Unconventional superfluidity in quasi-one-dimensional systems

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Abstract
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Reference

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We show that an unconventional superfluid triggered by spin-orbit coupling is realized for repulsively interacting quasi-one-dimensional fermions. A competition between spin-singlet and -triplet pairings occurs due to the breaking of inversion symmetry. We show that both superfluid correlations decay algebraically with the same exponent except for special coupling constants for which a dominant superfluid is controlled by the spin-orbit coupling. We also discuss a possible experiment to observe such phases with cold atoms.

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I. INTRODUCTION

Fermionic superfluids (SFs)—superconductors for charged particles—are ubiquitous in diverse fields ranging from condensed matter to elementary particle physics. The first realizations of such systems are described by the $s$-wave spin-singlet pairing picture of the celebrated BCS theory. Alternatively, fermionic SFs, which cannot be described by the standard BCS scenario, are called unconventional SFs. Such systems are quite rare and since the example of SF $^3$He [1] there have been very few new candidates. Unconventional superfluidity or superconductivity is thus one of the extremely challenging topics in many-body physics.

This interest has been revived with the arrival of cuprates as high-$T_c$ superconductors [2]. Superconductivity in noncentrosymmetric systems [3] has provided a novel route to unconventional superconductors. In these systems an admixture of spin-singlet and -triplet pairings is believed to be realized due to spin-orbit coupling (SOC) attributed to the breaking of inversion symmetry [4–7]. However, the presence of strong correlations in this class of materials makes it difficult to analyze theoretically such an admixture with a well-defined starting point.

When dealing with strong correlations, one- or quasi-one-dimensional systems provide some insight into their counterparts in higher dimensions by allowing the use of powerful analytical and numerical approaches [8]. For a standard SF, it is exactly known by use of the Bете ansatz that the evolution of the BCS state to Bose-Einstein condensation (BEC) is continuous [9,10]. For unconventional SFs, the suggestion of a resonant-valence-bond mechanism [2] and non-Fermi liquids [11] have been investigated as fermionic ladders. It has been established that in a ladder, a spin-singlet SF is present with purely repulsive interactions [12–17].

In addition to the realizations in condensed matter, ultracold atomic gases have offered an ideal playground for studying strongly interacting SFs due to the great controllability of the microscopic parameters and Hamiltonians [18]. By tuning an interatomic interaction with a Feshbach resonance, the BCS-BEC crossover has been achieved for two component fermions [19]. To go beyond the standard BCS scenario, experimental efforts [20–25] have been devoted to realizing $p$-wave SFs by using a $p$-wave Feshbach resonance [26] and Fulde-Ferrell-Larkin-Ovchinnikov [27,28] SFs by preparing spin-imbalanced gases. In addition, currently available synthetic gauge fields [29–41], where electromagnetic fields and SOC can be mimicked in both continuum and lattice spaces by using Raman lasers and driving an optical lattice, pave the way for the realization of nontrivial SFs in cold atoms. In fact, some theoretical works point out that such SFs emerge in higher dimensions [42–47]. However, it is also a challenging issue to demonstrate SFs with an admixture between spin-singlet and -triplet pairings in strongly correlated optical lattice systems.

In this paper, we propose a system of two fermionic chains with SOC, which breaks inversion symmetry. As a result, an unconventional SF with an admixture of spin-singlet and -triplet pairings emerges, which originates from the repulsive interaction and whose mechanism thus differs from a standard BCS scenario for attractive atomic gases with SOC in continuum space. Since such an admixture is controlled by the SOC, this model can also be used to realize an ideal spin-triplet SF in cold atoms. Furthermore, we show by using a mapping between attractive and repulsive situations that our findings are directly relevant for present experiments and discuss the observable consequences.

II. MODEL

We consider two coupled fermionic chains with SOC. There are two different ways to include the SOC effect: along the chain and along the rung directions. The SOC along the chain direction corresponds to a modification of the boundary condition of the chains [48] as long as the effect of the SOC is equal in the two chains. $^1$ Thus we focus on the case where the SOC is applied to the rung direction (Fig. 1), and the Hamiltonian is given as

$$H_{\Phi} = -t_x \sum_{j,p,\sigma} (c_{j,p,\sigma}^\dagger c_{j+1,p,\sigma} + \text{H.c.}) + U \sum_{j,p} n_{j,p \uparrow} n_{j,p \downarrow} - t_y \sum_{j,\sigma,\sigma'} \left[ (\epsilon^\Phi \vec{\sigma})_{\sigma,\sigma'} c_{j,\sigma}^\dagger c_{j,\sigma'} + \text{H.c.} \right],$$

(1)

where $p = \pm 1$ and $j$ are chain and site indices, respectively. The vector $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes Pauli matrices. The Peierls phase $\Phi \cdot \vec{\sigma}$ acts on the spin sector, and the direction of $\Phi$ determines the type of SOC: For $\Phi$ parallel and perpendicular

$^1$If the effect of the SOC is not equal in the two chains, nontrivial states may emerge since such an effect introduces a spin flux in the system. As for a charge flux, see, e.g., [49].
to a spin-quantization axis, the SOC effect works as a spin-dependent hopping and as a spin-flip hopping, respectively. We note that in the presence of the SOC, inversion symmetry along the rung direction is explicitly broken. The parameters $t_x$, $t_y$, and $U$, respectively, denote intra- and interchain hopping and the Hubbard interaction. In this paper we restrict ourselves to the following parameter regime in which the two-band nature of the ladder is relevant in the low-energy physics:

$$t_x \gg t_y, U \quad \text{or} \quad t_y \approx t_x \ll U.$$  

In addition, we consider only the incommensurate case for which a gapless charge excitation exists.

We briefly review the physics of a repulsively interacting fermionic ladder for $\Phi = 0$. In the presence of the rung hopping $t_r$, the two energy bands coming from the chains are split, and the two-band structure and their competition become crucial for the low-energy physics as long as the system is in the parameter regime (2). Correlations of interchain spin-singlet SFs are dominant. This result is well established from the analytical [8,12–17] and numerical [50–53] viewpoints. The interchain SF operator relevant for the repulsion is given by

$$O_{\text{SSC}}^{(c)} = c_{j,1\uparrow} c_{j,-1\downarrow} - c_{j,1\downarrow} c_{j,-1\uparrow},$$  

which is also called a $d$-wave-like SF operator because the symmetry resembles $d_{x^2-y^2}$ pairing [16], where in Fourier space the components with transverse wave vector, 0 and $\pi$, have opposite signs, and thus the sign of the operator changes with a $\pi/2$ rotation.

### III. UNCONVENTIONAL SUPERFLUID IN THE PRESENCE OF SPIN-ORBIT COUPLING

We now show the emergence in this model of an unconventional SF induced by the SOC. As mentioned, we have different types of SOC depending on the vector $\Phi$, and we discuss the three orthogonal cases: $\Phi_x = (0,0,\Phi)$, $\Phi_y = (0,\Phi,0)$, and $\Phi_z = (\Phi,0,0)$ where $-\pi \leq \Phi \leq \pi$.

Let us start with $\Phi = \Phi_z$. We use the following canonical transformation:

$$c_{j,\sigma} = d_{j,\sigma}, \quad c_{j,-\sigma} = e^{-i\phi} d_{j,-\sigma},$$  

which twists the spins in the $p = -1$ chain around the $\Phi_z$ direction. Due to Eq. (4), the phase $e^{-i\phi}$ is absorbed into $d_{j,\sigma}$, and thus the form of the Hamiltonian (1) becomes

$$H_{\Phi=0}^{(d)} \text{identical, in terms of the operators $d_{j,p,\sigma}$, to the original one (1) but for $\Phi = 0$. Thus the present problem is mapped onto one without the SOC, for which one can directly use the above solution for fermionic ladders. The ground state in the $d$ representation in our situation, incommensurate filling and the parameter regime (2), is thus dominated by the interchain SF pair correlation represented by the operator (3).}

We next consider how the ground state in the $d$ representation is described in the original $c$ representation via the transformation (4). In addition to the spin-singlet SF (3) we also look at the interchain spin-triplet SF along the $z$ direction, which is represented as

$$O_{\text{TSC}}^{(c)} = c_{j,1\uparrow} c_{j,-1\downarrow} + c_{j,1\downarrow} c_{j,-1\uparrow}.$$  

Due to Eq. (4), the operators $O_{\text{SSC}}^{(c)}$ and $O_{TSC}^{(c)}$ are transformed as follows:

$$O_{\text{SSC}}^{(c)} = \cos \Phi O_{\text{TSC}}^{(d)} + i \sin \Phi O_{\text{TSC}}^{(d)},$$  

$$O_{TSC}^{(c)} = \cos \Phi O_{TSC}^{(d)} + i \sin \Phi O_{SSC}^{(d)}.$$  

Note that both the spin-singlet and -triplet SF operators share $O_{\text{SSC}}^{(d)}$, and there is no other operator, including $O_{\text{TSC}}^{(d)}$, in the $c$ representation. Therefore, recalling that the $O_{\text{SSC}}^{(d)}$ correlation is dominant in the $d$ representation, the asymptotic form of the correlation of $O_{\text{SSC}}^{(c)}$ and $O_{TSC}^{(c)}$ is written as

$$\langle O_{\text{SSC}}^{(c)}(r) O_{\text{SSC}}^{(c)}(0) \rangle_c \sim \cos^2 \Phi \langle O_{\text{TSC}}^{(d)}(r) O_{\text{TSC}}^{(d)}(0) \rangle_d,$$  

$$\langle O_{\text{TSC}}^{(c)}(r) O_{\text{TSC}}^{(c)}(0) \rangle_c \sim \sin^2 \Phi \langle O_{\text{SSC}}^{(d)}(r) O_{\text{SSC}}^{(d)}(0) \rangle_d,$$

where $\langle \cdots \rangle_{c(d)}$ represents correlations in the $c$ ($d$) representations. As can be seen from Eqs. (8) and (9), the two different correlations in the original representation are described by the same asymptotic form except for the prefactor given by the SOC parameter $\Phi$. Since the algebraically decaying function $\langle O_{\text{SSC}}^{(d)}(r) O_{\text{TSC}}^{(d)}(0) \rangle_d$ is independent of $\Phi$, the relevancy of $\langle O_{\text{SSC}}^{(c)}(r) O_{\text{SSC}}^{(c)}(0) \rangle_c$ and $\langle O_{\text{TSC}}^{(c)}(r) O_{\text{TSC}}^{(c)}(0) \rangle_c$ is determined solely by the coefficients $\cos^2 \Phi$ and $\sin^2 \Phi$. For example, the interchain spin-singlet and -triplet pairing is dominant, respectively, for $0 \leq \Phi < \pi/4$ and for $\pi/4 < \Phi \leq \pi/2$. The ideal spin-singlet and -triplet pair states are realized only at $\Phi = 0$ and $\Phi = \pi/2$, respectively, and the spin-singlet and -triplet pairings are equally mixed at $\Phi = \pi/4$. In Fig. 2 we show the phase diagram as a function of $\Phi$. In the case of $\Phi = \Phi_z$, we first implement a global spin rotation as

$$f_{j,p,\sigma} = \sum_{n,q} e^{i\pi/4(n\sigma + q\bar{\sigma})} l_{n,q} c_{j,p,q'\bar{\sigma}}.$$  

The Hamiltonian is then transformed into the one for $\Phi_z$, which was already studied above. By using the same transformation (4) for $f_{j,p,\sigma}$, the Hamiltonian is reduced again to the one without SOC. The fermion operator in the consequent Hamiltonian is again $d_{j,p,\sigma}$. As a result the following fluctuation operators in the original $c$ representation are found to include the most relevant $O_{\text{SSC}}^{(d)}$ in the $d$
representation:
\[
O_{\text{SSC}}^{(d)} = \cos \Phi O_{\text{SSC}}^{(d)} + i \sin \Phi O_{\text{TSC}}^{(d)},
\]
(11)
\[
O_{\text{TSC}}^{(c)} = i \cos \Phi O_{\text{SSC}}^{(d)} - \sin \Phi O_{\text{TSC}}^{(d)},
\]
(12)
where the spin-triplet SF operator along the \( y \) direction has been defined as
\[
O_{\text{TSC}}^{(c)} = c_{j,\uparrow}^\dagger c_{j,-\uparrow} + c_{j,-\uparrow}^\dagger c_{j,\uparrow}.
\]
(13)

As in the case of \( \Phi_\perp \), the dominant fluctuations are found to be the spin-singlet and -triplet SF pair as a function of \( \Phi \), and their weights are determined by \( \Phi \), as illustrated in Fig. 2. A similar analysis can be applied for \( \Phi = \Phi_\parallel \), and shows that the spin-triplet SF along the \( x \) direction competes with the spin-singlet SF.

We can also discuss a case of arbitrary \( \Phi \) using the above result. Taking into account the SU(2) spin-rotational symmetry in the Hamiltonian of each chain, the appropriate spin rotation reduces the general \( \Phi \) problem to the \( \Phi_\parallel \) one, as seen in the discussion of the \( \Phi_\perp \) cases. Thus, it is concluded that admixture of the spin-singlet and -triplet pairings is also found to be induced by the SOC accompanying spin-singlet SF.

To summarize, unconventional spin-triplet SF pairs are found to be induced by the SOC accompanying spin-singlet SF pairs, which occurs for repulsive interaction. The weight of the spin-triplet pair is determined by the magnitude of the SOC parameters. The direction of the spin-triplet pairs, which is the so-called \( d \) vector in a spin-triplet \( p \)-wave SF [1], corresponds to that of the SOC vector \( \Phi \). The results are summarized in Table I. The most important point is that one can control the unconventional SF pair through the SOC.

Let us discuss the physical reason for the emergence of spin-triplet SF pairs. As mentioned above, the canonical transformation (4) physically rotates spins on the \( p = -1 \) chain around \( \Phi_\parallel \) by \( \Phi \), which is a direct way to change spin singlets to spin triplets. This can be easily confirmed by operating the spin rotation on one spin of a pair, \( Z_2(\theta) = e^{i\theta \sigma_\perp} \), on a singlet-spin pair \( |s\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle |\downarrow\rangle_2 - |\downarrow\rangle |\uparrow\rangle_2) \):

\[
Z_2(\theta) |s\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle |\downarrow\rangle_2 - e^{i\theta} |\downarrow\rangle |\uparrow\rangle_2),
\]
(14)

Finally let us comment on effects of disorder on the unusual SF described above. For the system without SOC, an analysis with bosonization and renormalization group methods predicts that in the presence of nonmagnetic disorder, the interchain SF realized for \( U > 0 \) is vulnerable to disorder while the intrachain SF realized for \( U < 0 \) is much more stable [54]. Since a nonmagnetic disorder favors SU(2) symmetry, these properties survive in the presence of the SOC. This is interpreted as the one-dimensional analog of the fact that an isotropic \( s \)-wave SF is robust against disorder (Anderson’s theorem) while an anisotropic SF could be destroyed by disorder.

### IV. ATTRACTIVE ROUTE

We now discuss the experimental possibilities for probing the unconventional SF predicted here. Because of the difficulty of confirming a SF directly in one-dimensional fermionic optical lattices, we alternatively employ the idea of the so-called attractive route to indirectly check its existence with a particle-hole transformation [55]. We first consider the \( \Phi = \Phi_\parallel \) case. The particle-hole transformation is well defined on a bipartite lattice as

\[
c_{j,p\uparrow} = h_{j,p\uparrow}, \quad c_{j,p\downarrow} = (-1)^{j+p} h_{j,p\downarrow}^\dagger,
\]
(14)

where \( \Theta \) is the Heaviside step function. Due to Eq. (14), \( n_{j,p\downarrow} \rightarrow -n_{j,p\downarrow} \), and the interchain hopping changes as

\[
-t_s \sum_{j,\sigma} \left( e^{i\Phi} \hat{h}_{j,\sigma}^\dagger \hat{h}_{j,-\sigma} + \text{H.c.} \right).
\]
(15)

Therefore the transformation (14) maps the Hamiltonian (1) on the one of the attractive fermionic ladders with a \( U(1) \) gauge field along the rung direction. While this type of \( U(1) \) gauge field already exists for \( ^{87}\text{Rb} \) Bose atom experiments [35,39],

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1While we can consider the same model with attractive \( U \) at incommensurate filling, where on-site spin-singlet SF correlation is dominant without the SOC, this on-site SF is always the dominant fluctuation in the presence of SOC. Namely, admixture between spin-singlet and -triplet pairings does not occur in this case.
this experimental technique can be applied in principle regardless of the atom statistics. In addition, the transformation (14) exchanges the roles of the particle filling and magnetization of a state: a spin-imbalanced state at half filling is transformed into a spin-balanced state away from half filling, and vice versa. (See Table II.) The attractive system after the mapping into a spin-balanced state away from half filling, and vice versa. The attractive system after the mapping into a spin-balanced state away from half filling, and vice versa.

From Eq. (14), the interchain spin-singlet SF is changed to

\[
\text{Re, Im} \left( O_{\text{SC}}^{(c)} \right) = (-1)^{j+1} \sum_{p \sigma} \rho(x,y)_{\sigma,-\sigma} h^\dagger_{j,p \sigma} h_{j,-p-\sigma},
\]

which is an operator for a staggered spin-flux phase on a plaquette in which there is no global spin flux but a local spin flux. This local staggered spin flux (16) is measurable because the technique to observe local phases on a four-square plaquette has been experimentally demonstrated in Refs. [33,41], and a scheme for the spatially resolved measurement of the current has also been proposed [56]. On the other hand, the interchain spin-triplet SF \( O_{\text{TSC}}^{(c)} \) is changed to

\[
\text{Re, Im} \left( O_{\text{TSC}}^{(c)} \right) = (-1)^{j+1} \sum_{p \sigma} (\sigma^a)_{\sigma,-\sigma} h^\dagger_{j,p \sigma} h_{j,-p-\sigma},
\]

which is an operator for a bond antiferromagnetic density wave along the rung direction. Such a correlation (17) may be measured in a similar way to the recently implemented measurement of nearest-neighbor spin correlations [57,58].

Next we consider the case of \( \Phi = \Phi_x \) or \( \Phi_y \). Then it turns out that Eq. (14) contains \( h_{j,1 \uparrow} h_{j,-1 \downarrow} \) from the rung hopping term, in which the number of particles is not conserved. Indeed, the off-diagonal spin operators, i.e., spin-flip hoppings, correspond to \( \eta \) pairings, which constitute the off-diagonal ones of SU(2) algebra in the charge sector [59]. Since Hamiltonians in cold atoms conserve the total number of particles, the attractive route would be much less useful in the \( \Phi = \Phi_x \) and \( \Phi_y \) cases.

Finally, we estimate the possible parameter regime in which to measure the states predicted in this paper. First we need a sufficiently low temperature \( T < t_x, t_y, U, \Delta \), where \( \Delta \) means the energy scale of gaps characterizing the realized state. If we choose \( t_x = t_y \) and \( 4 < U/t_x \leq 8 \) as a specific case in the parameter regime (2), this energy scale is estimated to be of the order of the exchange energy \( \Delta = 4t_x^2/U \) [50–53], and thus the necessary temperature will be \( \leq 10^{-1} t_x \). Let us note that this estimation works well even in the presence of a trap potential. From the point of view of the local density approximation, the realized state can form around the trap center, which can be regarded as bulk, while it should be broken on the edges. The contribution from such a bulklike regime must thus be non-negligible for the state to be observed.

This will be possible as long as the trapped system size is large enough, which should be possible given the existing experiments [58] and the proposal [60].

V. SUMMARY

We have analyzed two coupled fermionic chains with a repulsive interaction and found that an unusual SF emerges because of SOC along the rung direction. Properties such as the dominance and the \( d \) vector of a triplet pair are controllable. We have also discussed how to observe experimentally such a SF state via a particle-hole transformation and the use of attractive interactions. A staggered spin flux and bond antiferromagnetic correlations are possibly measurable observables associated with our predicted SF state. We finally point out that the discussion based on the canonical transformation (4) can be widely applied, e.g., to systems with a long-range interaction and with spin imbalance, which allows for further explorations of unconventional quantum states.

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