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Abstract

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Reference


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Proposal for exploring macroscopic entanglement with a single photon and coherent states

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I. INTRODUCTION

Why do we not easily observe entanglement between macroscopically populated (macro) systems? Decoherence is widely accepted as being responsible [1]. Loss or any other form of interactions with the surroundings destroys and more rapidly the quantum features of physical systems as their size increases. Technologically demanding experiments, involving Rydberg atoms interacting with the electromagnetic field of a high-finess cavity [2] or superconducting devices cooled down to a few tens of mK [3], have strengthened this idea.

Measurement precision is likely another issue [4]. In a recent experiment [5], a phase covariant cloner was used to produce tens of thousands of clones of a single photon belonging initially to an entangled pair. In the absence of loss, this leads to a micro-macro entangled state [6]. Nobody knows, however, how the entanglement degrades with a lossy amplification [7]. This led to a lively debate [8] concerning the presence of entanglement in the experiment reported in Ref. [5]. What is known is that under moderate coarse-grained measurements, the micro-macro entanglement resulting from a lossless amplification leads to a probability distribution of results that is very close to the one coming from a separable micro-macro state [9]. This suggests that even if a macro system could be perfectly isolated from its environment, its quantum nature would require very precise measurements to be observed.

Both for practical considerations and from a conceptual point of view, it is of great interest to look for ways that are as simple as possible to generate and measure macro entanglement, so that the effects of decoherence processes and the requirements on the measurements can all be studied together. In this paper, we focus on an approach based on linear optics only, where a single photon and a coherent state are combined on a mere beamsplitter. We show that the resulting path-entangled state [10] could be useful in quantum metrology for precision phase measurement. More importantly, it allows one to easily explore entanglement over various photon number scales, spanning from the micro to the macro domain, by simply tuning the intensity of the laser producing the input coherent state. We show that the entanglement is more and more sensitive to phase fluctuations between the paths when it grows. However, it features surprising robustness against loss, making it well suited to travel over long distances and to be stored in atomic ensembles. We further present a simple and natural method relying on local displacement operations in the phase space and basic photon detections to reveal the entanglement. Our analysis shows that the precision of the proposed measurement is connected to the limited ability to control the phase of the local oscillator that is used to perform the phase-space displacements. We also report on preliminary experimental results demonstrating that entanglement containing more than 1000 photons could be created and measured with currently available technologies.

II. CREATING MACRO ENTANGLEMENT BY COMBINING A SINGLE PHOTON WITH A BRIGHT COHERENT STATE ON A BEAMSPLITTER

A particularly simple way of generating entanglement is to use a beamsplitter. Consider a single photon sent through a 50:50 beamsplitter. It occupies the two output modes \( A \) and \( B \) with the same probability and creates a simple form of entanglement after the beamsplitter if and only if the input state of the form \( \rho_A \otimes |\beta\rangle \langle \beta|_B \) is nonclassical [12,13], i.e., it cannot be written as a mixture of coherent states [14]. Hence, a mere beamsplitter links two fundamental concepts of quantum physics: nonclassicality and entanglement. It also provides an attractive way to bring entanglement to a macroscopic level, as explained below.

Let us focus on the case in which the beamsplitter inputs are a single photon \( |1\rangle \) and a coherent state with \( 2|\alpha|^2 \) photons on average (see Fig. 1),

\[
|\psi_{in}\rangle = a^\dagger |0\rangle_A \otimes D_B(\sqrt{2}\alpha)|0\rangle_B.
\]

(2)

\( D_B(\sqrt{2}\alpha) = e^{\sqrt{2}\alpha b^\dagger - \sqrt{2}\alpha^* b} \) is the displacement operator generating a coherent state \( |\sqrt{2}\alpha\rangle \) from vacuum (in mode \( B \)), and \( a \) and \( b \) are bosonic annihilation operators associated with modes \( A \) and \( B \), respectively. A 50:50 beamsplitter transforms \( (a,b) \)
sensitivity is twice as large as a classical state. Moreover, in the
modes, each being individually a mixture of classical and
it possesses intriguing properties. It corresponds to a non-
displacement of the single-photon entanglement. Nevertheless,
entanglement by combining a single-photon Fock state |
and a coherent state on a beamsplitter, since the resulting state
variance of the photon number: The variance of
beats the coherent state sensitivity (3) to a phase variation
property distinguishes the state (3) from another path entangled
three times greater than the one characterizing
One easily shows that
for definition), decreases exponentially,
with size |N|2 and loss 1 −η [20]. In comparison, the state (3) exhibits a surprising robustness. Under the assumption
that mode B undergoes loss, |ψout⟩ becomes a statistical mixture of
and |ψout⟩ = Db(−√η/α)|0⟩B − √η/α)|1⟩B and
with weights 1+η α and 1−η α, respectively. After applying local displacements Da(−α) and
modes A and B, one finds that the negativity of
the resulting state is given by N = 2α. Since the entanglement
cannot increase through local operations, this provides a lower
bound for the entanglement before the displacements, i.e.,
between the macroscopically populated modes. Therefore, the
amount of entanglement in |ψout⟩ decays (at worst) linearly
with loss, independently of its size. This robustness may be
understood in light of the intimate link between nonclassicality
and entanglement at a beamsplitter, as mentioned before.
Indeed, loss, modeled by a beamsplitter, can be seen as
an interaction process entangling the nonclassical states and
the environment. However, the displacement is a classical
operation that does not promote the entanglement of a given
quantum system with its environment when it is amplified
|Db(α) → Db(√ηα)Dα(ejθ)[17]. The robustness of
the state (3) makes it well suited for storage in an atomic medium.
Entanglement between two ensembles, each containing a
macroscopic number of atoms, has been successfully created
by mapping a single-photon entanglement into two atomic
ensembles [21,22]. The storage of the displaced single-photon
entanglement |ψout⟩ would lead to a similar entanglement in
terms of the number of ebits [23], but it would contain a
macroscopic number of excited atoms.

III. PRECISION PHASE MEASUREMENT

The sensitivity s of a state ρ0 (describing the modes A and
B) to a phase variation Uρ = ejejρA can be estimated through
s = |< ρ0 |a⟩< a |ρ0 >|2 = ρ0 Uρ ρ0 Uρ† + |A⟩⟨A|, which reduces to
twice the variance of the photon number s = 2 var(a†a)
for pure states. Since the variance of the state (3) is twice as large
as a coherent state with the same mean photon number, its
sensitivity is twice as large as a classical state. Moreover,
in the presence of loss, the sensitivity of the displaced single-photon entanglement becomes s = 2η|α|2(1 − 3η + 4η2) + cte and
beats the coherent state sensitivity (sA = 2η|α|2) if the loss
1 − η does not exceed 1/2. For comparison, the sensitivity of the
NOON state s(N) = (N, 0) ideality scales as the square of
the photon number N, but it falls down below sA (for the
same photon number |α|2 = N/2) as soon as η < (1 − 1/2N−1). This makes the sensitivity of the NOON state very hard to use in practice [e.g., 1 − η has to be smaller than 2% (0.3%) for
N = 100 (1000) for NOON states to be useful] and highlights
the potential of the state (3) for precision phase measurement.
We now focus on the robustness of entanglement with respect
to loss and phase noise.

IV. ROBUSTNESS WITH RESPECT
to TRANSMISSION LOSS

In general, entanglement is seen to be increasingly fragile
to transmission loss as its size increases. Coherent state
entanglement |a⟩A|a⟩B + |−a⟩A|−a⟩B [17] provides a good example.
If the mode B is subject to loss (modeled by a beamsplitter with transmission coefficient η), the amount of entanglement, measured by the negativity (see [18,19]
for definition), decreases exponentially, N = 1/(2−η|α|2),
loss [25]. Nonetheless, the state becomes separable as
large loss [25].

FIG. 1. (Color online) Creation and detection of macro
Entanglement creation Entanglement detection

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following the lines presented in the previous paragraph, that the negativity of the displaced single-photon entanglement scales like $\mathcal{N} = -\frac{1}{2}[1 - \eta_c - \sqrt{1 - 2(1 - \eta_c)\eta_c}] \approx \frac{\eta_c}{2} + O(\eta_c^2) \geq 0$, where $\eta_c$ stands for the coupling efficiency of the input single photon. This robustness confers to the proposed scheme a great practical advantage over the one based on the universal cloner.

VI. ROBUSTNESS WITH RESPECT TO PHASE INSTABILITIES

Another decoherence process for path entanglement is associated with the relative phase fluctuations, due to, e.g., vibrations and thermal fluctuations. If the two optical paths corresponding to $A$ and $B$ acquire a phase difference $\varphi$, the displaced single-photon entanglement becomes $|\psi_{\text{out}}\rangle = \frac{1}{\sqrt{2}}[D_a(\alpha e^{i\varphi})|\alpha\rangle_B - |\alpha\rangle_A D_b(\beta)|\beta\rangle_B]$. Furthermore, if $\varphi$ varies from trial to trial, the state $|\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|$ has to be averaged over $\varphi$ with the probability distribution $p(\varphi)$ associated with the phase noise. The question of the sensitivity of the displaced single-photon entanglement with respect to phase instability thus reduces to a measure of the entanglement contained in $\rho_{\text{out}} = \int d\varphi p(\varphi)|\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|$. The negativity of this state can easily be obtained numerically by projecting $\rho_{\text{out}}$ into a finite-dimensional Hilbert space [see the solid lines in Fig. 2] for a Gaussian probability distribution $p(\varphi)$ with variance $\delta\varphi$. To derive an analytical lower bound on the negativity of this state, we first notice that for any density matrix $\rho$ and any vector $|v\rangle$, the following inequality holds: $\langle v|\rho|v\rangle \geq \lambda_{\text{min}} \geq -\mathcal{N}_0$, where $\Gamma$ denotes partial transposition and $\lambda_{\text{min}}$ is the smallest eigenvalue of $\rho^\Gamma$. For the state (3), where $p(\varphi) = \delta(0)$, it is easy to verify that the vector $|v\rangle$ saturating the inequality is $|v\rangle = \frac{1}{\sqrt{2}}D_a(\alpha)D_b(\beta)|0\rangle - |1\rangle|1\rangle$. For a general $\rho_{\text{out}}$, it is not optimal, however it provides a lower bound for the estimation of $\mathcal{N}$. We find $\mathcal{N}_{\text{est}}(\delta\varphi,|\alpha|^2) \geq \frac{1}{2} \int d\varphi p(\varphi)\ln \left(\frac{|\alpha|^2[1 - \cos(2\varphi)] - \cos\varphi}{\cos\varphi}\right)$ (the dashed line in Fig. 2 is obtained by performing the integral numerically for a Gaussian probability distribution). For a Gaussian probability distribution $p(\varphi)$ with variance $\delta\varphi$, the lower bound can be approximated by $\frac{2 - 2\varphi}{4(1 + 2|\alpha|^2\delta\varphi)^2}$ (see the dot-dashed line in Fig. 2) [26]. Figure 2 reveals what is expected from a macroscopic quantum state: the larger the size $2|\alpha|^2 + 1$ of the state is, the more it becomes sensitive to phase noise.

VII. REVEALING DISPLACED SINGLE-PHOTON ENTANGLEMENT

So far, we have discussed the properties of the state (1). We now present a simple way to reveal its entanglement. The basic idea is to displace each of the electromagnetic fields describing the modes $A$ and $B$ by $-\alpha$. Such a displacement in the phase space can be easily performed by mixing the mode to be displaced with an auxiliary strong coherent field (labeled as the local oscillator in the following) on a highly unbalanced beamsplitter [28], in a manner similar to homodyne measurements. Since $D_a(-\alpha) = D_a(\alpha)^{-1}$, the modes $A$ and $B$ ideally end up in the state (1), which can be revealed by tomography using single-photon detectors. Note that a similar approach was proposed in [29] to reveal entanglement in the scenario where a macroscopic state is created by phase-covariant cloning of an entangled photon pair. The authors proposed locally undoing the cloning before doing the measurement.

The approach developed in Ref. [30] does not require a full tomography after the local displacements. It gives a lower bound on the entanglement between the modes $A$ and $B$ from the estimation of the entanglement contained in the two-qubit subspace $\{|00\rangle, |01\rangle, |11\rangle, |10\rangle\}$. More precisely, the concurrence $C$ of the detected fields is bounded by $C \geq \max[0, V(p_{01} + p_{10} - 2\sqrt{p_{01}p_{10}})]$ [30], where $V$ is the visibility of the interference obtained by recombining the modes $A$ and $B$ on a 50:50 beamsplitter, and the coefficients $p_{mn}$ are the probabilities of detecting $m$ photons in $A$ and $n$ in $B$. This tomographic approach for characterizing single-photon entanglement is attractive in practice and already triggered highly successful experiments, demonstrating, e.g., heralded entanglement between atomic ensembles [21,22,30–32].

Note that the statistical fluctuations in the phase of local oscillators that are used to perform the displacements limit the precision of the measurement process. Under the assumption that the two local displacements $D_a(-\alpha), D_b(-\alpha)$ are performed with a common local oscillator, the measured state is of the form $\int d\varphi \rho(\varphi)D_a(-\alpha e^{i\varphi})D_b(-\alpha e^{i\varphi})\rho_{\text{out}}D_a(-\alpha e^{i\varphi})D_b(-\alpha e^{i\varphi})|\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|$. Let $V$ be the visibility of the interference that characterizes the phase stability of the local oscillator. One can show that for small imperfections $\epsilon = (1 - V) \ll 1$, the concurrence is bounded by $C \geq \max[0, 1 - 10(1 - V)|\alpha|^2]$. The necessary precision of the measurement thus scales as $\frac{1}{\epsilon} \approx 10\epsilon^{-1}$. This result strengthens the idea that precise measurements are generally essential for revealing quantum properties of macro systems.

VIII. PROPOSED EXPERIMENT

We now address the question of the experimental feasibility in detail. For concreteness, we focus on a realization of the
single-photon source from a pair source based on spontaneous parametric downconversion, the detection of one photon heralding the production of its twin. Filtering techniques must be used so that the mode of the heralded photon can be made indistinguishable from that of the coherent state produced. (In Ref. [33], we show how one can take mode mismatches into account.) Let η be the coupling efficiency of the single photon, and let η be the global detection efficiency, including the transmission from the 50:50 beamsplitter to the detector, as before. For small heralding efficiency, and if the parametric process is weakly pumped so that the detector, as before. For small heralding efficiency, and if the parametric process is weakly pumped so that the phase fluctuations with the same variance). To know the visibility that characterizes the phase stability of A, B and the local oscillator (we assume that the modes have independent phase fluctuations with the same variance). To know the value of the visibility that can be obtained in practice, we built a balanced Mach-Zehnder interferometer (see Ref. [33]). Using an active stabilization, we measured a visibility of V = 99.996 ± 0.001%. Assuming a coupling efficiency η = 50% and a detection efficiency η = 60%, the concurrence remains positive (C ≈ 0.01) for |α| = 28. This translates into entanglement populated by more than (2|α|^2 + 1) = 1500 photons.

**IX. CONCLUSION**

We have presented a scheme for creating and revealing macroscopic entanglement with a single photon, coherent states, and linear optical elements. The simplicity of our proposal is conceptually remarkable. On the one hand, it highlights the idea that although quantum systems are difficult to maintain and observe at macro scales, they can easily be created. On the other hand, it naturally raises the following question: is the resulting state really macroscopic? We have shown through experimental results that the entangled state that could be obtained with currently available technologies would involve a large enough number of photons to be seen with the naked eye [34]. This makes our approach satisfactory if macroscopicness is a notion related to size. We also mentioned that the components of the entangled state can easily be distinguished with a mere avalanche photodiode if one looks at the variance of the photon number distribution. This pleases those who believe that macro entangled states need to have components that can be easily distinguished. Although our study showed that the resulting state features an unexpected robustness against loss, we have shown that it is also more and more fragile under phase disturbance when its size increases. Our approach is thus also satisfactory if macroscopic means sensitive to decoherence, and it highlights the complexity of possible interactions between a given quantum system and its surroundings. We have also seen that the precision of the measurement that is required to reveal the quantum nature of the produced state increases with its size. This also makes our scheme satisfactory if macroscopicness is related to the requirement on the measurement precision. Note that there are many other candidates for macroscopicity measures [35]. Testing each of them is work for the future.

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[7] In the presence of loss before and after the phase covariant cloner, we do not know when the amplified state becomes separable.
[18] Negativity N [19] conveniently quantifies entanglement. It is defined by N = Σ λ, where λ are the eigenvalues of the partially transposed density matrix. According to this definition, the negativity of a two-qubit state is bounded by 1/2.
[20] Consider transmission efficiency η such that η|α|^2 ≫ 1. Furthermore, consider that the mode B undergoes loss. The coherent state entanglement becomes |α⟩_A (√|α|) (√|α|) |⟨−√|α|⟩_B |√I − |⟨−√|α|⟩_B | − |⟨−√|α|⟩_B | − |⟨−√|α|⟩_B | , where ℓ stands for the lost mode. Tracing η out, one gets

\[
\begin{align*}
\frac{1}{2} \left( (|α⟩_A (\sqrt|α|_A) (\sqrt|α|_A) + (−|α⟩_A) (−\sqrt|α|_A) (−\sqrt|α|_A) \right) \\
+ e^{−2(1−η)|α|^2} |α⟩_A (\sqrt|α|_A) (−|α⟩_A) (−\sqrt|α|_A) \\
− e^{−2(1−η)|α|^2} |−|α⟩_A) (−|α⟩_A) (−\sqrt|α|_A) \\
− e^{−2(1−η)|α|^2} |−|α⟩_A) (−|α⟩_A) (−\sqrt|α|_A) 
\end{align*}
\]

The smallest eigenvalue associated with its partial transpose is 

\[−e^{−2(1−η)|α|^2}.\]

Note that if multimode memories are used, the number of ebits could easily be increased by combining time-bin qudits with a bright coherent state on a beamsplitter.


In practice, the phase stability is usually characterized by the classical interferometric visibility \( \bar{V} \) that would be obtained by recombining modes \( A \) and \( B \) on a 50:50 beamsplitter [27]. Using the relation between the visibility and the phase fluctuation

\[
\bar{V} = e^{-\delta\bar{\phi}^2} / \left( 1 + e^{-\delta\bar{\phi}^2} \right),
\]

we obtain

\[
N_{\text{out}}(\bar{V}, |\alpha|^2) \geq \bar{V}^2 \left[ 1 + 4 |\alpha|^2 (1 - \bar{V}) \right]^{1/2}.
\]

In the limit \( |\gamma| \to \infty \) and \( r \to 0 \) such that \( \gamma r = -\alpha \), any input state \( \bar{\rho} \) evolves as \( D(-\alpha)\bar{\rho}D^\dagger(-\alpha) \), that is, it is displaced in the phase space by \(-\alpha\); see M. G. A. Paris, Phys. Lett. A 217, 78 (1996) for detailed proof. For small but nonzero reflectivity, \( \gamma \) can always be tuned to fulfill \( \gamma r = -\alpha \), so that the corresponding operation simply corresponds to a displacement \( D(-\alpha) \) followed by loss modeled with a beamsplitter with amplitude transmission \( r \).


