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Superconductor-to-insulator transition in linear arrays of Josephson junctions capactively coupled to metallic films

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I. INTRODUCTION

Low-dimensional superconductors are systems displaying a surprisingly complex and rich physics, allowing the study of paradigmatic phenomena in condensed-matter physics, like quantum phase transitions and quantum critical behavior, electronic localization, Coulomb blockade, etc. In particular, an intriguing superconductor-to-insulator phase transition (SIT) has been observed experimentally in superconducting films, wires, and in ultrasmall-capacitance Josephson-junction arrays (JJAs) in two and one dimensions. In the case of one-dimensional (1D) superconductors, one special kind of excitation, the so-called quantum phase slip (QPS) processes, have been the focus of an intense research due to both their putative role in the SIT in one dimension, as well as for their potential uses in novel qubit architectures, a fact that has motivated recent experimental research in 1DJJAs. More generally, a phase slip is a discrete process in a 1D superconductor in which the amplitude of the order parameter temporarily vanishes at a particular point, allowing the phase to change abruptly in units of $2\pi$. At sufficiently low temperatures, thermally activated phase slips are energetically forbidden, and only phase slips originating in macroscopic quantum tunneling (i.e., QPSs) are allowed.

On the other hand, since the seminal works by Caldeira and Leggett, it has been known that dissipation in macroscopic quantum systems plays a central role. For instance, in a two-dimensional JJA capactively coupled to a proximate two-dimensional electron gas (2DEG), Rimberg et al. observed a tunable SIT upon a variation of the back-gate voltage $V_g$ applied to the 2DEG. In that work, it was shown that $V_g$ has the effect of tuning the sheet resistance $R_\square$ in the 2DEG through the modulation of its electronic density, a fact that in turn modifies the electromagnetic environment of the JJA. It was argued later by Wagenblast et al. that the incomplete screening of the Coulomb interaction provided by the 2DEG renormalized the charging energy $E_C$ of the Josephson junction to higher values, driving a SIT whenever $E_J/E_C \sim 1$. While this scenario is reasonable in a 2D geometry, in a 1DJJA capactively coupled to a 2DEG, the relatively higher dimensionality of the 2DEG results in a very efficient screening of the Coulomb interaction, leading to the naive conclusion that the dissipation-controlled SIT should be hard to observe.

However, in a closely related Luttinger liquid system placed in proximity to a metallic plane, the prediction of a dissipation-driven quantum phase transition has been recently put forward. This transition originates in backscattering events having place in the Luttinger liquid under the effect of the dynamically screened Coulomb interaction. It is therefore interesting to study to what extent the same dissipative processes will affect the dynamics of QPSs in 1DJJAs. Generally speaking, the subject of dissipation in 1D superconductors is an active area of research, in which the available theories point toward the important role of intrinsic and/or extrinsic dissipation mechanisms in determining their $T = 0$ phase diagram.

In this paper we explore the possibility of a SIT in a 1DJJA capactively coupled to a proximate diffusive 2DEG. By means of bosonization methods, we derive the effective Euclidean action of the 1DJJA in the presence of the metallic plane, directly from a microscopic Hamiltonian. One important conclusion of our work is that for small values of the dissipative parameter $\eta$, the transition occurs always between a superconducting and an insulating phase. This is in contrast to other works predicting a more complex quantum phase diagram with superconducting, metallic, and mixed phases as the dissipative parameter is varied. We believe this to be a consequence of the different physical mechanisms producing dissipation. We find that, except for the experimentally challenging situation in which the Cooper-pair density in the array is not commensurate with the lattice, the SIT is always of the Berezinskii-Kosterlitz-Thouless (BKT) type and originates in the unbinding of QPSs. Dissipation in the dual field $\phi$ has the effect of quenching fluctuations of the superfluid density and consequently stabilizes an insulating ground state, a fact that could be observed experimentally in the dc resistivity of the 1DJJA. Specifically, we predict a resistivity of the form $\rho(T) \sim \Delta T + A T^{\nu}$ in the superconducting phase, and $\rho(T) \sim n_\phi T^{\nu} / \Delta$, in the insulating phase, with $\Delta$ the insulating gap and $n_\phi$ the dissipation parameter.
The paper is organized as follows. In Sec. II we derive the effective model for a 1DJJA coupled to a 2DEG, in Sec. III we derive the $T = 0$ phase diagram as a function of the parameters of the model, Sec. IV is devoted to the study of the experimental consequences of our results, and finally in Sec. V we present a summary and our conclusions.

II. MODEL

We start by considering an ideally isolated 1DJJA, with length $L \rightarrow \infty$. To simplify the analysis, we neglect in the following the fermionic degrees of freedom forming the Cooper pairs at a microscopic level. This “boson-only” approximation describes correctly the critical properties of a JJA at temperatures $T \ll T_c$, with $T_c$ the superconducting critical temperature in the bulk of the superconducting islands. The description of the 1DJJA is given in terms of the quantum phase model,2

\[
H_{\text{JJA}} = \frac{1}{2} \sum_{i,j} (n_i - \bar{n}) v_{ij} (n_j - \bar{n}) + \sum_{j} E_j (1 - \cos \theta_i - \theta_j). \tag{1}
\]

Here the dynamical variables are the number of Cooper pairs $n_i$ and the phase $\theta_i$ of the order parameter at the $i$th superconducting island in the array. These variables obey the usual phase-number commutation relation in the BCS ground state, i.e., $[\theta_i, n_j] = i \delta_{ij}$. The first term in Eq. (1) represents the charging energy, with $v_{ij}$ the unscreened Coulomb interaction [cf. Eq. (3)] between the excess charges at sites $i$ and $j$, and $\bar{n}$ corresponds to an average charge imposed, e.g., by external gate voltages. The second term is the Josephson energy contribution, parametrized by $E_j$. In the following we use the convention $\hbar = k_B = 1$.

The critical properties of model Eq. (1) are more conveniently studied with a field-theoretical approach, valid for fluctuations with wavelength much larger than the lattice parameter of the array $a$.25 We therefore introduce the coarse-grained superfluid density $\delta\rho(\mathbf{x})$, defined as $\delta\rho(\mathbf{x}) = (n_i - \bar{n})/a$, and we expand the Josephson term as $E_j \cos(\theta_i - \theta_j) \simeq E_j a^2 [\nabla \theta(\mathbf{x})]^2$, up to a constant term. The continuum limit of Eq. (1) reads

\[
H_{\text{JJA}} = \frac{1}{2} \int dx dx' \delta \rho(\mathbf{x}) v(x - x',0) \delta \rho(x') + \frac{1}{2} E_j a \int dx [\nabla \theta(\mathbf{x})]^2. \tag{2}
\]

Here the 1D superfluid interacts via the bare Coulomb potential, which we define for convenience as

\[
v(\mathbf{r},z) = \frac{e^2}{\varepsilon_r} \frac{1}{\sqrt{r^2 + z^2 + a^2}}. \tag{3}
\]

The variables $r = |\mathbf{r}|$ and $z$ are, respectively, the distance in the $xy$ plane and along the azimuthal direction between two point charges (cf. Fig. 1). The lattice parameter $a$ acts as the short-distance regularization of the interaction, and $\varepsilon_r$ is the permittivity of the insulating medium surrounding the islands. Note that in Eq. (2) we do not assume an a priori short-ranged, screened interaction as is usually done when dealing with JJAs.26 This will result as a natural consequence of the interaction with the 2DEG (see below). One problem of this field-theoretical approach is that Mott instabilities (crucial when the superfluid density is commensurate with the lattice) are lost in Eq. (2) after taking the continuum limit. One way to cure this problem is to introduce a phenomenological interaction term $H_{\text{int}} = -\int d\mathbf{r} V_{\text{int}}(\mathbf{r})$, where the effective superfluid density $\rho(\mathbf{x})$ [cf. Eq. (11)] couples to the phenomenological potential $V_{\text{int}}(\mathbf{x})$, having the same periodicity of the lattice.32

The electrons in the 2DEG are described by the Hamiltonian

\[
H_{\text{2D}} = \int d^2 r \sum_{\sigma} \left[ -\frac{1}{2m} \eta_{\sigma}^0(\mathbf{r}) \nabla^2 \eta_\sigma(\mathbf{r}) + V_{\text{imp}} \eta_{\sigma}^0(\mathbf{r}) \eta_\sigma(\mathbf{r}) \right] + \frac{1}{2} \int d^2 r d^2 r' \delta \rho_{2D}(\mathbf{r}) v(\mathbf{r} - \mathbf{r}',0) \delta \rho_{2D}(\mathbf{r}'). \tag{4}
\]

where the fermionic field operator $\eta_{\sigma}^0(\mathbf{r})$ creates an electron in the 2DEG with spin projection $\sigma$ at position $\mathbf{r} = (x,y)$, and $V_{\text{imp}} \equiv V_{\text{imp}}(\mathbf{r})$ represents a weak static impurity potential which provides a finite resistivity and dissipation in the metal. The density fluctuations relative to the average $\rho_{0,2D}$ are defined as $\delta \rho_{2D}(\mathbf{r}) \equiv \rho_{2D}(\mathbf{r}) - \rho_{0,2D}$, with $\rho_{2D}(\mathbf{r}) = \sum_{\eta} \eta_{\eta}^0(\mathbf{r}) \eta_{\eta}(\mathbf{r})$ the density operator in the 2DEG.

Finally, the interaction between the 1DJJA and the 2DEG placed at a distance $d$ (cf. Fig. 1) is described by the Hamiltonian

\[
H_{\text{int}} = \int d^2 r d^2 x' \delta \rho(x') v(\mathbf{x} - \mathbf{r},d) \delta \rho_{2D}(\mathbf{r}). \tag{5}
\]

Our goal in this section is to derive an effective model for the 1DJJA capacitively coupled to the 2DEG. To that end we introduce the partition function of the system26

\[
Z = \int D[\rho,\theta] D[\bar{\eta},\eta] e^{-S},
\]

where $S$ is the Euclidean action of the problem,

\[
S = S_{\text{JJA}} + S_{\text{2D}} + S_{\text{int}}, \tag{6}
\]

with

\[
S_{\text{JJA}} = \int_0^\beta d\tau \int dx \, i \partial_\tau \theta(x,\tau) \rho(x,\tau) + \int_0^\beta d\tau \, H_{\text{JJA}}(\tau),
\]

\[
S_{\text{2D}} = \int_0^\beta d\tau \left[ \int d^2 r \, \bar{\eta}(\mathbf{r},\tau) (\partial_\tau - \mu_{2D}) \eta(\mathbf{r},\tau) + H_{2D}(\tau) \right],
\]

\[
S_{\text{int}} = \int_0^\beta d\tau \, H_{\text{int}}(\tau).
\]
Here \( \mu_{2D} = k_F^2/2m - eV_g \) is the effective chemical potential in the metal, with \( k_F = |k_F| \) the Fermi wave vector, and \( V_g \) is the gate voltage applied to the 2DEG, which allows us to change the value of \( \rho_{0,2D} \) and therefore the sheet resistance \( R_{\perp} \).

The first step in the derivation of an effective model for the 1DJJA is to integrate out the fermionic degrees of freedom \( \eta(\mathbf{r}, \tau), \eta(\mathbf{r}, \tau) \) in the 2DEG. Within the random-phase approximation (RPA)\(^{32}\), in the 2DEG, this integration is done expanding \( \rho_{n,2D} \) to second order. After reexponentiation of the result, which amounts to the usual cumulant expansion, the following expression results:\(^{27,28}\)

\[
S_{\text{eff}} \simeq S_{\text{HA}} - \frac{1}{2} \int d\mathbf{r} d\mathbf{r'} \int d\mathbf{x} d\mathbf{x'} \delta \rho(\mathbf{x}, \tau) \times v_{\text{scr}}(\mathbf{x} - \mathbf{x'}, \tau - \tau') \delta \rho(\mathbf{x'}, \tau') \ . \tag{7}
\]

We do not provide here the details of this derivation, and we refer the interested reader to Refs. \(^{27} \) and \(^{28} \). In Eq. (7) we have introduced the 1D effective screening potential \( v_{\text{scr}}(\mathbf{x} - \mathbf{x'}, \tau - \tau') \), which encodes all the screening effects provided by the 2DEG. The screening potential writes more conveniently in Fourier representation as\(^{27}\)

\[
v_{\text{scr}}(k, \omega_m) = \frac{1}{L \sum} \left[ v_{2D}(k, \mathbf{d}) \right]^2 \chi_{0,2D}(k, \omega_m) \ . \tag{8}
\]

where \( \omega_m = 2\pi m / \beta \) are the bosonic Matsubara frequencies,\(^{37} \) and \( \mathbf{k} = (k_x, k_y) \) is the wave vector in two dimensions, where we have made explicit the component \( k_\perp \) in the 2DEG, perpendicular to the 1DJJA. The quantity \( v_{2D}(k, \mathbf{d}) = (2\pi e^2 / \epsilon_{\tau}) \exp(-|k| \sqrt{d^2 + a^2}) / |k| \) is the 2D Fourier transform of the Coulomb potential Eq. (3). We assume that the length of the array is \( L < \xi_{\text{loc}} \), with \( \xi_{\text{loc}} \) the Anderson localization length in the 2DEG, a reasonable situation for metals in the diffusive limit. In that case, and in the limit \( |\omega_m| \ll \tau_{\tau}^{-1}, k \ll \xi_{\text{loc}}^{-1} \), where \( \tau_{\tau} \) and \( \xi_{\text{loc}} \equiv \pi \tau_F \) are, respectively, the elastic scattering time and the mean free path in the 2DEG, the density-density response function, averaged over different configurations of the disorder potential \( V_{\text{imp}}(\mathbf{r}) \), writes \( \chi_{0,2D}(k, \omega_m) = 2N_{2D} k_F^2 / (Dk^2 + |\omega_m|) \), with \( D \) and \( N_{2D} \), the diffusion constant and the density of states (at the Fermi energy) per spin projection, respectively.\(^{36} \) In the limit \( |\omega_m| \ll Dk_F^2 / |k| \) (the most relevant for the present geometry\(^{35} \)), where \( k_{\text{TF}} \equiv 4\pi e^2 N_{2D} / \epsilon_{\tau} \) is the Thomas-Fermi wave vector in two dimensions, the screening potential can be approximated as

\[
v_{\text{scr}}(k, \omega_m) \simeq \frac{2e^2}{\epsilon_{\tau}} \left[ K_0(2kd) - \frac{\pi |\omega_m|}{2Dk_{\text{TF}} |k|} \right] , \tag{9}
\]

where \( K_0(z) \) is the zeroth-order modified Bessel function.\(^{39} \)

From Eq. (7), we can now define the total effective retarded interaction as

\[
v_{\text{eff}}(k, \omega_m) = v_{1D}(k, 0) - v_{\text{scr}}(k, \omega_m) \ , \tag{10}
\]

where \( v_{1D}(k, 0) = 2e^2 K_0(|k| a) / \epsilon_{\tau} \) is the Fourier transform of Eq. (3) in one dimension. Physically, the effective potential \( v_{\text{eff}}(k, \omega_m) \) describes the interaction among charges in the array, both via the (repulsive) instantaneous interaction \( v_{1D}(k, 0) \) arising from the bare intrawire Coulomb interaction, as well as indirectly via the coupling to the diffusive modes in the 2DEG, which corresponds to the (attractive) retarded screening potential \( v_{\text{scr}}(k, \omega_m) \).\(^{38} \) Note that in the static limit \( \omega_m = 0 \), the logarithmic divergences in the limit \( k \to 0 \) are compensated inside Eq. (10) due to the screening provided by the 2DEG, yielding \( \lim_{k \to 0} v_{\text{eff}}(k, 0) = 2e^2 \ln (2d/a) / \epsilon_{\tau} \).\(^{27}\)

We now introduce a more convenient representation of the superfluid density in the 1DJJA. To motivate our approach, we first note that in the absence of Josephson coupling [i.e., \( E_J = 0 \) in Eq. (1)], the Cooper-pair occupation number \( n_1 \) is a good quantum number in each island, fixed by \( \bar{n} \) through the application of an external gate voltage. Increasing \( E_J \) will evidently introduce fluctuations in \( n_1 \) due to the transfer of Cooper pairs between neighboring islands, and \( n_1 \) is no longer a good quantum number. However, we expect that in the experimentally interesting regime \( E_J / E_{C,0} \sim 1 \), where \( E_{C,0} \) is the characteristic charging energy in the island (see below), fluctuations of the charge \( \Delta n_1 \equiv n_1 - \bar{n} \) will be of order \( \Delta n_1 \sim \pm 1 \), and that all other charging states such that \( |\Delta n_1| \gg 1 \) will be energetically forbidden. We therefore truncate those states from our description and focus only on charge fluctuations involving \( \Delta n_1 = \pm 1 \). In terms of a continuous field \( \phi(x) \), which is slowly varying on the scale of \( a \), the superfluid density in this effective model is more conveniently written as\(^{40} \)

\[
\rho(x) = \left[ \rho_0 - \frac{1}{\pi} \nabla \phi(x) \right] \sum_p e^{i(2p|\pi\rho_0 x - \phi(x))} \ , \tag{11}
\]

where \( \rho_0 = 1/\pi \) is a parameter in the model, which can be interpreted as an effective average superfluid density in the 1DJJA. However, note that \( \rho_0 \) has no real physical meaning and cannot be interpreted as the real total physical superfluid density, in contrast to truly 1D systems.\(^{32} \) Only the fluctuations \( \delta \rho(\mathbf{x}) \equiv \rho(\mathbf{x}) - \rho_0 \) are physically meaningful in our model.

The contribution in square brackets in Eq. (11) describes long-wavelength density fluctuations around \( \rho_0 \), while in the last term, each contribution describes low-energy density fluctuations of momentum \( k \sim 2\pi p \rho_0 \), where \( p \) is an integer. In order to obey the original phase-number commutation relations, note that the field \( \phi(x) \) must verify

\[
[\theta(x), \nabla \phi(x')] = i\pi \delta(x - x') , \tag{12}
\]

characteristic of dual fields.\(^{32} \) Using the \( \phi \) representation for the density, the effective action Eq. (7) is expressed as

\[
S_{\text{eff}} = S_0[\theta, \phi] + S_1[\phi] + S_2[\phi] , \tag{13}
\]

where \( S_0[\theta, \phi] \) describes a Luttinger liquid,\(^{32} \)

\[
S_0[\theta, \phi] = \int dxd\tau \left[ i\theta \nabla \phi \right] + \frac{uK_0(\nabla \phi)^2}{2} + uK_0(\nabla \phi)^2 \right] \right) + \frac{1}{2\pi \beta} L \sum_{k,\omega_m} \eta_{\phi}(\omega_m) \left[ \chi_{0,2D}(k, \omega_m) \right] \rho(\mathbf{k}, \omega_m)^2 , \tag{14}
\]

resulting from the hydrodynamic sector (i.e., \( \omega_m \to 0 \)) of the theory. This model describes a 1D propagating plasma mode with linear dispersion relation and velocity

\[
u = a \sqrt{E_J / E_{C,0}} . \tag{15}
\]

Here \( E_{C,0} \) is the charging energy with respect to the ground. In order to make contact with the usual description of JJAs, note
that the (inverse) capacitance matrix \( [C^{-1}]_{ij} \) can be obtained comparing the electrostatic energy in Fourier space, with our Eq. (10) with \( \omega_{0n} = 0 \) (i.e., static screening). This allows us to obtain the relation

\[
[C^{-1}]_{k} = \frac{2}{(e^2a)^2} \varphi(k,0),
\]

where \( [C^{-1}]_{k} = \frac{1}{\pi^2} \sum_{i,j} e^{-\imath k (x_i - x_j)}/[C^{-1}]_{ij} \) and \( N \) is the number of junctions in the array. In the special case of an array with only ground and junction capacitances (\( C_g \) and \( C \), respectively), the above Eq. (16) enables us to extract the charging energy \( E_{C,0} \equiv \varphi(0) = 0 \)/\( e^2/2C_g \), with \( C_g = \epsilon_a/4 \ln(2d/a) \) encoding the effects of static screening by the 2DEG.\(^{41}\) Note that in the long-wavelength limit \( k \to 0 \), the junction capacitance (relevant for density fluctuations with momentum \( k \sim a^{-1} \)) drops off, and only \( C_g \) is relevant.

The parameter \( \lambda_0 \) is the dimensionless Luttinger parameter defined as

\[
\lambda_0 \equiv \pi \sqrt{\frac{E_I}{E_{C,0}}},
\]

Physically, \( \lambda_0 \) quantifies the competition between superconducting coherence and charging effects in the array, and therefore controls the decay of the power-law correlation functions \( \langle e^{\imath \alpha x} e^{-\imath \beta y} \rangle \sim x^{-1/2k_0^2}. \)

The parameter \( \eta_\phi \) is the dissipative parameter associated to the field \( \phi \), and is defined as

\[
\eta_\phi \equiv \frac{c_1 R_{cl}}{8\pi R_0},
\]

where \( R_{cl} = 1/(2e^2 DN a) \) is the sheet-resistance of the 2D film and \( c_1 \) is a numerical constant of order \( c \sim 1.0 \). Note that \( \eta_\phi \) is inversely proportional to the dissipation parameter \( \alpha \propto R_0/R_{cl} \) associated to the superconducting phase \( \theta \) in the usual analysis of Josephson junctions.\(^{1,2,26}\) Equation (14) with a nonvanishing \( \eta_\phi \) describes a 1D plasmon mode with a finite lifetime \( \Gamma \sim |k|/\eta_\phi \), as a consequence of the capacitive coupling to the diffusive modes in the 2DEG.\(^{27}\) Note that the term \( \sim \eta_\phi |\omega_{nm}|/|k| \) has scaling dimension 2 and consequently, the critical properties of the system are not modified, i.e., this term alone cannot drive a phase transition. For a metallic plane with \( R_{cl} \sim 0.1R_0 \) (cf. Ref. 12) we obtain an order-of-magnitude estimation \( \eta_\phi \sim 10^{-2} \ll 1 \). Consequently, the correlation function,

\[
I(x) = \langle T_x(\phi(x) - \phi(0))^2 \rangle = 2\pi \lambda_0 \sum_{\omega_{nm} \neq 0} \frac{1}{\omega_{nm}^2 u^2 + k^2 + \eta_\phi |\omega_{nm}|/2\pi c_1 u},
\]

can be approximated as \( I(x) \approx \lambda_0 \ln(x/a) - \eta_\phi \lambda_0 \ln(x/a)/2\pi^2 c_1 + O(\eta_\phi^2) \). Integrating out the phase field \( \theta \), this approximation allows us to recover a space-time (Lorentz-)invariant action

\[S[\phi] \sim \frac{1}{2\pi^2} \int \frac{d^2k}{k^2 u \omega_m^2 + k^2 K} \langle k,\omega_m \rangle|k,\omega_m \rangle^2, \]

with renormalized Luttinger parameter

\[K \equiv K_0 \left( 1 - \frac{\eta_\phi}{2\pi^2 c} \right),\]

The next term \( S_1 \) in Eq. (13) originates in the phenomenological potential \( V_I(x) \), and is physically related to the generation of QPSs in the array. Since \( V_I(x) \) has the same periodicity of the array, it can be decomposed in Fourier components as \( V_I(x) = \sum_n v_n \cos (Qnx) \), with \( Q = 2\pi/a \). In general, all terms other than \( p = n = 0 \) in Eq. (11) are rapidly oscillating and vanish under the integral sign. However, when the average density of bosons is commensurate with the lattice (i.e., or equivalently \( 2n\pi p = Qn \)), then the term \( \sim \int dx V_I(x) \rho(x) \) yields the nonoscillating contributions (in addition to terms with \( p = n = 0 \))

\[S_1[\phi] = -\frac{1}{\tau_0} \int dx d\tau \sum_{\nu > 0} \lambda_{n} \cos \left[ 2\phi(x,\tau) \right],\]

with \( \lambda_{n} \equiv V_n \tau_{0}. \)

The term \( V_0 \) can be reabsorbed in a redefinition of the chemical potential of the externally imposed charge \( \bar{n} \), and we will not consider it. Moreover, in the following we keep only the most relevant commensurability \( n = 1 \) in Eq. (21), and therefore set \( \lambda_{n} = 0 \) for all \( n > 1 \). Physically, the parameter \( \lambda \equiv \lambda_1 \) is related to the QPS rate in the Josephson junction by \( \Gamma_{QPS} = \lambda/\tau_0 \). Estimated experimental values for \( \Gamma_{QPS} \) are in the order of \( \sim 1 \) GHz,\(^{21}\) which yields \( \lambda \sim 0.06 \).

The final term in Eq. (13) comes from the dissipative part of \( \varphi_{\text{int}}(k,\omega_n) \). Due to the strongly oscillating factors \( \sim e^{-ie^{-\imath kx_0} \phi} \) in Eq. (11), it results in the local dissipative term

\[S_2[\phi] = -\frac{\eta_\phi}{a} \int dx d\tau d\tau' \sum_{\nu > 0} \cos \left[ 2\phi(x,\tau) - \phi(x,\tau') \right] \left( \tau - \tau' \right)^2.\]

This contribution is consistent with the derivation of Cazalilla et al.,\(^{28}\) obtained in the context of Luttinger liquids capacitively coupled to diffusive metals. Note that although Eq. (23) looks \textit{a priori} similar to the dissipative model derived by Ambegaokar, Eckern, and Schön,\(^{42}\) a crucial difference is that it depends on the \textit{dual field} \( \phi \), and not on \( \theta \). In order to make contact with other theoretical works on dissipative JJAs, note that in the absence of QPSs (i.e., \( S_1[\phi] = 0 \)), a model describing the array in terms of the field \( \phi \) can be obtained from an expansion in Gaussian fluctuations of Eq. (23), i.e., \( \cos [2\phi(x,\tau) - \phi(x,\tau')] \sim 1 - 2[\phi(x,\tau) - \phi(x,\tau')]^2 \), and a subsequent integration of the field \( \phi \) in favor of \( \theta \). This procedure yields the action

\[S_{\text{eff}}[\theta] = \frac{1}{2\pi^2} \int \frac{d^2k}{k^2 u \omega_m^2 + k^2 K + \eta_\phi |\omega_{nm}|} [\theta(k,\omega_m)]^2,\]

which is qualitatively similar to that obtained by Wagenblast et al.,\(^{26}\) for the case of a 2DJA, and shows that our bosonization approach allows a systematic derivation of the effective action.
that goes beyond the Gaussian approximation. In the following we study the critical properties and phases of the model obtained in Eq. (13).

III. PHASE DIAGRAM

A. Weak-coupling renormalization-group analysis

We perform a renormalization-group (RG) analysis of the model Eq. (13), assuming that \( S_1[\phi] \) and \( S_2[\phi] \) are weak perturbations to the Luttinger liquid action \( S_0[\phi] \) in Eq. (19). Since \( S_0[\phi] \) is Lorentz invariant in space and imaginary time, we adopt an RG procedure that rescales homogeneously space and time. As usual, we assume that the original theory is defined up to a certain momentum cutoff \( \Lambda(l) = \Lambda_0 e^{-l} \) (with \( \Lambda_0 \sim a^{-1} \)), and we study how the action \( S_0[\phi] \) is renormalized upon integration of high-energy modes in a window between \( \Lambda(l)/s < |q| < \Lambda(l) \), with \( s = e^{dl} \), where we have employed the compact notation \( q \equiv (k, -\omega_m) \) and \( x \equiv (x, ut) \).23,43

We obtain the perturbative RG-flow equations of the model by performing a one-loop correction in \( S_2[\phi] \) and a two-loop correction in \( S_1[\phi] \), and requiring that the term \( S_0[\phi] \) is invariant upon scaling.45 We obtain the RG-flow equations

\[
\frac{dK(l)}{dl} = \left[ -2\pi \eta_\phi(l) - (2\pi)^2 K(l) \lambda^2(l) C \right] \lambda^2(l),
\]

\[
\frac{du(l)}{dl} = -2\pi \eta_\phi(l) u(l) K(l),
\]

\[
\frac{d\lambda(l)}{dl} = [2 - K(l)] \lambda(l),
\]

\[
\frac{d\eta_\phi(l)}{dl} = [1 - 2K(l)] \eta_\phi(l),
\]

where the numerical constant \( C \) is of order unity.

Note that both \( S_1[\phi] \) and \( S_2[\phi] \) tend to destroy superconducting correlations in the Luttinger liquid phase, a fact that is reflected in Eq. (24) where the Luttinger parameter \( K(l) \) is renormalized to smaller values. Physically, the coupling to the diffusive degrees of freedom in the 2DEG via the Coulomb interaction reduces charge fluctuations in the 1DJJA, resulting in an enhanced effective charging energy \( E_{C, 0} \), and a lower \( K(l) \) in view of Eq. (17). In addition, since \( S_2[\phi] \) is the only term that breaks the Lorentz invariance of the theory, note that the RG-flow equation for the plasma velocity \( u(l) \) [cf. Eq. (25)] is proportional only to \( \eta_\phi(l) \), and is independent of \( \lambda(l) \).

When \( K(l) < 2 \), the perturbative parameter \( \lambda(l) \) flows to strong coupling [cf. Eq. (26)], and the perturbative RG procedure is no longer valid. In the limit \( \eta_\phi \to 0 \) we recover the usual Mott transition of the BKT type described by the sine-Gordon model, and below the critical value \( K_c = 2 \), the 1DJJA is in the insulating phase.2,3,24,45 Using Eq. (17), this means that in the absence of dissipation, the CIT occurs for \( E_f/E_{C, 0} = (2/\pi)^2 \). Note that our situation corresponds to the superfluid density in the 1DJJA being commensurate to the lattice, and is in clear distinction to the noncommensurate situation (i.e., \( \lambda = 0 \)), where the term \( S_2[\phi] \) becomes relevant for \( K(l) < 1/2 \), inducing a different kind of nonsuperconducting ground state.28

In the present case, the scaling dimension of the dissipative parameter \( \eta_\phi(l) \) is always smaller than that of \( \lambda(l) \), which means that for \( K(l) \approx 2 \), \( S_1[\phi] \) is a stronger perturbation as compared to \( S_2[\phi] \). Therefore one expects the nature of the nonsuperconducting ground state to be determined essentially by \( S_1[\phi] \). However, sufficiently near the CIT, the role of the dissipative term \( S_2[\phi] \) is non-negligible and enables an experimental control “knob” to study the CIT through its renormalization effect on \( K(l) \) [cf. Eq. (24)]. Experimentally, the magnitude of the dissipative parameter \( \eta_\phi(l) \) (and therefore of \( S_2[\phi] \)) could be controlled with the application of a gate voltage \( V_g \) on the 2DEG, which has the effect of changing the sheet resistance \( R_{\|} \).12 We illustrate this point in Fig. 2, where the quantum phase diagram of the 1DJJA in the plane \( K_0 - \eta_\phi(l) \) is obtained by integration of the RG flow Eqs. (24)–(27) up to a scale \( l_{\max} = \ln(\Lambda/\Lambda_0) = 5 \), using an initial parameter \( \lambda_0 \equiv \lambda(l = 0) = 10^{-3} \) and taking into account the additional renormalization of the bare value \( K_0 \) given by Eq. (20). This figure shows the stabilization of the insulating ground state given by an increase of \( \eta_\phi(l) \). Technically, the perturbative RG formalism imposes a constraint \( \eta_\phi(l) \ll 1 \) on the allowed values of the dissipative parameter, and consequently we have stopped the integration of the RG flow at the value \( \eta_\phi(l) \approx 0.02 \). Note that since the RG flow Eq. (24) is nonuniversal, our field-theoretical approach only allows for an order-of-magnitude estimation of \( K(l_{\max}) \), and more precise calculations would need other approaches taking into account the microscopic details of the theory. However, within order of magnitude, a value \( \eta_\phi(l) \approx 0.02 \) (representing an experimentally accessible value of \( R_{\|} \approx 3 \Omega \) in the film12) would induce a non-negligible renormalization \( \Delta K = K(l_{\max}) - K(0) \approx -0.15 \), and lends credence to the use of the sheet resistance \( R_{\|} \) as a control knob of the CIT in 1DJJAs.

In a first approximation, this effect is qualitatively similar to the dissipation-driven CIT observed in 2DJJAs capacitively coupled to a diffusive 2DEG.12,26,46 However, in that case Wagenblast et al. argued that the renormalization of the effective parameters \( E_f \) and \( E_C \) of the array was due to the incomplete screening of the Coulomb interaction in the frequency regime \( k_{2DJJA}/R_{\|}C_q \ll |\omega_m| < 1/R_{\|}C \) (with \( k_{2DJJA} \) the wave vector in the 2DJJA), where their effective action...
showed Ohmic dissipation. Physically, the slow diffusive response of the 2DEG cannot follow the faster dynamics of the 2D plasma mode, and cannot screen it efficiently. In contrast, in the 1D geometry the 1D plasmon is effectively very well screened by the 2DEG, and from here it could be naively concluded that no effects of dissipation near the SIT should be observed. However, this efficient screening effect is compensated by the presence of dynamically generated backscattering [i.e., the term \( S_2 \) in Eq. (23)], which renormalizes \( K(\lambda) \) and restores a dissipation-controlled SIT in the 1DJJA.

Although one expects the nature of the nonsuperconducting ground state to be of the Mott-insulating type, by analogy with the well-known results for the sine-Gordon model, strictly speaking we cannot extrapolate the results in this section to the strong-coupling situation, and a different method is needed in that regime.

B. Self-consistent harmonic approximation

To gain more insight into the phase in which the parameter \( \lambda(\lambda) \) flows to strong coupling, in this section we make use of the variational self-consistent harmonic approximation. This method consists in finding the optimal propagator \( g_{s}(q) \) of a Gaussian trial action describing the 1DJJA

\[
S_{\text{tr}} = \frac{1}{2\beta L} \sum_{q} g_{s}^{-1}(q) \phi(q)^{2},
\]

where \( \phi(q) \) is the Fourier transform of \( \phi(x,\tau) \). The idea is to minimize the variational free-energy

\[
F_{\text{var}} \equiv F_{u} + \frac{1}{\beta} \left( S_{\text{eff}} - S_{u} \right)_{\text{tr}},
\]

where the trial free energy \( F_{u} \) is

\[
F_{u} = -\frac{1}{2\beta L} \sum_{q} \ln \beta L g_{u}(q) .
\]

The factor \( 1/2 \) in Eqs. (28) and (30) comes from the constraint \( \phi^{*}(q) = \phi(-q) \) since \( \phi(x,\tau) \) is a real field, a fact that reduces the number of independent degrees of freedom.

The minimization of \( F_{\text{var}} \) in Eq. (29) with respect to \( g_{u}(q) \) yields the self-consistent equation for \( g_{u}(q) \),

\[
g_{s}^{-1}(q) = \frac{1}{\pi u K} \omega_{m}^{2} + \frac{u}{\pi K} k^{2} + \frac{4\lambda a}{\tau_{0}} \int_{0}^{\beta} e^{-2(\beta/2L)\sum_{q} \eta_{s}(q)}
\]

\[
\times \left[ \frac{8\eta \sqrt{2}}{a} \int_{0}^{\beta} d\tau \cos(\omega_{m}\tau) - 1 \right]
\]

\[
x \times e^{-\frac{4(1/\beta L)\sum_{q} \eta_{s}(1-\cos(\omega_{m}\tau))\eta_{s}(q)}} .
\]

In general, the solution of this equation has to be found numerically. However, for small \( \lambda \) and \( \eta_{\phi} \) the analytical solution

\[
g_{s}^{-1}(q) = g_{sL}^{-1}(q) + \frac{\zeta}{a} |\omega_{m}| + \frac{\Delta}{a\tau_{0}},
\]

is obtained. Here \( g_{sL}^{-1}(q) = \frac{1}{\pi u K} \omega_{m}^{2} + \frac{u}{\pi K} k^{2} \) is the Luttinger liquid propagator, corresponding to the action Eq. (14) (for

\[
\text{FIG. 3. (Color online) Dimensionless SCHA parameters } \Delta \text{ and } \zeta \text{ as a function of } K, \text{ calculated for the parameters } \eta_{\phi} = 0.01, \lambda = 0.05. \text{ We obtain nonvanishing values only in the region } K \lesssim 2 \text{ (note the abrupt increase in that region), consistent with the results of the RG analysis.}
\]

\[
\eta_{\phi} = 0 \). Parameters } \zeta, \Delta \text{ are found from the following set of nonlinear equations:
\]

\[
\zeta = 8\pi \eta_{\phi} \left( \frac{\zeta K \pi + 2\sqrt{K\pi}}{4} \right)^{2K}, \tag{33}
\]

\[
\Delta = 4\lambda \left( \frac{\zeta K \pi + 2\sqrt{K\pi}}{4} \right)^{K}, \tag{34}
\]

obtained replacing the solution Eq. (32) back into Eq. (31). Physically, Eq. (32) describes an insulator (given by a nonvanishing gap or “mass” term \( \Delta \)) with Ohmic-dissipative dynamics (encoded in a nonvanishing \( \zeta \)). Note that dissipation dominates for frequencies \( |\omega_{m}| \gtrsim \zeta/\tau_{0} \). Starting from the self-consistent solution of Eqs. (33) and (34) for \( \Delta \) in the absence of dissipation (i.e., \( \eta_{\phi} = 0 \)), we can study the regime \( \eta_{\phi} \ll \lambda \ll 1 \) perturbatively in \( \eta_{\phi} \), and we obtain the following estimate for the gap increase due to dissipative effects:

\[
\delta \Delta \simeq 2\pi^{2} \eta_{\phi} \zeta K \Delta_{0}^{2} / \lambda . \tag{35}
\]

This result is consistent with the fact that dissipation in the density (i.e., field \( \phi \)) quenches charge fluctuations, favoring an insulating ground state.

In Fig. 3 we show numerical results for \( \Delta \) and \( \zeta \) as a function of \( K \), calculated for the small \( \lambda = 0.05 \) and \( \eta_{\phi} = 0.01 \). Note the sharp increase of both \( \Delta \) and \( \zeta \) for \( K \lesssim 2 \). This result is consistent with the RG analysis, which predicts the breakdown of the Luttinger liquid phase for \( K \lesssim 2 \) in the weak-coupling regime. Within the SCHA, the physics of the strong-coupling fixed point is encoded in nonvanishing values of \( \zeta \) and \( \Delta \), providing a complementary description to the RG analysis.

IV. TRANSPORT PROPERTIES

In this section we concentrate on the dc conductivity of the 1DJJA, a quantity of central interest in experiments. We first focus on the current density \( j(x) \). Since the field \( \nabla \Theta(x)/\pi \) is the momentum of Cooper pairs [cf. Eq. (2)], the usual
minimal coupling procedure $\nabla \theta (x)/\pi \to [\nabla \theta (x) - 2eA(x)]/\pi$ (with $e$ the electron charge and $A$ the vector potential) in Hamiltonian Eq. (2) allows us to obtain the current as $j(x) \equiv - \delta H_{\text{JJA}}/\delta A(x)$. In our problem, it explicitly reads

$$j(x) = uK \left(\frac{2e}{\pi}\right)[\nabla \theta (x) - 2eA(x)].$$

The conductivity along the wire is obtained from the Kubo formula

$$\sigma(\omega) \equiv \frac{\chi^R_{jj}(0,\omega)}{i(\omega + i\delta)},$$

where $\chi^R_{jj}(k,\omega) \equiv \lim_{\omega_{\text{in}} \to \omega + i\delta} \chi_{jj}(\omega)$ is the retarded current-current correlation function and $\chi_{jj}(\omega) \equiv \langle j^+(\omega, q)j(\omega) \rangle = \delta^2 \ln Z/\delta A(q)\delta A^*(q)|_{A=0}$ is the current-current correlation function obtained in the linear-response regime. It is convenient to express this correlator as $\chi_{jj}(\omega) = \chi^{\text{d}}_{jj} + \chi^p_{jj}(\omega)$, where $\chi^{\text{d}}_{jj} \equiv -(2e^2/\pi)uK/\pi$ is the diamagnetic contribution and

$$\chi^p_{jj}(\omega) \equiv \left(\frac{2e^2}{\pi}\right)^2(uK)^2k^2(\theta(q) \theta(-q),$$

is the paramagnetic term. In absence of current-decaying mechanisms [i.e., $\lambda = \eta = 0$ in Eq. (6)], the conductivity reads

$$\sigma_0(\omega) = \frac{(2e)^2}{\hbar}uK \left[\delta(\omega) + iP\left(\frac{1}{\pi\omega}\right)\right],$$

where we have restored the Planck constant and where we have used that $\chi^{\text{d}}_{jj}(\omega) \to 0$ in the limit $k \to 0$. Note that the real part of $\sigma_0(\omega)$ consists of a Drude peak at $\omega = 0$, as expected for a superconductor. This result can be understood from the fact that the total charge current $J = \int dx \ j(x)$ is a conserved quantity in the absence of QPS and dissipation processes, i.e., it commutes with the Hamiltonian $H_{\text{JJA}}$.

The effect of a finite $\eta_\phi$ in the Gaussian sector of the theory [cf. Eq. (14)] has been studied in Ref. 27, and produces a broadening of the plasmon peak, whose width $\Gamma$ vanishes as $\Gamma \sim |k|$. Consequently, only taking into account this effect, a well-defined Drude peak in $\sigma(\omega)$ for $\omega \to 0$ is recovered, and the system should behave as a perfect conductor.

Let us now study the effects of the terms $S_1[\phi]$ and $S_2[\phi]$. When $\lambda$ and $\eta_\phi$ are irrelevant perturbations (in the RG sense), their effects on the conductivity can be studied within the theoretical framework of the memory function formalism. In this approach, the central assumption is that the Kubo formula for the conductivity Eq. (37) can be recast as

$$\sigma(\omega, T) = \frac{i(2e)^2}{\pi\hbar}uK \left(\frac{2e^2}{\pi}\right)^2 \frac{M(\omega, T)}{\omega + M(\omega, T)},$$

where $M(\omega, T)$ (i.e., the memory function) is a meromorphic function depending on the terms in the Hamiltonian responsible for degrading the current hence producing a finite resistivity. Current decay originated in QPS and in the coupling to the dissipative modes in the 2DEG induce finite resistivity in the 1DJJA for all temperatures $T < T_c$. In particular for temperatures $T \ll T_c$, and perturbatively in $\lambda$ and $\eta_\phi$, we obtain

$$\rho(T) = \frac{\hbar}{(2e)^2}[A_1T^{2\kappa_3} + A_2T^{2\kappa}],$$

where

$$A_1 \equiv \lambda^2 4\pi^3 \left[\cos\left(\frac{\pi K}{2}\right)B\left(\frac{K}{2}, 1 - K\right)\right]^2 \left(\frac{2\pi u}{\eta_\phi}\right)^{2\kappa_3},$$

and

$$A_2 \equiv \eta_\phi^2 2\pi \left[\cos[(1 + K)\pi]\right] T^{2\kappa} \left(\frac{2\pi u}{\eta_\phi}\right)^{2\kappa},$$

where the function $B(x, y)$ is defined as $B(x, y) \equiv \Gamma(x)\Gamma(y)/\Gamma(x + y)$, and $\Gamma(x)$ is the standard Euler’s $\Gamma$ function. The term $\sim T^{2\kappa}$ is consistent with the behavior predicted by Cazalilla et al. Note that at lowest order in $\lambda$ and $\eta_\phi$, the two contributions add up independently, indicating that for temperatures $T^* < T \ll T_c$, where $T^* \equiv \sqrt{A_1/A_2}/2\pi\tau_0$, the resistivity in the 1DJJA is dominated by Ohmic dissipation, while for $T < T^* < T_c$ the effect of QPS takes over.

The nontrivial effects due to the renormalization of the bare couplings can be taken into account using Eq. (20) and integrating the RG flow Eqs. (24)–(27). We then inject the result in the above Eqs. (40)–(42). We integrate the RG flow up to the scale given by the temperature $a(l) = a(0)\tau^l = u(l)/2\pi T$, and we use formula Eq. (40) with the parameters of the model calculated at the scale $a(l)$. This allows us to obtain $\rho[T(l)] \approx \rho[T]$.

In Fig. 4 we show the resistivity $\rho(T)$ of the 1DJJA, calculated for different values of the parameter $K$ and using the estimations for the bare parameters $\lambda_0 = 0.01$ and $\eta_{\phi, 0} = 0.01$. The results are normalized to a “high-temperature” resistivity $\rho(T_0)$, where $T_0 \approx \tau^{-1}_0$.

Note that for the values $K_0 = 2.5$ and $K_0 = 2.3$, the resistivity shows a monotonically decreasing behavior, indicating a superconducting ground state and consistent with

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{(Color online) dc resistivity $\rho(T)$ of the 1DJJA, normalized to a “high-temperature” value $\rho(T_0)$, as a function of $T/T_0$, calculated for the parameters $\lambda_0 = 0.01$ and $\eta_{\phi, 0} = 0.01$, and for different values of $K$. A low-temperature upturn of $\rho(T)$ signals the formation of the insulating phase.}
\end{figure}
driven SIT occurs upon increasing dissipation, the array is in the superconducting phase, a dissipation-driven SIT occurs upon increasing $\eta_\phi,0$, consistent with the results in Fig. 2. The curve $\varphi(T) \sim T^{2K-3}$ is shown for comparison.

The RG analysis of Sec. III A. We also note a small kink around $T_\ast \sim 0.4 T_0$, signaling the aforementioned crossover from dissipation-dominated to QPS-dominated resistivity. For $K_0 = 2.1$, the resistivity first decreases and then shows a low-temperature upturn, indicating that the array is near the quantum critical point $K_c$. Finally, for lower values of $K_0$, the insulating behavior in the 1DJJA is clear. Since both the integration of the RG-flow equations and the calculation of the memory-function formulas are perturbative, the calculation of the resistivity must be stopped whenever $\lambda(l)$ or $\eta_\phi(l)$ become of order unity.

In Fig. 5, we show the resistivity as a function of $T/T_0$, calculated for fixed $K_0 = 2.3$ and $\lambda_0 = 0.01$, and for different values of the parameter $\eta_\phi$. We see that for $\eta_\phi = 0$ (i.e., $R_Q = 0$ in the 2DEG), the array shows superconducting behavior, and the resistivity process is well described by the predicted power law $\varphi(T) \sim T^{2K-3}$, with $K = K(l \to \infty) \simeq 2.25$ the renormalized value predicted by Eq. (26). Upon increasing the parameter $\eta_\phi$, the resistivity of the array increases, developing the aforementioned kink, but most importantly, the low-temperature resistivity develops an upturn, indicating a phase transition to the insulating phase.

More insight into the insulating phase can be obtained using the Luther-Emery refermionization solution for $K = 1, 32,50$ In the absence of dissipation (i.e., $\eta_\phi = 0$) an exact solution is obtained in terms of noninteracting fermions, with a gap $\Delta \equiv \pi a \lambda$ in their spectrum of excitations. Using the Kubo formula, one obtains the following expression for the dc conductivity at low temperatures $T \ll \Delta$:

$$\sigma(\omega) \approx \frac{e^2}{\hbar} \frac{2\pi T}{\Delta} e^{-\Delta/T} \delta(\omega).$$

This contribution arises from the excited quasiparticles above the gap $\Delta$, which have an exponential population at low enough temperatures. This infinite conductivity occurs because in absence of dissipation, excited quasiparticles are infinitely long lived. Using the memory-function approach for the refermionized problem in the regime $\eta_\phi \ll \lambda, T \ll \Delta$, we find the analytical result

$$\sigma(\omega = 0) = \frac{e^2}{\hbar} \frac{c_2}{\eta_\phi^{1/2}} \frac{1}{\hbar T_0 t_0^{1/2}} \left(\frac{\Delta}{T}\right) e^{-\Delta/T},$$

where $c_2$ is a numerical coefficient $c_2 \simeq O(1)$. As expected, dissipation introduced a finite lifetime in the quasiparticles, and a finite resistivity is obtained at $\omega = 0$.

V. SUMMARY AND CONCLUSIONS

We have investigated the properties of a 1DJJA capacitively coupled to a proximate diffusive 2DEG. Using a bosonization approach, we have derived an effective model for the 1DJJA in terms of a field $\phi$, which is the dual to the superconducting phase field $\theta$, and have obtained its critical properties and phases at $T = 0$. This choice is motivated by the fact that the density-density interaction [cf. Eq. (5)], which mediates the coupling to the diffusive modes in the 2DEG, can be expressed formally exactly in the bosonization framework in terms of $\phi$ and enables a systematic derivation of the dissipative effective action of the 1DJJA.

Our main result is the prediction of a SIT tuned by the parameter $\eta_\phi \sim R_Q/R_G$ [cf. Eq. (18)] in a 1DJJA. Although this effect has been observed experimentally in 2DJJAs, in 1DJJAs the possibility of a SIT tuned through the sheet resistance of the 2DEG is still an open subject. The proposed experimental setup could be used to investigate systematically the superconductor-insulator transition in a 1DJJA under better controlled experimental conditions as compared to other setups used in the past.14–17 In addition, our results could also be applied to other 1D superconducting systems showing a similar behavior, such as ultrathin superconducting wires built by molecular templating8,10,51 or by e-beam lithography52 techniques. In those systems, it is experimentally challenging to vary in situ the parameters controlling the SIT, in contrast to 1DJJAs where the effective Josephson energy $E_J$ is tuned via external magnetic fields.2

Quite importantly, note that since the parameter $\eta_\phi$ is associated to the dual field $\phi$, its definition is inversely proportional to the parameter $\alpha \propto R_G/R_Q$, associated to the phase $\theta$ in the usual analysis of Josephson junctions.1,12,26

The properties of the resulting dissipative model for the 1DJJA [Eq. (13)] are studied with a weak-coupling RG analysis and a variational approach. We conclude that this SIT is in the BKT universality class and is mediated by unbinding of QPS/anti-QPS pairs, like in the dissipationless case.14 Near the critical line the effects of QPS are stronger than those originated in the dissipative term $S_2[\phi]$ [Eq. (23)]. This scenario is corroborated by a subsequent variational analysis of action Eq. (13), which suggests the formation of a gap $\Delta$ in the spectrum of excitations of the 1DJJA [cf. Eq. (32)], and excludes the stabilization of other mixed or metallic phases, as suggested in other theoretical works on dissipative 1DJJAs.29,30 We believe this discrepancy is originated in the different physical sources of dissipation: while in Refs. 29 and 30 their model describes most likely dissipation due to the effect of normal quasiparticles, in the present work it arises from the capacitive coupling to diffusive modes in the 2DEG. This points toward the importance of a correct identification.
and description of the physical sources of dissipation in a JJA.

We have shown that besides the more or less trivial static screening effect [which can be absorbed in the definition of the bare parameters of the theory, Eqs. (15) and (17)], the presence of a 2DEG also induces dissipative effects in the quantum dynamics of the 1DJJA. In particular, the dynamically screened Coulomb interaction Eq. (8), depending explicitly on the sheet of a 2DEG also induces dissipative effects in the quantum physics of the insulating phase, as can be seen in Figs. 2 and 5. This is due to the Luttinger liquid universality class, and the effects of QPS processes were absent, and it was dissipation itself that drove the quantum phase transition for the critical value $K_c = 1/2$.28

We have also studied the consequences on the temperature-dependent dc resistivity of the array $\rho(T)$. We have shown that a nonvanishing $R_{\parallel}$ induces a rich behavior of $\rho(T)$. In particular in the superconducting phase, where the 1DJJA is in the Luttinger liquid universality class, and the effects of QPS and dissipation are perturbative, the resistivity of the array $\rho(T)$ follows a power-law behavior $\rho(T) = A_1 T^{v_1} + A_2 T^{v_2}$, with exponents $v_1 = 2K - 3$ and $v_2 = 2K$ [cf. Eq. (40)] generated by QPS and dissipation, respectively. In the insulating phase, the low-temperature dc resistivity is expected to show thermally activated behavior.2,32 In particular for $K = 1$, the resulting model can be studied analytically with a refermionization approach, and results in a resistivity $\rho(T) \sim \eta_\phi T e^{3/7}/\Delta$, where the gap $\Delta$ depends also implicitly on $\eta_\phi$, via Eq. (35).

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Note, however, that in real-space representation \( v_{\text{eff}}(x,0) \) decays as \( \sim 1/x^3 \), i.e., an effective dipole-dipole interaction in one dimension as the result of the interaction with image charges in the 2DEG. In contrast, a capacitance matrix with only ground capacitance \( C_g \) and interjunction capacitance \( C \) induces an electrostatic potential that is exponentially screened in a length \( \Lambda_{\text{sc}} = \sqrt{C/C_g} \) (Ref. 2).

\[ \text{References} \]


41. Note, however, that in real-space representation \( v_{\text{eff}}(x,0) \) decays as \( \sim 1/x^3 \), i.e., an effective dipole-dipole interaction in one dimension as the result of the interaction with image charges in the 2DEG. In contrast, a capacitance matrix with only ground capacitance \( C_g \) and interjunction capacitance \( C \) induces an electrostatic potential that is exponentially screened in a length \( \Lambda_{\text{sc}} = \sqrt{C/C_g} \) (Ref. 2).


