Experimental constraints on the outgassing dynamics of basaltic magmas

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Abstract
The dynamics of separated two-phase flow of basaltic magmas in cylindrical conduits has been explored combining large-scale experiments and theoretical studies. Experiments consisted of the continuous injection of air into water or glucose syrup in a 0.24 m diameter, 6.5 m long bubble column. The model calculates vesicularity and pressure gradient for a range of gas superficial velocities (volume flow rates/pipe area, $10^{-2} – 10^2$ m/s), conduit diameters (100–2 m), and magma viscosities (3–300 Pa s). The model is calibrated with the experimental results to extrapolate key flow parameters such as Co (distribution parameter) and Froude number, which control the maximum vesicularity of the magma in the column, and the gas rise speed of gas slugs. It predicts that magma vesicularity increases with increasing gas volume flow rate and decreases with increasing conduit diameter, until a threshold value (45 vol.%), which characterizes churn and annular flow regimes. Transition to annular flow regimes is expected to occur at minimum gas volume flow rates of $103–104$ m$^3$/s. The vertical pressure gradient decreases with [...]

Reference

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Experimental constraints on the outgassing dynamics of basaltic magmas

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1. Introduction

[2] Basaltic eruptive activity is highly variable in intensity, ranging from low-energy lava effusion, fire fountaining, and intermittent Strombolian explosions, to more energetic ash-forming violent Strombolian, subplinian, and Plinian activity. Moreover, unlike silica-rich volcanism, there is no direct relationship between explosivity and magma flux, due to the complex interplay between gas segregation, initial gas content of the magma, and ascent rate. The low viscosity of basaltic magmas allows for separated two-phase (liquid and gas) flows, which play a fundamental role in the shallow conduit and explosive dynamics. In particular, bubble concentration (and its vertical and lateral gradients), bubble size distribution, and vertical pressure drop within the magma column at shallow levels control the gas rise dynamics and, consequently, define the eruptive style and explosivity. For example, intermittent explosions may be generated when the pattern is periodic (i.e., characterized by regular temporal and spatial fluctuations), and the pressure drop within the magma column and the rise rate (which controls degassing) also control overpressure buildup, together with near surface region processes [James et al., 2008].

[3] Previous studies have described the main categories of basaltic explosive eruptions (i.e., Hawaiian and Strombolian) in terms of different two-phase flow regimes in the volcanic conduit (bubbly, intermittent, and annular flow) [Vergniolle and Jaupart, 1986; Jaupart and Vergniolle, 1989; Parfitt and Wilson, 1995; Vergniolle and Mangan, 2000, Figure 1]. Each pattern is defined by a peculiar geometry of the distribution and dynamics of gas and liquid phases, which will be described in details in section 1.1. In particular, slug flow is expected for Strombolian eruptions; Hawaiian eruptions have been explained in terms of homogeneous two-phase flows (where the gas segregation is suppressed by high magma ascent velocity [Wilson and Head, 1981]) or annular flows [Vergniolle and Jaupart, 1986]. These studies were also confirmed by evidences from real eruptions [Blackburn et al., 1976; Mangan and Cashman, 1996; Vergniolle and Brandeis, 1996; Polacci et al., 2006]. Transition between conduit flow patterns is
also believed to determine sudden changes of eruptive style, switch from pulsatory to continuous activity, and variations in explosivity [Vergniolle and Mangan, 2000; Parfitt, 2004; Polacci et al., 2006; Parfitt and Wilson, 1995; James et al., 2009; Lyons et al., 2010].

[s] The stability and characteristics of two-phase flow patterns in low-viscosity magmatic flow is typically predicted based on fluid dynamical models developed from a combination of theoretical studies and experiments using very low viscosity (10^{-5}-10^{-6} Pa s) and low surface tension (∼10^{-2} N/m) liquids flowing along small diameters (a few centimeters) pipes [Seifried and Freundt, 2000; Jaupart and Vergniolle, 1989; James et al., 2004, 2009]. These experiments, however, due to incomplete scaling to magmatic systems, were appropriate only for the investigation of specific aspects of the flow dynamics.

[s] More general theoretical models on separated two-phase flow dynamics along vertical pipes have been implemented for engineering applications. They are based on models appropriate for inertially dominated flows and include turbulence effects, which influence coalescence and breakup rates and control the maximum size of stable bubbles [Brown, 1965; Wallis, 1969; Taitel et al., 1980; Mishima and Ishii, 1984; McQuillan and Whalley, 1985; Speeding et al., 1998].

[s] Other models also account for gas segregation mechanisms occurring in the upper portion of the magma chamber, leading to the formation of foam layers [Jaupart and Vergniolle, 1989], or at sills and dykes [Menand and Phillips, 2007; Pioli et al., 2009]. Gas segregation is mostly efficient at very low magma eruption rates, where the bubble slip velocity (i.e., the velocity of the bubble relative to the liquid) approaches or equals the average bubble velocity, favoring outgassing processes. In these conditions, separated two-phase flow dynamics is expected to control the volcanic activity, and, for constant magma and gas supply rates, eruptive style will be controlled by shallow conduit processes.

[s] These conditions are expected at several basaltic volcanoes, which are typically characterized by constant passive degassing from the central vent during intermittent times (Masaya, Miyakejima, Stromboli, Kilauea, Etna) [Allard et al., 1991, 1994; Burton et al., 2000; Kazahaya et al., 2004; Stevenson and Blake, 1998]. Open-conduit degassing from a single volcano is responsible of emission of 10^6 to 10^7 tons of gas per day, consisting in a mixture of H2O, CO2 S, Cl, and F rich gases [Aiuppa et al., 2006; Gerlach et al., 2002] and can represent a significant contribution on the global volcanic emission (for example, up to 8% by mass in the case of Etna volcano [Allard et al., 2002]).

[s] Recent improvements of thermal, video, radar, and acoustic monitoring techniques have provided new powerful tools for the study of outgassing processes and made available geophysical and geochemical data sets for many central volcanoes, such as Stromboli and Etna (Italy), Kilauea (Hawaii), and Villarrica (Chile). These studies reveal that outgassing is an unsteady, often pulsedatory process, characterized by fluctuations in both intensity and composition of the emitted gases [Harris and Ripepe, 2007; Ripepe et al., 2010, 2002; Edmonds and Gerlach, 2007]. However, unambiguous interpretation of infrasound and gas flux measurements in terms surface activity in terms of subsurface flow dynamics responsible is not yet possible due to the limited knowledge of the parameter values that control changes in the separated flow processes in magmas. As a result, it is still not possible to infer the conduit flow regimes and properties from volcano monitoring data and to predict the effect of abrupt shifts in gas and magma flow rates at open conduit volcanoes or during eruptive events in terms of conduit flow and eruption dynamics.

[s] To address this issue, we performed a series of large-scale laboratory experiments and develop a general model for gas rise dynamics into basaltic magmas in cylindrical conduits to gain further insights into (1) how coalescence and segregation of gas bubbles control the flow dynamics, (2) the expected vesicularity and conduit flow pattern characteristics and stability for a range of gas flow rates, and (3) the effect of these processes on volcanic degassing dynamics. These experiments were done within stagnant liquid (i.e., bubbly column dynamics) to ensure ideal conditions for separated flow and are ideally scaled to outgassing dynamics, but the flow general properties can be extrapolated also to regimes with liquid outflux [Mudde and Saito, 2001] and might potentially be applied to describe eruptive regimes characterized by separated two-phase flow dynamics.

[10] This is the first study coupling large-scale experiments and high viscosity (300 Pa s) liquids. These experiments allowed for accurate scaling to magmatic conditions (as explained in section 2.2), the calibration of fundamental parameters and constants and testing of the applicability of more general models [Zuber and Findlay, 1965; Wallis, 1969] to the two-phase flow dynamics of basaltic magmas in volcanic conduits (described in sections 4 and 5). In the next section, we summarize two-phase flow regimes in vertical pipe flow from engineering studies, before describing the experimental method in section 2. In section 3 we present the experimental results, and we present their analysis in section 4. In section 5 we describe the application to volcanic systems, before highlighting the main conclusions in section 6.

1.1. Two-Phase Flow Regimes in Vertical Conduits for Low-Viscosity Magmas

[11] When magma containing exsolved volatiles rises through the upper conduit, different flow regimes or patterns develop. Each pattern is defined by a peculiar geometry of the distribution and dynamics of gas and liquid phases. In silicic magmas, expected flow regimes are homogenous bubbly flow (characterized by noninteracting bubbles distributed in the liquid) and dispersed flow (i.e., liquid drops dispersed in the gas phase), separated by the fragmentation level [Sparks, 1978]. Permeable flow can also occur through interconnected gas bubbles [Jaupart and Allègre, 1991; Saar and Manga, 1999; Blower, 2001; Rust and Cashman, 2004; Melnik et al., 2005], favoring outgassing and effusive eruptions. Individual gas bubbles can separate from the liquid in low-viscosity magmas (e.g., mafic and carbonatitic) and generate a more complex series of flow patterns, which can be grouped into bubbly, intermittent, and annular flows [Vergniolle and Mangan, 2000]. These flow patterns have been originally described for water-air flows [Hewitt and Hall-Taylor, 1970, Figure 1].
The main factors controlling the two-phase flow patterns are the liquid and gas fluxes, conduit geometry, liquid properties, bubble coalescence, liquid and gas entrainment, but also the entrance conditions at the bottom of the pipe (Lucas et al., 2005; Delnoij et al., 1997; Kozma, 1995; Vijayan et al., 2001). Regular oscillations of flow properties (given by variations of gas volumetric fraction and instabilities at liquid/gas interfaces) are characteristics of all patterns and control the pattern transition mechanisms (James et al., 2004). The transition between flow patterns can be described not only in terms of volume or mass fluxes of the phases but also pressure drop, vertical and lateral variation of void fraction, and periodicities in the flow (Delnoij et al., 2005; Clift et al., 1996). The flow becomes unstable when the gas fraction is sufficient to allow local liquid “flooding” (i.e., the liquid is carried upward by the gas) (Wallis, 1969; McQuillan and Whalley, 1985; Guedes de Carvalho and Ferreira, 2000). Flows characterized by the simultaneous downward and upward motion of the liquid film at different points along the pipe walls are highly unstable and called churn flow. Further increases in gas flux promote annular flow, characterized by a central gas phase and upward movement of all the liquid film along the pipe walls because the gas phase has enough energy to fully support the liquid (Hewitt and Hall-Taylor, 1970; Barbosa et al., 2001). Liquid droplets are incorporated into the central gas flux by shearing at the liquid-gas interface. The competition between droplet entrainment and deposition determines the liquid film thickness along the walls (Azzopardi and Wren, 2004). It is important to notice that changes in gas composition should not relevantly affect the stability and characteristic of flow patterns, as most of the properties depend on the density difference between the magma and gas phase.

1.2. Regime Transitions

In separated two-phase flows, transitions between flow regimes are due to bubble migration and coalescence. For any given set of conditions, the bubble size distribution depends on bubble nucleation and growth by diffusion and expansion and the balance between bubble coalescence and breakup (Greveskott et al., 1996; Lehr et al., 2002). Bubble coalescence rate \( \Gamma \) is the sum of the contributions related to turbulence, buoyancy, and laminar flow effects, and for a population of bubbles of class sizes \( i, j \) it can be computed as (Prince and Blanch, 1990):

\[
\Gamma = \frac{1}{2} \sum_{i,j} \left\{ \frac{1}{t_i} + \frac{1}{t_j} + \frac{1}{t_{ij}} \times \exp\left(-t_{ij}/T \right) \right\} \tag{1}
\]

\( \Gamma \) is the bubble coalescence rate determined by the sum of the contributions related to turbulence, buoyancy, and laminar flow, respectively, \( t \) is the coalescence time, and \( T \) is the bubble contact time. Bubbles interact with each other due to velocity fluctuations (turbulence effects), differential rise velocity due to size differences (buoyancy effects), and lateral velocity gradients (laminar shear effects). In very general terms, the lower the viscosity of the fluid, the higher the collision rate, but the lower the contact time and hence coalescence rate. Bubble breakup rate depends on the combination of turbulence, shearing, and surface instability (Clift et al., 1978; Stein and Spera, 1992).

Bubble breakup determines the maximum stable bubble diameter \( (d_{b,\text{max}}) \) and is also important for stability of slug flow when the pipe diameter \( D \geq d_{b,\text{max}} \) (which was the case for our water-airflow experiments). When \( D \geq d_{b,\text{max}} \) the expected flow patterns, with increasing volume gas fluxes, are bubbly to churn to annular and dispersed (Ohnuki and Akimoto, 2000; Omebere-Iyari et al., 2007).

In high Reynolds number flows, bubble breakup is mainly related to turbulence processes, resulting in a maximum stable bubble size which depends on the size (and energy) of the turbulent eddies; the eddies can transport small bubbles, but break bubbles with diameters similar or slightly larger than their size, and do not affect larger bubbles (Hinze, 1955; Prince and Blanch, 1990; Hesketh et al., 1991; Coletta et al., 1999). Owing to the large experimental data set available for these types of flows from engineering studies, several numerical models have been developed for the quantification of this process (Coletta et al., 1999; Olmos et al., 2001; van den Engel et al., 2005). Shear related breakup requires high instantaneous velocity gradients and therefore is expected to be mostly relevant for small bubbles confined in the liquid phase.
film in annular and churn flow or at the sides of the gas slug in slug flow, and it likely plays a minor role in flow pattern stability. In low Reynolds numbers, separated two-phase flows, formation of Rayleigh-Taylor instabilities on the bubble-liquid interface is the main mechanism controlling maximum bubble size \cite{Clift1972, Clift1978}. These instabilities, in fact, form perturbations at the interface which grow deeper eventually leading to bubble splitting. In general, the maximum stable bubble size increases with increasing liquid viscosity and surface tension, which decreases Reynolds numbers and the wavelength of interface instabilities.

Unfortunately, existing models that describe the required conditions for bubble rupture do not take into account bubble interaction and wall effects and consequently might underestimate the maximum diameter. For example, \cite{Clift1978} suggest a maximum stable diameter of 4.9 cm for air bubbles in water but stable slug flow has been observed in vertical pipes with a diameter of 7.2 cm, and in a 20 cm diameter pipe, where the ratio between the pipe length and diameter $L/D$ exceeded 100 \cite{Guet2002, Shen2010}. The rise of a single gas slug was also observed in a 25 cm diameter pipe \cite{James2011}. Therefore it is not clear what the effect of pipe walls on the stability of large bubbles may be and what is the minimum pipe length required to reproduce stable slug flow under experimental conditions. More experimental work is needed to infer the maximum pipe diameter that allows for slug flow stability for very viscous fluids and magmas in vertical conduits. On the other hand, the minimum bubble diameters in the distributions depend not only on the breakup mechanisms (i.e., the size of the newly formed bubbles), and, in the case of noninteracting bubbles in dilute suspensions, to gas expansion effects but in the case of magmas, also to bubble nucleation dynamics.

Transitions between two-phase flow regimes were first quantified in terms of gas and liquid mass flux \cite{Hewitt1969} and then in terms of superficial liquid and gas velocities, which are the components of the velocity of the mixture $V_m$:

$$V_m = u_d + u_{sg} \quad (2)$$

where $u_d$ and $u_{sg}$ are the superficial gas and liquid velocities and are defined as:

$$u_s = \frac{Q}{A} \quad (3)$$

where $u_s$ is the superficial velocity, $Q$ is the volume flux, and $A$ is the pipe cross-sectional area. The real velocities of the phases are

$$u_g = \frac{u_{sg}}{\varepsilon_g} \quad (4)$$

$$u_l = \frac{u_d}{1 - \varepsilon_g} \quad (5)$$

where $u_{sg}$ are the real velocities of the gas and liquid and $\varepsilon_g$ is the average gas volume fraction. Superficial velocities have been used directly to model bubble concentration and flow dynamics \cite{Taitel1980, Mishima1984}.

2. The Experimental Apparatus and Flow Parameters

Experiments were conducted on a 6.5 m high cylindrical transparent acrylic resin bubble column with cross section diameter of 0.24 m (Figure 2). The gas inlet system consisted of 25, individually controlled, 5 mm diameter nozzles with circular cross section. The nozzles were positioned in three rings with 4, 8, and 12 nozzles each. The 25th was at the center of the column. The diameters of the rings were selected so that each nozzle was in the middle of 1/25th of the column cross-sectional area. Gas flow was measured using up to five rotameters to ensure equal distribution of gas at the bottom of the column during experiments using multiple nozzles. Absolute pressure was monitored using four, equally spaced, Keller PD-25 sealed gauge pressure transmitters installed along the column (Figure 2). The temperature of the mixture was also monitored at 1.5 and 2.9 m from the base to ensure that there were no thermal gradients since the viscosity of the liquids used changes with temperature. Average gas volume fraction of the column at stable flow conditions was calculated by comparing initial liquid height at zero gas flux and the final height (when the column height was oscillating, both minimum, maximum, and average height were measured) with the aid of a vertical scale which was attached to the column external wall.

Experiments consisted of generating a range of fluxes of air through stagnant liquid which was either water, glucose syrup (Cargill “C star sweet”), or a mixture of the two. Gas volume flux ranged from $\sim 10^{-4}$ to $10^{-1}$ m$^3$/s, corresponding to superficial gas velocities from $\sim 10^{-2}$ to 1.9 m/s for water experiments and $\sim 10^{-2}$ to 0.7 m/s for syrup experiments. The main physical characteristics of the liquids are summarized in Table 1. Pure glucose syrup viscosity has a shear thinning rheology, which was measured with a rotational viscometer. The shear rate in bubble columns is a linear function of superficial gas velocity \cite{Nishikawa1991} and, because of the limited range of gas flow rates in the experimental conditions, the syrup viscosity can be approximated to a constant value of 300 Pa s. The rheology of the mixtures of water and glucose syrup was Newtonian, and their bubble-free viscosity was measured after each experiment.

2.1. Initial Bubble Size and Entrance Effects

The initial dimension of a bubble generated from an orifice in a stagnant liquid depends on the gas volumetric flow rate and the physical properties of both liquid and gas phases. It can be calculated by balancing the bubble growth rate and the rise speed \cite{Davidson1960}. When compared with nozzle spacing for different configurations (with 25, 20, 10, or 5 active, equally spaced nozzles), the calculations for initial bubble diameters show that for $u_{sg} \geq 0.1$ m/s, and when all the nozzles are active, bubbles in syrup interact with each other in the inlet area and coalesce at the base of the column, forming a single large bubble. For lower $u_{sg}$, the bubbles generated are smaller than the spacing between nozzles. This phenomenon could slightly favor the stability of slug flow in the experimental conditions.
2.2. Applicability and Limitations of the Experiments to Magmatic Conditions

[21] The applicability of our experiments to describe outgassing of basaltic magma can be discussed in terms of geometric and dynamic similarity. Geometric similarity requires that flow scale (as quantified by pipe height/diameter ratio, bubble diameter/pipe diameter ratio) is large enough to be able to reproduce flow phenomena within the upper portion of the volcanic edifice. Dynamic scaling requires that the flow regime, as defined by the relative effects of the forces acting on the flow, matches the expected conditions for the natural system. In addition, the bubble size range (which depends on both initial bubble diameter and bubble interaction processes) controls bubble rise speed and pressure drop in the column (which depend on the column height and mixture density).

Table 1. Physical Characteristics of the Liquids Used Compared With Average Features Basaltic Magmasa

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Density (Kg/m³)</th>
<th>Rheology</th>
<th>Viscosity (Pa s)</th>
<th>Surface Tension (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1000</td>
<td>Newtonian</td>
<td>8.9 × 10⁻⁴</td>
<td>7 × 10⁻²</td>
</tr>
<tr>
<td>Glucose syrup</td>
<td>1450</td>
<td>Shear thinning</td>
<td>3.0 × 10²</td>
<td>8 × 10⁻²</td>
</tr>
<tr>
<td>Basaltic magma</td>
<td>~2800</td>
<td>Variable!</td>
<td>10⁰–10⁵</td>
<td>4 × 10⁻¹</td>
</tr>
<tr>
<td>Mixture 1</td>
<td>1350</td>
<td>Newtonian</td>
<td>6.5</td>
<td>8 × 10⁻²</td>
</tr>
<tr>
<td>Mixture 2</td>
<td>1420</td>
<td>Shear thinning</td>
<td>2 × 10²</td>
<td>8 × 10⁻²</td>
</tr>
<tr>
<td>Mixture 3</td>
<td>1410</td>
<td>Shear thinning</td>
<td>2.8 × 10²</td>
<td>8 × 10⁻²</td>
</tr>
</tbody>
</table>

aFor syrup, the value of viscosity refers to experimental conditions.
Regime transitions and stability of slug flow are also strong function on the experimental pipe length/diameter ratio ($L/D$); in fact, gas slugs are expected to form for any flow conditions in an infinite length pipe because bubbles will have an infinite time to interact and will eventually coalesce into slugs [Wallis, 1969]. However, stable conditions for regime transitions have been reached for water airflows in experiments with $L/D$ between 8 and 29.9 and greater [Lusca et al., 2005]. For the liquids used here with viscosities significantly higher than water, the minimum $L/D$ required for flow stability has not been investigated, but it is expected to be lower, due to increased coalescence efficiency. In our experiments, the maximum $L/D$ ranged from 9 to 25; within this interval, no variation of the stability of the flow patterns was observed in glucose syrup experiments, suggesting that they can be considered to be representative for longer pipes. Moreover, it is also important to consider that cylindrical volcanic conduit geometry is likely only for the shallow part of volcanic edifices [Keating and Valentine, 2008; Ryan, 1988]. For these reasons, the experiments can investigate the dynamics of gas rise only along the last few hundreds meters of the conduit. Also, the total pressure drop in our experiments is limited, and near surface gas expansion effects are not fully reproduced.

Relevant dimensionless parameters for the flow dynamics are bubble Eotvos number ($Eo$), which is the ratio between buoyancy and surface tension forces; Morton number ($Mo$), relating viscous to surface tension forces; Froude ($Fr$), which is the ratio of inertia and gravitational forces; and Reynolds number ($Re$) which is the ratio of inertial to viscous forces and dimensionless inverse viscosity ($N_r$), of large bubbles due to inertial effects and is in accordance with experimental observations of direct transition from bubbly to churn flow in vertical pipes with diameters larger than 10 cm and similar $L/D$ [Cheng et al., 1998; Ohmuki and Akimoto, 2000; Omebere-Iyari et al., 2007]. In all experiments, liquid circulation within the column was marked by turbulent structures developing at the conduit peripheries, clearly visible at the walls. The structures developed throughout the whole column height; the size of eddies (from a few to $>20$ cm) decreased with increasing gas flux. Churn flow conditions were marked by strong turbulence and up to 30 cm column height fluctuations, whereas in bubbly flow, column height oscillations were limited to a few centimeters. A mist formed by a liquid spray was produced at the top of the liquid column.

In water airflows, bubbly flow was observed for low $u_{sg}$, and transition to churn flow was observed at $u_{sg}$ $\sim$1.4 m/s. The absence of slug flow is related to instability of large bubbles due to inertial effects and is in accordance with experimental observations of direct transition from bubbly to churn flow in vertical pipes with diameters larger than 10 cm and similar $L/D$ [Cheng et al., 1998; Ohmuki and Akimoto, 2000; Omebere-Iyari et al., 2007]. In all experiments, liquid circulation within the column was marked by turbulent structures developing at the conduit peripheries, clearly visible at the walls. The structures developed throughout the whole column height; the size of eddies (from a few to $>20$ cm) decreased with increasing gas flux. Churn flow conditions were marked by strong turbulence and up to 30 cm column height fluctuations, whereas in bubbly flow, column height oscillations were limited to a few centimeters. A mist formed by a liquid spray was produced at the top of the liquid column.

In pure syrup and syrup mixtures-airflows, large bubbles formed at the base of the column when superficial gas velocities exceeded $5 \times 10^{-3}$ m/s. True slug flow conditions (i.e., formation of gas slugs) were attained for $u_{sg} \geq 10^{-2}$ m/s. With increasing gas flow rate, the length of the slugs increased and the spacing between two consecutive slugs decreased. Liquid at the pipe walls was driven downward by the rise of the slugs and decimeter- to meter-sized circulation cells developed leading to efficient recirculation along the whole column height. Transition to churn flow was
observed at $u_{sg} \sim 0.4$ m/s for pure syrup and $>0.9$ m/s for the lowest viscosity mixture 1 (Table 1). The transition was marked by the onset of instability of the liquid film surrounding the gas slugs, which was reduced to a few centimeters. The film was locally dragged upwards by the gas flux and more complex dynamics of the liquid circulation pattern developed, leading to discontinuous shearing and variable lengths of the gas slugs, which had heights between 2 m to several meters (for high gas volume rates they could also be as long as the column). Small quantities of fine mist were observed forming at the top of the column at the highest gas fluxes. The distance between two different slugs was always $<10$ cm. Very small (submillimeter to centimeter size) bubbles coexisted with the gas slugs in all of our experiments with glucose syrup. Most were too small to move independently from the liquid and were transported in the liquid circulation pattern and sheared at the pipe walls (Figure 3). To ensure that small bubbles were not inherited from trapping of air during pouring of syrup in the column, and to investigate their formation mechanisms, we performed some experiments injecting single gas slugs in bubble-free glucose syrup (which had been left to degas in the column for several weeks). During the experiments, small bubbles formed both from the top of the column during the burst of gas slugs (as already described by Pandit et al. [1987], Bird et al. [2010], and Seyfried and Freundt [2000]) and within the inner portion during coalescence of two consecutive gas slugs, due to the collapse of partially resorbed liquid film on the bottom of the newly formed bubble.

[28] Coalescence between gas slugs typically occurs due to entrainment phenomena [Manga and Stone, 1994] or wake effects [De Nevers and Wu, 1971; Bhaga and Weber, 1981; Colella et al., 1999]. This was also observed in our experiments. Interaction among small bubbles at the conduit walls was strongly affected by shearing; coalescence occurred mainly between large bubbles and was negligible between smaller, spherical bubbles (diameter $\leq 1$ mm). We observed two main mechanisms for bubble coalescence, suggesting that it was mostly controlled by buoyancy and shearing: (1) a large (centimeter size) bubble, decoupled from the liquid, intersects a smaller one with lower velocity and the horizontal film between them is rapidly resorbed; (2) two elongated bubbles come in contact during liquid shearing at the walls related to the passage of the gas slug. Coalescence occurs at the surface that is parallel to the shearing direction.

[29] In experiments starting with bubble-free syrup, the number of small bubbles was observed to increase with time until stationary conditions were attained and a stable population characterized each gas volume flow rate. An example of spatial distribution of small bubbles as seen from the pipe walls, calculated by processing random images captured during a run at $u_{sg} = 0.06$ m/s, is shown in Figure 4. Their average dimension decreased with increasing superficial gas velocity, ranging from centimeter to submillimeter size, due to increasing shearing-induced breakup in the liquid film at the pipe walls. After all the experiments, submillimeter bubbles remained trapped in the glucose syrup for several weeks to months. Their concentration was calculated measuring the density of the bubbly liquid at the beginning of any new set of experiments, and their contribution was considered in calculating the average gas volume fraction.

![Figure 3. Sheared bubbles visible at the pipe walls during a syrup-air experiment ($u_{sg} = 0.06$ m/s). Scale bar is 2 cm long.](image_url)

![Figure 4. Areal bubble size distribution as observed at the pipe wall during a syrup-air, 5 min long, experiment with gas superficial velocity of 0.06 m/s calculated from a series of five images taken at random intervals. The gray area represents the total distribution variability, and the dashed line is the average value. The images were selected at random from a high-definition (8.9 pixels per mm), 25 fps movie of the run. 850 to 500 bubbles were analyzed in each image through standard image analysis techniques.](image_url)
3.2. Average Volume Gas Fraction

The experiments showed a progressive increase of the average volumetric gas content $ɛ_g$ of the column with increasing gas flux, and no dependence on the initial bubble size (which varied with gas superficial velocity and the number of active nozzles) and the column height, suggesting that stationary conditions were reached during all the experiments (Figures 5 and 6). Pure syrup-air experiments show a higher dispersion at high superficial gas velocities (churn flow conditions), which is probably related to a less accurate estimation of the average column height due to larger and faster oscillations.

For all liquids, the curves show an asymptotic behavior toward high superficial gas velocities. This pattern marks the transition to churn flow regimes when the bubbly column approaches the gas volume fraction typical for annular flow conditions, which is the maximum value that can be attained before onset of dispersed flow. Our experiments suggest that this transition occurs at lower superficial gas velocities with increasing liquid viscosity. This means that for low superficial gas velocities, higher viscosity liquids can attain the highest gas volume fraction, whereas for high superficial gas velocities, lower liquid viscosity shows the higher gas volume fraction. In fact, higher gas fractions are expected under annular flow conditions for lower viscosity liquids, due to smaller thickness of the liquid film at the pipe walls, which is a power law function of the liquid film Reynolds number [Asali et al., 1985].

The initial amount of small bubbles dispersed in the syrup and mixtures (ranging from 2 to 6 vol. %, marked on Figure 6 as points at zero $u_{sg}$) does not affect the gas volume fraction pattern, suggesting that equilibrium dynamics was not significantly affected by initial conditions. This observation is also confirmed by the lack of any relevant hysteretic effects; the average gas fraction was measured for two consecutive series of experiments with increasing and decreasing gas fluxes, respectively, and the results indicate no significant variations at corresponding superficial gas velocities.

3.3. Pressure Gradient

Pressure gradients were computed from the instantaneous pressure data from two different sensors. In general, the time-averaged pressure gradient in the column decreases with increasing gas flux because of the reduced density of the mixture, but the amplitude of their oscillations increase (Figure 7), down to a minimum value, corresponding to the plateau in the $u_{sg}$/volume gas fraction plot. The pressure gradient is controlled by both liquid density and viscosity. At low gas superficial velocities, the gradient is higher for air-pure syrup or air-mixture experiments than for water-air experiments. It decreases more rapidly with increasing $u_{sg}$.

Figure 5. Gas volume fraction variations with superficial gas velocity ($u_{sg}$) for water-air experiments. Numbers in the legend indicate the number of active nozzles used to inject gas. Vertical error bars indicate variability of gas fraction calculated for maximum and minimum column height during fluctuations at equilibrium. Horizontal dimension of circles is larger than the expected error in determination of $u_{sg}$.

Figure 6. Gas volume fraction variation for different superficial gas velocities ($u_{sg}$). Error bars indicate gas fraction calculated for maximum and minimum column height during fluctuations at equilibrium. Horizontal dimension of circles and squares is larger than the expected error in determination of $u_{sg}$. The legend indicates the liquid used, whose characteristics are listed in Table 1. Horizontal dimension of symbols is larger than the expected error in determination of $u_{sg}$. 
In order to model gas flow dynamics (as described in the review paper of Woldesemayat and Ghajar), the gas volume fraction is a function of gas volumetric flux \( Q_g \) and gas velocity \( u_g \), which is the rise velocity of the gas bubbles within the liquid (i.e., the terminal rise velocity of a single bubble) [Zuber and Findlay, 1965; Wallis, 1969].

For water-air experiments, the regime is inertia-dominant (\( N_f > 300 \) and \( Eo > 10^7 \) which suggests that viscosity and surface tension effects can be neglected), and the Froude number is taken to have a value of 0.351 [Wallis, 1969]. In general, the Froude number can be expressed as a complex function of Eotvos and Reynolds numbers and has a value between 1.2 and 1.4 [Collins et al., 1978; Fabre and Liné, 1992; Viana et al., 2003; whereas for laminar flows it varies with conduit Eotvos number only according to the law derived by Bendiksen [1985]:

\[
C_0 = 2.29 \left( 1 - \frac{20}{Eo} (1 - e^{-0.0125Eo}) \right)
\]

where \( C_0 \) is the distribution parameter, a constant related to the shape of the axial profile of velocity in the conduit (Zuber and Findlay [1965], also discussed in the work of Pioli et al. [2009]). In turbulent flows, \( C_0 \) varies with Reynolds and Eotvos numbers and has a value between 1.2 and 1.4 [Collins et al., 1978; Fabre and Liné, 1992; Viana et al., 2003; whereas for laminar flows it varies with conduit Eotvos number only according to the law derived by Bendiksen [1985]:

\[
C_0 = 2.29 \left( 1 - \frac{20}{Eo} (1 - e^{-0.0125Eo}) \right)
\]

\[\text{(1 - } C_0 \varepsilon_g) \rightarrow 0 \]

so

\[
\varepsilon_{g, m} = \frac{1}{C_0}
\]

where \( \varepsilon_{g, m} \) is the asymptotic gas fraction value. For pure syrup, experimental results indicate that \( C_0 \approx 2.29 \) in agreement with the theoretical prediction (equation (9)), whereas for water \( C_0 = 1.4 \).

For bubbly flows of low viscosity liquids (0.001 Pa s), it has been suggested that the terminal bubble velocity (gas drift velocity) can be described by

\[
v_{gd} = K \left( \frac{\sigma g (\rho_l - \rho_g)}{\rho_l^2} \right)^{0.25}
\]

with \( K_1 = 1.53 \) [Harmathy, 1960]. For more viscous liquids, a lower value of \( K_1 \) has been suggested. For slug flow the drift velocity of a bubble is proportional to the square root of the pipe diameter \( D \) [White and Beardmore, 1962]:

\[
v_{gd} = Fr \sqrt{gD_p}
\]

For water-air experiments, the regime is inertia-dominant (\( N_f > 300 \) and \( Eo > 10^7 \) which suggests that viscosity and surface tension effects can be neglected), and the Froude number is taken to have a value of 0.351 [Wallis, 1969]. In general, the Froude number can be expressed as a complex function of Eotvos and Reynolds numbers. Using the model proposed by Viana et al. [2003], we calculated the expected Froude number for our glucose syrup-air experiments of 0.23.

4. Pressure Drop in the Column

Vertical pressure variations in the column can be calculated from the momentum equation. In the case of
steady flow, adopting cylindrical coordinates, the pressure variation can be simplified to the sum of the gravitational ($G$), friction ($F$), and acceleration ($A$) components [Wallis, 1969, p. 19]:

\[
\frac{dp}{dz} = \frac{-dp_G}{dz} + \frac{-dp_F}{dz} + \frac{-dp_A}{dz}
\]

which, for bubbly flows, can be rewritten as [Whalley, 1986]

\[
\frac{dp}{dz} = \left[ \epsilon_g \rho_g + (1 - \epsilon_g) \rho_l \right] \cdot g + \frac{4\tau}{d} \\
+ G^2 \frac{d}{dz} \left[ \frac{x^2}{\epsilon_g \rho_g} + \frac{(1-x)^2}{(1-\epsilon_g) \rho_l} \right]
\]

where the three terms on the right-hand side of the equation correspond to gravitational, friction, and acceleration contributions, respectively, and $\tau$ is the shear stress, $G$ is the total mass flux, and $x$ is the flow quality, i.e., the mass fraction of the flow rate which is gas. The total pressure drop can be calculated neglecting the acceleration component (which can be considered a valid assumption since the liquid outflux is zero, $x = 1$) and calculating the friction component for a bubble column following Guet and Ooms [2006]:

\[
p_z(h) = \int_0^h \left( \left( \epsilon_g \rho_g + (1 - \epsilon_g) \rho_l \right) g + \frac{(1-x)^2}{(1-\epsilon_g) \rho_l} f_m u_g^2 \right) dz
\]

where $f_m$ is the Darcy friction factor, which is a dimensionless coefficient relating the friction pressure loss to the volumetric flow rate, and equals $64/Re$ for laminar flows [Moody, 1944].

In water-air experiments, the measured pressure drop is coincident with the gravitational pressure drop as calculated from the average gas volume fraction and the friction component can be neglected (Figure 8a).

In slug flows, and therefore in our all syrup experiments, the gravitational component depends on the fractional slug length $\beta$ which is the ratio between the length of the gas slug and the total length of the slug unit (comprising the gas slug and the liquid between two consecutive slugs), and the acceleration component can be neglected because it represents only a small contribution when the liquid flow is zero [De Cachard and Delhaye, 1996]. The total pressure drop reduces to

\[
\frac{dp_G}{dz} - \frac{dp_F}{dz} = (1 - \beta) \rho g \left[ 1 + \frac{f_m u_g^2}{2gD} \right].
\]

The fractional slug length can be calculated for any superficial gas velocity solving a system of equations as shown in Appendix A. As before, the second term in the square brackets of the equation (17) can be neglected because of the low range of superficial gas velocities (Figure 8b). Moreover, using the model described above, the experimental data are best reproduced assuming a volume concentration of small bubbles in the liquid matrix of 0.10 (Figure 8b). This value is in good agreement with the volume percentage of bubbles calculated from the density of the bubbly liquid at the end of the experiments, which varied from 6 to 2 vol. %, measured a few hours to a few weeks after the experiments, respectively.

5. Application to Magmatic Systems

5.1. Expected Conditions for Degassing of Basaltic Magmas

Using the models described here, it is possible to infer the properties of a degassing column of stagnant basaltic magma for a range of conditions. To calculate gas fraction volume variations with the gas flux, it is necessary to estimate a value for the main coefficients: the distribution parameter $C_0$, which controls the maximum vesicularity of the column (and the thickness of the annular film) and the Froude number, which controls the rise velocity of gas slugs and the stability of slug flow. As none of these numbers have been directly measured for magmas, they must be estimated...
using more general models. $C_0$, as calculated from equation (9), is expected to have a value of approximately 2.29, whereas the Froude number is expected to change with magma viscosity and conduit diameter (Figures 9a and 9b) according to the experimental model proposed by Viana et al. [2003] basing on a wide data set on the rise of gas slugs within different liquids. This model computes the Froude number as a power law function of the Eotvos and buoyancy Reynolds number ($R$) and considers the contribution of inertial, viscous, and interfacial forces for a wide range of dynamic regimes.

The variations of magma average vesicularity for a range of conditions are estimated in Figure 10a; the vesicularity of the magma increases with increasing superficial gas velocity, reaching a maximum value of $\sim 0.45$, corresponding to the expected conditions for annular flow. An increase in viscosity of the magma increases the vesicularity of the mixture at low superficial gas velocities and decreases the stability of slug flow, favoring churn flow conditions, above which, higher vesicularities are expected for lower viscosities basaltic magmas. This can be explained with an increased coalescence efficiency, which favors formation of large bubbles and flooding. Also, the Bendiksen [1985] and Viana et al. [2003] models suggest that conduit diameter controls both the Froude number and $C_0$, decreasing the expected vesicularity for the formation of large vesicles in the conduit and the volume of gas required. These results suggest that when the conditions for separated two-phase flow are met, an increase in viscosity of the magma will favor bubble coalescence and increase the bubble size in the mixture. Also, with increasing conduit diameter, the effect of viscosity becomes less important and magmas with different viscosities show similar degassing dynamics. Figure 10a suggests that transition to annular flow (which is indicated by the asymptotic behavior in the vesicularity-superficial gas velocity curve) occurs at superficial gas velocities, which increase with increasing conduit diameter and decreasing magma viscosity. In terms of volcanic activity, transition to annular flow should determine the onset of gas-driven eruptions, as the liquid is dragged upward by the gas.

A more precise quantification of the threshold gas flow rate for the onset of annular flow dynamics was done by Zapke and Kröger [1996, 2000] which, based on experimental data, proposed that the threshold $u_{sg}$ is a function of the Ohnesorge number $Oh = \eta_l / (D \rho_l \sigma)^{0.5}$ [Ohnesorge, 1936], which relates viscous forces to inertial and surface tension forces and the densimetric gas Froude number $Fr_{HG} = Fr_{r_g} / (\rho_l \rho_g)$. When applying this model to magma, we find that the threshold $u_{sg}$ ranges from $\sim 28 \text{ m/s} \left(2 \times 10^4 \text{ m}^3/\text{s}\right)$ ($D = 1 \text{ m}, \eta_l = 300 \text{ Pa s}$) to $156 \text{ m/s} \left(1 \times 10^3 \text{ m}^3/\text{s}\right)$ ($D = 10 \text{ m}, \eta_l = 300 \text{ Pa s}$). These values (from $8.6 \times 10^7$ to $1.7 \times 10^9 \text{ m}^3/\text{day}$) can be considered unrealistic in passive degassing conditions [Ripepe et al., 2002, 2008; McGonigle et al., 2003; Aiuppa et al., 2008; Boichu et al., 2010; Sawyer et al., 2008], and more typical of eruptive events.

The pressure gradients expected for magmatic columns strongly depend on the flow regime. In fact, if the maximum stable bubble diameter is smaller than the conduit, no continuous slug flow is possible and the pressure gradient...
is a direct function of the magma average vesicularity (Figure 10b), but if the conditions for stable slug flow are met, the pressure gradient depends on the fractional slug length $\beta$ and for similar gas flow rates, lower values are expected (Figure 11). Considering both our direct observations and previous studies (mentioned in section 1.2), we can hypothesize that stable slug flow conditions (i.e., when, at constant flow rates, the gas rises as a continuous sequence of equally spaced slugs) are very likely to be met in small conduits (with $D$ up to a few m) and unlikely for low viscosity (a few to some tens of Pa s) magmas degassing in large conduits (with diameters of 10 to a few tens of meters). Stable slug flow requires noncontinuous, but regular, gas supply at the base of the conduit. These observations do not exclude the possibility of the rise of a single gas slug within a stagnant liquid, which can been experimentally reproduced within large diameters pipes even for very low viscosity liquids (i.e., within water, along a 0.24 m diameter pipe [James et al., 2011]). Moreover, it is important to remember that our experiments did not test the validity of these models for very large diameters and they should be considered only a first-order approximation until full validation. [45] When extrapolating the gradient to depth, it is necessary to take into account gas volume changes due to pressure variations, and their effect on the volume gas flow rate (and superficial gas velocity), as already suggested by James et al. [2008]. As the model does not consider the effects of gas exsolution and diffusion, we limit the calculations to a hundred meters below the surface. We assume isothermal expansion and mass conservation to calculate changes in the gas superficial velocity up the column. As an example, we computed the expected pressure variation in a degassing conduit fixing gas superficial velocity at the vent at atmospheric pressure. Superficial velocities at different depths, and hence different pressures were calculated in a stepwise approach. The approach was implemented using small step lengths (1 m). Results are shown in Figure 12 where the nonlinear trends in the curves show the gas expansion effects. In this it was assumed that the conditions for slug flow stability are met. The additional, near surface expansion dynamics demonstrated by James et al. [2008, 2009] were not taken into account.

5.2. Scale of Gas Transport Structures and Formation of Small Bubbles

46] The bubble size distribution within a degassing magma column derives from the balance of coalescence and breakup phenomena. Coalescence is a dynamic process depending not only on bubble size and magma properties but also on the vertical and lateral velocity distribution within the column. The breakup rate in magmatic flows is expected to be very low, due to complete suppression of turbulence phenomena, and is dominated by bubble instability. In the case of a homogenous population of small bubbles, the flow
5.3. Liquid Convection and Mixing

Magma outgassing should be considerably lower with small bubbles, we suggest that the contribution of small bubbles to gas transport is between 20 and 30% of the total mass outflux [Muller and Davidson, 1992] and decreases with increasing superficial gas velocity. Since an increase of viscosity increases the stability of large bubbles and decreases with increasing superficial gas velocity, determining a progressive decrease in the pressure gradient. It is also affected by the physical and rheological properties of the magma. The theoretical model predicts that for the same superficial gas velocity, it increases with decreasing conduit diameters and with increasing magma viscosity (this aspect is also confirmed by the experiments). The maximum vesicularity attained by magma, corresponding to churn and annular flow regimes, changes with the Eotvos number (equation (9)) and is expected to be around 0.45. The pressure gradient in the magmatic column also decreases with increasing superficial gas velocity, with increasing magma viscosity and with decreasing conduit diameter.

Figure 12. Expected pressure variations in a degassing magma column for superficial velocities of 1, 5, 10 m/s measured at the vent; conduit diameter of 5 m and magma viscosity of 300 Pa s; and magma density of 2600 kg/m$^3$.

The expected pressure variations in a degassing magma column are shown in Figure 12. The pressure decreases with increasing superficial gas velocity, reaching a value of 0.3 MPa at 10 m/s. This indicates that the gas flow rate is much lower than the effect of viscosity on the pressure gradient, even over two orders of magnitude, is much lower than the effect of gas flow rate. This implies that the effects of these processes (which would affect liquid viscosity) on pressure profiles might not be large.

6. Conclusions

As our experiments did not reproduce the effect of magma cooling and crystallization and the formation of thermal gradients within the pipe, more dedicated studies are necessary to quantify the efficiency of liquid mixing for a range of gas flow rates. However, the effect of viscosity on pressure gradient, even over two orders of magnitude, is much lower than the effect of gas flow rate. This implies that the effects of these processes (which would affect liquid viscosity) on pressure profiles might not be large.
bubbles, also favored by large L/D ratios, which are expected in natural systems. Degassing processes also determine the formation of circulation cells within the conduit and continuous mixing, favoring magma homogeneity.

[55] Our experiments suggest that the bubble size distribution in the flow is not controlled only by bubble interaction processes, but the coalescence of large bubbles and their breakup at the magma free surface is forming a stable population of smaller bubbles, deriving from incomplete collapse of bubble ruptured films. These bubbles are too small to move within the liquid and circulate with it. They are also not significantly contributing to outgassing. It is interesting to note the occurrence of small bubbles during stable slug flow and liquid mixing was already noticed in the experiments of Seyfried and Freundt [2000], which used water-salt solutions (viscosities comprised between 10^6 and 10^4 mPa s) and air. We observe, however, that most of these small bubbles likely originated in the wakes behind the gas slugs due to the low viscosity of the liquid used and not necessarily only by air trapping during the bubbles burst. In any case, they observed that for continuous gas supply, the bubbles rapidly distributed along the entire column due to backflow in the liquid.

[54] These observations suggest that more specific studies are necessary to evaluate the significance of the vesicularity and texture of scoria samples erupted during low explosivity (Strombolian) regimes where gas segregation processes are of primary importance. In particular, the permeability of degassing magma should be controlled by large-scale structures and not by the microscale bubble distribution that reflects the bubble content of small parcels of magma and is controlled by the dynamics of the liquid motions.

[55] Annular flow conditions are unlikely to be met unless the conduit diameter is very small (a few meters).

**Appendix A: Calculation of Fractional Slug Length**

[56] The fractional slug length $\beta = l_{TB}/l$ is the ratio between the length of the gas slug ($l_{TB}$) and the total length of the slug unit ($l$), which also comprises the liquid separating two consecutive bubbles (Figure A1). Fernandes et al. [1983] introduced this parameter in a general model describing the hydrodynamics of slug flow. To calculate $\beta$, we adapt and solve simultaneously a system of equations derived from their model.

[57] The average gas volume fraction in the slug unit can be considered equal to the average gas fraction in the entire column. It is also equal to the sum of the gas volume fraction in the gas slug area ($e_{TB}$) and the gas volume fraction in the film between two consecutive gas slugs ($e_{LS}$) multiplied by their relative lengths:

$$e_g = \beta e_{TB} + (1 - \beta) e_{LS}. \quad (A1)$$

Mass balance requires that the superficial gas velocity equals the sum of the average velocity of the gas in the slug ($u_{GTB}$) and the average velocity of the small bubbles in the liquid between two consecutive gas slugs ($u_{GLS}$):

$$u_g = \beta e_{TB} u_{GTB} + (1 - \beta) e_{LS} u_{GLS}. \quad (A2)$$

Also, the net liquid velocity is zero, then:

$$(1 - \beta)(1 - e_{LS}) u_{LLS} - \beta (1 - e_{TB}) u_{LTB} = 0 \quad (A3)$$

where $u_{LLS}$ is the velocity of liquid in between two gas slugs and $u_{LTB}$ is the velocity of the liquid in the film around the gas slugs.

[58] At equilibrium conditions, the continuity equations for liquid and gas are

$$1 - e_{LS} (u_g - u_{LLS}) = (1 - e_{TB}) (u_g - u_{LTB}) \quad (A4)$$

$$e_{LS} (u_g - u_{GLS}) = e_{TB} (u_g - u_{GTB}), \quad (A5)$$

respectively.

[59] The rise velocity of the small bubbles in the liquid between two consecutive gas slugs can be calculated after Harmathy [1960] and Zuber and Hench [1962]:

$$u_{GLS} = u_{LLS} + K_1 \left( \frac{\rho_g - \rho_b}{\rho_b^2} \right) 0.25 \left(1 - e_{LS}\right)^{0.5}. \quad (A6)$$

Finally, the downward velocity of the liquid in the film around gas slugs was calculated by De Cachard and Delhaye [1996], basing on the thickness of the film as modeled by Wallis [1969] and Nusselt [1916]:

$$V_{LTB} = -11.2 [gD(1 - e_{TB}^{0.5})]^{0.5} \quad (A7)$$

that is valid for turbulent regimes, and

$$u_{LTB} = -0.333 \left( \frac{gD^2}{\eta_l} \right) \left(1 - e_{TB}^2\right)^2 \quad (A8)$$

for laminar regimes.

[60] As explained in section 4.2, the rise velocity of gas $u_g$ is

$$u_g = C_0 u_{GTB} + Fr \sqrt{gD}. \quad (A9)$$

There are in total seven variables, $\beta, e_{TB}, e_{LS}, u_{GTB}, u_{GLS}, u_{LLS}, u_{LTB}$ which can be calculated solving simultaneously equations from (A1) to (A7) or (A8) and calculating the gas velocity from equation (A9).

**Notation**

- $A$: pipe cross-section area (m²).
- $C_0$: distribution parameter.
- $D$: pipe diameter (m).
- $d_b$: bubble diameter (m).
- $d_{b,\text{max}}$: maximum stable bubble diameter (m).
- $f_m$: friction factor.
- $G$: mass flow rate (kg/s).
- $G_g$: gas mass flow rate (kg/s).
- $L$: pipe length (m).
- $Q_g$: gas volume flux (m³/s).
- $t_c$: coalescence time between bubbles of diameter $i$ and $j$ (s).
Figure A1. Schematic section of a slug unit.

- $T_g$: bubble contact time between bubbles of sizes i and j (s).
- $u_g$: gas velocity (m/s).
- $u_{GLS}$: average velocity of the gas in the slug (m/s).
- $u_{LLS}$: average velocity of the small bubbles in the liquid between two consecutive gas slugs (m/s).
- $u_{LLT}$: velocity of liquid in between two gas slugs (m/s).
- $u_l$: liquid velocity (m/s).
- $u_{sg}$: superficial gas velocity (m/s).
- $u_{sl}$: superficial liquid velocity (m/s).
- $v_{gd}$: gas drift velocity (m/s).
- $V_m$: mixture velocity (m/s).
- $x$: flow quality = $G_{ig}/G_l$.
- $\beta$: fractional slug length.
- $\Gamma$: bubble coalescence rate (1/m$^3$s).
- $\epsilon_g$: gas volume fraction.
- $\epsilon_{TB}$: gas volume fraction in the gas slug volume.
- $\epsilon_{LS}$: gas volume fraction in the liquid between two gas slugs.
- $\eta$: viscosity (Pa s).
- $\Theta_i$: collision frequency related to turbulence, buoyancy, laminar effects (1/m$^3$s).
- $\rho_g$: gas density (kg/m$^3$).
- $\rho_l$: liquid density (kg/m$^3$).
- $\tau$: wall shear stress (Pa).
- $\sigma$: surface tension (N/m).
- $Eo$: Eotvos number = $g(\rho_g - \rho_l)h^{2}/\sigma$.
- $Fr$: Froude number = $u_{g}^{2}/(gD)^{0.5}$.
- $Fr_{HG}$: densimetric gas Froude number = $\rho_{g}^{0.5}u_{g}^{2}/[gD(\rho_{g} - \rho_{l})]$.
- $Mo$: Morton number = $g\rho_{l}^{4}(\rho_{l} - \rho_{g})h^{2}\sigma^{3}$.
- $Ne$: dimensionless viscosity = $\eta_{l}/(\rho_{l}\sigma)^{0.25}$.
- $Oh$: Ohnesorge number = $\eta_{l}/(D \rho_{l} \sigma)^{0.5}$.

R Buoyancy Reynolds number = $(D^{3}/g(\rho_{l} - \rho_{g})\rho_{l}^{0.5})/\eta_{l}$.
Re Reynolds number = $\rho_{l}d_{b}\nu/\eta_{l}$.
Re$e$ Reynolds number of the liquid film (in annular and slug flow).

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References


