On the possibility of ferromagnetic, antiferromagnetic, ferroelectric, and ferroelastic domain reorientations in magnetic and electric fields

SCHMID, Hans

Abstract
By combining the thermodynamic classification of primary, secondary, and tertiary ferroics and the corresponding symmetry-allowed or symmetry-forbidden driving forces for domain switching and reorientation, with a classification of Aizu's 773 species (prototype/ferroic phase point group pairs) into 30 ensembles of property combinations, permitting to evaluate presence or absence of coupling between the different primary ferroic spontaneous quantities, symmetry conditioned possibilities of ferroic domain switching and reorientation have been evaluated. Main accent is put on cross-effects, such as magnetic field induced reorientation of antiferromagnetic, para- or diamagnetic ferroelectric and/or ferroelastic domains, and the equivalent electric field induced effects. Possibilities of poling antiferromagnetic domains have been evaluated. Examples referring to different types of ensembles are given.

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On the Possibility of Ferromagnetic, Antiferromagnetic, Ferroelectric, and Ferroelastic Domain Reorientations in Magnetic and Electric Fields

HANS SCHMID

University of Geneva, Department of Inorganic, Analytical and Applied Chemistry, 30, quai Ernest-Ansermet, CH-1211 Geneva 4, Switzerland

By combining the thermodynamic classification of Primary, Secondary and Tertiary Ferroics and the corresponding symmetry-allowed or symmetry-forbidden driving forces for domain switching and reorientation, with a classification of Aizu's 773 species (prototype/ferroic phase point group pairs) into 30 ensembles of property combinations, permitting to evaluate presence or absence of coupling between the different primary ferroic spontaneous quantities, symmetry conditioned possibilities of ferroic domain switching and reorientation have been evaluated. Main accent is put on "cross"-effects, such as magnetic field induced reorientation of antiferromagnetic, para- or diamagnetic ferroelectric and/or ferroelastic domains, and the equivalent electric field induced effects. Possibilities of "poling" antiferromagnetic domains have been evaluated. Examples referring to different types of ensembles are given.

Keywords: Ferroic domain reorientation; ferroelectrics; ferroelastics; ferromagnetics; antiferromagnetics

INTRODUCTION

The primary ferroic switching of spontaneous magnetization \( ^{3}M_{i} \), spontaneous polarization \( ^{3}P_{j} \) and spontaneous deformation \( ^{3}e_{ij} \) by means of the driving forces magnetic field \( H_{i} \), electric field \( E_{i} \) and mechanical stress \( \sigma_{ij} \), respectively, is rather well documented in literature\(^{[1,2]} \). The following "cross-effects", however, seem to have received less attention:

i) reorientation of the spontaneous magnetization \( ^{3}M_{i} \) by means of an electric field \( E_{i} \) or mechanical stress \( \sigma_{ij} \), ii) reorientation of the spontaneous polarization \( ^{3}P_{j} \) by means of a magnetic field \( H_{i} \) or mechanical stress \( \sigma_{ij} \) and iii) reorientation of the spontaneous deformation \( ^{3}e_{ij} \) by means of a magnetic or an electric field. With a view to exploring all potential cases, we are using two tools, first, the thermodynamic classification into primary, secondary and tertiary ferroics giving us the different potential symmetry allowed terms of the free energy and the corresponding driving forces for domain switching and reorientation\(^{[1,2,3,4]} \), and second, Aizu's classification\(^{[5]} \) of the prototype/ferroic
phase point group pairs ("species"), giving information on full, partial or no coupling between the different types of domain during switching. In an attempt at classifying the possibilities of contrast formation in polarized light between ferroelectric and ferroelastic domains, the 212 gray species, covering the paramagnetic and diamagnets, have been split into 9 ensembles, defined by the matrix Fully ferroelectric, Partially ferroelectric, Non-ferroelectric on the one hand and Fully ferroelastic, Partially ferroelastic, Non-ferroelastic (co-elastic) on the other hand. By extending these 9 ensembles to antiferromagnetic, fully and partially ferromagnetic ones, i.e. to 773 species, 36 ensembles of species are obtained. For symmetry reasons ensembles 4, 5, 6, 7, 8 and 20 are empty, leading to a total of 30 possible ensembles (Tables I, II).

POSSIBILITIES OF REORIENTATION AND SWITCHING

1. Reorientation of ferromagnetic, antiferromagnetic and ferroelectric domains by magnetic or electric fields is possible in ferroelastic phases only.

2. For ferroelastic domain reorientation the driving potential for wall motion is \( \Delta G \propto \Delta \varepsilon_{ij} \sigma_{ij} \), but ferrobimagnetic \( \Delta G \propto \Delta \chi_{ij} \mathbf{H}_i \mathbf{H}_j \) or ferrobielectric \( \Delta G \propto \Delta \chi_{ij} \mathbf{E}_i \mathbf{E}_j \) reorientation is possible also in case of sufficient anisotropy of magnetic and electric susceptibility, \( \Delta \chi_{ij} \) and \( \Delta \chi_{ij} \), respectively. Partial ferroelastics, however, can not be made single domain. In full ferroelastics/full ferroelastics the ferroelastic, ferrobimagnetic and ferrobimagnetic driving forces allow control both of direction and sense of \( \mathbf{P}_1 \). If being i) fully ferromagnetic, the direction, but not the sense of \( \mathbf{M}_i \), and if ii) partially ferromagnetic, the direction of \( \mathbf{M}_i \), but not the domain state can be controlled.

3. The driving potential for ferromagnetic wall motion, \( \Delta G \propto \Delta \mathbf{M}_i \mathbf{H}_i \), allows 180° reversal of \( \mathbf{M}_i \) in ferromagnetic non-ferroelastics (co-elastics) and inside ferroelectric and ferroelastic domains. In ferromagnetic fully and partially ferroelastic phases it permits also reorientation of the ferromagnetic/ferroelastic domains due to coupling of the orientation of \( \mathbf{M}_i \) with that of \( \varepsilon_{ij} \). Ferrobimagnetic reorientation \( \Delta G \propto \Delta \chi_{ij} \mathbf{H}_i \mathbf{H}_j \) and a magnetostrictively induced effect \( \Delta G \propto \Delta \varepsilon_{ij} (\lambda \gamma_{ijk}/s_{ijkl}) \mathbf{H}_i \mathbf{H}_j \); without summation; see item 6), may contribute, too. In full ferroelastics/full ferroelastics the magnetic field controls all ferromagnetic and ferroelectric/ferroelastic states.

4. The driving potential for reversing \( \mathbf{P}_i \) by 180° in co-elastic ferroelastics and inside ferroelastic domains, and for reorientation of \( \mathbf{P}_i \) (direction and sense) and of \( \varepsilon_{ij} \) in fully ferroelastic/fully ferroelastics is \( \Delta G \propto \Delta \varepsilon_{ij} \mathbf{E}_i \mathbf{E}_j \). Reorientation by ferrobielectric \( \Delta G \propto \Delta \varepsilon_{ij} (\gamma_{ijk}/s_{ijkl}) \mathbf{E}_i \mathbf{E}_j \); without summation; \( \gamma_{ijk} \) = electrostriction coeff., \( s_{ijkl} \) = elastic compliance) is also possible. For a crystal of Ensemble 1 (Tab.1,II) an electric field can command all fully coupled \( \mathbf{P}_i \mathbf{E}_j \) states, but \( \mathbf{M}_i \) being invariant under space inversion, it can control only the direction of \( \mathbf{M}_i \) and not its sense.
Probably nickel iodine boracite Ni$_3$B$_7$O$_{13}$I is so far the only material, on which an electric field induced reorientation (rotation of 90°) of $^5$M$_i$ and $^6$B$_{ij}$ and a electric field induced reorientation of $^5$P$_i$ (by ~180°) was realized (below 61K)[8,9]. In first experiments species 43m1Fm'm2 was mimicked[8,9] due to layered monoclinic domains. Later the true species was determined as 43m1Fm[10,11]. Table II/Ensemble 1 gives other potential boracite candidates.

5. Potential reorientation and spin reversal of antiferromagnetic domains.

5.1 Reorientation of antiferromagnetic domains by magnetic field, electric field, or mechanical stress, and herewith reorientation of their spin directions, is obligatorily linked with ferroelasticity and may in principle operate via ferroelectric, ferroimagnetic, ferroelastic, magneto-, or electrostrictively induced or ferroelectric domain reorientation, in the latter case in fully or partially coupled ferroelectric/ferroelastic phases. The ferroimagnetic driving potential $\Delta G = \Delta \chi_{ij} H_i H_j$ may reorient antiferromagnetic/ferroelastic domains and herewith their spin directions. For NiO (m 31F 3 3 m [4x2]: at T<T$_N$=525K[1,2]), DyVO$_4$ (4/mmm 1Fmm'm'(p) [2x2]?): T<T$_N$=3.04K[13], *T* $\perp$ m') and TbPO$_4$[14] 4/mmm 1F2/m's) [4x2]? (Ensemble 21) such magnetic field induced reorientations have been achieved. However, so far it is not clear, to what proportions ferroimagnetic and/or magnetostrictive (see item 6) interactions were responsible.

5.2 Antiferromagnetic domain "poling" (180°-spin reversal) in non-ferroelastic (co-elastic) phases or inside antiferromagnetic/ferroelastic domains is possible in 39 antiferromagnetic Shubnikov point groups, (among a total of 59, see e.g. Table II/ref. [15]), permitting the magnetoelectric $\alpha_{ij} E_i H_j$ term and the driving potential $\Delta G = \Delta \alpha_{ij} E_i H_j$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching). For four (6', 6'm2, 6'2'm, 6'm2) of the remaining 20 groups, switching is in principle possible with the magnetoelectric $\gamma_{ijk} H_i E_j E_k$ term and the driving potential $\Delta G = \Delta \gamma_{ijk} E_i H_j E_k$ ("(ferro)magnetoelastic" switching).
In a spin ordered phase allowing $E_i H_j H_k$, the contribution of the spin system would be expected to be opposite in sign for spin reversed domains, but smaller or larger than the non-reversible lattice contribution.

6. Potential reorientation of para- or diamagnetic, ferroelectric and ferroelastic domains. i) A magnetic field alone can in principle act only via the ferrobiomagnetic driving potential $\Delta G \propto \chi_{ij} H_i H_j$ for reorienting the fully or partially ferroelastic domains of full and partial ferroelectrics and non-ferroelectrics. The most interesting species are found in the fully ferroelectric/fully ferroelastic Ensemble 28, where the ferroelectric domains are identical with the ferroelastic ones, thus the magnetic field has in principle full control both over the orientation of all ferroelastic domains and due to the full coupling, even over the sense of $'P_i$. The only materials on which such an experiment has been realized, seem to be the phosphates $\text{Tb}_2(\text{MoO}_4)_3$ and $\text{TbGd}(\text{MoO}_4)_3$ (species 42m1'Fmm21'[2]). On the former one the application of a magnetic field of 10 Tesla (at 78K in the paramagnetic phase) along the orthorhomic b-axis leads to alignment of the a-axis along the field and 180°-reversal of $'P_i$ along the c-axis. The hysteresis cycle had been repeated by interchanging the direction of the magnetic field by 90°. The phenomenon was attributed to a difference in magnetostriction along the a- and b-axes. The tertiary ferroic magnetostriction driving potential is $\Delta G \propto \Delta \lambda_{ij} \sigma_{ij} H_i H_j$, requiring application of both magnetic field and stress. However, since a magnetic field alone reorienting the domains, the secondary ferroic driving potential involving magnetostriction, $\Delta G \propto \Delta \varepsilon_{ijkl}(\lambda_{ijkl}/s_{ijkl}) H_i H_j$ ($\lambda_{ijkl}$ = magnetostriction coefficient, $s_{ijkl}$ = elastic compliance), must have been responsible in the case of $\text{Tb}_2(\text{MoO}_4)_3$.

CONCLUSION

The symmetry possibilities have been evaluated for magnetic field induced reorientation of ferroelectric and ferroelastic domains, and for electric field induced reorientation of magnetic domains. Fully controlled reorientation and switching of domains is possible in the ensembles of species Fully ferroelectric / Fully ferroelastic on the one hand and Fully ferromagnetic, Antiferromagnetic or Para-/or diamagnetic on the other hand. For such species a magnetic field alone can in principle control all possible states of $'M_i$, $'P_i$ and $'\varepsilon_{ij}$, but not the sense of the spins of antiferromagnetic domains, and an electric field can control the orientation of all ferroelectric/ferroelastic states and the orientation, but not the sign of the spins in the ferro- or antiferromagnetic domains. In certain species (e.g. Ensemble 1 / 43m1'Fmm21'[6×2]) the reorientation of H by 90° can reverse $'P_i$ by 180°. An electric field, however, can never reverse $'M_i$ by 180°. So far there is great paucity of experiments on H-induced reorientation of $'P_i$ and $'\varepsilon_{ij}$, and on E-induced reorientation of $'M_i$. 
Table I Ensembles of species with particular ferroic property combinations

<table>
<thead>
<tr>
<th>Ensemble Type</th>
<th>Fully ferroelectric</th>
<th>Partially ferroelectric</th>
<th>Non-ferroelectric</th>
<th>Number of species</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ensemble No.</td>
<td>Number of species</td>
<td>Ensemble No.</td>
<td>Number of species</td>
</tr>
<tr>
<td>Fully ferromagnetic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fully ferroelastic</td>
<td>1</td>
<td>45</td>
<td>2</td>
<td>6</td>
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<td>4</td>
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<td>7</td>
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<td>8</td>
<td>None</td>
</tr>
<tr>
<td>Partially ferromagnetic</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Fully ferroelastic</td>
<td>10</td>
<td>18</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>Partially ferroelastic</td>
<td>13</td>
<td>50</td>
<td>14</td>
<td>31</td>
</tr>
<tr>
<td>Non-ferroelastic</td>
<td>16</td>
<td>18</td>
<td>17</td>
<td>8</td>
</tr>
<tr>
<td>Antiferromagnetic</td>
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<tr>
<td>Fully ferroelastic</td>
<td>19</td>
<td>4</td>
<td>20</td>
<td>None</td>
</tr>
<tr>
<td>Partially ferroelastic</td>
<td>22</td>
<td>9</td>
<td>23</td>
<td>5</td>
</tr>
<tr>
<td>Non-ferroelastic</td>
<td>25</td>
<td>11</td>
<td>26</td>
<td>3</td>
</tr>
<tr>
<td>Para- or diamagnetic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fully ferroelastic</td>
<td>28(1)#</td>
<td>42</td>
<td>29(IV)</td>
<td>6</td>
</tr>
<tr>
<td>Partially ferroelastic</td>
<td>31(II)</td>
<td>31</td>
<td>32(III)</td>
<td>17</td>
</tr>
<tr>
<td>Non-ferroelastic</td>
<td>34(VI)</td>
<td>15</td>
<td>35(VIII)</td>
<td>8</td>
</tr>
</tbody>
</table>

Number of species:
- Fully ferroelastic: 243
- Partially ferroelastic: 90
- Non-ferroelastic: 440
- Total: 773

#) Roman numbers refer to the corresponding numbers in Table I a), b), c), d) of H. Schmid, in: N. Setter and E.L. Colla, Eds, Ferroelectric Ceramics, Monte Verità, Birkhäuser Verlag, Basel, 1993, pp. 107-126/112
Table II  Examples of materials in Ensembles of Species

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Fully ferromagnetic/Fully ferroelectric/Fully ferroelastic</strong>&lt;br&gt;4m1Fm'm2' [6x2]: Ni3B2O13Cl[11], Ni3B2O13Br (at 29K&gt;T&gt;21K)[11], Co3B2O13Br[11], Co3B2O13I[11], Co3B2O13NO3(?) [11], Mn3B2O13Cl[20], Mn3B2O13Br[20], Mn3B2O13J[20,21]. Ferrotoroidal, ( T \parallel ) a-axis, i.e. ( \perp m' )&lt;br&gt;4m1Fm' [12x2]: Ni3B2O13[10], Ferrotoroidal, ( T \parallel b )-axis, i.e. ( \perp m' )&lt;br&gt;4m1F1 [24x2]: Ni3B2O13Br at T&lt;21K[11], Ferrotoroidal, ( T ): arbitrary</td>
</tr>
<tr>
<td>2</td>
<td><strong>No examples known</strong></td>
</tr>
<tr>
<td>3</td>
<td><strong>Fully ferromagnetic/Non-ferroelectric/Fully ferroelastic</strong>&lt;br&gt;m 3m 1F4/mm'm'[3x2]: ( \alpha )-Fe (( \alpha )-iron) at T&lt;( T_c = 1042 )K&lt;br&gt;m 3m 1F 3m' [4x2]: magnetite Fe3O4. ( T_e=858K&gt;T&gt;T_{Verwey}=119K) [11]&lt;br&gt;m 3m 1F2/m [3x2]: hematite ( \alpha )-Fe2O3 (( T_e=950K&gt;T&gt;250K) [19]</td>
</tr>
<tr>
<td>4, 5, 6, 7, 8</td>
<td><strong>Not allowed</strong></td>
</tr>
<tr>
<td>9</td>
<td><strong>Fully ferromagnetic/Non-Ferroelectric/Non-ferroelastic</strong>&lt;br&gt;4/mmm1F4/mm'm'[2]: CrO2, 6/mmm1F6/mm'm'[2]: BaO.6Fe2O3&lt;br&gt;m 2m1Fm'2'm [2]: Ga2Fe6O3 pyroelectric, ferrotoroidal[22], ( T \parallel m' )&lt;br&gt;10</td>
</tr>
<tr>
<td>11, 12</td>
<td><strong>No examples known</strong></td>
</tr>
<tr>
<td>13</td>
<td><strong>Partially ferromagnetic/Fully ferroelectric/Partially Ferroelastic</strong>&lt;br&gt;m 3m 1F1[48x2]: Magnetite Fe3O4 at T&lt;T_{Verwey} = 119K[11], Ferrotoroidal, ( T ): arbitrary direction</td>
</tr>
<tr>
<td>14, 15, 16, 17, 18</td>
<td><strong>Not allowed</strong></td>
</tr>
<tr>
<td>19</td>
<td><strong>Antiferromagnetic/Fully ferroelectric/Fully ferroelastic</strong>&lt;br&gt;3m1Fmm2 [6x2]: Cr3B2O13Cl 13.5K&gt;T&gt;9.7K[11,24], Ferrotoroidal, ( T \parallel c )&lt;br&gt;<strong>Not allowed</strong></td>
</tr>
<tr>
<td>20</td>
<td><strong>Antiferromagnetic/Non-ferroelectric/Fully ferroelastic</strong>&lt;br&gt;m 3m 1F 3m [4x2]: NiO at T&lt;( T_N = 525K ) [12]&lt;br&gt;4/mmm1Fmm'm'(p) [2x2] (?): DyVO4 (T&lt;( T_N = 3.04K ) [12], ( T \parallel m' )&lt;br&gt;4/mmm1F2/m'(s) [4x2] (?): TbPO4[13] (( T_N = 2.28K&lt;T&lt;T_{N2} = 2.13K ))</td>
</tr>
<tr>
<td>22</td>
<td><strong>Antiferromagnetic/Fully ferroelectric/Partially ferroelastic</strong>&lt;br&gt;m 3m 1F&quot;3m1&quot; [6x2]: BiFe3O5, antiferromagnetic/INc.[25,26], see env. 31</td>
</tr>
<tr>
<td>23</td>
<td><strong>Antiferromagnetic/Partially ferroelectric/Partially ferroelastic</strong>&lt;br&gt;No example known</td>
</tr>
<tr>
<td>24</td>
<td><strong>Antiferromagnetic/Non-ferroelectric/Partially ferroelastic</strong>&lt;br&gt;m 3m 1F4/m [6x2]: Garnet Ca3Mn2Ge2O12[27]</td>
</tr>
<tr>
<td>Number</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
</tbody>
</table>
| 25     | *Antiferromagnetic/Fully ferroelectric/Non-ferroelastic*  
6/mmm 1 'F6' mm'  
(YMnO₃ and REMnO₃ (RE = rare earth)[15]) |
| 26     | *Antiferromagnetic/Partially ferroelectric/Non-ferroelastic*  
No example known |
| 27     | *Antiferromagnetic/Non-ferroelectric/Non-ferroelastic*  
3 m 1 'F3' m'  
(hematite α-Fe₂O₃ at T< Tₕ=250K[12,19]) |
|         | 4/mmm 1 'F4' /m'm'  
(CoF₂[19] (α-Fe₂O₃ and CoF₂ are piezomagnetic) |
| 28     | "(I)" *Para- or diamagnetic/Fully ferroelectric/Fully ferroelastic*  
4 2/m Fmm21 [2]: GMO(Gd₂(MoO₄)₃)-type²⁹, Tb₂(MoO₄)₃[14];  
KDP-type²⁹, TANANE³⁰, RblLiMoO₄[11];  
4 3/m F3m1 [4]: Fe₃B₇O₁₃Cl[6], Fe₃B₇O₁₃Br[6], Fe₃B₇O₁₃I[6];  
Co₃B₇O₁₃Cl[6], CsLiMoO₄[6,3¹], CsLiWO₄[6,3¹], RblLiMoO₄[6,3¹];  
4 3/m F3m1 [12]: Fe₃B₇O₁₃X (X=Cl, I)[6], M₂B₇O₁₃Cl (M=Co, Zn)[6] |
| 29     | "(IV)" *Para- or diamagnetic/Partially ferroelectric/Fully ferroelastic*  
No example known |
| 30     | "(V)" *Para- or diamagnetic/Non-ferroelectric/Fully ferroelastic*  
4/mmm 1 'F4' Fmm21 [2]: Ge-fresnoite Ba₂TiGe₂O₈[6], polar only  
4/mmm 1 'F2' /m(s) [4]: VO₂[6];  
4/mmm 1 'F2' /m(p) [4]: K₂Fe₃F₁₅[13²]  
4/mmm 1 'Fmm1' [2]: YBa₂Cu₃O₆[3³] |
| 31     | "(II)" *Para- or diamagnetic/Fully ferroelectric/Partially ferroelastic*  
m 3 m 1 'F3' m'  
(Perovskite BiFeO₃ (pseudo-3m1'-behavior, due to incommensurate antiferromagnetic structure at T< Tₕ=653±3K[2²5,26])  
m 3 m 1 'F4' m'm' [6] and  m 3 m 1 'F3' m' [8]: BaTiO₃[3⁴];  
m 3 m 1 'Fmm21' (s) [12]: BaTiO₃[3⁴], Al-Sodalite Sr₃(Al₁₂O₃₄)(CrO₄)₂[3⁵] |
| 32     | "(III)" *Para- or diamagnetic/Partially ferroelectric/Partially ferroelastic*  
m 3 m 1 'F4' [12] and m 3 m 1 'F3' [12]: Pyrochlore Cd₂Nb₂O₇[3⁶]  
m 3 m 1 'F4' [12]: YBa₂Cu₃O₆[3³] |
| 33     | "(VII)" *Para- or diamagnetic/Non-ferroelectric/Partially ferroelastic*  
No example known |
| 34     | "(VI)" *Para- or diamagnetic/Fully ferroelectric/Non-ferroelastic*  
61 'F3' [2]: Pb₅Ge₃O₁₁, electro-ambidextrous[3⁷]  
2/m1 'Fm' [2]: LiH₃(SeO₃)₂[3⁸], pseudo-electro-ambidextrous[3⁸]  
m 3 m 1 'Fmm21': NaNO₃[3⁸], pseudo-electro-ambidextrous[3⁸] |
| 35     | "(VIII)" *Para- or diamagnetic/Partially ferroelectric/Non-ferroelastic*  
No example known |
| 36     | "(IX)" *Para- or diamagnetic/Non-ferroelectric/Non-ferroelastic*  
621 'F321' [2]: α-quartz, ferrobielastic[1];  
m 3 m 1 'F4' 3m1' [2]: NH₄Cl, ferroelastoelectric[1] |
Acknowledgements
The author thanks Jean-Pierre Rivera for helpful advice.

References