2} \sin^2 \left( \frac{\omega t}{2} \right) \right] S_2(\omega), \]

(11)

where we have introduced the noise power, \( S_\alpha(\omega) = \int |d\phi_\alpha(\omega)|^2/2 \pi \), and the time delay between the wave packets, \( t_D = D/v - D/u \).

Since the transport through the injecting QPC is not affected by interactions, the free-fermion scattering approach [9] may be used to obtain

\[ S_\alpha(\omega) = S_\eta(\omega) + T_\alpha(1 - T_\alpha) S_n(\omega), \]

(12)

where \( S_\eta(\omega) = \omega \theta(\omega)/2 \pi \) is the ground-state (Fermi sea) contribution and \( S_n(\omega) = \sum_\mu [S_\mu(\omega + \Delta \mu) - S_\mu(\omega)] \) is the nonequilibrium part. Therefore, in the expression (11), the ground-state and nonequilibrium contributions separate, \( \ln K(t) = -\ln \epsilon_F + \ln K_n(t) \), and the noise temperature (4) may be presented as

\[ \Theta = -\frac{1}{2\pi \ln 2} \int_0^\infty \frac{dt}{t - i\eta} K_n(t), \quad \eta \to 0, \]

(13)

where the nonequilibrium contribution reads

\[ \ln K_n(t) = -4T(1 - T)(\lambda_1/\pi)^2 \]
\[ \times \int_0^1 \frac{dz}{z^2} (1 - z) \sin^2 \left( \frac{\Delta \mu z}{2} \right) \sin^2 \left( \frac{\Delta \mu z}{2} \right). \]

(14)

We note that the ground-state contribution to the correlator \( K \) is always Gaussian and is independent of the interactions because the effect of the injecting QPC on the states below \( \epsilon = \epsilon_F \) is simply a unitary transformation.

Next, we focus on the weak injection regime \( T \ll 1 \), verify the validity of the perturbation approach with respect to weak tunneling, and show that it may fail. It turns out that the expansion of \( K_n \) with respect to \( T \) as \( K_n = 1 + \ln K_n + \cdots \) is dangerous because \( \ln K_n \) diverges at large \( t \) and \( t_D \). More precisely, at distances \( D \gg D_{ex} \), where \( D_{ex} \equiv u/[\mu - \mu_\alpha \Delta \mu] \) is the characteristic length of the energy exchange between edge channels [6], its asymptotic reads \( \ln K_n = -\langle \lambda_1^2/2\pi \rangle T \Delta \mu \min(t, t_D) \). Therefore, to leading order in tunneling at the first QPC, the time integral in Eq. (13) diverges logarithmically. At
the short-time limit, this integral should be cut at \( t \sim 1/\Delta \mu \), where it behaves regularly. At the upper limit, it is cut at either \( t \sim 1/(T \Delta \mu) \), where \( \ln K_\alpha \) is not small and the perturbation approach fails, or at \( t \sim t_D \), where \( \ln K_\alpha \) takes a constant value smaller than 1 if \( T \Delta \mu t_D \ll 1 \). Thus, for \( T \ll 1 \), the noise temperature reads

\[
\frac{\Theta}{\Delta \mu} = \frac{\lambda^2 T}{2\pi \ln^2(1/T)} \ln(1/t), \quad \text{if } D \gg D_{ex}. \tag{15}
\]

We recall the notations \( t_D = D/v - D/u \) and \( D_{ex} = uv/[(u - v)\Delta \mu] \).

This regime can be viewed as perturbative with respect to both tunneling and interactions. Only in this regime does our result for the noise temperature agree with the one of Ref. [11].

**Non-Gaussian noise: Exact results.—**To complete our analysis, we investigate the situation, where even at long distances, \( D \ll D_{ex}/T \), the fluctuations of the edge fields remain non-Gaussian. At such distances, two plasmon modes, arriving with the time delay \( t_D \) longer than the correlation time \( 1/\Delta \mu \) of boundary currents (see Fig. 1), separate the injected charges \( Q \) into Eq. (10a) into uncorrelated terms. Therefore, the correlation function \( K \) splits in the product of four terms

\[
K(t) = \chi^\dagger(\lambda_1, t)\chi^\dagger(-\lambda_1, t)\chi(\lambda_2, t)\chi(-\lambda_2, t), \tag{17}
\]

each taking the form of the generator of full counting statistics (FCS) [17]:

\[
\chi_\alpha(\lambda, t) = \langle e^{i\lambda Q_\alpha(t)} e^{-i\lambda Q_\alpha(0)} \rangle. \tag{18}
\]

The correlation function \((17)\) is independent of \( D \); i.e., in the limit \( D \gg D_{ex}/T \), electrons in the inner channel do indeed reach a nontrivial stationary state.

We note that the FCS generator of the inner channel at the edge \( M \) contains only the Gaussian contribution from the Fermi sea, \( \ln \chi^\dagger(\lambda, t) = -(\lambda^2/4\pi^2) \ln \epsilon_F t \), while the FCS generator at the outer channel, being perturbed by a QPC, acquires an additional non-Gaussian part from the transport electrons, \( \ln \chi(\lambda, t) = -(\lambda^2/4\pi^2) \ln \epsilon_F t + \ln \chi_\alpha(\lambda, t) \). This leads to the expression (13) for the effective noise temperature with

\[
K_\alpha(t) = \chi^\dagger(\lambda_1, t)\chi^\dagger(-\lambda_1, t)\chi(\lambda_2, t)\chi(-\lambda_2, t). \tag{19}
\]

We stress that in the limit \( \Delta \mu \ll \epsilon_F \) the nonequilibrium FCS generator \( \chi_\alpha \) depends on time only via the dimensionless combination \( \Delta \mu t \), which is the consequence of a free-fermion character of the electron transport through a single QPC. Therefore, at distances \( D \gg D_{ex}/T \), the noise temperature is always linear in applied voltage bias \( \Delta \mu \), independently of details of the interaction.

In the following, we concentrate on the realistic case of a Coulomb interaction screened at distances \( D \gg a \), where \( a \) is the distance between edge channels. Therefore, one may approximate \( U_{ab} = \pi \mu \), where \( u/v_F \sim \ln(d/a) \ll 1 \), so that \( \theta = \pi/4 \) and \( \lambda_1 = \pi \) [8]. The dimensionless function \( \chi_\alpha(\pi, t) \) can be represented as a determinant of a single-particle operator [17] and calculated numerically [18]. The result for the normalized noise temperature \( \Theta/\Delta \mu \) as a function of transparency \( T \) of the injecting QPC is shown in Fig. 3. We also plot the normalized temperature \( \Theta_{eq}/\Delta \mu \) of an equilibrium distribution reached by electrons in the inner channel at \( D \to \infty \),

\[
\Theta_{eq}/\Delta \mu = \sqrt{3T(1 - T)/2\pi^2}, \tag{20}
\]

which is found by comparing the energy flux of electrons \( \pi^2 \Theta_{eq}/6 \) to the half of the heat flux \( \Delta \mu T(1 - T)/2 \) injected at the first QPC.

One can see in Fig. 3 a singular behavior of \( \Theta \) at \( T \to 0 \) and \( T \to 1 \). In order to describe it analytically, we recall the FCS generator for the tunneling process: \( \ln \chi_\alpha(\lambda_1, t) = (\Delta \mu t/2\pi) \langle e^{\lambda Q \alpha(t)} - 1 \rangle/2 \). Note that this FCS generator is universal; i.e., it does not require an assumption of free-electron transport at the QPC, and reflects the simple fact that tunneling is a Poisson process with all the current cumulants equal to the average current. Substituting this expression into Eqs. (13) and (19) and setting \( \lambda_1 = \pi \), we find the noise temperature at \( T \ll 1 \) in the non-Gaussian noise regime

![FIG. 3 (color online). The normalized effective noise temperature at the detector QPC is plotted as a function of the transparency \( T \) of the source QPC, generating a non-Gaussian noise. Solid blue line: The exact value of \( \Theta/\Delta \mu \) for a nonequilibrium stationary state at \( D \gg D_{ex}/T \), evaluated with the help of the determinant representation of the FCS generator [17]. Dashed blue line: Its asymptotic behavior (21) for \( T \ll 1 \) and similar results for \( 1 - T \ll 1 \). Dotted red line: The temperature (20) of a locally equilibrium state at \( D \to \infty \), extracted from the energy flux in the inner edge channel.](image-url)
\[ \Theta / \Delta \mu = (2 / \pi^2 \ln 2) T \ln(1/T). \quad D \gg D_{ex}/T. \quad \text{(21)} \]

It differs from the one for the Gaussian noise, Eq. (15), only by a numerical prefactor.

To summarize, we have investigated the effects of the integrability of the charge dynamics at the QH edge at a filling factor of \( \nu = 2 \), where two chiral edge channels coexist. We have found that the double-step electron distribution, created in one of the channels with the help of a voltage-biased QPC, evolves via several intermediate regimes to a nonequilibrium stationary state. Measuring the backscattering current noise in the second, copropagating channel reveals a nontrivial effect of the integrability and strong interchannel Coulomb interactions: The effective noise temperature \( \Theta \) of this stationary state is a nonanalytical function of the transparency \( T \), which scales as \( \Theta \sim T \ln(1/T) \) at \( T \ll 1 \).

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[14] For arbitrary \( T' \), the exact calculations show that the noise temperature is equal to \( 1 - T' \) times its perturbative value. Therefore, to avoid unnecessary complications, we use the perturbative approach.