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Abstract

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Reference


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Numerical Simulation Evidence of Dynamical Transverse Meissner Effect and Moving Bose Glass Phase

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We present 3D numerical simulation results of moving vortex lattices in the presence of 1D correlated disorder at zero temperature. Our results with field tilting confirm the theoretical predictions of a moving Bose glass phase, characterized by transverse pinning and dynamical transverse Meissner effect, the moving flux lines being localized along the correlated disorder direction. Beyond a critical transverse field, vortex lines exhibit along all their length a "kink" structure resulting from an effective static "tin roof" pinning potential in the transverse direction.

Depinning and dynamical ordering of periodic lattices, such as vortex lattices (VL) driven by an external force in the presence of static point disorder has been widely studied these past years. A large velocity \( v \) expansion [1] showed that the motion reduces the effects of disorder. In this approximation the effects of disorder on moving VL results only in an additional "shaking" temperature \( T_{sh} \approx 1/v \) with expectation of moving crystalline VL at large \( v \). It was however shown in [2] that due to the disorder Fourier components transverse to motion a static disorder persists even at large \( v \), and leads to a moving glass phase (MG). The motion of particles in the MG occurs along rough static channels. Depending on the velocity the channels can be coupled or decoupled due to dislocations, thus giving two generic moving glasses. For weak disorder or large \( v \) in \( d = 3 \), channels are elastically coupled leading to a topologically ordered structure: the moving Bragg glass (MBG) [2]. For larger disorder or in \( d = 2 \), channels are decoupled by dislocations giving a moving transverse glass (MTG) or smectic [2,3]. The predicted channel structure and dynamical ordering was confirmed by numerical [4–7] and further analytical [8] studies and observed experimentally [9].

Within the MG model [2], the situation for correlated disorder such as heavy-ion columnar tracks has been recently investigated [10]. For all velocities at \( T = 0 \) or for weak velocities at \( T > 0 \) a moving Bose glass phase (MBG) characterized by a transverse critical force and a diverging tilt modulus due to localization effect arising from the columnar pins was predicted [11]. This novel feature specific to correlated disorder results in a vanishing tilt response below a critical transverse field. This so-called dynamical transverse Meissner effect (DTME) is a crucial consequence of the MG theory; its observation should provide an unambiguous signature of the MBoG phase. However, until now there was no experimental or numerical simulation evidence for this phase.

This Letter reports 3D numerical simulation of field tilting in a moving layered vortex system at zero temperature in a random pinning landscape of columnar defects. The elastically moving vortex lines confirm the existence of MBoG displaying DTME. Above a critical transverse field, the vortex lines exhibit a kink structure along their length. This results from the existence of an effective static pinning landscape which is almost periodic in the transverse direction to the vortex flow. Finite thickness effects are discussed within a simple model of a single flux line in a static periodic potential.

We model a stack of \( N_z \) Josephson-coupled parallel superconducting planes of thickness \( d \) with interlayer spacing \( s \). Each layer in the \( (x, y) \) plane contains \( N_s \) pancake vortices interacting with a random pinning background of 1D correlated disorder, namely, \( N_p \) columnar pins parallel to the \( z \) direction. At \( T = 0 \) the overdamped equation of motion of a pancake \( i \) in position \( \mathbf{r}_i(z) \) reads

\[
\eta \frac{d \mathbf{r}_i}{dt} = -\sum_{j \neq i} \nabla_i U^{uv}(\mathbf{r}_{ij}, z_{ij}) - \sum_p \nabla_i U^{vp}(\mathbf{r}_{ip}) + \mathbf{F}^L + \mathbf{F}^{\text{full}}(z),
\]

where \( \mathbf{r}_{ij} \) and \( z_{ij} \) are the components of \( \mathbf{r}_i = \mathbf{r}_j - \mathbf{r}_i \) in cylindrical coordinates, \( \mathbf{r}_{ip} \) is the in-plane distance between the pancake \( i \) and a pinning site, and \( \nabla_i \) is the 2D gradient operator acting in the \( (x, y) \) plane. The viscosity coefficient is \( \eta \), \( \mathbf{F}^L = F^L \dot{\mathbf{x}} \) is the Lorentz driving force due to an applied current, and \( \mathbf{F}^{\text{full}}(z) \) is the surface force due to the field tilting away from the \( z \) axis in the \( y \) direction. This force acts as a torque on each flux line, i.e., \( \mathbf{F}^{\text{full}}(z = 0) = -\mathbf{F}^{\text{full}}(z = N_s s) = \mathbf{F}^{\text{full}} \dot{\mathbf{y}} \) and \( \mathbf{F}^{\text{full}}(z = 0) \) for pancakes in the bulk. The tilting force strength is defined by \( F^{\text{full}} = e^2 \phi_0 H_s/4\pi = 8\pi e^2 \epsilon_0 \lambda_{ab}^2 H_s / \sqrt{3} a_0^2 H_s \), where \( \epsilon_0 = (\phi_0 / 4\pi \lambda_{ab}^2)^2 \), \( \lambda_{ab} \) is the in-plane magnetic penetration depth, \( a_0 \) is the average vortex distance, \( H_s \) is the field transverse component, and \( \epsilon \) is the anisotropy parameter. The intraplane vortex-vortex pairwise repulsive interaction is given by a modified Bessel function \( U^{uv}(\mathbf{r}_{ij}) = 2\epsilon_0 d K_0(\mathbf{r}_{ij}/\lambda_{ab}) \) [12]. The interplane attractive interaction between pancakes in adjacent layers of
altitude \( z \) and \((z + s)\) reads \( U^{\nu}(\rho_{ij}, z_{ij} = s) = (2s\varepsilon_{0}/\pi) \times [1 + \ln(\lambda_{ab}/s)][(\rho_{ij}/2\rho_{s})^{2} - 1] \) for \( \rho_{ij} \leq 2\rho_{s} \) and \( U^{\nu}(\rho_{ij}, z_{ij} = s) = (2s\varepsilon_{0}/\pi)[1 + \ln(\lambda_{ab}/s)][\rho_{ij}/\rho_{s} - 2] \) otherwise; in this expression \( r_{s} = \xi_{ab}/\varepsilon \), where \( \xi_{ab} \) is the in-plane coherence length. This pairwise interaction describes pancake interaction resulting from both the electromagnetic and the Josephson coupling [13]. Finally, the attractive pinning potential is given by \( U^{\nu}(\rho_{ij}) = -\alpha A_{p} e^{-\rho_{ij}/R_{p}} \), where \( A_{p} = (\varepsilon_{0}d/2)\ln[1 + (R_{p}^{2}/2\xi_{ab}^{2})] \) as proposed in [14] and \( \alpha \) is a tunable parameter. We consider periodic boundary conditions of \((L_{x}, L_{y})\) sizes in the \((x, y)\) plane while open boundaries are taken in the \( z \) direction. All details about our method for computing the Bessel potential with periodic conditions can be found in [15]. Molecular dynamics simulation is used for \( N_{v} = 30 \) vortex lines in a rectangular basic cell \((L_{x}, L_{y}) = (5.6\sqrt{3}/2)\lambda_{ab} \), and for a number of layers varying from \( N_{z} = 19 \) up to \( N_{z} = 149 \). The number of columnar pins is set to \( N_{p} = 30 \). We consider the London limit \( \kappa = \lambda_{ab}/\xi_{ab} \approx 90 \), with an average vortex distance \( a_{0} = \lambda_{ab} \), and \( d = 2.83 \times 10^{-3}\lambda_{ab} \), \( s = 8.33 \times 10^{-3}\lambda_{ab} \), \( R_{p} = 0.22\lambda_{ab} \), \( \varepsilon = 0.01 \), \( \eta = 1 \). The choice of the second-order Runge-Kutta algorithm time iteration step \( \delta t \) is dictated by the dominant force \( F^{L} \), and we take \( \delta t = 10^{-2}t_{0} \), where \( t_{0} = \eta\xi_{ab}/F^{L} \). We choose the tunable pinning parameter \( \alpha = 1/25 \) so that the maximum pinning force is \( F_{\max}^{\nu} \approx F_{0}/5 \) where \( F_{0} = 2\varepsilon_{0}d/\lambda_{ab} \) is the unit force defined by the Bessel interaction. Finally, the driving force applied along a principal vortex lattice direction \( x \) is set to \( F^{L} \approx 2.8F_{0} \). This corresponds to the fully elastic flow limit since \( F^{L} \sim 25F_{c}^{L} \), where \( F_{c}^{L} \) is the critical Lorentz force along \( x \). In this dynamical regime and for all tilting angles reported here, no flux line cutting and decoupling is observed.

The “experimental” procedure is the following: we start in 2D with the \((x, y)\) plane by randomly throwing \( N_{v} \) pancakes and \( N_{p} \) Gaussian pins, and relaxation with zero Lorentz force yields a vortex structure with dislocations. The Lorentz force is then slowly increased far in the elastic phase up to \( F^{L} \sim 2.8F_{0} \) with steps \( \delta F^{L} \sim 3.5 \times 10^{-3}F_{0} \) every \( 3.6 \times 10^{4}t_{0} \). The successive dynamical regimes observed in our 2D vortex system are the following: (i) pinned regime where all vortices have zero velocity; (ii) plastic channels flowing through pinned regions and with periodic motion as seen in [5,6]; (iii) plastic turbulent flow with no stationary vortices as seen in [6,7]; (iv) weakly decoupled channels named MTG since quasi-long-range longitudinal order exists in each channel; (v) elastic phase where all vortices have the same average velocity and named MBG since no dislocation appears and the motion occurs through rough static channels even in the high driving phase \( F^{L} \sim 25F_{c}^{L} \) studied below. Our 2D numerical results are therefore in agreement with the moving glass theory developed in [2].

We now focus on the fully elastic flow obtained for \( F^{L} \sim 25F_{c}^{L} \). We first extend the vortices and the pins in the \( z \) direction in order to obtain 3D straight vortex lines moving elastically along the \( x \) direction through a random columnar pin landscape. We then slowly increase \( F^{\text{fil}} \), i.e., \( H_{z} \) the field component transverse to the vortex flow. For several system thickness, Fig. 1 displays the vortex line response to low field tilting, i.e., the average vortex line inclination \( \tan\theta_{B} = B_{y}/B_{z} \) versus the field inclination \( \tan\theta_{H} = H_{y}/H_{z} \) compared with the linear response of the pinning free vortex system. It shows a partial screening of \( H_{z} \), which tends to be total for infinite thickness since the slopes tend towards zero when \( N_{z} \to \infty \). This transverse screening is followed by a transition at a critical field \( H_{c}^{z}/H_{z} \sim 0.50 \), above which the average vortex line inclination recovers the vortex line inclination obtained without any pinning. Therefore, Fig. 1 shows the existence of DTME which is the very signature of the MBoG phase as predicted in [10]. As a good illustration of DTME, we show in the inset of Fig. 1 the vortex line configuration of the MBoG compared with the nondisordered lattice obtained for the same field tilting. One clearly sees that below the transition the transverse field enters at the surface without penetrating into the bulk, in opposition to the pinning free vortex system. Thus, below the critical transverse field the vortex lines remain parallel to the columnar defects even though they are moving at enough large velocity.

We now examine the situation above the critical transverse field. Far above \( H_{c}^{z}/H_{z} \) and whatever \( N_{z} \), is the
average vortex line inclination recovers the vortex line inclination obtained without any pinning (not shown). A more interesting case happens in the intermediate transverse field range above the transition and shown for $N_z = 49$ layers in Fig. 2. Increasing the transverse field above its critical value $H^e_y/H_z \sim 0.50$ gives rise to plateaus in the $\tan \theta_B$ vs $H_y/H_z$ curve. A consequence of these plateaus is that decreasing the transverse field generates hysteresis in the vortex inclination. A typical vortex line configuration above the transition is shown for $N_z = 149$ layers in Fig. 3(a). A modulated structure $z(y)$ appears along their length. This modulation is more obvious looking at the derivative $dz/dy$ shown for all the vortex lines in Fig. 3(b). By using the minima and maxima of the derivative we may roughly define two parts of small and big inclinations in the vortex lines with respect to $z$. The small ones are termed “pinned” regions and are linked by “unpinned” regions where the vortex inclination is bigger. The materialization of the pinned regions is shown in grey in Fig. 3(a). We visualize in this way a static pinning landscape in the transverse direction $y$. The effective columnar pin radius corresponds to the real pinning “radius” $R_p$ and the spacing between the effective pins is given by the channel spacing $\sim a_0$. Each plateau of Fig. 2 can be thereby labeled with an odd number corresponding to the number of pinned regions passed through by each vortex line. In this way, while increasing the transverse field above the transition, the vortex lines for $N_z = 49$ first pass through three pinned regions for a given range of $H_y$ defining the “P3” plateau, then the system jumps to the following plateau “P5” where the vortex lines pass through five pinned regions, and so on. The same process holds while decreasing $H_y$, thereby generating hysteresis. Finally, the central point displayed in Fig. 3(a) is that the vortex lines which are moving in the $x$ direction see an effective static tin roof potential along $y$ which generates kinks. This is a direct consequence of the transversely ordered static pinned channels, combined with the localization effects due to correlated disorder [2,10]. It clearly shows that the action of the disorder in the moving system cannot be simply reduced [1] to a temperature. Note that this effective potential is not strictly periodic because of the channel roughness.

We now turn to the discussion of the finite thickness effects in our simulation by considering the tilt response of a single static elastic vortex line in a tin roof pinning potential at $T = 0$. Such a simple model allows one to gain valuable insight into the transverse properties of the moving vortex lattice in the presence of correlated disorder, e.g., the transverse Meissner effect (TME) and the kink structure. The energy of a line of length $L$ is given by

$$E(u) = \int_0^L dz \left[ \frac{c}{2} \left( \frac{du}{dz} \right)^2 + V_{\text{eff}}(u) \right] + f(u(L) - u(0)), \quad (1)$$

where $u(z)$ is the 1D displacement field along $y$, $c = e^2\epsilon_0$ is the elastic constant, $f \propto H_y$ is a surface force, and $V_{\text{eff}}(u) = g(1 - \cos(4\pi u/\sqrt{3}a_0))$ with $g$ being a constant. We calculate the tilt response of the vortex line defined by $\tan \theta = [u_L(L) - u_L(0)]/L$. In the limit $L \rightarrow \infty$, we find a critical force $f_c = 4\sqrt{gc}/\pi$ below which $u_{L=\infty, f} \equiv 0$, giving $\tan \theta = 0$ and hence TME, as already

![FIG. 2. Plateaus and hysteresis observed above the transition for $N_z = 49$. The origin of each plateau is connected to a given number of pinned regions passed through by each vortex line. For details about the plateau labels and the pinned regions defining the effective static pinning landscape, see Fig. 3 and the text. The plateaus and hysteresis disappear in the infinite thickness limit $N_z \rightarrow \infty$.](image)

![FIG. 3. (a) Projection in the $(y, z)$ plane of the 30 moving vortex lines composed of $N_z = 149$ layers and obtained for $H_y/H_z \sim 0.53$ while increasing the transverse field. In grey are materialized the pinned regions which define an effective pinning landscape which is static and almost periodic. (b) Derivative $dz/dy$ for all the 30 vortex lines shown in (a).](image)
found in [16]. For \( f > f_c \), the vortex line is tilted and its ground state \( u_{L-\infty,f}(z) \) is expressed in terms of Jacobian elliptic functions which generate kink structures as observed in our simulation [see Fig. 3(a)]. The tilt response is shown in the inset of Fig. 4 together with the pure elastic response, i.e., \( g = 0 \). The kink structures obviously rely only on the tin roof potential. Note that \( f^* = \sqrt{3} f / 8 \pi c \) is the equivalent quantity of \( H_y/H_z \). For a finite size line and \( f = 0 \), the minimum energy solution is \( u_{L,f=0} \equiv 0 \), i.e., the line is pinned. If \( f \) is nonzero but sufficiently weak, both extremities of the line are pulled away from its minimum energy configuration in a continuous way, yielding a nonzero tilt response as observed in our simulation (see Fig. 1 and inset). Furthermore, for given values of \( f \), the vortex line abruptly stretches to a more favorable position resulting in jumps in the tilt response. We show in Fig. 4 the tilt response computed for \( L = 0.4d_0 \), which is the equivalent size of the 49 layers in our simulation results shown in Fig. 2. We find plateaus and hysteresis which are very close to those observed in our simulation and shown in Fig. 2.

To conclude, we find evidence of a MBoG phase with DTME below a critical transverse field, as predicted in [10]. Above \( H^*_c \), we find some dynamical effects due to correlated disorder. First, we find a transverse static tin roof potential giving rise to a kink structure. Its origin may be understood in terms of combined effects between the interactions generic of the MG and those specific to correlated disorder. For \( T > 0 \), this potential would persists at weak \( v \) and would disappear above a critical velocity [10]. Finally, we predict for thin film superconductors the existence of plateaus in the linear tilt response with hysteresis behaviors. These effects should be observed in local Hall probe experiments and magnetization vector measurements in the presence of an applied current.

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[12] For all tilting angles, the reordered dislocation free vortex structure we obtain at \( F^L \sim 25 F^L_c \) (see below) shows that the long range effect of the more suited logarithmic intraplane potential would not bring out crucial changes to our findings.