Anisotropic mass loss and stellar evolution: from Be Stars to Gamma Ray Bursts

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Abstract

The treatment of the rotation in stellar evolution codes had considerably improved the predictive capacity of such codes. One of the effects of rotation is the modification of various surface properties of the star. In particular, the stellar winds, which are spherical without rotation, become ever more anisotropic when rotation progressively increases. It is thus important to account precisely for this effect, to obtain a good estimation of the final angular momentum content of the star, which is decisive for the determination of the final fate of the star. In this thesis, we explore the various effects of such anisotropic winds on stellar evolution.

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Anisotropic Mass Loss and Stellar Evolution: From Be Stars to Gamma Ray Bursts

THÈSE

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Ce travail de thèse a donné lieu à des publications dont la liste se trouve à la page 153.
Thesis Outline

Since a dozen of years, it appeared that stellar evolution is strongly affected by the interplay of two processes: rotation and mass loss. Their account in stellar evolution codes has considerably improved the predictive power of the models generated by such codes, and has allowed a better fit of various observational features, as the metallicity dependence of the number of various supernova types and of the WR star populations, or the surface chemical enrichments.

Rotation affects the mass loss mainly through two ways. First, the total amount of matter lost by the star is increased by rotation. Second, rotation modifies the global shape of the stellar winds. From a spherically symmetric wind without rotation, the mass flux becomes anisotropic as the rotation velocity increases. On the other hand, the mass loss directly influences rotation, by spinning down the stellar surface and strengthening the differential rotation. In some extreme cases, the star reaches the critical rotation: the external layers in the equatorial regions are no more bound to the star, and an equatorial disc may form.

The aim of this thesis work is to study this interplay, with an emphasis on the development of strongly anisotropic winds when the star reaches very high rotation velocities. Chapter 1 gives a short introduction to stellar evolution. In Chapters 2 and 3, I present the physical processes which are treated in the Geneva stellar evolution code, and the modifications which were brought during this work. The effects of rotation on the convection near the surface of fast rotating, hot massive stars are described in Chapter 4. In Chapter 5, the impact of anisotropic radiative winds and equatorial mass loss on stellar evolution is discussed. We focus on two concrete cases: on the one hand, Gamma Ray Burst progenitors and on the other hand, Be stars, for which estimated mean mass loss rates and total mass lost in the equatorial disc are evaluated. The various type of supernova progenitors, and the effect of metallicity on the supernova type produced by a given star are presented in Chapter 6. Chapter 7 explains why homogeneous stars have a critical, maximal mass (called the $\mu^2M$-limit), and how the reaching of this $\mu^2M$-limit should trigger mass loss for those stars. Accounting for it, its effects on very massive star evolution are described. In Chapter 8, I take stock of the situation of a major project of the Geneva stellar evolution group: the computation of large rotating stellar model grids. A short description of a work outside the main topics of this thesis (the development of a simple planetary structure model), and of a work to which I only marginally contributed is presented in Chapter 9. Finally, conclusions and future perspective are given in the last chapter of this thesis.
Résumé

Perte de masse anisotrope et évolution stellaire :
Des étoiles Be aux sursauts gamma

Introduction

Le but de ce travail de thèse est l’étude de l’interaction entre la rotation des étoiles massives, et le phénomène de perte de masse que ces étoiles subissent (que l’on appelle « vents stellaires »). La rotation modifie de manière notable la forme, ainsi que certains paramètres, de la surface de l’étoile. En effet, sous l’effet de la force centrifuge, l’étoile devient de plus en plus oblate à mesure que sa rotation s’accélère. Ceci implique notamment une variation latitudinale de la température effective de la surface, ainsi que du flux lumineux : il devient plus important aux pôles que dans la région équatoriale. La manière dont l’étoile perd sa masse est directement reliée à ce flux ; ainsi, les vents stellaires deviennent eux-aussi anisotropes.

La perte de masse agit en retour sur la rotation, en soustrayant du moment cinétique de l’étoile, freinant ainsi la vitesse de rotation de sa surface. Ce freinage influence le mélange des espèces chimiques à l’intérieur de l’étoile, ainsi que la distribution interne de moment cinétique. Rotation et perte de masse sont donc deux phénomènes intimement liés, et une prise en compte détaillée de ces deux effets est extrêmement importante pour comprendre l’évolution des étoiles massives en rotation, particulièrement pour les rotateurs les plus rapides, pour lesquels les effets d’anisotropie deviennent significatifs.

Modifications du code d’évolution stellaire de Genève

Durant ce travail, le code de Genève a été modifié de manière substantielle afin de prendre en compte de nouveaux effets liés à la rotation. Premièrement, le traitement de la convection dans les zones externes des étoiles en rotation rapide a été modifié, incluant dorénavant le critère de Sølberg-Hoiland. Ceci permet maintenant de décrire la structure externe de l’étoile différentes à latitudes.

**Résumé**

Finalement, une nouvelle prescription de perte de masse, basée sur la relation masse-luminosité et la limite d’Eddington, a été dérivée: la limite $\mu^2 M$. Cette prescription est valable uniquement pour des étoiles homogènes.

**Principaux résultats**

L’influence de la rotation sur les zones convectives externes d’une étoile de $20 M_\odot$ en rotation rapide a été étudiée. On trouve que la rotation modifie notablement ces zones : l’extension spatiale des zones convectives augmente entre le pôle et l’équateur. De plus, leur taille s’accroît : elle est trois fois plus importante dans les zones équatoriales du modèle en rotation que pour le modèle sans rotation.

L’effet des vents anisotropes sur l’évolution des étoiles massives a aussi été évalué en détail. De même que dans le cas le plus extrême, les vents anisotropes ne permettent pas au modèle d’étoile de conserver beaucoup plus de moment cinétique que dans le cas où ces effets sont négligés (au mieux, il y a 30% de différence entre les deux modèles). Le contenu en moment cinétique n’étant pas grandement affecté par la prise en compte des effets de l’anisotropie, le destin des étoiles massives n’est pas modifié : la production de sursauts gamma, notamment, n’est pas changée.

Une étude du phénomène d’étoiles Be (présentant une rotation rapide ainsi qu’un disque équatorial) a également été menée. Pour la première fois, une estimation des taux de pertes de masse dans le disque, ainsi que de la masse totale perdue mécaniquement à l’équateur de telles étoiles a été obtenue. Les effets liés à la masse initiale sont discutés : on trouve que la perte de masse équitoriale est plus importante pour des étoiles entre $7 – 9 M_\odot$. Malgré le traitement approximatif de la perte de masse mécanique dans le code de Genève, les taux de perte de masse de ces modèles de Be sont comparés avec des taux mesurés observationnellement, et sont en bon accord avec ceux-ci.

De récentes observations à grande échelle des supernovae apportent de nouvelles contraintes aux modèles d’évolution stellaire. Nous avons étudié le lien entre certains sous-types de supernova et son étoile progénitrice. Dans le cadre des modèles d’étoiles simples, les supernovae de type Ic proviennent toutes d’étoiles de type WC ou WO. Les supernovae de type Ib proviennent des étoiles WNL les plus massives, des étoiles WNE ou des étoiles WC les moins massives. Finalement, les supernovae de type II proviennent majoritairement d’étoiles supergéantes (rouges ou bleues). On trouve aussi que les supernovae de type Ibc deviennent de plus en plus nombreuses à mesure que la métallicité augmente, à cause de l’efficacité de plus en plus importante des vents stellaires à plus haute métallicité, favorisant la mise à nu des couches internes de l’étoile. En supposant que toutes les étoiles massives produisent une supernova observable à la fin de leur vie (y compris celles dont le rémanent est un trou noir), les modèles d’étoiles simples reproduisent bien les rapports observés des supernovae de type Ib ou Ic sur les supernovae de type II, à toutes les métallicités. Cependant, si la création d’un trou noir lors de l’effondrement de l’étoile empêche une supernova observable d’avoir lieu, alors les modèles d’étoiles simples ne parviennent plus à reproduire les tendances observationnelles à basse métallicité, où d’autres scénarios doivent être envisagés (notamment le scénario des étoiles binaires).

Finalement, l’évolution d’étoiles proches de la limite $\mu^2 M$ a été analysée. Cette loi de perte de masse simple permet d’expliquer plusieurs aspects observationnels liés aux étoiles de type Wolf-Rayet. Les taux de perte de masse, l’évolution dans le diagramme HR ainsi que la relation masse-luminosité sont par exemple très similaires à ceux obtenus avec les pertes de masse standards. On s’attend cependant à des différences qui peuvent être importantes au niveau de l’enrichissement chimique du milieu interstellaire par les vents de ces étoiles. Malgré cela, nos résultats montrent que les étoiles Wolf-Rayet évoluent simultanément très près de leur limite $\mu^2 M$ et de leur luminosité d’Eddington.
Remerciements

Paradoxalement, les remerciements, que l’on écrit pratiquement en dernier quand on estime que le reste de la thèse tient à peu près la route, ou qu’il ne reste de toute façon plus assez de temps pour mieux faire, figurent toujours en début de thèse. Celle-ci ne va pas faire exception.

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Introduction

Non ! croiriez-vous, je viens de le voir en tombant,
Que Sirius, la nuit, s’affuble d’un turban ?
Confidentiel. L’autre Ourse est trop petite encor pour qu’elle morde !
Riant. J’ai traversé la Lyre en cassant une corde !
Superbe. Mais je compte en un livre écrire tout ceci,
Et les étoiles d’or qu’en mon manteau roussi
Je viens de rapporter à mes périls et risques,
Quand on l’imprimera, serviront d’astérisques !

E. Rostand, Cyrano de Bergerac

1.1 General context

The stars are the constitutive bricks of the Universe. They are the main sources of radiation in the universe, and thus, of energy. They play a key role in the planet formation process, are responsible for the chemical evolution of the Universe since the Big Bang, and have produced all the elements we are made of, during their life or their explosive death. The comprehension of the galactic dynamics, and of the chemical evolution of the galaxies depends also strongly on the knowledge of the stars themselves. As fundamental objects in the Universe, their study is important for the understanding of our world, and is useful for many fields in astronomy.

1.1.1 A brief review of the stellar evolution history

The stellar evolution is the study of the successive changes that a star undergoes from its birth to its death, and of its internal constitution. Historically, the first star which was studied was our closest neighbour, the Sun. The source of its energy was puzzling for a long time. In 1862, Lord Kelvin examined the possible energy sources for the Sun in the light of the knowledge at that time. He concluded that the Sun had an age of at most 3’000 years if its energy source was from chemical origin, and between 20 and 100 million years if this source was the infall of meteorites on its surface in past times. This result was unfortunately below the Earth age estimated from geological discoveries at the same time.

An important step was accomplished around 1910, with the parallel works of Ejnar Hertzsprung and Henry Norris Russell. They plotted the absolute magnitude of stars as a function of their spec-
1.1. General context

They showed that the stars were placed in some particular regions of the plot, rather than randomly distributed. Note that they proposed at that time an evolutive scenario, where giant stars contract and progressively reach the dwarf’s region (later called the **Main Sequence**), scenario that was abandoned a few years later.

![Hertzsprung-Russell diagram](http://www.atlasoftheuniverse.com/hr.html)

**Figure 1.1:** Hertzsprung-Russell diagram. The position of 23000 stars is shown. Source: *An Atlas of the Universe*, [http://www.atlasoftheuniverse.com/hr.html](http://www.atlasoftheuniverse.com/hr.html).

In 1924, the work of Eddington on the internal structure of the stars allowed to derive for the first time the mass–luminosity relation for perfect gas stars. This relation was in good agreement with the observed one, and it is probably one of the first confirmed results validating the theoretical work on stellar structure. The set of four equations of the internal structure of stars was complete, and astronomers had then a coherent description of the interior of stars. The last problem remained the source of energy, which was still not understood.

In 1929, Gamow proposed for the first time that the fusion of hydrogen atoms could provide the needed energy. This idea was developed later and independently by Hans Bethe and Carl von Weizsäcker in 1938. In their work, they describe the basic nuclear reactions occurring during most of the stellar lifetime and converting hydrogen into helium: the pp-chain reactions and the CNO cycle. The comprehension of the nucleosynthesis in the stars was improved in 1953, with the work of Hoyle. He predicted a resonant reaction, called the $3 - \alpha$ process, which allows three atoms of...
helium to be transformed into one atom of carbon. The final stone added to this preliminary works on nucleosynthesis was the paper by Burbidge et al. (1957), which described all the nuclear processes occurring in stars, from the hydrogen burning phase up to the fusion of the heavy elements, which takes place in the core of the most massive stars.

From that time, the general scheme of the stellar evolution was quite well understood. During the first stage of its history, a star burns hydrogen into helium. This phase, corresponding to the Main Sequence on the Hertzsprung-Russell diagram, is the longer phase of stellar evolution, which spreads over 90% of the total stellar lifetime. After the exhaustion of this reaction, the stellar core contracts, increasing the central temperature and density. It allows the ignition of the helium-burning reactions. For stars less massive than $8 - 9$ solar masses, the exhaustion of helium marks the end of the story. The star expels its external layers, producing a planetary nebulae. Only the core remains, mainly composed of helium, carbon and oxygen. It slowly cools down with time, and is called a white dwarf.

For the more massive stars, however, the core becomes sufficiently hot and dense to go through other successive burning phases: carbon burning, neon photodisintegration, oxygen burning, and eventually silicon burning. This last reaction produces an iron core in the centre of the star. Exothermal nuclear reactions are no more possible, the star cannot sustain its own gravity and the core collapses suddenly. During the collapse, the external layers are ejected in an event called a supernova. The various chemical elements produced during the star lifetime are dispersed in the interstellar medium. During the supernova explosion, another nucleosynthetic process occurs ($r$-process), producing some heavier nuclei by neutron captures.

1.1.2 Mass loss

The first real evidence that stars lose some amount of matter came at the beginning of the 20th century, when Beals (1929) suggested that a continuous wind of matter could explain the spectra of Wolf-Rayet (WR) stars. Few years later, Kosirev (1934) made some first estimates of the mass loss rate of WR stars: $10^{-5} \, M_\odot \cdot yr^{-1}$, in very good agreement with the values inferred recently. Mass loss from other type of stars was progressively stated: supergiant stars (Deutsch 1956), Sun (Neugebauer & Snyder 1962), ... The fact that stars indeed lose mass across the HR diagram was progressively established.

The mass loss is a complex process, which involves several physical mechanisms. It is thus not easy to give a simple description of that mechanism. Several theories were developed to explain how the particles of matter are lost from the stellar surface, and how they are accelerated up to their terminal velocity. Among them, we can list the coronal wind theory, the sound wave driven wind theory, the dust driven wind theory, the line driven wind theory, the magnetic rotator theory, ...

The main process operating at the surface of hot massive stars is the line driven wind theory. In a few words, this theory, initially developed by Lucy & Solomon (1970) and Castor et al. (1975), describe the interaction between the absorption lines present in the stellar spectrum, and the radiation field. The huge luminosity of massive stars generates a large number of energetics photons, which interact with the spectral lines when escaping from the star. This interaction pushes the ions away, producing an outwards mass flux forming the stellar winds.

1.1.3 Stellar rotation

The stars are born from gas and dust clouds (called molecular clouds) present in the galaxies. The typical size of such clouds ranges from a few tenths of a parsec to a few parsecs ($10^{12} - 10^{13}$ km). A star on the main sequence has a radius between $1/10 \, R_\odot$ (where the label $\odot$ refers to the Sun) and $10 \, R_\odot$ ($10^5 - 10^6$ km). According to the conservation of angular momentum law $\Omega r^2 = cst$, even a very small rotation movement in a molecular cloud implies a large amount of angular momentum, due to its large size. During the formation process, this angular momentum is distributed in the
star. It explains why the stars must rotate. Actually, the angular momentum content of the cloud is even larger than the sum of the angular momenta of all the stars that are formed from its matter. A still open question is how this amount of angular momentum is dissipated through the star formation process.

Why is rotation important for stellar evolution? Because it intervenes in many aspects of stellar physics. The first puzzling point, shown by von Zeipel (1924), is that a rotating star cannot be at the thermal equilibrium. In order to restore the equilibrium, some large scale currents of matter take place in the star: the Eddington-Sweet circulation, or meridional circulation. These currents of matter carry chemical species and angular momentum through the stellar interior, modifying the chemical composition and the internal rotation profile.

A second point is that rotation induces a new force to account for in the structure equations: the centrifugal force. Contrarily to the gravity force, which is always directed toward the centre of the star, the centrifugal force is parallel to the equatorial plane, and is directed outward with respect to the rotation axis. The effective gravity, which is the composition of the previous forces, is thus generally not a radial force. This will modify the shape of the star, because the equatorial regions are sustained against a weaker effective gravity than the polar regions.

The stars do not rotate as a solid body, but present a differential rotation, i.e. the angular velocity is a function of the radius. This favours frictions between neighbour layers in the star, producing turbulences. These turbulences increase the mixing in the interior.

Rotation also modifies how much mass is removed from the surface by the stellar winds, and where the mass is lost. Rotation tends to increase, in some cases dramatically, the amount of matter rejected in the interstellar medium through stellar winds, because of the centrifugal force, which decreases the strength of the ties holding the matter back. A rotating star develops anisotropies in their winds: the mass loss preferentially occurs in the polar regions.

In some extreme cases, a star can reach its critical rotation velocity: the rotation is so fast that at the equator, the centrifugal force compensates for the gravity, and the effective gravity vanishes. The external layers of the star can easily be carried away, producing a mechanical mass loss in the equatorial plane. This is though to be related to the Be phenomenon: these hot stars show emission lines in their spectra, probably due to the presence of a surrounding disc.

1.1.4 Present state-of-the-art

In the early times of stellar evolution study, it was not easy to obtain quantitative results, because of the difficulty to solve the internal structure equations. Without the help of computers, it was only possible to find some solutions based on simplifications of the equations. However, the development of the computer science allowed progressively the numerical treatment of this problem, and the simultaneous improvement of the numerical methods have made possible the writing of stellar evolution codes, ever more complete and refined. To render a better account of the observations of main sequence stars, the overshooting was added at the beginning of the 70s.

An important improvement of the evolution codes was the addition of the mass loss mechanisms, during the 80s. Considering that stars lose mass during their life considerably modifies their evolution, and it is of prime importance to account for it. A further step at the beginning of the 90s was some theoretical works on stellar rotation which permitted to add it in the stellar codes.

Nowadays stellar evolution codes allow to account for several physical process occurring in stellar interiors. Some of them are listed below, but of course, this list is not exhaustive:

- various mass loss prescriptions;
- various treatments of the rotation;
• nuclear networks allowing to follow the evolution of the star up to the end of its nuclear lifetime;

• mixing processes (rotationally induced mixing, microscopic diffusion, gravitational settling,...);

• treatment of the internal magnetic field;

The physical content of evolution codes is of course closely bound to the knowledge of those processes. Some of them are still only roughly approximated and parametrised, like the treatment of the convection. Others are regularly updated, like the nuclear reaction rates, or the opacity tables.

The inclusion of rotation and mass loss in the stellar evolution codes have greatly improved the quality of the models, which are now in better agreement with the observations than before. The chemical enrichments, the observed ratio of blue- to red-supergiants, the ratio of Wolf-Rayet stars to O-type stars, the evolution of the type Ibc to type II supernova ratio as a function of the metallicity, and the pulsation rate of pulsars (among other things) are well reproduced (see e.g. Meynet & Maeder 2000; Heger et al. 2000; Heger & Langer 2000; Maeder & Meynet 2001; Meynet & Maeder 2003, 2005; Heger et al. 2005).

1.2 Thesis outline

The aim of this thesis is to explore the interplay between rotation and mass loss. Both ingredients are of primordial interest for the predictions provided by stellar evolution codes. At the beginning of this thesis, the Geneva evolution code already included them, but as it was implemented, an accurate conservation of the angular momentum during the mass loss process was not ensured. This part was considerably improved and allows now more precise computations, even with the account of the wind anisotropy.

In Chapter 2, I give a review of the state of the Geneva code at the beginning of this thesis. I focus particularly on the physics of rotation: I summarise the treatment of the internal transport of angular momentum, and of chemical species. The surface effects induced by rotation are then described. It concerns mainly the deformation of the surface, as well as the latitudinal variations of the effective temperature and the luminous flux. I also expose the physics leading to the existence of two critical velocities that a star may encounter during its evolution. The increase of the total mass loss rates because of rotation is discussed in details, particularly the assumptions which are made, and their domain of validity.

Chapter 3 contains the description of the modifications which were implemented in the Geneva stellar evolution code during this work. The first point concerns the effects of rotation on the convection in the external layers of massive stars. Three effects are presented: the Solberg-Høiland criterion for convection, which replaces the classical Schwartzschild criterion, the modification of the radiative gradient, and finally, the modification in the mixing-length theory.

The development of anisotropic winds is exposed, and their impact on the stellar surface is studied through a semi-analytical model. Some first quantitative results are given. I expose then extensively the numerical modifications that I implemented in the Geneva code in order to solve the problem of the non-conservation of angular momentum. The inclusion of the effects of anisotropic winds in this scheme is presented. The computational method allowing for an estimation of the amount of mass lost in an equatorial disc when the star reaches the critical rotation is also discussed.

During this thesis, a new mass loss prescription was developed, based on considerations on the mass-luminosity relation and the Eddington luminosity, leading to a critical mass for homogeneous stars: the $\mu^2 M$-limit. The way this mass loss recipe is obtained is described, and its validity discussed.
1.2. Thesis outline

Finally, the current state of the Geneva code is exposed: mass loss and rotation prescriptions, abundances, opacities and nuclear reaction rates. I also give a list of other minor modifications improving and facilitating the usage of the Geneva code.

After these first parts describing the programming contribution, the next parts of this thesis are dedicated to the applications of the new implementations to various astrophysical contexts. In Chapter 4, the treatment of the convection in massive stars is applied to the case of a very fast rotating $20\,M_\odot$ model. I show how the external convective zones are affected by rotation.

The core of this work is the study of the effects of anisotropic winds on massive star evolution. This point is treated in Chapter 5. I show how anisotropic winds and equatorial mass loss affect two types of objects which are thought to originate from fast rotation: Gamma Ray Bursts progenitors and Be stars.

Chapter 6 explores the link between a given type of supernova, and the type of its progenitor star. The variation of the ratios of various subtypes of supernova with metallicity is presented, and compared with recent observational data. In Chapter 7, I show how the reaching of the $\mu^2 M$-limit affects the evolution of very massive fast rotating stars.

An on-going work involving the whole internal structure group is briefly described in Chapter 8: the computation of new stellar grids. The calibrations which were performed to choose the rotation prescriptions reproducing at best some important observational features, are shown. Chapter 9 gives a brief summary of a couple of works to which I marginally contributed.

My conclusions, and future perspectives, are given in Chapter 10.
The Geneva evolution code for rotating stars

Before describing the different improvements I brought to the Geneva code, it is useful to give a short outlook of the main features which were already implemented at the beginning of this thesis. The code was in its Origin 2006 version. I don’t want to describe here precisely all the physical processes which are accounted for. The interested reader can find additional information in Ekström (2008) and Eggenberger et al. (2008).

The last major improvement of the Geneva code, at the end of the 90’s, was to progressively introduce the effects of rotation on the stellar evolution, with a more and more accurate description of rotating stars. In this section, I briefly summarise some important points about the treatment of the rotation in the Geneva code. Of course, the aim is not to give a complete lecture on the rotation effects: this would fill a whole book. However, it is important to recall some general principles. The first part is a small overview of the structure of the stellar models, as computed by the Geneva evolution code, with or without rotation. Then, the more important points on rotation physics are exposed.

2.1 Structure of the stellar models

In the Geneva code, the star is divided in three main zones, with a specific treatment for each of them: the atmosphere, the envelope and the interior. The general structure is shown on Fig. 2.1 on next page.

**The atmosphere** In this zone, we suppose that the gravity and the opacity are constant. The integration variable is the optical depth \( \tau \) defined as \( d\tau = -\kappa \rho dr \). The hydrostatic equilibrium equation is solved from \( P = 0 \), down to an optical depth \( \tau = \frac{2}{3} \), where the temperature is by definition the effective temperature of the star. All the boundary conditions are thus known at that optical depth: the luminosity \( L \), the effective temperature \( T_{\text{eff}} \), the pressure \( P \) and the radius \( R_\star \). For more details, see Maeder (2009).

**The envelope** The envelope spreads from the bottom of the atmosphere, down to a given mass fraction of the star, called the fitting mass \( \text{FITM} \). In the envelope, we suppose that there is no energy production by the nuclear reactions. The three remaining structure equations are integrated down to \( \text{FITM} \). Partial ionisation is accounted for, and the convection is treated non-adiabatically.
2.2 A short outline of the physics of rotation

(see Section 3.1 on page 27). If the rotation is accounted for, we suppose that the envelope rotates at constant angular velocity, equal to the angular velocity of the first layer of the interior. The transport of angular momentum equation (see the Section 2.2) is not applied in this zone, which can entail problems, as we will see later. To ensure a better follow-up of the rotation, $\text{FITM}$ is set to 0.9999 when rotation is accounted for. For the case of no- (or slow) rotation, or when an important external convective zone appears, it is decreased to 0.98.

**The interior** It is the main zone of the star, fully ionised, where energy generation occurs by nuclear reactions, and where the effects of rotation are carefully accounted for. The medium is supposed to be fully ionised, and the convection adiabatic. The full set of the structure equations is solved here. The numerical method used is a relaxation method, first described in the case of the stellar physics by Henyey et al. (1964).

![Figure 2.1: Schematic view of the three different zones considered in the Geneva evolution code. The surface of the star is up. Under the surface, the envelope spreads down to the fitting mass, $\text{FITM}$. Then we find the successive layers of the star, down to the centre.](image)

### 2.2 A short outline of the physics of rotation

The rotation induces some modification to the internal structure of the stars. There are three main effects, that we shortly discuss below:

1. The hydrostatic equilibrium is modified, due to the additional support of the centrifugal force.

2. The local thermal equilibrium is broken. Large scale currents (the meridional circulation) develop. They contribute to the transport of angular momentum and of the chemical species directly, through advection processes, or indirectly, producing differential rotation which triggers shear instabilities. Differential rotation can also be produced by other means than meridional circulation, for instance changes in the structure (contraction of the core, expansion of the envelope).

3. The surface properties are modified. The shape of the star becomes oblate, and the effective temperature varies with the colatitude. It produces a change of the mass loss rates and induce anisotropies of the winds.

#### 2.2.1 Hydrostatic equilibrium and modified structure equations

The main assumption made in the Geneva code to treat the effects of rotation is the hypothesis of shellular rotation, i.e, the angular rotation velocity is constant on an isobar. This is justified if we suppose that there is a strong horizontal diffusion on the isobar, which homogenises the angular velocity. This assumption is reasonable, since the temperature gradient in the vertical direction tends to stabilise the matter in that direction, whereas the motions are less restricted horizontally (see Zahn 1992; Maeder 2003).

For a star in shellular rotation, one can show that the surface of constant $\psi$, defined as

$$\psi = \phi - \frac{1}{2} \Omega^2 r^2 \sin^2(\theta) = \text{const}, \quad (2.1)$$

$\phi$ is the function of stellar rotation that is constant on the isobar.
is an isobaric surface ($\phi$ is the gravitational potential). Following Kippenhahn & Thomas (1970), Meynet & Maeder (1997) have shown that the structure equations of the star can be expressed with respect to isobaric surfaces with only small changes, compared to the classical non-rotating case. They define a new radial coordinate, $r_P$, as:

$$V_P = \frac{4\pi}{3} r_P^3,$$  \hspace{1cm} (2.2)

with $V_P$ the volume surrounded by the isobar labelled by $P$. The hydrostatic equilibrium then writes

$$\frac{dP}{dM_P} = -\frac{GM_P}{4\pi r_P^2} f_P.$$  \hspace{1cm} (2.3)

The factor $f_P$, which accounts for all the effects of rotation, is:

$$f_P = \frac{4\pi r_P^4}{GM_P S_P} \frac{1}{\langle g_{\text{eff}}^{-1} \rangle}.$$  \hspace{1cm} (2.4)

$S_P$ is the total surface of the considered isobar, and the mean value $\langle \rangle$ is an average over the whole surface. Without rotation, we see that $f_P$ equals $1$, and the hydrostatic equilibrium (2.3) is then similar to the classical case.

It is also convenient to determine the continuity equation as a function of the same variables. The result is:

$$\frac{dr_P}{dM_P} = \frac{1}{4\pi r_P^2 \bar{\rho}}.$$  \hspace{1cm} (2.5)

Here, $\bar{\rho}$ is the mean density between two isobars (be careful, it is not strictly equal to $\langle \rho \rangle$).

With the use of the equation of state, it is now possible to define a mean temperature $\bar{T}$ using the pressure $P$ and the density $\bar{\rho}$. With a few more simplifying assumptions (see Meynet & Maeder 1997), it is possible to write the energy conservation equation as:

$$\frac{dL_P}{dM_P} = \epsilon_{\text{nucl}} - \epsilon_{\nu} + \epsilon_{\text{grav}},$$  \hspace{1cm} (2.6)

with $\epsilon_{\text{nucl}}$ the nuclear energy production rate, $\epsilon_{\nu}$ the energy rate removed by the neutrinos, and $\epsilon_{\text{grav}}$ the energy rate due to contraction or expansion.

The transport of energy can also be rewritten as a function of the same variables:

$$\frac{d\ln (\bar{T})}{dM_P} = -\frac{GM_P}{4\pi r_P^2} f_P \min \left( \nabla_{\text{ad}}, \nabla_{\text{rad}} \frac{f_T}{f_P} \right),$$  \hspace{1cm} (2.7)

where

$$f_T = \left( \frac{4\pi r_P^2}{S_P} \right)^2 \frac{1}{\langle g_{\text{eff}} \rangle \langle g_{\text{eff}}^{-1} \rangle}.$$  \hspace{1cm} (2.8)

The main point here is that the set of structure equations with rotation (2.3), (2.5), (2.6) and (2.7) are similar to the classical structure equation without rotation, except that the variables are slightly different and that there are some additional factors. The numerical scheme to solve them is thus the same, and, above all, rotation in stellar interiors can be treated by an uni-dimensional code like most stellar evolution codes.

2.2.2 Angular momentum transport by meridional circulation and diffusion

How does rotation evolve? Starting from a flat angular momentum profile\(^1\) on the Zero Age Main Sequence (ZAMS), what processes modify that profile? To date, the treatment of the internal

---

\(^1\)Works by Heger et al. (2000) show that the mixing processes are strong enough during the pre-MS to ensure solid-body rotation in the ZAMS.
2.2. A short outline of the physics of rotation

rotation of stars contains still a lot of parameters which are theoretically or experimentally not well known, in particular the diffusion coefficients (see below). However, the general frame is well described under the assumption of shellular rotation. The main points are summarised here. To have a general view of the various processes discussed below, you can refer to Zahn (1992), Maeder & Zahn (1998) and Maeder (2009).

Under the assumption of shellular rotation, and in lagrangian coordinates, the equation of transport of angular momentum can be written as:

$$\rho \frac{\partial}{\partial t} \left( r^2 \Omega \right) = \frac{1}{5r^2} \frac{\partial}{\partial r} \left( \rho r^4 \Omega U^2 \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho D_v r^4 \frac{\partial \Omega}{\partial r} \right).$$

(2.9)

The velocity of the motion $\vec{U}$ is decomposed on the Legendre polynomials: $\vec{U} = U_2(r) P_2(\cos(\theta)) \vec{e}_r + V_2(r) \frac{\partial P_2(\cos(\theta))}{\partial \theta} \vec{e}_\theta$, where $P_2$ is the second Legendre polynomial. $U_2$ is thus the radial component of the velocity. $D_v$ is the diffusion coefficient in the radial direction. This equation of transport tells us that the time-variation of the specific angular momentum is produced by two processes. The first term on the right-hand side is an advection term, and is related to the velocity field in the star. The second one is a diffusion term, and corresponds to the transport of angular momentum by diffusive processes.

Two ingredients are needed to solve the equation of transport (2.9): the radial velocity $U_2(r)$ and the diffusion coefficient $D_v$. In a rotating star, the thermal balance cannot be locally satisfied. In order to ensure the conservation of energy, some large scale currents must exist. These currents, known as the meridional circulation or Eddington-Sweet circulation form the velocity field appearing in eq. 2.9. The derivation of a computable expression for this velocity field is long and complex. The first complete one was established by Zahn (1992), and it was improved by Maeder & Zahn (1998). The expression of $U_2$ can be found in Maeder (2009).

Vertical turbulence is induced by instabilities driven by the shear due to the differential rotation. Different prescriptions exist for the diffusion coefficient $D_{\text{shear}}$ related to this instability (Maeder 1997; Talon & Zahn 1997). The Talon & Zahn (1997) prescription also depends on the horizontal diffusion coefficient $D_h$. The expression of $D_h$ is uncertain, and different formulae have been proposed (Chaboyer & Zahn 1992; Zahn 1992; Maeder 2003; Mathis et al. 2004). The different prescriptions which can be used in the Geneva evolution code are presented in Section 3.4.3 on page 59.

The numerical method used in the Geneva code to solve the angular momentum transport (2.9) is a little particular. The whole equation is not solved in one block. It is divided in two parts, the advection and the diffusion. Each of them are solved separately and alternatively (the diffusion at one time step, the advection at the next time step, ...), with a time step twice bigger. The diffusion equation is a “simple” first order partial derivative equation, solved with an implicit finite difference method (Press et al. 2007). This method is very robust and very accurate (Meynet et al. 2004). The advection part is more difficult. The velocity term $U_2(r)$ contains a term of third order $\partial^3 \Omega/\partial \theta^3$. Thus, the advection equation is of fourth order, which complicates the resolution. In that case, a relaxation method is used, as it is done to solve the internal structure equations.

2.2.3 Transport of chemical elements

Basically, the equation of transport for the chemical species is identical to the angular momentum transport (2.9):

$$\rho \frac{\partial X_i}{\partial t} + \rho \vec{U} \nabla X_i = \nabla \left( \rho \vec{D} \nabla X_i \right),$$

(2.10)

where $X_i$ is the mass fraction of the element $i$, $\vec{U}$ the velocity field (due to the meridional circulation), and $\vec{D}$ the diffusion tensor. However, contrarily to the angular momentum transport
equation, one can simplify that equation to a pure diffusion equation (Chaboyer & Zahn 1992):

\[
\rho \frac{\partial X_i}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[ \rho r^2 \left( D_v + D_{\text{eff}} \right) \frac{\partial X_i}{\partial r} \right], 
\]

(2.11)

with \(X_i\) the mean mass fraction of the element \(i\) over the isobar, and \(D_{\text{eff}}\) the effective diffusion coefficient, which includes the effects of the meridional circulation:

\[
D_{\text{eff}} = \frac{(rU_2)^2}{30D_h}. 
\]

(2.12)

We see that the meridional circulation favours the mixing of chemical species, while the horizontal turbulence inhibits it.

2.2.4 Treatment of the internal magnetic field

The Geneva code allows also the computation of stellar models accounting for the effects of internal magnetic fields on the transport of angular momentum and chemical species. This process is based on the Tayler-Spruit dynamo (Spruit 2002). The basic idea is that, if a small radial magnetic field initially exists in the star, the differential rotation will wind up the field in the azimuthal direction \(\vec{e}_\phi\). It enforces the field in this direction, until it encounters the Tayler instability, which generates a small turbulence in the radial direction. It creates a new small radial magnetic field, which winds up, etc... Eventually, the field is largely amplified in the horizontal direction (which contributes to the maintenance of the shellular rotation), and a small radial field is created, which takes part in the transport process. However, the question of the closing of the dynamo loop is still debated. Some authors (Braithwaite 2006) reproduced the complete dynamo loop in numerical simulation. However, Zahn et al. (2007) expressed a doubt on the validity of the heuristic approach of Spruit, claiming that the dynamo-loop as presented by Spruit was not realistic. They proposed another type of dynamo-loop, but their numerical simulations did not show any evidence of dynamo-loop closure.

The implementation of the transport of chemical species and angular momentum produced by the internal magnetic field in the Geneva evolution code is described in Maeder & Meynet (2003, 2004, 2005) and Eggenberger et al. (2005).

2.2.5 Modifications at the surface of the star

The only (non asteroseismological) informations that observations of stars bring are their surface properties. It is thus important to have the best possible description of the stellar surface, particularly for rotating stars, which interests us in this thesis. In this small section, we concentrate on the modifications induced by rotation to the physical properties of the stellar surface. The peculiar case of the anisotropic winds will be discussed in more details in Chapter 5 on page 69.

Shape of the surface

Following the approach of Maeder (2009), the first thing to do is to find the shape of a rotating star. It is given by the following relation, indicating that for any displacement \(d\vec{s}\) on the surface, the work of the effective gravity has to be 0:

\[
\vec{g}_{\text{eff}} \cdot d\vec{s} = 0. 
\]

(2.13)

By definition, \(\vec{g}_{\text{eff}}\) is the sum of the gravitational acceleration \(\vec{g}_{\text{grav}}\) and of the centrifugal acceleration \(\vec{g}_{\text{cen}}\) (see the schema on Fig. 2.2 on next page). We thus need first an expression for the centrifugal acceleration. In spherical coordinates, it is:

\[
\vec{g}_{\text{cen}} = \Omega^2 r \sin(\theta) \left( \sin(\theta)\vec{e}_r + \cos(\theta)\vec{e}_\theta \right), 
\]

(2.14)
2.2. A short outline of the physics of rotation

Figure 2.2: Shape of a star rotating at the critical velocity, viewed equator-on. The black thick line is the stellar surface. The axis are in units of polar radius. The black arrow shows the direction of the gravitational acceleration, and the red one the centrifugal acceleration for the same position on the surface. The blue arrow represents the effective gravity, which is the sum of the two previous accelerations.

with \( \Omega \) the angular velocity of the surface (supposed to be constant in the frame of the shellular rotation approximation), and \( (r, \theta) \) the usual spherical coordinates, where \( \theta \) is the colatitude (\( \theta = 0 \) at the pole). Including the gravitational acceleration, the effective gravity becomes:

\[
\vec{g}_{\text{eff}} = \left( -\frac{GM}{r^2} + \Omega^2 r \sin^2(\theta) \right) \vec{e}_r + \Omega^2 r \sin(\theta) \cos(\theta) \vec{e}_\theta
\]

\[
= \left( -\frac{\partial \Phi}{\partial r} + \Omega^2 r \sin^2(\theta) \right) \vec{e}_r + \Omega^2 r \sin(\theta) \cos(\theta) \vec{e}_\theta,
\]

where \( \Phi = -\frac{GM}{r} \) is the gravitational potential\(^2\). As we can see, the effective gravity is no more directed in the radial direction, except at the poles and at the equator. The angle \( \epsilon \) between \( \vec{g}_{\text{eff}} \) and the radial direction is:

\[
\cos(\epsilon) = \frac{\vec{g}_{\text{eff}} \cdot \vec{e}_r}{|\vec{g}_{\text{eff}}|}.
\]

It is interesting to express the effective gravity as a function of the isobaric surfaces defined in (2.1). We can see that

\[
\vec{g}_{\text{eff}} = -\nabla \Psi - r^2 \sin^2(\theta) \Omega \nabla \Omega.
\]

In the case of solid-body rotation, the last term vanishes, and the effective gravity is derived from a potential. This is not true in the general case presented here. Let us calculate the scalar product (2.13). We obtain:

\[
- \frac{\partial \Psi}{\partial r} dr - \frac{1}{r} \frac{\partial \Psi}{\partial \theta} d\theta - r^2 \sin^2(\theta) \Omega \frac{\partial \Omega}{\partial r} dr - r^2 \sin^2(\theta) \frac{\Omega}{r} \frac{\partial \Omega}{\partial \theta} r d\theta.
\]

The assumption of shellular rotation makes the last term to vanish. We can integrate (2.18) to obtain the equation of the stellar surface:

\[
- \frac{GM}{r(\theta)} - \frac{1}{2} \Omega^2 r^2(\theta) \sin^2(\theta) + \sin^2(\theta) \int_{R_P}^{r(\theta)} r^2(\theta) \Omega \frac{d\Omega}{dr} dr = -\frac{GM}{R_P},
\]

where \( R_P \) is the polar radius of the star. This is the most general equation of the surface in the case of shellular rotation. In the Geneva code, we suppose that the envelope rotates at a constant

---

\(^2\)This assumption on the gravitational potential, called the Roche approximation, supposes that the mass inside the considered isobar is spherically distributed, or, at least, that the mass can be considered as point-like.
angular velocity (see Sect. 2.1 on page 7). In that case, the last equation reduces to:

\[ \frac{GM}{r(\theta)} + \frac{1}{2} \Omega^2 r^2(\theta) \sin^2(\theta) = \frac{GM}{R_P}. \]  

(2.20)

with \( R_P \) the polar radius at the angular velocity \( \Omega \).

We introduce here the concept of critical angular velocity \( \Omega_{\text{crit}} \). It is defined as the angular velocity at which the effective gravity at the equator becomes 0. From (2.15), we have:

\[ \Omega_{\text{crit}}^2 = \frac{GM}{R_{E,\text{crit}}^3}, \]  

(2.21)

where \( R_{E,\text{crit}} \) is the equatorial radius when the star is at the critical angular velocity. Introducing that velocity in (2.20), and evaluating it at the equator leads to the ratio of the polar to equatorial radius for a critically rotating star:

\[ \frac{R_{E,\text{crit}}}{R_{P,\text{crit}}} = \frac{3}{2}. \]  

(2.22)

At that point, it is convenient to introduce some dimensionless variables. We define the rotation parameter:

\[ \omega = \frac{\Omega}{\Omega_{\text{crit}}}, \]  

(2.23)

and a new radial variable

\[ x(\theta) = \frac{r(\theta)}{R_{P,\text{crit}}}. \]  

(2.24)

Then the simplified surface equation becomes

\[ \frac{1}{x} + \frac{4}{27} \omega^2 x^2 \sin^2(\theta) = \frac{R_{P,\text{crit}}}{R_P(\omega)}. \]  

(2.25)

Writing the ratio \( \frac{R_{E,\text{crit}}}{R_E} = T \), we can write the stellar surface as:

\[ \theta(x) = \arcsin \left( \sqrt{\frac{27(Tx - 1)}{4\omega^2 x^3}} \right) \quad \text{if } \omega \neq 0 \]

\[ x = \frac{R_P(\omega)}{R_{P,\text{crit}}} \quad \text{if } \omega = 0. \]  

(2.26)

If the numerator vanishes, the solution is for the pole \( \theta = 0 \), thus, for \( x = \frac{1}{T} \). The equatorial radius is obtained when the square root equals 1. This is equivalent to the following third order equation:

\[ f(x) \equiv 4\omega^2 x^3 - 27Tx + 27 = 0. \]  

(2.27)

Ekström et al. (2008b) have shown that the polar radius varies only slightly as a function of \( \omega \). To simplify the previous equation, we can thus approximate \( T \approx 1 \). With this assumption, we see that the shape of the surface (2.25) is only dependent on \( \omega \)! Several stars with different values of mass, luminosity, ... but rotating at the same rotation parameter \( \omega \) will have the same shape, and particularly the same ratio \( R_E/R_P \) (with \( R_E \) the actual equatorial radius).

Before continuing, let us check that putting \( T = 1 \) does not greatly affect the stellar shape. The equatorial radius \( R_E \) is given by solving eq. (2.27), for a given \( \omega \) and \( T \). We take here the data from Fig. 2 in Ekström et al. (2008b), for the two extreme cases of \( T \): their 60 M_⊙ and 1 M_⊙ models. From the data, we compute the real ratio \( R_P/R_E \), and the ratio we obtain under the assumption that the polar radius is insensitive to the rotation (putting \( T = 1 \) in (2.27)). Fig. 2.3 shows the curves of \( f(x) \) for the four models from Ekström et al. (2008b) (delimiting the grey area), and the curve for
2.2. A short outline of the physics of rotation

Table 2.1: Comparison between values of the ratio $R_E/R_P$ obtained with and without assuming $T = 1$ (see text). The first column indicates the mass of the model, the second one the rotation parameter $\omega$, and the third the ratio $R_{E\text{crit}}/R_P(\omega)$, from Ekström et al. (2008b). The fourth column gives the ratio $R_E(\omega)/R_P(\omega)$ obtained with the real polar radius, and the fifth one the same ratio, under the assumption that the polar radius $R_P(\omega)$ does not depend on the rotation parameter $\omega$.

<table>
<thead>
<tr>
<th>mass [$M_\odot$]</th>
<th>$\omega$</th>
<th>$R_{P\text{crit}}/R_P(\omega)$</th>
<th>$R_E(\omega)/R_P(\omega)$</th>
<th>$R_E(\omega)/R_P(\omega)_{T=1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.9</td>
<td>1.02</td>
<td>1.191</td>
<td>1.215</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1.05</td>
<td>1.036</td>
<td>1.042</td>
</tr>
<tr>
<td>1</td>
<td>0.9</td>
<td>0.985</td>
<td>1.239</td>
<td>1.215</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.95</td>
<td>1.050</td>
<td>1.042</td>
</tr>
</tbody>
</table>

$T = 1$ (in the centre of the grey area). The results are shown in Tab. 2.1 for two different rotation parameter: $\omega = 0.9$ and $\omega = 0.5$. The ratio $R_E(\omega)/R_P(\omega)$, computed with account for the polar radius variation, is indicated (column 4). We also give the same ratio, computed assuming that $T = 1$, in the last column. We see that the differences remain very small (a few percents), and thus, that the shape of the star is only marginally affected by the variation of the polar radius (even for a smaller or larger polar radius compared to the critical polar radius, the global shape is preserved, the main effect is an homothety).

We can thus adopt the simplification that the polar radius of the star is independent of rotation, and put $T = 1$ in previous equations. The stellar surface can be expressed with a very good approximation as:

$$\frac{1}{x} + \frac{4}{27}\omega^2x^3\sin^2(\theta) = 1,$$

or, in a simpler form:

$$\theta(x) = \arcsin\left(\sqrt{\frac{27(x-1)}{4\omega^2x^3}}\right) \quad \text{if } \omega \neq 0$$

$$x = 1 \quad \text{if } \omega = 0.$$

The shape is represented on Fig. 2.4. Without rotation (red area), the star is perfectly spherical. When the ratio $\omega$ increases, the stars becomes more and more oblate (black curves, $\omega$ is indicated...
at the bottom of each of them). When the star rotates exactly at the critical angular velocity $\Omega_{\text{crit}}$, the equatorial radius reaches its maximal extension (blue area). The shape of the star at the equator is always smooth (i.e. the local vertical direction is continuous crossing the equatorial plane), except when $\omega = 1$, where the local vertical has a discontinuity at the equator.

Figure 2.4: Shape of the stellar surface, as seen from an equator-on point of view (the $x$-axis is in the equatorial plane, and the $y$-axis is along the rotation axis). The unit of both axis is the polar radius of the star. The red curve shows the spherical shape of a non-rotating star, the blue curve, the shape of a critically rotating star. The black curves are the shape for other $\omega$ values, labelled on the plot.

**Effective gravity**

Once the shape of the surface is known, we can easily compute the value of the effective gravity along the surface, according to eq. (2.15). On the left panel of Fig. 2.5 on next page, we show the variation of the intensity of $\vec{g}_{\text{eff}}$, normalised by the polar one as a function of the colatitude. The red curve represents the non-rotating case, the black curves some intermediate values of $\omega$ (labelled on the plot), and the blue curve the critical rotation case ($\omega = 1$). When the rotation parameter increases, $g_{\text{eff}}$ progressively decreases towards the equator, until it vanishes when the angular velocity reaches the critical one.

The angle $\epsilon$ defined in (2.16) is shown on the right panel of Fig. 2.5 on next page as a function of $\theta$. The curves have the same signification than on the left panel. Without rotation, the angle between $\vec{g}_{\text{eff}}$ and the radial direction is of course $0$ at every colatitudes. Increasing $\omega$ produces a growing angle $\epsilon$, whose maximal value progressively shifts towards the equator. At the critical velocity, because of the shape of the star which is discontinuous at the equator (see Fig. 2.4), there is a sudden discontinuity. $\epsilon$ is very large just above the equatorial plane, and vanishes when we are exactly on the equator.

**von Zeipel theorem and effective temperature variations**

In the most general way, the luminous flux at a given colatitude and angular velocity can be written as:

$$\vec{F}(\Omega, \theta) = -\chi \nabla T(\Omega, \theta),$$

with $\chi = \frac{4\pi R^2}{g \sigma}$. In case of solid body rotation, we can show that on an equipotential (which is also an isobar), the temperature and the density are constant (von Zeipel 1924). This implies that the previous relation can be expressed as follow:

$$\vec{F}(\Omega, \theta) = -\chi \frac{dT}{dP} \nabla P(\Omega, \theta) = -\rho \chi \frac{dT}{dP} \vec{g}_{\text{eff}}(\Omega, \theta).$$
2.3. The two critical velocities

As $\rho$, $\chi$ and $dT/dP$ are constant on an isobar, the flux is proportional to the effective gravity. Before going further, we have to remind that by definition, the total luminosity is given by the integration of the flux over the surface of the isobar. This allows to determine $\rho \chi dT/dP$ more conveniently, and we finally obtain the following expression (Maeder 2009):

$$\vec{F}(\Omega, \theta) = -\frac{L}{4\pi GM^*} \vec{g}_{\text{eff}}(\Omega, \theta),$$

(2.32)

with $M^*$ given by

$$M^* = M \left(1 - \frac{\Omega^2}{2\pi G \rho_M}\right),$$

(2.33)

where $\rho_M$ is the mean density inside the considered isobar. The luminous flux behaves like $\vec{g}_{\text{eff}}$ (see the left panel of Figure 2.5). In the case of shellular rotation, this result is almost correct. There is an additional multiplying term $(1 + \zeta(\theta))$ in (2.32) (see Maeder 1999), which is usually small.

As $T_{\text{eff}}$ is related to the flux through the Stefan-Boltzmann law $F = \sigma T^4$, we immediately get an expression for the local effective temperature:

$$T_{\text{eff}}(\Omega, \theta) = \left(\frac{L}{4\pi \sigma GM^*}\right)^{\frac{1}{4}} (g_{\text{eff}})^{\frac{1}{4}}.$$

(2.34)

Figure 2.6 on next page shows the variation of the $T_{\text{eff}}$ as a function of the colatitude for a $20 \, M_\odot$ model at $Z = 0.02$ (from Ekström et al. 2008b). We see that the larger the rotation parameter, the hotter the poles and the cooler the equatorial regions become.

2.3 The two critical velocities

The concept of critical velocity is often used in the literature. However, its definition is rarely precisely defined, and we can find a variety of such velocities. In order to be clear in this work, we
shortly give a definition of the two critical velocities which are used in this work. We follow here the derivation of Maeder & Meynet (2000).

We consider the star at the critical velocity when the total force applied at a point of the stellar surface vanishes. In a rotating star, there are three forces to account for: the gravitational force, the centrifugal force and the radiative force. The total acceleration $\vec{g}_{\text{tot}}$ is thus:

$$\vec{g}_{\text{tot}} = \vec{g}_{\text{grav}} + \vec{g}_{\text{cen}} + \vec{g}_{\text{rad}},$$

(2.35)

where $\vec{g}_{\text{grav}}$, $\vec{g}_{\text{cen}}$ and $\vec{g}_{\text{rad}}$ are respectively the gravitational, centrifugal and radiative accelerations. As above, we introduce the effective gravity $\vec{g}_{\text{eff}} = \vec{g}_{\text{grav}} + \vec{g}_{\text{cen}}$ to obtain

$$\vec{g}_{\text{tot}} = \vec{g}_{\text{eff}} + \vec{g}_{\text{rad}}.$$  (2.36)

The radiative acceleration at a given colatitude $\theta$ and angular velocity $\Omega$ is:

$$\vec{g}_{\text{rad}}(\theta, \Omega) = \frac{\kappa(\theta, \Omega)}{c} \vec{F}(\theta, \Omega),$$

(2.37)

with $\kappa$ the opacity and $\vec{F}$ the radiative flux. With (2.32), the total acceleration becomes:

$$\vec{g}_{\text{tot}} = \left[ 1 - \frac{\kappa(\theta, \Omega)L}{4\pi c GM \left( 1 - \frac{\Omega^2}{2\pi G \rho_M} \right)} \right] \vec{g}_{\text{eff}}.$$  (2.38)

The condition $\vec{g}_{\text{tot}} = 0$ thus defines a local limiting flux $\vec{F}_{\text{lim}}$:

$$\vec{F}_{\text{lim}}(\theta, \Omega) = -\frac{c}{\kappa(\theta, \Omega)} \vec{g}_{\text{eff}}(\theta, \Omega).$$  (2.39)

As in the classical non-rotating case, we can now define the local Eddington factor $\Gamma_{\Omega}$, as the ratio of the actual local flux to the limiting flux:

$$\Gamma_{\Omega}(\theta) = \frac{\vec{F}(\theta, \Omega)}{\vec{F}_{\text{lim}}(\theta, \Omega)} = \frac{\kappa(\theta, \Omega)L}{4\pi c GM \left( 1 - \frac{\Omega^2}{2\pi G \rho_M} \right)}$$

(2.40)
Without rotation, $\Gamma_\Omega \rightarrow \Gamma_{Edd} = \frac{\kappa L}{4\pi c GM}$, as expected. For hot enough stars, the opacity is dominated by the electron scattering opacity, which does not depend on $\theta$. For such stars, the local Eddington factor is constant over the whole stellar surface. For cooler stars, $\kappa$ can vary over the surface, and be higher at the equator, due to the higher effective temperature. The limiting flux is thus reached in the equatorial regions first.

The condition $\Omega \Gamma = 1$ defines the maximal luminosity $L_{\Gamma \Omega}$ for a rotating star:

$$L_{\Gamma \Omega} = \frac{4\pi c GM}{\kappa(\theta)} \left( 1 - \frac{\Omega^2}{2\pi G \rho M} \right).$$  \hspace{1cm} (2.41)

In the non-rotating case, we find that $L_{\Gamma \Omega} \rightarrow L_{Edd}$. An interesting point is that for a rotating star, the maximal allowed luminosity is decreased compared to the case of no rotation (Glatzel 1998; Maeder & Meynet 2000).

Expression (2.40) for $\Gamma_\Omega$ allows us to write $\vec{g}_{tot}$ (see (2.38)) simply as:

$$\vec{g}_{tot} = \left[ 1 - \Gamma_\Omega \right] \vec{g}_{eff}.$$  \hspace{1cm} (2.42)

As we said at the beginning of this section, the critical velocities are given by the condition $\vec{g}_{tot} = 0$. We see here that this equation has two roots, explaining why we have two different critical velocities, depending on which of the terms vanishes first.

The first critical velocity is given by $\vec{g}_{tot} = 0$, where $\vec{g}_{eff}$ is given by (2.15). The equator first reaches the critical velocity, we can thus evaluate $\vec{g}_{eff} = 0$ at $\theta = 90^\circ$, to obtain the critical angular velocity, as we did in Section 2.2.5 on page 11:

$$\Omega_{crit} = \sqrt{\frac{GM}{R_{E,crit}^3}} \quad \text{(2.43)}$$

This angular velocity leads to the following expression for the first critical velocity:

$$v_{crit,1} = \Omega_{crit} R_{E,crit} = \sqrt{\frac{2GM}{3R_{P,crit}}}.$$  \hspace{1cm} (2.44)

A star at the first critical velocity is said to be at the $\Omega$-limit.

The second root of equation (2.42) gives the second critical velocity. Relation (2.40) implies that (in what follows, the subscript $\max$ means that the value is evaluated for the star rotating at the second critical velocity):

$$\frac{\kappa(\theta, \Omega)L}{4\pi c GM} = 1 - \frac{\Omega_{\max}^2}{2\pi G \rho M}.$$  \hspace{1cm} (2.45)

The left-hand side corresponds to the classical Eddington factor $\Gamma_{Edd}$. The right-hand side depends only on the geometrical properties of the surface of the star for a given $\Omega$ through the term $\rho M$. We can write:

$$\frac{\Omega_{\max}^2 V_{\max}}{2\pi GM} = 1 - \Gamma_{Edd},$$  \hspace{1cm} (2.46)

where we have replaced the mean density by the ratio $M/V_{\max}$. Using the dimensionless notation $V_{\max} = V_{\max}/R_{E,crit}^3$ and introducing $\omega_{\max}$ with the help of relation (2.43) leads to:

$$\frac{4\omega_{\max}^2 V_{\max}^3}{27\pi} = 1 - \Gamma_{Edd}.$$  \hspace{1cm} (2.47)

$\frac{4\omega_{\max}^2 V_{\max}^3}{27\pi}$ can be evaluated by numerical integration of the surface of the star. Its representative curve as a function of $\omega$ is shown on Figure 2.7 on next page. We can see that the value of $4\omega_{\max}^2 V_{\max}^3/(27\pi)$ is smaller than 1, even when $\omega = 1$. As a consequence, relation (2.47) cannot
be satisfied for any value of $\Gamma_{\text{Edd}}$. The maximal value is $4\omega_{\text{max}}^2 V_{\text{max}}/(27\pi) = 0.3607$, and thus, the previous relation is meaningful only for stars with $\Gamma_{\text{Edd}} \geq 0.6393$. For other stars, the first critical velocity (i.e. $\tilde{g}_{\text{eff}} = 0$) is reached before $1 - \Gamma_{\Omega} = 0$.

A star rotating at the second critical velocity has an angular velocity given by (using equation (2.46)):

$$\Omega_{\text{max}}^2 = \left(1 - \Gamma_{\text{Edd}}\right) \frac{2\pi G M}{V_{\text{max}}}.$$  (2.48)

We can now derive an expression for the second critical velocity, as:

$$v_{\text{crit},2} = \frac{\Omega_{\text{max}}^2 R_{E,\text{max}}}{R_{P,\text{crit}}^2} = \frac{2\pi GM}{R_{P,\text{crit}}^2} \frac{R_{E,\text{max}}^2 (\omega_{\text{max}})(1 - \Gamma_{\text{Edd}})}{V_{\text{max}}}.$$  (2.49)

A star rotating at that critical velocity is said to be at the $\Omega\Gamma$-limit.

An amazing result is that we can define a dimensionless variable to express both critical velocities as well as the equatorial velocity of the star. Actually, we can write:

$$\frac{v_{\text{crit},1}}{v_{\text{esc},P}} = \sqrt{\frac{2}{3}},$$

$$\frac{v_{\text{crit},2}}{v_{\text{esc},P}} = \sqrt{\frac{2\pi R_{E,\text{max}}^2 (1 - \Gamma_{\text{Edd}})}{R_{P,\text{crit}}^2 V_{\text{max}}}} = \sqrt{\frac{2\pi x_{\text{max}}^2 (\theta = 90^\circ)(1 - \Gamma_{\text{Edd}})}{V_{\text{max}}}},$$

$$\frac{v_{\text{eq}}}{v_{\text{esc},P}} = \omega \frac{R_E}{R_{P,\text{crit}}} \sqrt{\frac{8}{27}} = \sqrt{\frac{8}{27}} \omega x(\theta = 90^\circ),$$  (2.50)

where $v_{\text{esc},P} \equiv \sqrt{\frac{GM}{R_{P,\text{crit}}}}$ is the escape velocity at the poles when the star is at the critical angular velocity $\Omega_{\text{crit}}$, and $x_{\text{max}}(\theta = 90^\circ)$ is defined in equation (2.24) and is a pure geometrical variable. We have thus a way to express the different velocities implied in stellar evolution independently

\[^3\text{As we have seen above, the polar radius depends only slightly on } \omega, v_{\text{esc},P} \text{ is thus also roughly the escape velocity at the pole for any rotation parameter.}\]
of the stellar properties as mass, luminosity, etc., but only as a function of the rotation parameter $\omega$, which completely defines the shape of the surface, and on the Eddington factor of the star.

The results are shown in Figure 2.8. The first critical velocity is shown in dashed red, and is independent of the rotation parameter $\omega$. The second critical velocity is in solid blue, and is represented for various values of $\Gamma_{\text{Edd}}$ (indicated along the curves). The position where a star can lie during its evolution is along the black curve. For a star with a low Eddington factor ($\Gamma_{\text{Edd}} < 0.639$), we see that if the rotation parameter increases, the equatorial velocity increases too, and encounters no obstacle until it reaches the first critical velocity, of course when $\omega = 1$.

For Eddington factors $\Gamma > 0.639$, we see that the star will reach the second critical velocity before the first one, and the permitted range of $\omega$ for that star is reduced. There is thus a zone (red area) of the diagram where it is impossible to find a star, on the bottom-right corner. Such a star would be over-critically rotating. We also show the part of the curve (in dark green) where anisotropic winds are noticeable (i.e. with a polar mass flux twice as large as the equatorial one, see Section 3.2 on page 31 for more details).

To summarise the main features discussed in this section:
CHAPTER 2. THE GENEVA EVOLUTION CODE FOR ROTATING STARS

• There are two important different critical velocities for stellar evolution;

• For stars with $\Gamma_{\text{Edd}} < 0.639$, the only critical velocity is the first one. The full range of $\omega$ is allowed, from 0 to 1.

• For stars with $\Gamma_{\text{Edd}} > 0.639$, the second critical velocity is smaller than the first one. $\omega$ is limited to a reduced range, which depends on the value of $\Gamma_{\text{Edd}}$ (the larger $\Gamma_{\text{Edd}}$, the narrower is the range of allowed values for $\omega$). For those stars, the surface effects, which only depend on $\omega$, are also limited by the maximal authorised value of the rotation parameter.

2.4 Rotationally induced increase of the total mass loss rate

2.4.1 Theoretical derivation

According to the CAK-theory for radiative winds, we can express the mass loss rate of a star by (see Section 3.4.1 on page 48, particularly equation (3.80) for more details)

$$\dot{M} = \frac{4\pi}{c} \frac{k \alpha \kappa_{\text{es}} L}{4 \pi c} \left( \frac{1}{\alpha} \right)^{1-\alpha} \left( \frac{1 - \alpha}{\alpha} \right) \frac{(GM (1 - \Gamma_{\text{Edd}}))^{\alpha-1}}{\alpha}. \quad (2.51)$$

Expressing it for a surface unit $d\sigma$ leads to (we replace the Eddington factor by its local version, $\Gamma_{\Omega}$):

$$\frac{d\dot{M}(\theta)}{d\sigma} \sim \left( \frac{1}{T_{\text{eff}}(\theta)} \right)^{\frac{1}{2}} (k \alpha)^{\frac{1}{2}} \left( \frac{1}{\alpha} \right)^{1-\alpha} \frac{g_{\text{eff}}(\theta)}{(1 - \Gamma_{\Omega}(\theta))^\frac{1}{2}-1}, \quad (2.52)$$

with $F(\theta)$ the luminous flux, and $k$ and $\alpha$ the force multiplier parameters (see Section 3.4.1 on page 48). This relation was first established by Maeder & Meynet (2000), but the term in $T_{\text{eff}}^{-1/2}$ was missing. The correction presented here is given in Maeder (2009). With the expression of the effective temperature (2.34) and of the radiative flux (2.32), the local mass flux can be rewritten:

$$\frac{d\dot{M}(\theta)}{d\sigma} \sim A \frac{1}{\left( \frac{L}{\pi c G M^*} \right)^{\frac{1}{2}}} \frac{L}{\left( \frac{GM}{\pi c G M^*} \right)^{\frac{1}{2}}} \frac{g_{\text{eff}}}{(1 - \Gamma_{\Omega}(\theta))^\frac{1}{2}-1}. \quad (2.53)$$

with $A = (k \alpha)^{\frac{1}{2}} \left( \frac{1-\alpha}{\alpha} \right)^{1-\alpha}$. Gathering the similar terms, we finally obtain:

$$\frac{d\dot{M}(\theta)}{d\sigma} \sim A \left( \frac{L}{4\pi G M^*} \right)^{\frac{1}{2}} \frac{1 - \frac{1}{2} g_{\text{eff}}^{\frac{1}{2}}}{(1 - \Gamma_{\Omega}(\theta))^{\frac{1}{2}-1}}. \quad (2.54)$$

In the frame of the radiative wind theory, the opacity is expressed through the force multiplier parameters as a function of the electron scattering opacity. We have thus to use $\kappa_{\text{es}}$ in the expression (2.40) for $\Gamma_{\Omega}$. As mentioned before, for hot enough stars, this opacity is constant over the stellar surface, and the only term which depends on the colatitude is $g_{\text{eff}}$. For cooler stars, the variation of the opacity with the colatitude should be accounted for.

To obtain the total mass loss rate, we need to average relation (2.54) over the whole stellar surface. As a first approximation, we will consider that $\Gamma_{\Omega}$ is constant over the surface. We have thus:

$$\int_{\Sigma} \frac{d\dot{M}}{d\sigma} d\sigma \sim A \left( \frac{L}{4\pi G M^*} \right)^{\frac{1}{2}} \frac{1}{(1 - \Gamma_{\Omega})^{\frac{1}{2}-1}} \frac{\int_{\Sigma} g_{\text{eff}} d\sigma}{\Sigma}. \quad (2.55)$$
In Maeder (2009), the assumption that \( \frac{J_\text{c}}{\Sigma g^7_8 \text{d} \sigma} = \left( \frac{J_\text{c}}{\Sigma g \text{d} \sigma} \right)^7 \) is done, which simplifies considerably the evaluation. Let us assume the validity of this hypothesis, which we will discuss later. We can show that at the first order (see e.g. Zahn 1992)

\[
\iint \vec{g}_{\text{eff}} \cdot \text{d} \vec{\sigma} = \iiint (-\Delta \Phi + 2\Omega^2) \text{d}V = 4\pi GM^*.
\]

Thus, putting all together, we obtain for the total mass loss rate:

\[
\dot{M} \sim \frac{AL^{\frac{1}{\alpha} - \frac{1}{k}} \Sigma^k}{\left( 4\pi GM \left( 1 - \frac{\Omega^2}{2\pi G \rho M} \right) \right)^{\frac{1}{\alpha} - 1} (1 - \Gamma)^{\frac{1}{\alpha} - 1}}.
\]

(2.57)

We can now express the ratio of the mass loss rate of a rotating star to the mass loss rate of a non-rotating star lying at the same position of the HRD:

\[
\frac{\dot{M}(\Omega)}{\dot{M}(0)} = \frac{(1 - \Gamma_{\text{Edd}})^{\frac{1}{\alpha} - 1}}{(1 - \frac{\Omega^2}{2\pi G \rho M})^{\frac{1}{\alpha} - 1} (1 - \Gamma)^{\frac{1}{\alpha} - 1}}.
\]

(2.58)

Of course, this ratio tends to 1 when \( \omega \to 0 \). We see that rotation induces an enhanced mass loss compared to the non-rotating case.

Figure 2.9: Top panel: Mean effective gravity over the stellar surface. The blue solid curve is the average value \( \langle g^7_8_{\text{eff},1} \rangle \). The (nearly superposed) red dashed curve shows \( \langle g^7_8_{\text{eff},2} \rangle \). Bottom panel: Relative difference between the two computation methods.

Let us briefly discuss the validity of the above assumption about the average \( g^7_8_{\text{eff}} \) over the stellar surface. We note:

\[
\langle g^7_8_{\text{eff},1} \rangle = \frac{\int_{\Sigma} g^7_8_{\text{eff}} \text{d} \sigma}{\Sigma},
\]

\[
\langle g^7_8_{\text{eff},2} \rangle = \left( \frac{\int_{\Sigma} g^7_8_{\text{eff}} \text{d} \sigma}{\Sigma} \right)^7.
\]

(2.59)
<g_{\text{eff},1}> is the correct way to compute the average value over the stellar surface. <g_{\text{eff},2}>> is the way followed by Maeder (2009), which leads to a simplified expression. In Figure 2.9 on previous page, we show in the upper panel <g_{\text{eff},1}> (in solid blue line) and <g_{\text{eff},2}> (in red dashed line). The bottom panel shows the relative error committed if we use the simplifying hypothesis. We see that the error remains small. Even for the highest rotation parameter, it does not exceed 3%. We can thus adopt this simplification, which allows to compute the effect of the rotation on the total mass loss rate without using a numerical integration of the surface.

2.4.2 Quantitative results

![Figure 2.10: Increase of the mass loss rate induced by rotation (in a logarithmic scale), vs $\omega$. Each curve corresponds to a different value of the Eddington factor $\Gamma_{\text{Edd}}$ (labelled along the curve).](image)

In this small section, we discuss qualitatively the effect of rotation on the total mass loss rate. Starting from relation (2.58), introducing the expression for $\Gamma_\Omega$ (2.40) and using the definition of $\omega$ and the relation for $\Omega_{\text{crit}}$ (2.43), we can write:

$$\frac{\dot{M}(\Omega)}{\dot{M}(0)} = \frac{(1 - \Gamma_{\text{Edd}})^{\frac{1}{\alpha} - 1}}{\left(1 - \frac{4\omega^2\gamma}{2\pi} - \Gamma_{\text{Edd}}\right)^{\frac{1}{\alpha} - 1}}.$$ (2.60)

Again, the increase of the mass loss rate due to rotation depends only on geometrical factor and on the Eddington factor of the star. As the condition that the denominator vanishes is completely equivalent to the condition that $\Gamma_\Omega = 1$, we conclude that the denominator can vanish only for stars with an Eddington factor $\Gamma_{\text{Edd}} > 0.639$, for which the mass loss rate diverges when the star reached the second critical velocity. For stars with $\Gamma_{\text{Edd}} < 0.639$, the denominator never vanishes, and the mass loss rate remains finite, even for stars at the first critical velocity.

For stars with $\Gamma_{\text{Edd}} > 0.639$, It is possible to rewrite relation (2.60) as a function of the ratio
2.4. Rotationally induced increase of the total mass loss rate

\( v_{\text{eq}}/v_{\text{crit},2} \) (for lower Eddington factors, we recall that the second critical velocity is not defined):

\[
\frac{\dot{M}(\Omega)}{\dot{M}(0)} = \frac{(1 - \Gamma_{\text{Edd}})^{\frac{1}{\alpha} - 1}}{(1 - \Gamma_{\text{Edd}}) \left( 1 - \frac{\nu_{\text{eq}}}{\nu_{\text{crit},2}} \frac{\nu_{\text{max}}}{\nu_{\text{E,max}}} \right)^{\frac{1}{\alpha} - 1}}.
\]

(2.61)

Since \( V \to V_{\text{max}} \) and \( R_{\text{E}} \to R_{\text{E,max}} \) when \( v_{\text{eq}} \to v_{\text{crit},2} \), we conclude that \( v_{\text{crit},2} \) is the interesting quantity governing the mass loss rate, while \( v_{\text{crit},1} \) is governing the shape of the stellar surface.

Table 2.2: For various values of the Eddington factor \( \Gamma_\Omega \) (columns 1 and 4), we give the maximum rotation parameter that the star is allowed to reach (columns 2 and 5), and the ratio \( \dot{M}(\omega)/\dot{M}(\omega = 0) \) (columns 3 and 6). The values are computed for a force multiplier parameter \( \alpha = 0.43 \).

<table>
<thead>
<tr>
<th>( \Gamma_{\text{Edd}} )</th>
<th>( \omega_{\text{max}} )</th>
<th>( \dot{M}(\omega)/\dot{M}(\omega = 0) )</th>
<th>( \Gamma_{\text{Edd}} )</th>
<th>( \omega_{\text{max}} )</th>
<th>( \dot{M}(\omega)/\dot{M}(\omega = 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>1.810</td>
<td>0.6</td>
<td>1.0</td>
<td>21.696</td>
</tr>
<tr>
<td>0.1</td>
<td>1.0</td>
<td>1.972</td>
<td>0.639</td>
<td>1.0</td>
<td>( \infty )</td>
</tr>
<tr>
<td>0.2</td>
<td>1.0</td>
<td>2.214</td>
<td>0.7</td>
<td>0.969</td>
<td>( \infty )</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0</td>
<td>2.612</td>
<td>0.8</td>
<td>0.861</td>
<td>( \infty )</td>
</tr>
<tr>
<td>0.4</td>
<td>1.0</td>
<td>3.383</td>
<td>0.9</td>
<td>0.659</td>
<td>( \infty )</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>5.444</td>
<td>0.95</td>
<td>0.484</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

We show on Figure 2.10 on previous page the ratio \( \log \left( \frac{\dot{M}(\Omega)}{\dot{M}(0)} \right) \) as a function of the rotation parameter \( \omega \) for various values of \( \Gamma_{\text{Edd}} \). We used here a typical value of 0.43 for \( \alpha \), valid for effective temperature \( 4.05 < \log(T_{\text{eff}}) < 4.3 \). We see that for low values of \( \Gamma_{\text{Edd}} \), the stars can reach the first critical velocity (\( \omega = 1 \)), with a finite enhancement of the mass loss rate. For high \( \Gamma_{\text{Edd}} \) values, the star encounters the second critical velocity first, and the mass loss rate diverges at a rotation parameter \( \omega \) smaller than 1. In Table 2.2, we summarise for various Eddington factors the increase of the mass loss rate when the star reaches the first or second critical velocity, and the maximum value of \( \omega \) that a star with a given \( \Gamma_{\text{Edd}} \) can reach.

2.4.3 Case of opacity variations with the colatitude

If the effective temperature shows strong variations with the colatitude, as expected for the fastest rotators, we should account for the induced variation of the opacity over the surface. In the radiative wind theory, the opacity is expressed as a function of the electron scattering opacity (see Section 3.4.1 on page 48). Its variation is thus accounted for through the modification of the force multiplier parameters \( \alpha \) and \( k \), and through \( A \). In that case, we normally have to compute numerically the average value of equation (2.55) over the surface, each term varying with the colatitude.

This exact method is not applied in the Geneva code, where we suppose that \( \alpha \) and \( k \) are given by the mean \( T_{\text{eff}} \) of the star. We probably underestimate the radiative mass loss for these fast rotators. However, this phenomenon is only expected for a) the less massive hot stars (around \( 5 - 10M_{\odot} \)), where the effective temperature is lower, and b) for massive stars very close to the first critical velocity, where large \( T_{\text{eff}} \) gradient could exist on the surface. In the first case, the radiative winds are negligible during the MS. In the last case, through evolution processes, the star will encounter the critical velocity, and begin to lose mass mechanically at the equator (see Section 3.2). This equatorial mass loss is now accounted for in the Geneva code (see Section 3.2.3 on page 39), and thus includes the effects of the enhancement of the mass loss rate due to opacity variations over the surface, at least roughly.
To estimate this effect, we consider the following (academic) case of three identical 60 M\(_{\odot}\) stars, with a luminosity \(L = 10^6 L_{\odot}\), an effective temperature of \(\log (T_{\text{eff}}) = 4.75\), and an Eddington factor \(\Gamma_{\text{Edd}} = 0.4\) (corresponding to a surface hydrogen mass fraction of \(\sim 0.56\)). One model is considered as non rotating, one rotating at \(\omega = 0.995\) and the last as critically rotating. For the rotating ones, the critical angular velocity on the surface is \(\Omega \simeq 1.088 \cdot 10^{-4} \text{ s}^{-1}\). For the cases with rotation, we numerically integrated equation (2.54) over the surface, accounting for the deformation. The non rotating case is trivial: the local mass flux being constant, one needs only to multiply it by the stellar surface, obtained from the values of \(L\) and \(T_{\text{eff}}\).

The results are presented in Table 2.3. For the model with \(\omega = 0.995\), the agreement between the full integration over the surface and the averaged expression is good. However, the model at the \(\Omega\)-limit shows a strong discrepancy. The region close to the equator has a strong enhancement of the local mass flux, which considerably increases the total mass loss rate compared to the one obtained with the “averaged” expression: the real increase is more than 9 orders of magnitude above the increase estimated with the simplified expression. This is produced by the very low effective temperature in that region, implying the use of different force multiplier parameters. Notice the following points:

- This effect only occurs extremely close to the critical limit.
- Numerical integration shows that the term responsible for the huge increase in relation (2.54) is the term containing \(L\) and \(M^*\). The equatorial enhanced mass loss will thus occur preferentially in the most massive stars.
- The strong mass flux is concentrated in the equatorial regions.

As we will discuss later, for numerical reasons, the maximal rotation parameter allowed during our calculations is \(\omega = 0.99\) (see Section 3.2.3 on page 39). The strong equatorial mass flux will be absorbed in the mechanical mass loss computed to maintain the star at that rotation parameter.

Table 2.3: Increase of the mass loss rate induced by rotation, taking the force multiplier parameters given by the mean effective temperature, or integrating the local mass flux over the whole stellar surface, for the 60 M\(_{\odot}\) models discussed in the text. The first column gives \(\omega\), the second one the ratio \(\dot{M}(\Omega)/\dot{M}(\Omega = 0)\) using equation (2.60) (we use \(\alpha = 0.6\) here, according to the high value of the effective temperature), and the last one the ratio \(\dot{M}(\Omega)/\dot{M}(\Omega = 0)\) obtained integrating relation (2.54) over the surface.

<table>
<thead>
<tr>
<th>(\omega)</th>
<th>(\dot{M}(\Omega)/\dot{M}(\Omega = 0)) (2.60)</th>
<th>(\dot{M}(\Omega)/\dot{M}(\Omega = 0)) (2.54)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.995</td>
<td>1.774</td>
<td>1.868</td>
</tr>
<tr>
<td>1.000</td>
<td>1.846</td>
<td>3.4 \cdot 10^9</td>
</tr>
</tbody>
</table>
2.4. Rotationally induced increase of the total mass loss rate
Improvement of the Geneva code: the origin 2009 version

In this chapter, we describe all the modifications brought to the Geneva code during this thesis work. Four main axis were developed. First, the addition of the Solberg-Høiland criterion for the convection in the stellar envelope. Second, a complete rewriting of the treatment of the wind anisotropies, which now ensures a more precise conservation of the angular momentum during the stellar evolution, and includes the mechanical equatorial mass loss for stars rotating at the critical velocity. Third, the add-on of a new mass loss prescription based on stability considerations, and at last, a huge rejuvenation of all the Geneva code, in order to allow its compatibility with the Linux operating system. This long-term work was mainly done with the precious collaboration of Sylvia Ekström.

3.1 Convection in rotating stars

3.1.1 Solberg-Høiland criterion

In a rotating star, the centrifugal force induced by rotation modifies the classical Schwartzschild (or Ledoux) criterion for stability in the stellar medium. We can derive a new stability criterion, the Solberg-Høiland criterion, which accounts for rotation. We follow here the approach of Maeder (2009), in the simplified case of cylindrical symmetry.

We consider the situation showed on Figure 3.1 on next page. A fluid element is displaced from its equilibrium position (blue), at the coordinates \((r_0, \theta_0)\) (at a distance \(\varpi_0 = r_0 \sin(\theta)\) from the rotational axis), to another close position (red, at a distance \(\varpi\) from the rotational axis). In the rotating frame, the equation of motion of the fluid element is:

\[
\frac{d^2 r}{dt^2} = -\frac{g}{\rho_{\text{int}}} (\rho_{\text{int}} - \rho_{\text{ext}}) + \varpi \left( \Omega_{\text{int}}^2 - \Omega_{\text{ext}}^2 \right),
\]  

(3.1)

where \(\rho_{\text{int}}\) respectively \(\rho_{\text{ext}}\) is the density of the fluid element, respectively of the external medium, and \(\Omega_{\text{int}}\) respectively \(\Omega_{\text{ext}}\) is the angular velocity of the fluid element, respectively of the surrounding medium. The quantities \(\rho_{\text{intr}}, \rho_{\text{ext}}, \Omega_{\text{int}}\) and \(\Omega_{\text{ext}}\) are evaluated in \(\varpi\), as in \(\varpi_0\), one has \(\rho_{\text{int}} = \rho_{\text{ext}}\) and \(\Omega_{\text{int}} = \Omega_{\text{ext}}\). The first term in the right-hand side of (3.1) is the buoyancy acceleration, while the second one is the centrifugal acceleration.
3.1. Convection in rotating stars

Figure 3.1: Schematic representation of the Solberg-Høiland criterion. The fluid element, originally in an equilibrium position \((r_0, \theta_0)\) (in blue) is displaced (red). \(\omega_0\) is the distance to the rotational axis: \(\omega_0 = r_0 \sin(\theta)\), and the angular velocity is \(\Omega\). Adapted from Maeder (2009).

We introduce the specific angular momentum \(j = \varpi^2 \Omega\) in the previous relation, to obtain:

\[
\frac{d^2 r}{dt^2} = -\frac{g}{\rho_{\text{int}}} (\rho_{\text{int}} - \rho_{\text{ext}}) + \frac{1}{\omega^3} (\dot{j}^2_{\text{int}} - \dot{j}^2_{\text{ext}}). \tag{3.2}
\]

Assuming that the fluid element conserves its specific angular momentum content and that its density remains constant during the small displacement leads at the first order to:

\[
\frac{d^2 r}{dt^2} + \left[ \frac{g}{\rho_{\text{int}}} \left( \frac{d \rho_{\text{int}}}{dr} - \frac{d \rho_{\text{ext}}}{dr} \right) + \frac{1}{\omega^3} \frac{d (\Omega^2 \varpi^4)}{d \varpi} \sin(\theta) \right] (r - r_0), \tag{3.3}
\]

where the derivatives are evaluated in \(r = r_0\). This is the equation of an harmonic oscillator, whose frequency (or growing time-scale) \(N\) is given by:

\[
N^2 = \frac{g}{\rho_{\text{int}}} \left( \frac{d \rho_{\text{int}}}{dr} - \frac{d \rho_{\text{ext}}}{dr} \right) + \frac{1}{\omega^3} \frac{d (\Omega^2 \varpi^4)}{d \varpi} \sin(\theta). \tag{3.4}
\]

Using the general equation of state\(^1\) \(\frac{dP}{\rho} = \alpha \frac{dP}{T} - \delta \frac{dT}{T} + \varphi \frac{d\mu}{\mu}\) and the definition of the pressure scale height \(H_P = -\frac{dP}{dP} P\), we can rewrite (3.4) like:

\[
N^2 = N^2_T + N^2_\mu + N^2_\Omega \sin(\theta) \quad \text{with}
\]

\[
N^2_T = \frac{g \delta}{H_P} (\nabla_{\text{int}} - \nabla)
\]

\[
N^2_\mu = \frac{g \varphi}{H_P} \nabla_{\mu}
\]

\[
N^2_\Omega = \frac{1}{\omega^3} \frac{d (\Omega^2 \varpi^4)}{d \varpi} \equiv \frac{g \delta}{H_P} \nabla_{\Omega}. \tag{3.5}
\]

\(^1\)We define here \(\alpha = \left( \frac{\partial \ln(\rho)}{\partial \ln(P)} \right)_{T,\mu}\), \(\delta = -\left( \frac{\partial \ln(\rho)}{\partial \ln(T)} \right)_{P,\mu}\) and \(\varphi = \left( \frac{\partial \ln(\rho)}{\partial \ln(\mu)} \right)_{P,T}\), such as for a perfect gas, we have \(\alpha = \delta = \varphi = 1\).

with the usual notations \( \nabla_{\text{int}} = \frac{\text{d} \ln(T_{\text{rad}})}{\text{d} \ln(P)} \), \( \nabla = \frac{\text{d} \ln(T_{\text{rad}})}{\text{d} \ln(P)} \) and \( \nabla_{\mu} = \frac{\text{d} \ln(\mu_{\text{int}})}{\text{d} \ln(P)} \), and the definition of a new term, \( \nabla_{\Omega} = \frac{H_P}{g_{\delta}} \frac{1}{\text{d} \ln(P)} \frac{d \Omega^2}{d \ln(P)} \). \( N^2 > 0 \) implies a stable medium, and \( N^2 < 0 \) an unstable medium, where convection develops. As in this work, we are interested in the hot massive star envelopes, where there is no chemical species gradient, the stability criterion with respect to convection becomes:

\[
\nabla_{\text{ad}} - \nabla_{\text{rad}} + \nabla_{\Omega} \sin(\theta) > 0,
\]

known as the Solberg-Høiland criterion. If this criterion is not fulfilled, the medium is convective.

As we generally assume that the envelope rotates as a solid-body, the expression for \( \nabla_{\Omega} \) simplifies into:

\[
\nabla_{\Omega} = \frac{H_P}{g_{\delta}} \frac{1}{\text{d} \ln(P)} \frac{d \Omega^2}{d \ln(P)} \frac{4}{\rho g_{\text{eff}}},
\]

where we used the definition of \( H_P \) given above and the hydrostatic equilibrium accounting for rotation \( \frac{dP}{dr} = -\rho g_{\text{eff}} \). As we can see, in the case of solid-body rotation, \( \nabla_{\Omega} \) is positive, and thus, rotation tends to inhibit convection.

Rotation does not modify the adiabatic gradient \( \nabla_{\text{ad}} = \frac{P_{\delta}}{c_P r^2} \). However, the radiative gradient \( \nabla_{\text{rad}} \) is changed in a rotating star, as we shall see below.

### 3.1.2 Rotational increase of the radiative gradient

To derive the radiative gradient in a rotating medium, we have to start from the basic expressions of the local radiative flux and hydrostatic equilibrium:

\[
\vec{F} = -\chi \nabla T
\]

\[
\nabla P = -\rho g_{\text{eff}}.
\]

(see eq. (2.30) for the definition of \( \chi \)). The radiative gradient becomes:

\[
\nabla_{\text{rad}} = \frac{dT}{dn} \frac{P}{dT} = \frac{F}{\chi \rho g_{\text{eff}} T} \frac{1}{\frac{dT}{dn}},
\]

the derivative being expressed along the direction \( \vec{n} \), perpendicular to the isobars. Using relations (2.32) and (2.33) leads to:

\[
\nabla_{\text{rad}} = \frac{3}{16\pi a c G} \frac{\kappa L(P)}{M^*} \frac{P}{T^4}.
\]

We see that except \( L(P) \), all the terms are local and can vary with \( r \) or \( \theta \). Thus, \( \nabla_{\text{rad}} \) also varies as a function of the colatitude, and the convection criterion may change between the equatorial regions and the poles.

### 3.1.3 Non-adiabatic convection with rotation

In the outer layers of a star, the assumption of adiabatic convection is no longer valid, and we have to compute properly the external temperature gradient \( \nabla \) to integrate the envelope structure. We follow here the approach of Kippenhahn & Weigert (1990, see also Maeder 2009). The general frame is the mixing length theory, which assumes that the convective fluid elements move over an average distance \( l \) (the mixing length) which is defined with respect to the pressure scale height:

\( l = \alpha H_P \), where \( \alpha \) is a proportionality factor.

We write that the total energy flux \( F_{\text{tot}} \) is the sum of the energy flux carried by radiation \( F_{\text{rad}} \), and by convection \( F_{\text{conv}} \). The radiative gradient is defined as the gradient which would be necessary for the flux to be completely radiative:

\[
F_{\text{tot}} = F_{\text{rad}} + F_{\text{conv}} = \frac{4a c G M^*_r T^4}{3 \kappa P r^2} \nabla_{\text{rad}}.
\]
The real temperature stratification of the star is given by the radiative flux:

\[ F_{\text{rad}} = \frac{4acG}{3} \frac{M_r T^4}{\kappa Pr^2} \nabla. \]  

(3.12)

Let us consider a convective element with an average temperature excess \( \Delta T \) with respect to the surrounding medium. Its velocity is \( v \) and we assume that it is in pressure equilibrium (i.e. its velocity is inferior to sound speed). The convective energy flux carried by that element is:

\[ F_{\text{conv}} = \rho cvT. \]  

(3.13)

We express the variation of temperature of the element with respect to the external medium after it has moved over half of the distance \( l \):

\[ \frac{\Delta T}{T} \approx \frac{1}{T} \frac{\partial (T_{\text{int}} - T_{\text{ext}})}{dr} \frac{l}{2} = \frac{1}{H_P} (\nabla - \nabla_{\text{int}}) \frac{l}{2}. \]  

(3.14)

The next step is to consider the specific work of the buoyancy force \( -g_{\text{eff}} \Delta \rho / \rho \) over this motion. In the mixing length theory, the average force during the motion is estimated as half of the force estimated in \( l/2 \). We have, using the general equation of state for a medium in pressure equilibrium and without \( \mu \) gradient\(^2\) and (3.14):

\[ -\frac{1}{2} g_{\text{eff}} \frac{\Delta \rho}{\rho} \frac{l}{2} = \frac{g_{\text{eff}}}{H_P} (\nabla - \nabla_{\text{int}}) \frac{l^2}{8}, \]  

(3.15)

If roughly half of this energy is converted into kinetic energy, we obtain an expression for the velocity of the fluid element:

\[ v^2 = \frac{g_{\text{eff}}}{H_P} (\nabla - \nabla_{\text{int}}) \frac{l^2}{8}. \]  

(3.16)

Inserting (3.14) and (3.16) into relation (3.13) allows to express the convective energy flux as:

\[ F_{\text{conv}} = \rho cP T \sqrt{\frac{g_{\text{eff}}}{H_P}} \frac{l^2}{4\sqrt{2}} \left( \frac{\nabla - \nabla_{\text{int}}}{H_P} \right)^{3/2}. \]  

(3.17)

Eventually, we consider the temperature change in the fluid element during its motion. There are two causes of temperature variations in the element: first, the adiabatic contraction or expansion, and second, the radiative losses. The radiative energy losses by time unit can be expressed as:

\[ \lambda = \frac{8acT^3}{3\kappa \rho} \Delta T \frac{S}{d}, \]  

(3.18)

with \( S \) and \( d \) the surface and diameter of the element. The energy losses by length unit is thus \( \frac{\lambda}{\rho V cP v} \), and we can write:

\[ \frac{dT_{\text{int}}}{dr} = \frac{dT_{\text{ad}}}{dr} - \frac{\lambda}{\rho V cP v}, \]  

(3.19)

(the index “ad” refers to adiabatic). We have thus the following relation:

\[ \nabla_{\text{int}} - \nabla_{\text{ad}} = \frac{\lambda H_P}{\rho V cP v T}. \]  

(3.20)

Using (3.14), and writing the geometric factor \( \frac{IS}{V^2} = \frac{9}{2l} \), this last relation leads to:

\[ \nabla_{\text{int}} - \nabla_{\text{ad}} = \frac{6acT^3}{\kappa \rho^2 cP v l v}. \]  

(3.21)

\(^2\)In that case, the equation of state reduces to \( \frac{\Delta \rho}{\rho} = -\delta \frac{\Delta T}{T} \).
Equations (3.11), (3.12), (3.13), (3.16), and (3.21) form a complete set of equations for the five unknown variables $F_{\text{rad}}$, $F_{\text{conv}}$, $v$, $\nabla$ and $\nabla_{\text{int}}$. With the dimensionless quantities

$$U = \frac{3acT^3}{c_P \rho^2 \kappa t^2} \sqrt{\frac{8HP}{g_{\text{eff}} \delta}}$$

$$W = \nabla_{\text{rad}} - \nabla_{\text{ad}}$$

$$\xi = \sqrt{U^2 + \nabla - \nabla_{\text{ad}}}$$

the whole set of equations simplifies in one third order equation:

$$(\xi - U)^3 + \frac{8U}{9} (\xi^2 - U^2 - W) = 0,$$

which can be solved by numerical methods. Once the temperature gradient of the medium $\nabla$ is known, the envelope can be integrated.

During this thesis work, we applied this theoretical frame to the case of the envelopes of rotating hot massive stars, solving the above set of equations. The results are presented in Chapter 4 on page 63.

### 3.2 Anisotropic mass loss in rotating stars

As we have seen in Chapter 2 on page 7, rotation has many effects on the surface of a star, particularly for very fast rotators. In this chapter, we will first discuss the particularities of the mass loss for such rotating stars, especially the development of anisotropies in the stellar winds. In a second part, we also discuss the modifications of the Geneva code which were needed to implement properly the effect of such anisotropies.

#### 3.2.1 Anisotropic winds: theoretical context

The basic relation which shows that stellar winds must be anisotropic is the equation (2.54) on page 21, that we recall here:

$$\frac{dM(\theta)}{d\sigma} \sim A \left( \frac{L}{4\pi GM^*} \right)^{\frac{1}{2}} \frac{g_{\text{eff}}^{1-\frac{1}{8}}}{(1 - \Gamma_{\Omega}(\theta))^{\frac{1}{2}-1}}.$$  (3.24)

As already mentioned, in the frame of the radiative wind theory, we must consider the electron scattering opacity in the expression of $\Gamma_{\Omega}$, which is thus constant over the surface of the star (actually, the line opacity is accounted for in the force multiplier parameters, see Section 3.4.1 on page 48). The terms $L$ and $M^*$ are global parameters of the star. However, the remaining terms $A$ and $g_{\text{eff}}$ vary as a function of the colatitude $\theta$. These simple considerations explain completely why we expect anisotropies in the radiative winds for rotating stars.

### Anisotropic mass flux

Maeder & Meynet (2000) distinguish two effects, related to each of the two varying terms:

- **The $g_{\text{eff}}$-effect**: Contrary to what one might thing, the mass flux is (almost) proportional to the effective gravity. One expects an enhanced mass flux at the poles, due to the stronger $g_{\text{eff}}$ in those regions.

---

3Here, we have to consider the radiative gradient accounting for rotation (see Section 3.1.2 on page 29).
3.2. Anisotropic mass loss in rotating stars

Figure 3.2: Local mass loss flux $\frac{dM}{d\sigma}$ over the stellar surface. The $x$-axis represents the colatitude $\theta$. The poles are on the left, and the equator on the right. The $y$-axis corresponds to the rotation parameter $\omega$. The non-rotating case is on the bottom, and the critical-rotation one on the top. The colour scale on the right shows the intensity of the mass loss flux. Red is for the larger one, and blue for the smaller one. The mass loss rate is given in units of $\dot{M}_\star/(4\pi R_\star^2)$, where $\dot{M}_\star$ is the total mass loss rate.

Figure 3.3: Left panel: Local mass flux for various colatitudes, at a rotation parameter $\omega = 0.7$. The shape of the surface, seen equator-on, is represented by the thick black curve, the $x$- and $y$-axis being in units of the polar radius. The local mass flux is represented by the blue arrows. The length of the arrow is proportional to the intensity of the wind at that point, normalised to the polar wind intensity. Right panel: Same as left panel, but for a rotation parameter $\omega = 1$. 
• **The \( \kappa \)-effect**: Due to variations of the force multiplier parameters \( \alpha \) and \( k \) with the effective temperature, \( A \) will vary over the stellar surface, if a strong \( T_{\text{eff}} \) gradient (as a function of the colatitude) is present. In the most extreme cases, \( A \) is larger by a factor of \( \sim 50 \) between the poles and the equator. The variation of \( \alpha \) also contributes to the equatorial mass flux through the terms containing \( L/M^* \) and \( \Gamma_\Omega \).

In a first study, we shall consider a constant value of \( \alpha \) and \( k \) over the whole surface. We use the surface model presented in Section 2.2.5 on page 11, where the geometrical factors are normalised with respect to the polar radius of the star. To allow easy comparison between various models with different different surface parameters \( (L, T_{\text{eff}},...) \), we assume that the total mass loss rate of the star is \( 4\pi \), in order to have a local mass flux of 1 in the non-rotating case. Note that these normalisations are of course not applied in the computation of our stellar models. They are made here just for a purpose of making the comparison between various models easier and illustrate the effects of rotation on the wind anisotropy. The local mass flux is shown in Figure 3.2 on previous page.

In the non-rotating case, the mass flux is of course constant at each colatitude. With increasing rotation parameters, the mass loss becomes more and more polar. At the critical limit, the equatorial mass loss becomes even zero, due to the vanishing of the effective gravity in that region. In Figure 3.3 on previous page, we show for two rotation parameters \( (\omega = 0.7 \text{ and } \omega = 1) \) the behaviour of the local mass flux for different colatitudes. We see that for \( \omega = 0.7 \) (left panel), the anisotropic effect, even if they are already noticeable, remains modest. At the first critical velocity (right panel), the wind is clearly polar. However, we recall that we have considered here that the force multiplier parameters remain constant, even for low effective temperature. In a more realistic treatment, the equatorial mass flux would be enhanced due to opacity variations. This effect is accounted for in the Geneva code, as we will see below.

![Figure 3.4: Fraction of the total mass loss contained in a cone aligned with the rotation axis and of semi-aperture \( \theta \). The red curve shows the non-rotating case, and the blue one, the critically rotating case \( \omega = 1 \). The intermediate black curves are, from bottom to top, for \( \omega = 0.5, 0.75, 0.9, 0.96, 0.98, 0.99, 0.995 \).](image-url)
3.2. Anisotropic mass loss in rotating stars

It is interesting to consider the integration of the local mass flux up to a given colatitude $\theta$, and to compare it with the total mass loss of the star:

\[
\text{fraction of total mass loss} = \frac{\int_0^\theta \int_0^{2\pi} \frac{dM}{\sigma} r^2 \sin(\theta) d\theta d\varphi}{\dot{M}}. \tag{3.25}
\]

Figure 3.4 on previous page represents this fraction of the total mass loss, as a function of the colatitude, and for various rotation parameter. In the case of a non-rotating star, we see that half of the total mass loss is lost in a cone of semi-aperture $\theta = 60^\circ$. At the first critical velocity, this angle reduces to around $50^\circ$: the mass loss is more concentrated along the rotation axis.

Another quantity which could be measurable through interferometric observations is the equatorial to polar flux ratio. If we assume that the star is not extremely close to the first critical velocity (and thus, that the force multiplier parameters over the surface are constant enough), a measurement of such a ratio is a way to determine the rotation parameter of the star. Moreover, if the equatorial velocity of the star is measurable by other means, the comparison with the expected value is a direct test of the model described in this work. In Figure 3.5, we show this ratio, as a function of the rotation parameter $\omega$. The polar flux becomes twice as large as the equatorial one at $\omega \sim 0.8$. The variation becomes very important close to the critical rotation parameter, where the anisotropic effects are more marked. Amazingly, the variation of the ratio $F_{\text{equator}}/F_{\text{pole}}$ as a function of the rotation parameter $\omega$ is almost a perfect circle! This ratio can thus be very well approximated by:

\[
\frac{F_{\text{equator}}}{F_{\text{pole}}} \approx \sqrt{1 - \omega^2}. \tag{3.26}
\]

The error is less than 1% for $0 < \omega < 0.95$. 

Figure 3.5: Equatorial to polar mass flux ratio, as a function of $\omega$. Initially isotropic, the flux becomes more and more polar as $\omega$ increases.
Angular momentum loss

An important thing for stellar evolution is the angular momentum content of the stars. As the local mass flux becomes anisotropic under the effect of rotation, the angular momentum lost is modified, as more mass is lost along the polar axis, carrying away less angular momentum. Once we know the local mass flux $\frac{dM}{d\sigma}(\theta)$ and the shape of the surface, we are able to compute the local angular momentum lost $\frac{d\dot{L}}{d\sigma}$:

$$\frac{d\dot{L}}{d\sigma}(\theta) = \frac{d\dot{M}}{d\sigma}(\theta)\Omega \varpi^2(\theta),$$  \hspace{0.5cm} (3.27)

where $\varpi$ is the distance to the rotation axis. We have immediately that:

$$\frac{d\dot{L}}{d\sigma}(\theta) = \frac{d\dot{M}}{d\sigma}(\theta)\Omega r^2(\theta) \sin^2(\theta).$$  \hspace{0.5cm} (3.28)

In order to compare stars with different angular velocities, it is more convenient to compute the variation of the ratio $\frac{\dot{L}}{\Omega}$, which is independent of $\Omega$:

$$\frac{d}{d\sigma} \left( \frac{\dot{L}}{\Omega} \right)(\theta) = \frac{d\dot{M}}{d\sigma}(\theta) r^2(\theta) \sin^2(\theta).$$  \hspace{0.5cm} (3.29)

The expression of the surface element $d\sigma$ is simply the classical spherical surface element, corrected by $\frac{1}{\cos(\epsilon)}$, where $\epsilon$ is the angle between the local normal direction, and the radial direction:

$$\cos(\epsilon) = \frac{\hat{g}_{\text{eff}} \cdot \hat{e}_r}{|\hat{g}_{\text{eff}}|}.$$  \hspace{0.5cm} (3.30)

We have thus: $d\sigma = \frac{r^2 \sin(\theta)}{\cos(\epsilon)} d\theta d\varphi$. The geometry of the problem being axisymmetric, we can integrate that loss over a ring centred on the rotation axis at the colatitude $\theta$. This leads to:

$$\frac{d}{d\theta} \left( \frac{\dot{L}}{\pi} \right)(\theta) = 2\pi \frac{d\dot{M}}{d\sigma}(\theta) r^4(\theta) \frac{\sin^3(\theta)}{\cos(\epsilon)}.$$  \hspace{0.5cm} (3.31)

This expresses the variation or the momentum of inertia of an infinitesimal ring at the colatitude $\theta$ due to the mass loss. The results are shown on Figure. 3.6 on next page. Without rotation (or with very small rotation), the ratio of the angular momentum to the surface angular velocity, or, as the surface is assumed to rotate at a constant angular velocity, the angular momentum, is lost mainly at the equator. The mass flux being constant over the surface, the loss occurs where the distance to the rotation axis is the larger. Increasing the rotation parameter $\omega$, two counterbalancing effects appear:

- the modification of the shape of the star, which makes the equatorial radius larger, and thus increases the angular momentum lost at the equator;
- the modification of the local mass flux (see Figure 3.2 on page 32), which favours polar mass loss, and thus tends to decrease the equatorial loss of angular momentum.

Up to a rotation parameter $\sim 0.85$, the first effect is dominant: the maximum contribution to the total angular momentum lost is at the equator, and increases with increasing $\omega$. Above that value, the anisotropy in the winds becomes significant, and the second effect begins to influence the distribution of angular momentum loss. We see that the location of the maximum contribution to the angular momentum loss is progressively shifted towards lower colatitudes, up to $\theta \sim 70^\circ$.
3.2. Anisotropic mass loss in rotating stars

Figure 3.6: Distribution of the loss of angular momentum \( \frac{d(\dot{L}/\Omega)}{d\theta} \) over the stellar surface. The \( x \)-axis is the colatitude (the poles are on the left, the equator on the right), and the \( y \)-axis the rotation parameter \( \omega \). The colour scale on the right indicates the intensity of \( \frac{d(\dot{L}/\Omega)}{d\theta} \), in units of \( \dot{M}R_p^2/(4\pi) \). The highest values are in red, the lowest in blue.

For a critically rotating star. In the same time, the contribution of the equatorial region strongly decreases, becoming even zero\(^4\) when \( \omega = 1 \).

Once we know the distribution of the angular momentum loss \( d\dot{L}/d\theta \), we can integrate it over the colatitudes to obtain the total angular momentum loss of the star for a given rotation parameter. To better understand the effects of the anisotropic winds on the angular momentum content of the star, we have to compare different cases. We distinguish the following ones:

- **Case 1:** It is the method used in most of the stellar evolution codes to compute the loss of angular momentum. Both stellar shape deformation and anisotropic winds are neglected, and the stellar radius is defined with the help of the mean effective temperature and luminosity: 
  \[ r_{\text{mean}} = \sqrt{\frac{L}{4\pi \sigma (T_{\text{eff}})^4}}. \]
  The angular momentum brought away by the stellar winds is computed as 
  \[ \dot{L} = \frac{3}{2} M \Omega r_{\text{mean}}^2. \]

- **Case 2:** Only the effects of rotation on the stellar surface shape are accounted for. The wind anisotropies are neglected. This case is useful to highlight the effect of the anisotropy, comparing with case 3, or the effects of the shape of the surface only, comparing with case 1.

- **Case 3:** All the effects discussed above are accounted for. The angular momentum loss is computed integrating equation (3.31) from the poles to the equator.

On Figure 3.7 on next page, we show the differences between these three cases. The loss of momentum of inertia (or, equivalently, the angular momentum loss) is plotted as a function of \( \omega \). Case 1 is in red (long-dashed line), case 2 in black (solid line) and case 3 in blue (short dashed line). In case 1, the angular momentum loss increases with increasing \( \omega \). This is due to the fact that the mean effective temperature of a fast rotating star is lowered, producing an increase of the deduced mean stellar radius \( r_{\text{mean}} \). In case 2, the effects of the stellar surface deformation are maximal. The equatorial radius becomes very large at high rotation parameter, producing a strong increase of

\[^4\text{We recall here that the assumption of the constancy of the force multiplier parameters } \alpha \text{ and } k \text{ over the whole surface for a star at the first critical velocity is probably not valid, since the effective temperature in the equatorial region strongly decreases in that case.}\]
the angular momentum loss. Due to the anisotropic winds, case 3 shows a different behaviour: the modification of the shape is counterbalanced by the development of the polar winds, which reduce the angular momentum loss. At very high rotation parameter (> 0.95), the drop of the mass flux at the equator is strong enough to produce a decrease in the angular momentum loss.

![Figure 3.7](image.png)

Figure 3.7: Variation of the ratio of the angular momentum to $\Omega$ (or, after multiplication by the angular velocity, angular momentum loss), as a function of the rotation parameter $\omega$, for the three cases discussed in the text. Case 1 is in red (long-dashed line), case 2 in black (solid line) and case 3 in blue (short dashed line).

To estimate the error in terms of angular momentum content in the star in the "classical" calculation (case 1), we compute the ratio of the time-variation of the ratio $\frac{\dot{\mathcal{L}}}{\Omega}$ (case 1) to $\frac{\dot{\mathcal{L}}_{\text{ani}}}{\Omega}$ (case 3, all effects accounted for). The result is shown on Figure 3.8 on next page. We see that for not extreme rotators, the error is smaller than 5%. For rotation parameters above 0.9, the difference between the two cases increases more, but even at the critical velocity, it is never larger than 25%.

As in most of the cases, stars are not close to the critical velocity, neglecting the anisotropic correction is a fairly good approximation. For the faster rotators, a detailed study has to be made (see Section 5.1 on page 69). However, as a first guess, we expect that most of the stars reaching such high rotation parameters will also reach the critical velocity. For these stars, an equatorial mass loss appears, which removes angular momentum from the star more efficiently than the radiative winds.

### 3.2.2 The angular momentum problem in the Geneva code: first estimates

In this section, we discuss two important points related to anisotropic winds and angular momentum conservation in the Geneva evolution code. First, we do some order-of-magnitude estimates about the amount of angular momentum removed by the stellar winds. Second, we show why the treatment of the angular momentum was not accurate enough in origin 2006 and previous versions to properly account for anisotropic winds.

If we consider a standard 25 M$_\odot$ star at solar metallicity, a typical value for $\dot{M}$ during the MS is $\log (\dot{M}) \simeq -6.5$ and for the luminosity and effective temperature, $\log (L) \simeq 5$ and $\log (T_{\text{eff}}) \simeq 4.5$.
3.2. Anisotropic mass loss in rotating stars

The angular velocity at the surface is around $\Omega \sim 10^{-5} \, \text{s}^{-1}$. This leads to a mean radius $r_{\text{mean}} \simeq 2.5 \cdot 10^{12} \, \text{cm}$. The typical angular momentum lost by time unit is given by $\dot{L} \simeq \frac{2}{3} M \Omega r_{\text{mean}}^2 \simeq 10^{38} \, \text{g cm}^2 \, \text{s}^{-2}$. A characteristic time step for such a star in our stellar models is $\sim 100 \, \text{yr}$. Thus, on a time step, the star loses roughly $4 \cdot 10^{47} \, \text{g cm}^2 \, \text{s}^{-1}$. Initially, a $25 \, \text{M}_\odot$ star with an initial velocity of 40% of the critical velocity contains $\sim 5 \cdot 10^{52} \, \text{g cm}^2 \, \text{s}^{-1}$.

On a time step, a star loses thus typically $10^{-5}$ times its total content of angular momentum. Anisotropic effects, as we saw before, have an impact of a few percents, let us say, 1% to 10% of the angular momentum loss computed neglecting the anisotropic effects. Thus, to follow accurately a stellar evolution sequence accounting for the effects of wind anisotropies, we must ensure a conservation of the angular momentum in the system “star + stellar winds” of the order of $10^{-7}$ to $10^{-6}$ times the total content at each time step!

As we mentioned before (see Section 2.2 on page 8), the transport of angular momentum is only treated in the stellar interior in the Geneva code. The envelope is “floating” above the first layer, and is assume to rotate at the same angular velocity. This induces some variations in the angular momentum content. Actually, even with a fitting mass as high as $\text{FITM} = 0.9999$, due to the large radius of the envelope, its angular momentum content is $\sim 10^{-3}$ the total amount of angular momentum in the whole star. If the angular velocity of the first layer of the star is modified, for example because of internal transport processes, it produces a variation of the angular momentum of the envelope, which is not considered in the conservation of that quantity. Even if that variation is small, it needs not to be larger than $10^{-4} - 10^{-3}$ of the angular velocity variation produced by the stellar winds to completely drown the anisotropic effects.

For these reasons, the Geneva code in its 2006 version was not able to follow carefully the effects of anisotropic winds, and it was necessary to bring some modifications to ensure a good conservation of the angular momentum. The method which was developed during this thesis work is described in the next section.
3.2.3 New treatment of the anisotropic winds

The actual structure of the Geneva code does not allow a simple treatment of the rotation in the envelope, which would be the best solution to treat properly the angular momentum conservation. We chose another approach, which consists in correcting the angular velocity profile on a given number of layers, in order to account for the angular momentum removed by the stellar winds. This method has the advantage of easily allowing the estimation of the mechanical equatorial mass loss for stars reaching the first critical velocity. The new method is extensively described in this section.

The general scheme of the treatment of mass loss and rotation in the Geneva code

Figure 3.9 on next page shows the general organisation of the treatment of the mass loss and rotation in the Geneva code. Let us briefly describe here the main steps, indicated in the blue squares. The green squares represent some stages which do not appear at each time step. The orange diamonds show the main conditional tests. The most important modifications were brought at the points marked with a red circle. The number therein will serve as a reference for a much detailed explanation below.

The progress of the treatment of mass loss and rotation follows the following way:

- Initially, the computation of the structure at a given time step starts on the basis of an approximate model, estimated on the converged models of the previous time steps.
- From the estimated stellar parameters, the non-rotating mass loss rate is computed, according to one of the mass loss prescriptions available (see Section 3.4.1 on page 48).
- The modification of the mass loss rate induced by rotation is computed (see Section 2.4 on page 21).
- The real radiative mass loss rate is thus obtained.
- If one wants to consider the effects of the wind anisotropy, its effect on the angular momentum loss is computed (see Point 1 below).
- The corresponding loss of angular momentum by radiative wind is computed.
- The new angular velocity of the surface after the angular momentum loss is estimated (see Point 2 below).
- The condition that the estimated new angular velocity is lower than a maximal allowed fraction of the critical velocity is checked.
- If the surface is found to be over-critically rotating, the mass which needs to be removed and lost in an equatorial disk is computed, as well as the corresponding angular momentum loss. In the following of this work, this phenomenon is called indifferently “mechanical” mass loss, or “equatorial” mass loss (see Point 3 below).
- The total mass of the star and the total angular momentum loss during the time step are updated.
- If the diffusion is applied during the current time step, the $\Omega$-profile is corrected by the $\Delta\Omega$-correction, computed at the previous time step.
- Entrance in the structure and transport processes resolution loop. The new structure is computed first.
3.2. Anisotropic mass loss in rotating stars

Figure 3.9: General scheme of the Geneva evolution code. The blue squares represent the main stages of the treatment, the orange diamonds the points where a condition is tested, the green ones the stages where one does not pass at each time step. The numbers in the red circles indicates the part which were significantly modified during this work, and which are described in more details below (see text).

- On the new structure, the transport of chemical species and angular momentum is computed.

- If the conservation of the angular momentum is not accurate enough during the mixing process, a correction is applied to the whole \( \Omega \)-profile (see Point 4 below).

- Test of the convergence of the current model. If the convergence criterion are not fulfilled, the resolution loop is repeated.

- Once the model has converged, a correction of the angular velocity \( \Delta \Omega_{\text{corr}} \) is computed for a given number of layers \( N_{\text{corr}} \). This correction is applied during the next time step where the diffusion is applied (for robustness considerations). The correction ensures a complete conservation of the angular momentum of the system “star + stellar winds” during the time step (see Point 5 below).

- As the correction is applied only when diffusion occurs, the angular momentum loss during the “advective” time steps has to be saved for the next time step. It allows to account for that loss in computing the correction of the \( \Omega \)-profile during the next time step, and thus to ensure the conservation of angular momentum.

- A new model begins.

1 Anisotropic correction

The anisotropic correction is computed by integrating equation 3.24 over the stellar surface. As this relation is not an equality, but only a proportion, we can not obtain the total mass loss rate at each colatitude, but the relative fraction lost as a function of \( \theta \). Computing the mass loss rate of the star by one of the various implemented prescriptions allows to scale the relative anisotropic mass loss obtained to a real local mass loss.

Once we know the shape of the surface and the local mass loss, we can determine two quantities: the angular momentum loss accounting for the anisotropic effects \( \Delta L_{\text{ani}} \), and the isotropic angular momentum loss \( \Delta L_{\text{iso}} \), assuming that the mass loss is equally distributed over the surface, but accounting for the shape of the star.

The technical description of this modification is given in Appendix A.1 on page 147.

2 Estimation of the variation of the surface angular velocity due to mass loss

In order to estimate the effect of the radiative winds on the surface velocity, we use the fact that the angular momentum content of the star after the mass loss process should equal the initial one, minus the angular momentum removed by the stellar winds. As a first attempt, we supposed that all the angular momentum carried by stellar winds were extracted from the envelope and the first layer of the star. This solution was inefficient, due to the very small time steps needed.

To allow the increase of the time step, we consider that the angular momentum is removed from a certain number of layers \( N_{\text{corr}} \) under the stellar surface. Typically, for models with a total number of layers around 1000 – 1500, we chose \( N_{\text{corr}} = 200 \). It is roughly the depth which is sensitive to what happens on the surface under the influence of angular momentum transport processes. For example, typical value for the meridional circulation velocity near the surface of a 7\( \text{M}_\odot \) star is of the order of \( \sim 1 - 5 \cdot 10^{-5} \text{ km s}^{-1} \), and the depth of the 200th layer below the first one is \( \sim 5 \cdot 10^{10} \text{ cm} \). The characteristic time of the angular momentum transport between those layers is thus of the order of \( \sim 10^{10} \text{ s} \sim 300 \text{ yr} \). With a time step of \( \sim 500 - 1000 \text{ yr} \) for this model, the chosen value for \( N_{\text{corr}} \) is justified. The schema on Figure. 3.10 on next page clarifies the notation used below.
3.2. Anisotropic mass loss in rotating stars

Figure 3.10: Schematic view of the first layers under the surface. Each layer (as well as the envelope) has a given mass, radius, angular velocity and angular momentum. We assume here that the envelope has all its mass concentrated at the radius of the first internal layer, and that it rotates at the same angular velocity than the first layer.

<table>
<thead>
<tr>
<th>Layer</th>
<th>m_i</th>
<th>r_i</th>
<th>Ω_i</th>
<th>L_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Envelope</td>
<td>m_e</td>
<td>r_1</td>
<td>Ω_1</td>
<td>L_e</td>
</tr>
<tr>
<td>Layer 1</td>
<td>m_1</td>
<td>r_1</td>
<td>Ω_1</td>
<td>L_1</td>
</tr>
<tr>
<td>Layer 2</td>
<td>m_2</td>
<td>r_2</td>
<td>Ω_2</td>
<td>L_2</td>
</tr>
<tr>
<td>Layer 3</td>
<td>m_3</td>
<td>r_3</td>
<td>Ω_3</td>
<td>L_3</td>
</tr>
<tr>
<td>Layer 4</td>
<td>m_4</td>
<td>r_4</td>
<td>Ω_4</td>
<td>L_4</td>
</tr>
<tr>
<td>Layer ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The most important point is that we assume that the envelope rotates at the same angular velocity and has the same radius than the first layer of the interior. The first hypothesis is sustained by the strong mixing below the surface, which favours solid-body rotation. Moreover, in the Geneva evolution code, rotation is not followed in the envelope. The second one is justified by the strong concentration of the envelope mass just above the first layer of the interior. A future improvement could be the computation of the real momentum of inertia of the envelope during its integration, but it is not implemented yet.

The initial angular momentum content \( L_{\text{ini}}^{\text{tot}} \) of the \( N_{\text{corr}} \) layers is:

\[
L_{\text{ini}}^{\text{tot}} = \Omega_1 I_e + \sum_{i=1}^{N_{\text{corr}}} \Omega_i I_i,
\tag{3.32}
\]

where \( I_i \), respectively \( I_e \), is the momentum of inertia of the \( i \)th layer, respectively of the envelope, and \( \Omega_i \) the actual angular velocity. Assuming that during the mass loss process, the radius of each shell remains the same, and that the mass is lost in the envelope, we can write the new angular momentum content as:

\[
L_{\text{fin}}^{\text{tot}} = \Omega_1^{\text{new}} I_e \frac{m_e - \Delta M_{\text{rad}}}{m_e} + \sum_{i=1}^{N_{\text{corr}}} \Omega_i^{\text{new}} I_i,
\tag{3.33}
\]

with the superscript “new” applying to values estimated after the mass loss occurs, and \( \Delta M_{\text{rad}} \) the radiative mass loss.

Knowing the angular momentum \( \Delta L_{\text{rad}} \) removed by the stellar wind (with \( \Delta L_{\text{rad}} = \Delta L_{\text{ani}} \) or \( \Delta L_{\text{rad}} = \Delta L_{\text{iso}} \), according to the choice to follow or not the effects of anisotropic mass loss, see Point 1 above), we must have:

\[
L_{\text{tot}}^{\text{fin}} = L_{\text{tot}}^{\text{ini}} - \Delta L_{\text{rad}}.
\tag{3.34}
\]

We suppose that the new angular velocity can be written as:

\[
\Omega_i^{\text{new}} = \Omega_i (1 + q_{\text{corr}}),
\tag{3.35}
\]

where \( q_{\text{corr}} \) is a numerical factor, similar for the \( N_{\text{corr}} \) shells. Putting equations (3.32)-(3.35) together, one obtains an expression for \( q_{\text{corr}} \):

\[
q = \frac{L_{\text{fin}}^{\text{ini}} \Delta M_{\text{rad}}}{L_{\text{fin}}^{\text{ini}} - L_{\text{ini}}^{\text{ini}} \Delta M_{\text{rad}} / m_e}.
\tag{3.36}
\]
With relation \((3.35)\), it is now possible to compute the new angular velocity of the surface, only due to the radiative mass loss. This estimate account of course not for the internal transport of the angular momentum. However, it gives some indications about how the surface velocity is affected by the stellar winds.

The technical description of this modification is given in Appendix A.2 on page 147.

3 Mechanical mass loss computation

We are now able to (roughly) determine if the radiative mass loss through stellar winds is strong enough to keep the star at a sub-critical rotation parameter, or not. For numerical reasons, it is not possible, or very difficult, to compute the evolution when the star is at the critical velocity. We thus define a maximal fraction of the critical velocity \(R_{\Omega,\text{max}}\), defined as:

\[
R_{\Omega,\text{max}} = \frac{\Omega_{\text{max allowed}}}{\Omega_{\text{crit}}},
\]

above which the star is considered to be critically rotating. For stars with relatively low Eddington factor, \(R_{\Omega,\text{max}}\) can be as high as 0.99. For stars near the \(\Omega\Gamma\)-limit, the value of \(R_{\Omega,\text{max}}\) must be lower (around 0.95).

If the ratio \(\frac{\Omega_{\text{new}}}{\Omega_{\text{crit}}} > R_{\Omega,\text{max}}\), an additional contribution to the mass loss rate must be computed. As the star is critically rotating, we assume that this mass is lost mechanically at the equator, helped by the very weak effective gravity. We define a limit factor \(q_{\text{lim}}\), defined by:

\[
\Omega_{\text{max,allowed}} = \Omega_1 (1 + q_{\text{lim}}).
\]

which immediately leads to the definition of the limit angular velocity \(\Omega_{\text{lim}}^1\) for the other layers:

\[
\Omega_{\text{lim}}^i = \Omega_i (1 + q_{\text{lim}}) = \Omega_i \frac{\Omega_{\text{lim}}^1}{\Omega_1},
\]

where we write \(\Omega_{\text{max,allowed}} \equiv \Omega_{\text{lim}}^1\).

It is thus now possible to compute the actual angular momentum of the star with the angular velocity profile \(\Omega_i\), and the angular momentum that the star should have if it was critically rotating, with the values of \(\Omega_{\text{lim}}^i\). We assume that the difference between these two values is lost mechanically in an equatorial disc. We have:

\[
\Delta L_{\text{tot}} = L_{\text{ini}} - \Delta L_{\text{rad}} - \Delta L_{\text{mec}},
\]

where \(\Delta L_{\text{mec}}\) is the angular momentum lost in the form of an equatorial disc:

\[
\Delta L_{\text{mec}} = \Delta M_{\text{mec}} R_E^2 \Omega_1 (3.41)
\]

with \(\Delta M_{\text{mec}}\) the amount of mass lost during the time step in the disc, and \(R_E\) the equatorial radius of the star (see Section 2.2 on page 8). We have thus:

\[
\Omega_{\text{lim}}^1 T_e m_e - \Delta M_{\text{rad}} - \Delta M_{\text{mec}} + \sum_{i=1}^{N_{\text{corr}}} \Omega_{\text{lim}}^i T_i = \Omega_1 T_e + \sum_{i=1}^{N_{\text{corr}}} \Omega_i T_i - \Delta L_{\text{rad}} - \Delta M_{\text{mec}} R_E^2 \Omega_1. (3.42)
\]

This last equation leads to an expression for \(\Delta M_{\text{mec}}\), after rearranging the terms:

\[
\Delta M_{\text{mec}} = \frac{L_{\text{ini}} (1 - \frac{\Omega_{\text{lim}}^1}{\Omega_1}) + \rho_{\text{ini}} \frac{\Omega_{\text{lim}}^1}{\Omega_1} \Delta M_{\text{rad}} - \Delta L_{\text{rad}}}{R_E^2 \Omega_1 - \rho_{\text{ini}} \frac{\Omega_{\text{lim}}^1}{\Omega_1} m_e}. (3.43)
\]

All the terms of this relation being known, we have a way to compute the equatorial mass loss, and also the mechanical angular momentum loss.

The technical description of this modification is given in Appendix A.2 on page 147.
3.2. Anisotropic mass loss in rotating stars

4 The angular momentum conservation in the stellar interior during advection/diffusion

As already discussed in Section 2.2 on page 8, the diffusion and the advection of the angular momentum (equation (2.9)) are applied separately and one after the other: the advection during one time step, and the diffusion during the next one. The diffusion is treated with an implicit finite difference method (Meynet et al. 2004). This method is very robust and accurate. In most of the case, the difference between the initial angular momentum content of the stellar interior (before the mixing process) \( L_{\text{ini,mix}} \) and after the mixing process \( L_{\text{fin,mix}} \) is of the order \( L_{\text{ini,mix}} / L_{\text{fin,mix}} \sim 10^{-6} \) (which does not include the envelope!).

On the other hand, the advection part of the transport equation is much more difficult to solve, and need the use of a relaxation method. During the advective process, the conservation of the angular momentum is more poorly ensured: \( L_{\text{ini,mix}} / L_{\text{fin,mix}} \sim 10^{-4} - 10^{-3} \).

Even if the conservation of angular momentum during the diffusion is (marginally) compatible with the effects of the anisotropic winds, the conservation during the advection is still problematic. In order to ensure that conservation during both processes, we apply the following method. We compute a corrective factor \( C_{\text{tran}} \) as:

\[
C_{\text{tran}} = \frac{L_{\text{fin,mix}}}{L_{\text{ini,mix}}}. \tag{3.44}
\]

Once the whole structure has converged, we multiply the internal angular velocity profile by that factor:

\[
\Omega_i' = C_{\text{tran}} \Omega_i, \tag{3.45}
\]

where \( \Omega_i \) the angular velocity of the \( i \)th layer of the star, \( \Omega_i' \) the corrected one, and the shell number \( i \) ranges the whole interior. This computation ensures that the angular momentum at the end of the whole structure and transport computation is the same than its initial value.

The technical description of this modification is given in Appendix A.3 on page 148.

5 Correction of the angular velocity profile and angular momentum conservation

Following the algorithm presented above, once the current model has converged, a check of the angular momentum conservation is done, and, if necessary, a correction over the \( N_{\text{corr}} \) first layers is computed. The correction is applied at the beginning of the next time step where the diffusion is computed. If the advection is applied on the next time step, the correction is stored, and applied later. This method is chosen because of the bigger robustness of the diffusion: the correction is smoothed during the diffusion process, allowing an easier computation of the advection the next time step.

Once the radiative and mechanical mass loss is known, we can define the total loss of angular momentum during the time step:

\[
\Delta L = \Delta L_{\text{rad}} + \Delta L_{\text{mec}}. \tag{3.46}
\]

From the initial angular momentum content of the star \( L_{\text{ini}} \), one knows that the final one must be:

\[
L_{\text{fin}} = L_{\text{ini}} - \Delta L, \tag{3.47}
\]

the envelope being accounted for.

On the other hand, we have the final content of angular momentum \( L_{\text{fin}} \) as obtained after the whole computation of the model. Generally, \( L_{\text{fin}} \) and \( L_{\text{fin}} \) are not the same, due to the fact that the envelope is not accounted for in the angular momentum transport process. To ensure a right conservation of the angular momentum, we have to remove an amount \( \Delta L_{\text{diff}} \) of angular momentum from the converged model:

\[
\Delta L_{\text{diff}} = L_{\text{fin}} - L_{\text{fin}}. \tag{3.48}
\]
In the same way as above, we are looking for a correction factor \( q_{\text{corr}} \) that is applied to the rotation profile of the converged model:

\[
\Omega_i^{\text{corr}} = \Omega_i (1 + q_{\text{corr}}),
\]

(3.49)

with \( \Omega_i \) the converged rotation profile and \( i \) between 1 and \( N_{\text{corr}} \).

The problem is quite similar to the estimation of the new surface angular velocity discussed above in Point 2). The conservation of the angular momentum in the \( N_{\text{corr}} \) corrected layers is:

\[
\Omega_1^{\text{corr}} \mathcal{I}_e + \sum_{i=1}^{N_{\text{corr}}} \Omega_i^{\text{corr}} \mathcal{I}_i = \Omega_1 \mathcal{I}_e + \sum_{i=1}^{N_{\text{corr}}} \Omega_i \mathcal{I}_i - \Delta L_{\text{diff}}.
\]

(3.50)

Introducing relation (3.49), we can extract \( q_{\text{corr}} \):

\[
q_{\text{corr}} = \frac{-\Delta L_{\text{diff}}}{\Omega_1 \mathcal{I}_e + \sum_{i=1}^{N_{\text{corr}}} \Omega_i \mathcal{I}_i}.
\]

(3.51)

The correction which has to be applied to the converged rotation profile is then:

\[
\Delta \Omega_i^{\text{corr}} = \Omega_i q_{\text{corr}}.
\]

(3.52)

Let us finished with a few remarks about that correction:

- As the correction is applied only each two models (where diffusion occurs), the initial angular momentum content \( L_{\text{ini}}^{\text{tot}} \) in equation (3.47) can be the initial angular momentum content of the previous model.

- If there are some convective zones completely or partially contained in the \( N_{\text{corr}} \) corrected layers, the \( \Omega \)-profile is homogenised in that zones, in order to have a flat rotation profile.

- If a large external convective zone appears (in particular when the star is in the red part of the HRD), the correction is computed only in the convective zone and set to 0 elsewhere. It is done for convective zones containing more than 100 layers. Otherwise, \( N_{\text{corr}} = 200 \).

The technical description of this modification is given in Appendix A.4 on page 148.

This new computational method, allowing an estimation of the mechanical mass loss at the equator when the star reaches the critical velocity, and the account for the anisotropy in the radiative winds generated by rotation, has been applied to several different cases of stellar evolution. The results are presented in Chapter 5 on page 69.

### 3.3 The \( \mu^2 M \)-limit for the mass of the stars

A part of this thesis work was dedicated to the study of hot massive stars following an homogeneous evolution. Fast rotation, helped by magnetic coupling, favours a strong mixing in the stellar interior, reducing considerably the \( \mu \)-gradient through the star. For those stars, an upper mass limit can be estimated from basic relation, as we show in this section.
3.3. The $\mu^2 M$-limit for the mass of the stars

3.3.1 Mass–luminosity relation

We start with the three following equations: the radiative equilibrium, the hydrostatic equilibrium, and the equation of state for a mixture of a perfect gas and radiation:

$$\frac{L}{4\pi r^2} = -\frac{ac}{3k\rho} \frac{dT^4}{dr}$$
$$\frac{dP}{dr} = -\rho g$$
$$P = P_{\text{gas}} \frac{k}{\mu m_p} \rho T,$$

(3.53)

where $P_{\text{gas}}$ is the gas contribution to the total pressure $P$ and $m_p$ is the proton mass. From the hydrostatic equilibrium, we find a relation estimating the central pressure $P_c$:

$$P_c \sim \frac{M^2}{R^4},$$

(3.54)

with $M$ and $R$ the total mass and radius of the star. The equation of state leads to:

$$T_c \sim \frac{\mu \beta_c}{\rho_c} P_c.$$

(3.55)

As we are interested to homogeneous hot stars, $\mu$ is constant and thus does not need the subscript c. Replacing $P_c$ by equation (3.54), and replacing $M/R^3$ by $\bar{\rho}$ the mean density of the star, we obtain:

$$T_c \sim \frac{\mu \beta_c M}{\rho_c \bar{\rho} R^3}.$$

(3.56)

On the other hand, the radiative equilibrium says that:

$$L \sim \frac{R}{\kappa \rho} T_c^4,$$

(3.57)

with $\kappa$ the mean value of the opacity in the star. Inserting equation (3.56) leads to:

$$L \sim \mu^4 \beta_c^4 \left( \frac{\bar{\rho}}{\rho_c} \right)^4 \frac{M^3}{\kappa}.$$

(3.58)

In what follows, it is convenient to express this relation with respect to a reference homogeneous model, which is indicated with the subscript R:

$$\frac{L}{L_R} = \frac{\mu^4 \beta_c^4}{\mu_R^4 \beta_{cR}^4} \frac{M^3}{\rho_c^4 \rho_R^4} \frac{\bar{\rho}}{\kappa_R}.$$

(3.59)

3.3.2 The Eddington limit

In the external layers of the star, the gravity is counterbalanced by two forces: the gas pressure gradient and the radiation pressure gradient. If for some reason, the radiation pressure increases in the outer layer, the gas pressure will first decrease, since less force is needed to compensate for the gravity. At a given moment, the radiation pressure alone will be sufficient to sustain the outer layers. The star is said to be at the Eddington luminosity. If it increases above that point, “... the radiation observed to be emitted [by stars] must work its way through the star, and if there were too much obstruction it would blow up the star”, as written by Eddington (1926) in his book.

This condition is expressed as:

\[
\frac{1}{\rho_s} \frac{dP_{\text{rad}}}{dr} = \frac{\kappa_s}{c} \frac{L_{\text{Edd}}}{\kappa_s L_R} = \frac{GM}{R^2},
\]

(3.60)

where the subscript s indicates the surface values. Normalised to the reference luminosity, the Eddington limit is:

\[
\frac{L}{L_R} \leq \frac{L_{\text{Edd}}}{L_R} = \frac{4\pi c GM}{\kappa_s L_R}.
\]

(3.61)

3.3.3 Upper limit for homogeneous hot stars

From the Eddington limit (3.61), and including relation (3.59), we obtain the following relation:

\[
\mu^2 \frac{M}{M_\odot} \leq \left[ 4\pi G \frac{\mu_R^4}{\kappa_R} \frac{M_R}{L_R} \right]^{1/2} \frac{M_R}{M_\odot} \left[ \frac{\beta_R c}{\beta_c} \frac{\rho_R}{\rho_c} \right]^2 \equiv C_{\text{Edd}}.
\]

(3.62)

Since in hot massive stars, the dominant opacity is the free electron scattering opacity, and since the chemical composition of the star is supposed to be homogeneous, we have simplified the mean opacity and the surface one.

In order to estimate the constant \(C_{\text{Edd}}\), we use a homogeneous model of a 60 M_\odot star, when its actual mass is 30 M_\odot (see Section 7 on page 115). At that point, this model shows the following properties: \(L_R = 684000\), \(L_\odot, \beta_R c/\beta_c = 0.33\), \(\kappa_R = 0.19\) and \(\mu_R = 1.21\), and the last relation becomes:

\[
\mu^2 \frac{M}{M_\odot} \leq 1.9 \left( \frac{1}{\beta_c} \right)^2 \left( \frac{\rho_c}{\rho} \right)^2.
\]

(3.63)

To be useful, \(C_{\text{Edd}}\) should be constant, or at least, only weakly dependent on the mass. However, to show that this relation has a universal signification, we can not test it on the 60 M_\odot model discussed above. Actually, as this model was computed with the mass loss rate obtained directly applying relation (3.62), this ensures, by construction, that \(C_{\text{Edd}}\) is constant for that model. To check the constancy, we use a 120 M_\odot model computed with classical mass loss rates from the literature, during the whole phase where it is homogeneous. Looking at Table 3.1, we see that for this model, \(\beta_c\) is nearly constant. This is also true for the ratio \(\frac{\rho_c}{\rho}\). The result for \(C_{\text{Edd}}\) is shown on Figure 3.11 on next page. We see that the constancy of \(C_{\text{Edd}}\) is well verified: \(C_{\text{Edd}} \sim 30\) M_\odot, with variations smaller than 7%.

Table 3.1: Numerical values used to compute \(C_{\text{Edd}}\) for the eight points of Figure 3.11.

<table>
<thead>
<tr>
<th>(M) [M_\odot]</th>
<th>14.8</th>
<th>15.0</th>
<th>15.4</th>
<th>15.8</th>
<th>17.3</th>
<th>18.8</th>
<th>20.4</th>
<th>21.5</th>
<th>22.4</th>
<th>22.8</th>
<th>25.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_c)</td>
<td>0.50</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
</tr>
<tr>
<td>(\bar{\rho})</td>
<td>1304.9</td>
<td>392.2</td>
<td>294.7</td>
<td>262.9</td>
<td>204.2</td>
<td>176.4</td>
<td>154.4</td>
<td>140.9</td>
<td>130.2</td>
<td>125.2</td>
<td>112.9</td>
</tr>
<tr>
<td>(\rho_c)</td>
<td>4054.1</td>
<td>1193.5</td>
<td>892.2</td>
<td>794.8</td>
<td>615.8</td>
<td>533.2</td>
<td>467.7</td>
<td>427.4</td>
<td>394.2</td>
<td>377.7</td>
<td>339.7</td>
</tr>
</tbody>
</table>

Following other ways, many authors found the same limit \(\mu^2 M \leq \text{cst}\), starting from stability analyses. Ledoux (1941, 1970) found a value of around 40 M_\odot for the constant. Schwarzschild & Härm (1958) a smaller value of 22 M_\odot. Once the star reaches the \(\mu^2 M\)-limit, some instabilities are expected, which remove some mass from the star, keeping it at the limit. Taking the derivative of relation (3.62), we can easily obtain a mass loss rate such that the star is maintained at the \(\mu^2 M\)-limit.

\[
\dot{M}_{\text{Edd}} = \frac{2M}{\mu^2}. \quad \text{(3.64)}
\]
3.4 The origin 2009 version of the Geneva code

3.4.1 Mass loss in the Geneva code

As the aim of this thesis is not to give a precise description of all the various processes leading to mass loss at the surface of stars, we will only focus on the mass loss from hot, massive stars, which interests us in this work. The winds from such stars are best described by the line driven wind theory, and we give here a brief introduction on that topic. For the description of other mass loss processes, we let the interested reader refer to Lamers & Cassinelli (1999).

Line driven wind theory

The basis of this theory were laid by Lucy & Solomon (1970) and Castor et al. (1975) (these last authors gave their names to the theory, also known as the CAK theory for Castor, Abbott and Klein). In a previous work, Sobolev (1960) had showed that in a moving stellar envelope, and with the assumption that the line profile can be approximated by a Dirac δ–function, the optical depth of the medium is a function composed of two parts: the first one contains the physics of the line, like the statistical weights, the oscillator strength, etc; the second one contains the physical properties of the stellar wind, and depends on the wind density $\rho$ and on the velocity profile of the wind $\frac{dr}{dv}$.

In the CAK theory, the optical depth of a line is given in terms of a reference optical depth, which depends only on the structure of the wind (see Lamers & Cassinelli 1999):

$$ t = \kappa_{es} v_{th} \rho \frac{dr}{dv}, $$

(3.65)

This mass loss rate does not directly depend on the constant $C_{\text{Edd}}$. In Section 7 on page 115, we study in details the evolution of stars undergoing such a mass loss rate.

Figure 3.11: $C_{\text{Edd}}$ as a function of the stellar mass, for the $120 M_\odot$ model discussed in the text. The numerical values used to compute it at the indicated points are summarised in Table 3.1.

where $\kappa_{es}$ is the electron scattering opacity, $v_{th}$ is the mean thermal velocity of the protons at the surface of the star:

$$v_{th} = \sqrt{\frac{2k_B T_{\text{eff}}}{m_p}},$$

(3.66)

with $k_B$ the Boltzmann constant, $m_p$ the proton mass and $T_{\text{eff}}$ the effective temperature of the stellar surface.

The total radiative acceleration is given by the sum of the acceleration $g_{es} = \frac{\kappa_{es} L}{4\pi r^2 c}$ (where $L$ is the stellar luminosity) produced by the electron scattering in the continuum and of the acceleration $g_L$ due to the line absorption. In the CAK theory, this last term is expressed as a function of the electron scattering acceleration, multiplied by a factor called force multiplier $M(t)$:

$$g_L = g_{es} M(t).$$

(3.67)

The exact value of $M(t)$ depends on the chemical composition of the wind, and on the ionisation. However, it can be approximated at a good precision level as a power law:

$$M(t) = k t^{-\alpha} \left(\frac{10^{-11} n_e}{W}\right)^\delta,$$

(3.68)

where $n_e$ is the electron density and $W$ the geometrical dilution factor. $k$, $\alpha$ and $\delta$ are called the force multiplier parameters. We give typical values (from Pauldrach et al. 1986), and the value used in the present version of the code (Lamers, 2008, priv. comm.) in Table 3.2. As we can see, the value of $\delta$ remains small. In what follows, we will thus neglect the term in brackets in eq. (3.68), and consider that $M(t) = k t^{-\alpha}$.

Table 3.2: Force multiplier parameters for various effective temperature. The left column gives values from (Pauldrach et al. 1986). The right column gives the values used in the Geneva code. $k$ is from Pauldrach et al. (1986), and $\alpha$ from Lamers (2008, priv. comm.).

<table>
<thead>
<tr>
<th>$T_{\text{eff}}$ [K]</th>
<th>$k$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$T_{\text{eff}}$ [K]</th>
<th>$k$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20000</td>
<td>0.320</td>
<td>0.565</td>
<td>0.02</td>
<td>&lt; 11500</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>30000</td>
<td>0.170</td>
<td>0.590</td>
<td>0.09</td>
<td>11500 – 20000</td>
<td>0.32</td>
<td>0.43</td>
</tr>
<tr>
<td>40000</td>
<td>0.124</td>
<td>0.640</td>
<td>0.07</td>
<td>20000 – 30000</td>
<td>0.32</td>
<td>0.6</td>
</tr>
<tr>
<td>50000</td>
<td>0.124</td>
<td>0.640</td>
<td>0.07</td>
<td>30000 – 40000</td>
<td>0.17</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>&gt; 40000</td>
<td>0.124</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Typical values for $M(t)$ are between $\sim 100$ and $\sim 1000$ at low optical depth (i.e. far out from the stellar surface). It means that in these regions, the opacity in the lines dominates the electron scattering opacity. Near the star, at higher optical depth, $M(t)$ is around $\sim 0.1 – \sim 1$. In that case, the line opacity is comparable to the electron scattering opacity. Computed values of $M(t)$ for various optical depth are shown on Fig. 3.12 on next page.

To determine the mass loss rate, we start with the momentum equation, taking into account the acceleration due to the electron scattering opacity, and to the line opacity:

$$\frac{v}{r} \frac{dv}{dr} = -\frac{GM}{r^2} + \frac{1}{\rho} \frac{dp}{dr} + g_{es} + g_L.$$

(3.69)

We introduce here the Eddington factor $\Gamma_{\text{Edd}}$ for the electron scattering opacity, which is the ratio of the stellar luminosity to the Eddington luminosity:

$$\Gamma_{\text{Edd}} = \frac{L}{L_{\text{Edd}}} = \frac{\kappa_{es} L}{4\pi c GM}.$$

(3.70)
3.4. The origin 2009 version of the Geneva code

Figure 3.12: Force multiplier $M(t)$ value as a function of the optical depth, for various effective temperatures. The dotted line shows the power-law fit (Lamers & Cassinelli (1999), with data from Abbott (1982) and Shimada et al. (1994)).

Neglecting the pressure gradient (which is a reasonable assumption in the supersonic wind region, where the radiative acceleration dominates), and introducing eq. (3.68), the momentum equation rewrites:

$$v \frac{dv}{dr} = -\frac{GM}{r^2} (1 - \Gamma_{\text{Edd}}) + \frac{\kappa_{\text{es}} L}{4\pi r^2 c} k t^{-\alpha}. \quad (3.71)$$

We will briefly discuss further the case where the pressure gradient is not neglected. Using the mass conservation equation

$$\dot{M} = 4\pi r^2 \rho v = \text{const} \quad (3.72)$$

and introducing it in the optical depth definition (3.65) leads to the following relation (see Maeder 2009):

$$C D^\alpha = D + GM (1 - \Gamma_{\text{Edd}}), \quad (3.73)$$

where

$$C = \frac{\kappa_{\text{es}} L k}{4\pi c} \left( \frac{4\pi}{M v_{\text{th}, \kappa_{\text{es}}}} \right)^\alpha \quad \text{and}$$

$$D = r^2 v \frac{dv}{dr}. \quad (3.74)$$

$C$ and $GM (1 - \Gamma_{\text{Edd}})$ being constant, this relation is fulfilled at each radius $r$ only if $D = \text{const}$. In Fig. 3.13 on next page, the left-hand side of eq. (3.73) is plotted as a function of $D$ for three different values of $C$ (thick solid line and dashed lines), as well as the right-hand side (thin solid line). We see that eq. (3.73) has none, one or two solutions. Actually, we would like a unique solution for the mass loss rate, and thus for $C$. We write the last equation as $D + GM (1 - \Gamma_{\text{Edd}}) - C D^\alpha$. $C$ is given by its minimum:

$$C = \frac{D^{1-\alpha}}{\alpha}. \quad (3.75)$$
Replacing this value in eq. 3.73 allows to find $D$:

$$D = r^2 v \frac{dv}{dr} = \frac{\alpha}{1 - \alpha} GM (1 - \Gamma_{Edd}).$$

(3.76)

Integrating with the initial condition $v(R_\star) = 0$, i.e. that the initial velocity of the wind near the stellar surface is negligible, leads to the following expression for the wind velocity:

$$v(r) = \left[ \frac{\alpha}{1 - \alpha} 2GM (1 - \Gamma_{Edd}) \left( \frac{1}{R_\star} - \frac{1}{r} \right) \right]^{\frac{1}{2}}.$$  

(3.77)

We can then easily compute the terminal velocity of the stellar wind, far from the star, doing $r \rightarrow \infty$ in the previous relation:

$$v_\infty = \frac{\alpha}{1 - \alpha} \frac{2GM (1 - \Gamma_{Edd})}{R_\star} = \frac{\alpha}{1 - \alpha} v_{esc},$$

(3.78)

where $v_{esc}$ is the escape velocity, accounting for the radiation pressure. We see that the terminal velocity of the wind is directly proportional to the escape velocity, and is independent of $k$. We can now rewrite eq. (3.77) as:

$$v(r) = \sqrt{\left( \frac{1}{R_\star} - \frac{1}{r} \right)} v_\infty.$$  

(3.79)

From the definition of $C$ (3.74), and eqs. (3.75), (3.76) and (3.79), we can obtain an expression for the mass loss rate of the star:

$$\dot{M} = \frac{4\pi}{\kappa_{es} v_{th}} \left( \frac{k\alpha \kappa_{es} L}{4\pi c} \right)^{\frac{1}{2}} \left( \frac{1 - \alpha}{\alpha} \right)^{\frac{1 - \alpha}{\alpha}} \left( GM (1 - \Gamma_{Edd}) \right)^{\frac{\alpha - 1}{\alpha}}.$$  

(3.80)
3.4. The origin 2009 version of the Geneva code

This simplified relation for the mass loss rate of the star will be useful in order to study the development of anisotropic wind in Section 5.1 on page 69.

The CAK-theory briefly described above provides very good predictions for the mass loss rates of hot massive stars. The comparison with the observations generally agrees well for stars of spectral types from O to A (see Ahmad et al. 1997; Puls et al. 2008). However, some recent works point a possible overestimation of the mass loss rates for late-type O8-O9 stars (Bouret et al. 2008; Marcolino et al. 2009).

Discussion of the approximations

Account for the pressure gradient  The account for the pressure gradient in the momentum equation (3.69) complicates the resolution, but an analytical solution is still possible. For the mass loss rate, Castor et al. (1975) found the following relation (see also Cassinelli 1979; Lamers & Cassinelli 1999):

\[
\dot{M} = \left( \frac{4\pi}{\kappa_{\text{es}} v_{\text{th}}/c} \right) \kappa_{\text{es}} L^\frac{\alpha}{2} \left( \frac{10^{-11} n_e}{W} \right)^\frac{\alpha}{2} \left( GM (1 - \Gamma_{\text{Edd}}) - 2a^2 r_c \right)^{-\frac{1}{2}} \\
\cdot \left[ \frac{\alpha}{\alpha - 1} \left( GM (1 - \Gamma_{\text{Edd}}) - a^2 r_c \right) + a^2 r_c \right].
\]  

(3.81)

where \(a\) is the isothermal sound velocity and \(r_c\) is the radius of the critical point. The critical point is defined as the point where subsonic and supersonic solution of the equation link smoothly, and its value is roughly \(r_c \simeq 1.5R_\star\). The effect of the terms depending on \(\delta\) are also accounted for in this relation.

The behaviour of the full solution discussed here is close to the approximate one we show in the last section for the following reasons. First, the radiative acceleration in much greater than the pressure gradient acceleration in the winds of hot massive stars. Second, the wind velocity becomes larger than the sound speed very close to the stellar surface, and we can thus neglect the corresponding terms in a good approximation.

Finite stellar size  To derive the previous relations, we supposed that the radiative acceleration was produced by a point-source star, and thus, that it was directed only in the radial direction. It is indeed a good approximation far from the star, but closer, the effect of the finite size of the star can reach several tenths of percents. According to Lamers & Cassinelli (1999), it can be expressed as an additional correction factor multiplying the force multiplier:

\[
\mathcal{M}'(t) = \mathcal{M}(t) D_f,
\]  

(3.82)

where \(\mathcal{M}'(t)\) is the corrected force multiplier, and \(D_f\) is called the finite disk correction factor:

\[
D_f = \left( 1 + \sigma \right)^{\alpha+1} - \left( 1 + \sigma \mu^2 \right)^{\alpha+1} \frac{\alpha}{1 - \mu^2} \frac{\alpha}{(\alpha + 1) \sigma (1 + \sigma)},
\]  

(3.83)

with \(\sigma = \frac{r}{v} \frac{dr}{d\sigma} - 1\) and \(\mu^2 = 1 - \left( \frac{R_\star}{r} \right)^2\).

Multiple scattering  In the theory presented above, we supposed that a photon can be scattered only once. However, it is possible that multiple scattering occurs in stellar winds, which modifies the mass loss rate and the terminal velocity of the wind. The wind momentum \(M v_\infty\) can be increased by a factor of 2 for the O-stars, and up to 6 in the wind of WR stars (Gayley et al. 1995).
Wind blanketing If multiple scattering occurs in the wind, some photons can be scattered back in the stellar photosphere. Its properties is thus modified, as well as the emitted energy distribution. This modifies the mass loss rates and the terminal velocities of computed winds.

Continuum driven wind

In some cases, the star reaches a very high luminosity, and its $\Gamma_{\text{Edd}}$ factor approaches unity, or even exceeds it. For such star, the acceleration of the wind is no more due to the line opacity, but mainly by the continuum opacity. In recent work, Owocki et al. (2004) improved the analytical description of the CAK–theory to better account for the continuum driven wind, modifying the treatment of the lines approximation.

Super-Eddington stars (i.e. with $\Gamma_{\text{Edd}} > 1$) are possible if we consider a clumpy wind (Shaviv 1998). The wind is no long considered as homogeneous, but contains some clumps and some more diffuse area. The Eddington luminosity is exceeded in the clumps, but not in the low density regions, where the photons can more easily escape. The luminosity can thus in theory exceed the averaged Eddington luminosity.

Another effect which has to be accounted for is the photon tiring. Looking at eq. (3.80), we see that the mass loss rate scales as $1/(1 - \Gamma_{\text{Edd}})^{1-\alpha}$. We thus obtain a divergent mass loss rate when $\Gamma_{\text{Edd}}$ tends to 1. However, the maximum mass loss rate $\dot{M}_{\text{tir}}$ is set by the finite energy available, which comes from the stellar luminosity. We have:

$$\dot{M}_{\text{tir}} = \frac{L}{v_{\text{esc}}^2/2} = \frac{LR_*}{GM}.$$  
(3.84)

The available power at a given distance $r$ of the star is the stellar luminosity, minus the power used by the stellar wind to raise the gravitational potential:

$$L(r) = L_* - M \left[ \frac{v_{\text{esc}}^2}{2} + \frac{GM}{R_*} - \frac{GM}{r} \right],$$  
(3.85)

where $L_*$, $M$ and $R_*$ are respectively the stellar luminosity, mass and radius. Introducing the dimensionless variables $x = 1 - \frac{r}{R_*}$, $w = \frac{v_{\text{esc}}^2}{2}$ and $m = \frac{M}{M_{\text{in}}}$, this last relation rewrites:

$$L(r) = L_* \left( 1 - m(w + x) \right).$$  
(3.86)

Defining the ratio $\Gamma_c = \frac{\kappa_c L_*}{4\pi GM_{\odot}c}$, where $\kappa_c$ is the opacity of the continuum, we can compute the tiring-corrected Eddington factor:

$$\Gamma_{\text{rad}}(x) = \Gamma_c(x) \left( 1 - m(w + x) \right).$$  
(3.87)

To account for the photon tiring effect, the $\Gamma_{\text{Edd}}$ term in the momentum equation (3.71) has to be replaced by the previous relation.

van Marle et al. (2008) numerically solved the continuum wind problem, with account for the clumps (porosity of the wind), and the tiring photon effect. The mass loss rate they obtained are very big, of the order of $10^{-4}$ to $10^{-2}$ $M_{\odot}$yr$^{-1}$. These values are comparable to the observed mass loss rate of the Luminous Blue Variables (LBVs), these super-luminous stars (as an example, the most famous LBV is $\eta$ Car).

Mass loss prescriptions used in this work

Many mass loss prescriptions are available in the literature. In this section, we describe those that we used in this thesis, and under which conditions. In order to be easily implemented in an evolution code, they have to be determined from basics stellar parameters. Some are inspired from the CAK–theory described in section 3.4.1 on page 48, and others are empirical laws fitting the observations.
3.4. The 2009 version of the Geneva code

**Vink et al. (2000, 2001) prescription**  This mass loss prescription is based on the CAK–theory, but with account for the metallicity and the multiple scattering. The authors parametrise the mass loss rate for massive stars as a function of the characteristic wind density \( \langle \rho \rangle \), the stellar mass \( M_\star \) and luminosity \( L_\star \), the effective temperature \( T_{\text{eff}} \), the metallicity \( Z \) and the ratio \( v_{\text{esc}}/v_\infty \).

Due to transitions in the ionisation levels of the iron atoms, we notice the existence of two bistability jumps: the first one, at an effective temperature of roughly \( \sim 25000 \) [K], is produced by the recombination of Fe IV to Fe III; the second one, at around \( 15000 \) [K], is due to the recombination of Fe III to Fe II. Figure 3.14 (Vink et al. 2000) shows the theoretical mass loss rate computed for a \( 60 \) \( M_\odot \) star, with a luminosity of \( L = 10^6 \) \( L_\odot \). The three different symbols correspond to three different values of the ratio \( v_{\text{esc}}/v_\infty \). The bistability jumps are clearly visible.

The characteristic wind density is defined by a linear fit, as a function of \( \Gamma_{\text{Edd}} \) as defined in (3.70), and of the metallicity:

\[
\log (\langle \rho \rangle) = -14.94 (\pm 0.54) + 3.2 (\pm 2.2) \Gamma_{\text{Edd}} + 0.85 (\pm 0.10) \log (Z/Z_\odot). \tag{3.88}
\]

The position of the bistability jumps are computed with:

\[
\begin{align*}
T_{\text{jump}, 1} & = 61.2 (\pm 4.0) + 2.59 (\pm 0.28) \log (\langle \rho \rangle) \\
T_{\text{jump}, 2} & = 100 + 6 \log (\langle \rho \rangle). \tag{3.89}
\end{align*}
\]

The mass loss rates are computed with the following relations, depending on the effective temperature of the star and the position of the bistability jumps.
\begin{itemize}
  \item For $T_{\text{eff}} > T_{\text{eff}}^{\text{jump},1}$:
    \[
    \log \left( \frac{\dot{M}}{\text{M} \odot \cdot \text{yr}^{-1}} \right) = -6.697 + 2.194 \log \left( \frac{L_*}{10^5 L_\odot} \right) - 1.313 \log \left( \frac{M_*}{30 \text{M}_\odot} \right) \\
    - 1.226 \log \left( \frac{v_*}{v_{\text{esc}}} \right)^2 + 0.993 \log \left( \frac{T_{\text{eff}}}{40000} \right) \\
    - 10.92 \log^2 \left( \frac{T_{\text{eff}}}{40000} \right) + 0.85 \log \left( \frac{Z}{Z_\odot} \right). \quad (3.90)
    \]

    In this range of effective temperatures, the mean galactic ratio $v_*/v_{\text{esc}} = 2.6$, as determined by Lamers et al. (1995).

  \item For $T_{\text{eff}}^{\text{jump},1} > T_{\text{eff}} > T_{\text{eff}}^{\text{jump},2}$:
    \[
    \log \left( \frac{\dot{M}}{\text{M} \odot \cdot \text{yr}^{-1}} \right) = -6.688 + 2.210 \log \left( \frac{L_*}{10^5 L_\odot} \right) - 1.339 \log \left( \frac{M_*}{30 \text{M}_\odot} \right) \\
    - 1.601 \log \left( \frac{v_*}{v_{\text{esc}}} \right)^2 + 1.07 \log \left( \frac{T_{\text{eff}}}{40000} \right) + 0.85 \log \left( \frac{Z}{Z_\odot} \right). \quad (3.91)
    \]

    For these effective temperatures, the mean galactic ratio $v_*/v_{\text{esc}} = 1.3$.

  \item For $T_{\text{eff}} < T_{\text{eff}}^{\text{jump},2}$:
    \[
    \log \left( \frac{\dot{M}}{\text{M} \odot \cdot \text{yr}^{-1}} \right) = -5.99 + 2.210 \log \left( \frac{L_*}{10^5 L_\odot} \right) - 1.339 \log \left( \frac{M_*}{30 \text{M}_\odot} \right) \\
    - 1.601 \log \left( \frac{v_*}{v_{\text{esc}}} \right)^2 + 1.07 \log \left( \frac{T_{\text{eff}}}{40000} \right) + 0.85 \log \left( \frac{Z}{Z_\odot} \right). \quad (3.92)
    \]

    For these low effective temperatures, the ratio $v_*/v_{\text{esc}} = 0.7$, according to Lamers et al. (1995).
\end{itemize}

As this mass loss prescription is mainly based on the CAK–theory, its field of application is the same. In the Geneva evolution code, it is used for stars with a mass larger than $15 \text{M}_\odot$ and an effective temperature $\log (T_{\text{eff}}) > 3.9$.

\textbf{Gräfener & Hamann (2008) prescription} This prescription is based on a full calculation of the structure of the wind of WNL stars (to have more informations on Wolf-Rayet (WR) stars and their subtypes, see the Section 6.1 on page 95). The mass loss rate are computed with the Potsdam Wolf-Rayet model atmosphere code, which includes non-LTE treatment of the wind, line blanketing, and the structure of the wind, obtained by solving the hydrodynamic equations. Clumping effects in the wind are also accounted for. As the mass loss rates are computed for very particular cases, this prescription is valid only in a relatively small domain of effective temperature: $30000 \text{[K]} < T_{\text{eff}} < 70000 \text{[K]}$, and the validity domain in metallicity is restricted to $10^{-3}Z_\odot < Z < 2Z_\odot$.

The Gräfener & Hamann (2008) prescription is a function of stellar parameters: the effective temperature $T_{\text{eff}}$, the luminosity $L$, the surface hydrogen mass fraction $X$ and the ratio $\Gamma_{\text{Edd}} = L/L_{\text{Edd}}$ (see relation 3.70 on page 49) computed with the electron scattering opacity. The mass loss rate is given by:

\[
\log \left( \frac{\dot{M}}{\text{M} \odot \cdot \text{yr}^{-1}} \right) = -3.763 + \beta(Z) \log (\Gamma_{\text{Edd}} - \Gamma_0(Z)) - 3.5 (\log (T_{\text{eff}}) - 4.65) \\
+ 0.42 \left( \log \left( \frac{L}{L_\odot} \right) - 6.3 \right) - 0.45 (X - 0.4), \quad (3.93)
\]
where $\beta(Z)$ and $\Gamma_0(Z)$ are defined as follow:

$$
\beta(Z) = 1.727 + 0.25 \log\left(\frac{Z}{Z_\odot}\right)
$$

$$
\Gamma_0(Z) = 0.326 - 0.301 \log\left(\frac{Z}{Z_\odot}\right) - 0.045 \log^2\left(\frac{Z}{Z_\odot}\right).
$$

(3.94)

In the Geneva code, this mass loss rate is used for late WN stars with metallicity and effective temperature in the range of allowed values discussed above.

**de Jager et al. (1988) prescription** This prescription covers a very wide area in the Hertzsprung–Russel Diagram (HRD) with data from stars whose effective temperatures covers the range between $3.3 < \log(T_{\text{eff}}) < 4.8$ and with luminosities between $2.5 < \log(L/L_\odot) < 6.7$. It is not derived from physical considerations, but it is a compilation of a large number observed mass loss rates from the literature: the sample is composed by 271 stars from O-type to M-type stars of population I.

The authors fit the mass loss rates of this large sample as a function of only two parameters: the effective temperature $T_{\text{eff}}$ and the luminosity $L$. The analytical expression of the fit is performed on the base of the Chebychev polynomials, defined as:

$$
T_j(x) = \cos(j \arccos(x)).
$$

(3.95)

According to the authors, these polynomials better fit the data at the edges of the ranges. The fit function is:

$$
-\log\left(\frac{\dot{M}}{M_\odot \cdot \text{yr}^{-1}}\right) = \sum_{n=0}^{N} \sum_{i=0}^{n} \sum_{j=n-i}^{i} a_{ij} T_i \left(\frac{\log(T_{\text{eff}}) - 4.05}{0.75}\right) \times T_j \left(\frac{\log(L/L_\odot) - 4.6}{2.1}\right),
$$

(3.96)

and the $a_{ij}$ are computed with a $\chi^2$-minimising method. The results are given in Table 3.3.

Table 3.3: Coefficients $a_{ij}$ given the best fit for the mass loss prescription of de Jager et al. (1988) (see eq. (3.96)).

<table>
<thead>
<tr>
<th>$i$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.34916</td>
<td>-5.04240</td>
<td>-0.83426</td>
<td>-1.13925</td>
<td>-0.12202</td>
</tr>
<tr>
<td>1</td>
<td>3.41678</td>
<td>0.15629</td>
<td>2.96244</td>
<td>0.33659</td>
<td>0.57576</td>
</tr>
<tr>
<td>2</td>
<td>-1.08683</td>
<td>0.41952</td>
<td>-1.37272</td>
<td>-1.07493</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.13095</td>
<td>-0.09825</td>
<td>0.13025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.22427</td>
<td>0.46591</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.11968</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the Geneva evolution code, this prescription is used during the whole stellar evolution for stars with mass between $7 M_\odot$ and $15 M_\odot$, and during the red–supergiant (RSG) phase for the more massive ones.

**Nugis & Lamers (2000) prescription** This prescription is an empirical law, based on the observed winds of 44 WR stars, with account for the clumping in the determination of the mass loss rate. The sample is composed of 24 WN stars, 18 WC stars and 2 WO stars. It allows to determine the mass loss rates for two sub-samples: the WN stars sample, and the WC + WO stars sample.
To determine the WR subtype, the same method as in Maeder & Meynet (1994) is adopted (see also the discussion on that point in Section 6.1 on page 95). In addition to the Nugis & Lamers (2000) mass loss prescription, a metallicity dependence is added, according to Eldridge & Vink (2006), which was initially not included in the original work. As the WR winds are thought to be mainly driven by the line acceleration, and as the elements which are principally responsible of that acceleration are the Fe-peak elements (see Abbott 1982), the metallicity used for computing this dependence is not the actual metallicity of the surface $Z$, but the initial metallicity $Z_{\text{ini}}$. Actually, stellar evolution does not modify the surface abundance of Fe-peak elements, whereas the total metallicity $Z$ is increased by mixing processes and by the mass loss, which reveals enriched layers of the stellar interior.

The metallicity dependence is accounted for in the following way:

- for $Z_{\text{ini}} > 0.002$:
  \[
  \frac{\dot{M}}{M_{\odot} \cdot \text{yr}^{-1}} \sim \left( \frac{Z_{\text{ini}}}{Z_{\odot}} \right)^{x_1},
  \]

- for $Z_{\text{ini}} < 0.002$:
  \[
  \frac{\dot{M}}{M_{\odot} \cdot \text{yr}^{-1}} \sim \left( \frac{0.002}{Z_{\odot}} \right)^{x_1} \left( \frac{Z_{\text{ini}}}{0.002} \right)^{x_2}.
  \]

The values of $x_1$ and $x_2$ are given in Table 3.4.

Table 3.4: Values of $x_1$ and $x_2$ used in the metallicity scaling relation for the mass loss rate of WR stars, given by eqs (3.97) and (3.98) (from Eldridge & Vink 2006).

<table>
<thead>
<tr>
<th>type</th>
<th>$Z_{\text{ini}}$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WN</td>
<td>-</td>
<td>0.85</td>
<td>-</td>
</tr>
<tr>
<td>WC</td>
<td>&gt; $Z_{\odot}$</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$0.002 &lt; Z_{\text{ini}} &lt; Z_{\odot}$</td>
<td>0.66</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$&lt; 0.002$</td>
<td>0.66</td>
<td>0.35</td>
</tr>
</tbody>
</table>

The fit is performed as a function of the stellar luminosity $L$, the surface helium mass fraction $Y$ and the surface metallicity $Z$. The mass loss rates become:

- For WN stars:
  \[
  \log \left( \frac{\dot{M}}{M_{\odot} \cdot \text{yr}^{-1}} \right) = -13.6 + 1.63 \log \left( \frac{L}{L_{\odot}} \right) + 2.22 \log (Y) + 0.85 \log \left( \frac{Z_{\text{ini}}}{Z_{\odot}} \right). \tag{3.99}
  \]

- For WC and WO stars:
  \begin{itemize}
  \item For $Z_{\text{ini}} > 0.002$:
    \[
    \log \left( \frac{\dot{M}}{M_{\odot} \cdot \text{yr}^{-1}} \right) = -8.3 + 0.84 \log \left( \frac{L}{L_{\odot}} \right) + 2.04 \log (Y) + 1.04 \log (Z) + x_1 \log \left( \frac{Z_{\text{ini}}}{Z_{\odot}} \right), \tag{3.100}
    \]
  \item For $Z_{\text{ini}} < 0.002$:
    \[
    \log \left( \frac{\dot{M}}{M_{\odot} \cdot \text{yr}^{-1}} \right) = -8.3 + 0.84 \log \left( \frac{L}{L_{\odot}} \right) + 2.04 \log (Y) + 1.04 \log (Z) + x_1 \log \left( \frac{0.002}{Z_{\odot}} \right) + x_2 \log \left( \frac{Z_{\text{ini}}}{0.002} \right). \tag{3.101}
    \]
  \end{itemize}
In the Geneva evolution code, this prescription is used for the WR stars in the MS which are out of the ranges of the Gräfener & Hamann (2008) prescription, and for all the WR stars which are in their He-burning phase.

**Reimers (1975) prescription**  This prescription is based on (quite) old observations of the mass loss rate of red giant stars. The idea is that the mass loss rate must depend on basic stellar parameters, and that the easiest way to obtain the unity \( [g \cdot s^{-1}] \) is to combine \( \frac{LR_s}{g} \), with \( g \) the gravitational acceleration at the surface of the star. A linear fit is performed on the observations, which leads to:

\[
\dot{M} = \eta_N 4 \cdot 10^{-13} \frac{LR_s}{g}.
\] (3.102)

The numerical factor \( 4 \cdot 10^{-13} \) comes from the original version of the prescription (Reimers 1975). However, in following works, this factor was corrected (see e.g. Reimers 1977). We add thus here a factor \( \eta_N \) to correct that point.

In the code, this prescription is used, for stars with mass lower than \( 7 M_\odot \), during the Asymptotic Giant Branch (AGB) phase, and during the RGB phase, with \( \eta_N = 0 \). For the \( 7 M_\odot \) models, we use \( \eta_N = 0.6 \) during the RGB phase.

**LBV prescription**  The physical definition of the LBV phase is difficult, there is to date no clear criterion to decide if a star is a LBV or not. In our works, we use the following criteria: i) a star is in its LBV phase if it has super-Eddington layers in its envelope, and ii) the link between the interior of the star and the bottom of the envelope is not satisfying (see the Section 2.1 on page 7).

During that phase, we apply an unique mass loss rate, compatible with observational and theoretical considerations (see Section 3.4.1 on page 53):

\[
\dot{M} = 10^{-3} \left[ M_\odot \cdot yr^{-1} \right]
\] (3.103)

### 3.4.2 Abundances, nuclear reaction rates and opacities

**Abundances**

The determination of the solar abundances has undergone a lot of modifications the last decade, with the apparition of more accurate 3D models for the solar atmosphere. Asplund et al. (2005) obtained a lower value of the metal content in the Sun, compared to the previous measurements, with \( X = 0.7392, Y = 0.2486 \) and \( Z = 0.122 \). However, these new abundances modify some helioseismological data (Antia & Basu 2005; Bahcall et al. 2005a,b). Some authors suggest to increase the neon abundance, to conciliate the Asplund abundances with the helioseismology (Antia & Basu 2005; Bahcall et al. 2005c). Some recent measurements of the Ne abundance seem indicate that the neon abundance is indeed higher than the Asplund one (Cunha et al. 2006; Landi et al. 2007).

Since the origin 2006 version of the Geneva evolution code, the Asplund et al. (2005) abundances are used, with the high Ne abundance of Cunha et al. (2006).

**Reaction rates**

The reaction rates were updated for the origin 2006 version of the code. A complete description of the rates which are used now, and of the effects of the new rates on stellar evolution can be found in Ekström (2008).
Opacities

In the origin 2006 version, the opacities corresponding to the Asplund et al. (2005) and Cunha et al. (2006) were included, computed with the OPAL group web interface. To ensure the possibility to compute comparative models with older versions of the code, we include in the origin 2009 version a new parameter which allows the computation also with the Grevesse & Noels (1993) chemical composition.

The technical description of this modification is given in Appendix A.5 on page 149.

3.4.3 Rotation prescriptions

The evolution of stars is very sensitive to the chosen rotation prescription. In the Geneva Code, three prescriptions for the horizontal diffusion coefficient \( D_h \) (see Section 2.2 on page 8) are implemented, and two prescriptions for the shear diffusion coefficient \( D_{\text{shear}} \). All the combination between these two parameters are possible.

For the horizontal diffusion coefficient, the first prescription is the rough approximation given in Zahn (1992):

\[
D_h = \frac{1}{c_h} r |2V_2 - \alpha U_2|, \tag{3.104}
\]

where \( c_h < 1 \) is a coefficient, \( V_2 \) and \( U_2 \) the vertical and horizontal component of the meridional circulation, and \( \alpha = \frac{1}{2} \frac{d \ln (r^2 \Omega)}{d \ln (r)} \).

Another estimate for the horizontal diffusion coefficient is given in Maeder (2003), and is based on energetic considerations:

\[
D_h = Ar (r^2 \Omega |2V_2 - \alpha U_2|)^{\frac{1}{3}}, \tag{3.105}
\]

with \( \alpha < 0.1 \).

Finally, some indications come from laboratory experiments, and Mathis et al. (2004) determine a coefficient inspired by such experiments:

\[
D_h = \left( \frac{\beta}{10} \right)^{\frac{1}{2}} (r^2 \Omega)^{\frac{1}{2}} |r |2V_2 - \alpha U_2||^{\frac{1}{2}}, \tag{3.106}
\]

where \( \beta \sim 10^{-5} \).

The diffusion coefficient related to the shear mixing exists in two versions, one accounting for the effects of horizontal turbulence on the shear diffusion, and the other not. The first case leads to the following \( D_{\text{shear}} \) (Talon & Zahn 1997):

\[
D_{\text{shear}} = \frac{1}{4} \left( \frac{r \frac{d \Omega}{d r}}{N_T} \right)^2, \tag{3.107}
\]

with \( N_T \) the thermal part of the Brunt-Väisälä frequency, \( N_\mu \) the chemical part (see relations (3.5)), and \( K \) the thermal diffusivity. If the horizontal turbulence is not accounted for, Maeder (1997) found:

\[
D_{\text{shear}} = \frac{1}{4} K \left( \frac{r \frac{d \Omega}{d r}}{N_T} \right)^2, \tag{3.108}
\]

The technical description of this modification is given in Appendix A.6 on page 150.
3.4. The origin 2009 version of the Geneva code

3.4.4 Improvements of the code and minor modifications

A large part of this thesis work was done in close collaboration with Sylvia Ekström, and was to adapt, clean and rejuvenate the whole code, to allow the compilation and the computation on Linux machines. The aim of this section is to briefly describe the small modifications which were brought during this work. However, as all the details are not relevant, we put here only the modifications which improved the code at some point.

**FITM change and angular momentum conservation**

In the Geneva code, the value of the fitting mass $FITM$ is suitable to change. If that change is large enough, some external layers of the star are removed from the interior, and passed in the envelope. That is the case, in particular when the star crosses the HRD towards the red side. To ensure that the bottom of the external convective envelope is completely ionised, $FITM$ has to be lowered down to around $0.98$.

To preserve the angular momentum conservation, we have to account for the removal of these layers. A new routine $fitmshift$ was added, which computes the new angular velocity of the envelope during that process. The technical description of this modification is given in Appendix A.7 on page 150.

**Automatic FITM change**

During the crossing of the HRD or during the blue loops, $FITM$ has to be modified to keep a good link between the envelope and the first shell. In the previous versions of the Geneva evolution code, these changes had to be made by hand. With the help of Raphael Hirschi, this procedure is now automatised, in four different versions:

- the fitting mass follows the evolution of the external convective zone, and is set at $3/4$ of the extensions of this zone,
- the fitting mass depth is computed as a mix between the extension of the external convective zone, and the depth of the fully ionised medium,
- a “hand-like” method, which mimics what was done before by hand, and applied at each time step,
- a “hand-like” method, which mimics what was done before by hand, and applied once by sequence of computation.

The advantage of this automatisation is to make that process user-independent, and completely reproducible. The technical description of this modification is given in Appendix A.8 on page 151.

**Automatic parameter change**

During the computation of the stellar models, some parameters have to be changed at some given points. Most of them occur at a predictive time, and can thus be automatised. This new feature is now implemented in origin 2009. There is three levels of automatisation:

0: the changes are not automatised;

1: only the obligatory changes are automatised. It concerns, among others, the mass loss rate changes, the rotation prescription changes, and some convergence and discretisation parameters.
2: if the current model encounters convergence difficulties, the code tries to change some parameters to help it. It concerns, among others, the values of the width and height of the triangle used to compute the envelope.

The technical description of this modification is given in Appendix A.9 on page 151.

Ω- and \( \mu \)-profile smoothing

The resolution of the complex advection equation (see Section 2.2.2 on page 9) needs a particular care. To facilitate the numerical solution, the \( \frac{\partial \ln(\Omega)}{\partial \ln(r)} \) profile and the mean molecular weight profile are smoothed through the whole star using a polynomial fit over the nearest neighbours of each layers. The various parameters of used by this method (the degree of the polynomial interpolation, the number of considered points) were set to provide a good fit in the case of “standard” case, which consisted in 200 – 300 shells.

The modifications implemented during this thesis require a larger number of layers (more than 800 on the ZAMS, and this number tends to increase during the stellar evolution). The fit provided by the previous settings was no more accurate (indeed, the \( \mu \)-profile fit was so bad that it prevented the mixing in the star). We thus modified these settings, and added a check of the quality of the fit by computing the mean and maximal distance between fit and points. In case of a bad fit, the program modifies the values of the parameters, trying to find a better solution. If the fit remains not acceptable, the computation stops.

The technical description of this modification is given in Appendix A.10 on page 152.

Removal of all commons

In addition, to increase the legibility of the code, a huge work was to remove all the shared memory through the common structure, and to include them in a more modern way in the code, with the module structures. The variables are now sorted by type (structure variables, envelope variables, rotation variables, ...), and declared properly.
3.4. *The origin 2009 version of the Geneva code*
Chapter 4

Convective envelopes in rotating hot massive stars

In the stellar evolution lectures, one often finds the following general frame for the internal structure of the stars: “low mass stars have a radiative core and an external convective envelope, while high mass stars have a convective core, and a radiative envelope”. If this is roughly correct, massive stars have in fact a (very) small external convective envelope. Their mass content could seem too small for playing a major role in the evolution of the massive stars, however it could play an active part to the mass loss process of such stars. Fast rotation decreases the effective gravity near the equator, and the velocity of the convective shells could be sufficient to help some matter to leave the stellar surface. We examined the properties of the convective envelope of massive O and B-type stars in the following work (see on page 179):

**CONVECTIVE ENVELOPES IN ROTATING OB STARS**
A. Maeder, C. Georgy & G. Meynet
2008 A&A 479, L37

In this section, we describe the main results concerning the properties of the convective envelope in our rotating models. We begin with a description of the numerical method we used to do this work, before discussing the results. The physical processes used in this work are described in Section 3.1 on page 27.

### 4.1 Numerical method

As an initialisation, we perform a standard envelope integration accounting for rotation. This gives us an average structure of the envelope. The aim of this first integration is to obtain, for each isobar, the corresponding mass coordinate $M_r$ (we recall that the integration variable in the envelope is the pressure). As we will see below, there is no simple local equation for the mass conservation, and it is much more convenient to integrate it through the whole stars.

Once this first iteration is done, we perform some new local integrations at various colatitudes $\theta$ and not accounting for the rotation. The effects of rotation are taken into account indirectly:

- by the boundary conditions at the surface ($T_{\text{eff}}$ varies with $\theta$);
- by the $g_{\text{eff}}$ term appearing in the structure equations (see below);
4.2. Convective envelope in hot, massive stars

- by the modification of \( \nabla_{\text{rad}} \).

The structure equations are slightly modified:

\[
\frac{d \ln (r)}{d \ln (P)} = -\frac{P}{r \rho g_{\text{eff}}},
\frac{d \ln (T)}{d \ln (P)} = \min (\nabla_{\text{rad}}, \nabla),
\frac{d \ln (M_r)}{d \ln (P)} = \frac{4 \pi r^4 P}{GM_r^2 \frac{1}{f_P}}.
\] (4.1)

As we can see, the first two are completely local, and do not depend on global quantities. The third one is more problematic, as the mass contained in the isobar is a global variable. For this equation, we use the results of the first global integration with rotation. We obtain a set of envelope structures at various colatitudes, allowing us to examine the colatitude dependence of the external convective zones.

4.2 Convective envelope in hot, massive stars

Table 4.1: Summary of the properties of the external convective zones of a 20 M\(_\odot\) star at the end of the MS. The first column indicates the model (“Sch” is used for Schwartzschild criterion for convection, and “SH” for Solberg-Høiland one). The second column gives the position of the convective zone, the third one, the extent in radius of the zone at the pole, the fourth the same extent, but in percent of the total stellar radius. The fifth and sixth columns are similar to the two previous ones, but at the equator. The last one indicates the fraction of the total stellar mass contained in the convective zone.

<table>
<thead>
<tr>
<th>Model</th>
<th>CZ</th>
<th>( \frac{r_{\text{top}} - r_{\text{bottom}}}{R} )</th>
<th>% (pole)</th>
<th>( \frac{r_{\text{top}} - r_{\text{bottom}}}{R} )</th>
<th>% (equator)</th>
<th>( M_{\text{CZ}}/M_{\text{tot}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega/\Omega_{\text{crit}} = 0 )</td>
<td>upper</td>
<td>0.992 - 0.999</td>
<td>0.7</td>
<td>0.992 - 0.999</td>
<td>0.7</td>
<td>2.5 \cdot 10^{-9}</td>
</tr>
<tr>
<td>Sch</td>
<td>lower</td>
<td>0.915 - 0.962</td>
<td>4.7</td>
<td>0.915 - 0.962</td>
<td>4.7</td>
<td>7.4 \cdot 10^{-7}</td>
</tr>
<tr>
<td>( \Omega/\Omega_{\text{crit}} = 94 )</td>
<td>upper</td>
<td>0.987 - 0.999</td>
<td>1.2</td>
<td>0.958 - 0.988</td>
<td>3.0</td>
<td>1.3 \cdot 10^{-8}</td>
</tr>
<tr>
<td>Sch</td>
<td>lower</td>
<td>0.836 - 0.936</td>
<td>10.0</td>
<td>0.727 - 0.862</td>
<td>13.5</td>
<td>2.8 \cdot 10^{-6}</td>
</tr>
<tr>
<td>( \Omega/\Omega_{\text{crit}} = 94 )</td>
<td>upper</td>
<td>0.987 - 0.999</td>
<td>1.2</td>
<td>0.960 - 0.988</td>
<td>2.8</td>
<td>1.2 \cdot 10^{-8}</td>
</tr>
<tr>
<td>SH</td>
<td>lower</td>
<td>0.836 - 0.936</td>
<td>10.0</td>
<td>0.727 - 0.859</td>
<td>13.2</td>
<td>2.7 \cdot 10^{-6}</td>
</tr>
</tbody>
</table>

We compute the complete envelope structure for various colatitudes for a 20 M\(_\odot\) star, at a metallicity of \( Z = 0.020 \), at the end of the MS. We consider the case of a non-rotating model, and the case of a model rotating at \( \Omega/\Omega_{\text{crit}} = 94\% \). For the rotating one, we compute two sets of envelope structure: one where the convective zones are determined with the classical Schwartzschild criterion \( \nabla_{\text{ad}} - \nabla_{\text{rad}} > 0 \), and one where we apply the Solberg-Høiland criterion discussed above.

Without rotation, we find two convective zones in the envelope of the star. The first one is located very close to the surface, and contains very little mass: only 2.5 \cdot 10^{-9} of the total stellar mass (which is 19.97 M\(_\odot\) at the end of the MS). It ranges between \( r/R = 0.992 \) and 0.999, where \( R \) is the total stellar radius. The second one is deeper, and lies from \( r/R = 0.915 \) to 0.962, with a total mass fraction of 7.4 \cdot 10^{-7}. These properties are summarised in Table 4.1. On Figure 4.1 on next page, we show in red the opacity \( \kappa \) through the envelope. We notice two peaks in opacity:

\(^1\)As the chemical composition is supposed to be constant through the envelope, this is equivalent to the Ledoux criterion for convection.
one just below the surface, produced by a variation in the ionisation of helium, which modifies the opacity, and a second one, located slightly deeper in the envelope, produced by iron opacity (Cantiello et al. 2009). As $\nabla_{\text{rad}}$ depends on the opacity, we find the corresponding two peaks in the long-dashed blue curve. The dot-dashed blue curve is the adiabatic gradient. The two peaks in $\nabla_{\text{rad}}$ are large enough to overpass the value of $\nabla_{\text{ad}}$, producing the two external convective zones.

To show the effect of the modification of $\nabla_{\text{rad}}$ induced by rotation, we compute a model where this effect is accounted for. The results are in Table 4.1 on previous page. The effect is important: the sizes of both convective zones are doubled at the poles, and tripled at the equator! The mass content also increases, by a factor of $\sim 4 - 5$ for both zones. In the equatorial region, the convective zone covers more than 13% of the stellar radius. The position of the convective zones are shown on Fig. 4.2 on next page. From equation (3.10), we see that the local $\nabla_{\text{rad}}$ depends on the local variables $P$ and $T$. On an isobar, however, $P$ is of course constant, as $T$ varies only slightly (see Zahn 1992; Maeder & Zahn 1998). This means that the pressure (and the mass coordinate through relation (4.1)), at the top and the bottom of the convective zones are the same, i.e. that the border of the convective zones are on the same isobar at each colatitude.

The next step is to add the Solberg-Høiland criterion to determine the position of the convective zones. Comparing with the model computed with the Schwartzschild criterion only, we expect no difference at the poles (due to the term $\sin(\theta)$ in (3.6) which vanishes there), and a maximum effect in the equatorial plane. The values of $\nabla_{\text{ad}} - \nabla_{\text{rad}}$ and $\nabla_{\text{SH}} \equiv \nabla_{\Omega} \sin(\theta)$ at the equator are plotted in Figure 4.3 on next page. We see that $\nabla_{\text{ad}} - \nabla_{\text{rad}}$ is generally much bigger than $\nabla_{\text{SH}}$. This implies that the account of the Solberg-Høiland criterion for convection has only modest effects on the position of the convective zones. As mentioned before, it reduces slightly the size of both convective zones: the reduction is of the order of a few thousandths (see Tab. 4.1 on previous page).

To conclude, we see that rotation modifies strongly the size of the external convective zones of massive stars. The main effect is produced by the modification of the radiative gradient under the action of rotation. Due to its relative weakness with respect to the difference $\nabla_{\text{ad}} - \nabla_{\text{rad}}$, the Solberg-Høiland term in the convection criterion has only a minor impact on the size and position of these zones.
4.2. Convective envelope in hot, massive stars

Figure 4.2: Position of the convective zones of a $20\,M_\odot$ model rotating at $0.94\,\Omega_{\text{crit}}$. The two external convective zones are shown in red, and the convective core in blue. The rotation axis corresponds to the $y$-axis, and the $x$-axis lays in the equatorial plane. Both axis are in units of centimetres.

Figure 4.3: $\nabla_{\text{ad}} - \nabla_{\text{rad}}$ (solid line) and $\nabla_{\text{SH}} \equiv \nabla \Omega \sin(\theta)$ (short-dashed line) through the envelope at the equator of a $20\,M_\odot$ star. The zero level is indicated by a long-dashed line.
4.3 Link between convective envelopes and mass loss

The mass loss rate of the $20 \, M_\odot$ star considered here is $\dot{M} = 6.2 \cdot 10^{-7} \, M_\odot \, yr^{-1}$. With such a rate, the star loses in one year about 4 times the mass contained in the thin external convective zone. We have thus a complex interaction between the mass loss process and the convective zones: the loss of mass is eased by the velocity acquired in the convective motions. Acoustic waves generated in the convective zone should also be amplified in the winds, favouring the mass loss (Maeder 2009). Several processes are able to trigger mass loss with the help of convective zones (see Cantiello et al. 2009): microturbulence and non-radial oscillations excited by convection, or wind clumping produced by inhomogeneities in the density at the surface of the star, which could be a consequence of the convective motions under the stellar surface. Cantiello et al. (2009) use as a criterion that the mean velocity of the convective cells must be of the order of $2.5 \, km \, s^{-1}$ for these processes to be enhanced. In our models, we obtain velocities up to $20 \, km \, s^{-1}$ in the largest convective zone, ten times higher this last criterion. We expect thus a large contribution of the external convective zones of fast rotating massive stars to the global mass loss process.
4.3. Link between convective envelopes and mass loss
Chapter 5

Stellar evolution with anisotropic winds

5.1 Anisotropic radiative winds and stellar evolution

In Sections 2.2.5 and 3.2, we have shown how various surface phenomena appear with increasing surface velocities, and more particularly the development of anisotropic winds, and what are their effects on the angular momentum content of the star. Since we have now studied carefully the theoretical aspects, it is time to apply this theory to complete stellar evolution models, in order to evaluate is the impact of such anisotropies on stellar evolution. The results presented here have been submitted to Astronomy & Astrophysics (see the current submitted version on page 183):

Effects of anisotropic winds on massive stars evolution
C. Georgy, G. Meynet & A. Maeder
Submitted to A&A

5.1.1 Case study: evolution at constant mass loss rate

In a first step, we consider the case of a $9 \, M_\odot$ star, at the Small Magellanic Cloud metallicity ($Z = 0.002$). In order to shed light only on the anisotropic wind effects, the stellar models are computed with a constant mass loss rate of $10^{-9} \, M_\odot \, yr^{-1}$, independent of the surface stellar parameters (as the luminosity or the effective temperature). Moreover, we do not take into account the increase of the mass loss rate induced by rotation. We also neglect the variation of the force multiplier parameters in this first study. We keep them constant, with values $\alpha = 0.43$ and $k = 0.124$. This allows for a direct comparison with the semi-analytical results obtained in section 3.2 on page 31.

In this first stage, a set of two models is computed: one where the effects of anisotropic winds are not accounted for (only the deformation of the surface), and one where both effects are fully accounted for. The initial rotation parameter is set at $\omega_{\text{ini}} = 0.8$. To obtain the largest possible anisotropic effects, both models are computed with the internal transport produced by magnetic fields, according to the Tayler-Spruit dynamo (see Section 2.2.4 on page 11). This ensures a strong coupling between the surface and the core of the star, so the rotation profile remains nearly flat during the MS (Maeder & Meynet 2005; Petrovic et al. 2005). The loss of angular momentum is thus propagated in the whole star very quickly, and the surface is less spun down than in the classical case, where the internal magnetic fields are not accounted for. The evolution is followed until the moment the surface reaches the first critical velocity.
5.1. Anisotropic radiative winds and stellar evolution

**Evolution in the HRD**

The evolution in the HRD is shown in Figure 5.1. Starting on the bottom-left, with a rotation parameter \( \omega = 0.8 \), the evolution proceeds towards the top-right, where the surface reaches the first critical velocity. The red curve represents the isotropic model, and the blue one, the anisotropic case. As the surface rotation parameter progressively increases, we see that the anisotropic wind model evolves at an effective temperature slightly smaller than the isotropic one. Actually, the anisotropic model rotates appreciably faster (see Figure 5.3 on page 72). This induces the surface to become slightly larger, and thus the effective temperature somewhat lower for a given luminosity.

![HR diagram of the two 9 M_☉ models](image)

However, the effect on the tracks in the HRD remains extremely small, and the inclusion of the anisotropic winds does not significantly affect the stellar evolution, at least in this first simplified case (see below for a more realistic case).

**Angular momentum content and surface velocity**

This set of models allows us to verify that the modifications implemented in the Geneva stellar evolution code to ensure the angular momentum conservation are efficient. In Figure 5.2 on next page, we show in the upper panel the rate of angular momentum loss of the star, removed by the radiative winds. As the mass loss rate of these models is constant, the only variations of that rate come from the variation of the surface angular velocity. As shown below, the surface velocity increases as evolution proceeds, and thus also the rate of angular momentum loss. On the Zero Age Main Sequence (ZAMS), the rotation parameter \( \omega \) equals 0.8. According to Figure 3.7 on page 37 (compare case 2 and case 3), we should have a difference of roughly 10% between the models losing mass anisotropically and isotropically. This last value corresponds well to the one we find on the ZAMS. At the other extremity of the tracks, when the star rotates at the critical velocity, \( \dot{L}_{\text{iso}} / \dot{L}_{\text{ani}} \) should be \( \simeq 1.5 \) (always according to Figure 3.7). It is also the value we obtain comparing the two tracks of Figure 5.2. This confirms that the implementation of the anisotropic effect
CHAPTER 5. STELLAR EVOLUTION WITH ANISOTROPIC WINDS

computation in the Geneva code provides results in agreement with the semi-analytical approach followed previously.

The lower panel of Figure 5.2 illustrates, for each time step, the sum of the actual angular momentum contained in the star $L_{\text{star}}$, and the total angular momentum brought away by the stellar winds during the lifetime of the star, from the ZAMS to its actual age. If our calculation ensures a complete angular momentum conservation, this value must then be constant over time. The first calculations presented here indicate that it is indeed the case, and validate the implementation of the modifications ensuring that conservation in the code.

In the upper panel of Figure 5.3 on next page, we show how the rotation parameter evolves as a function of time. The anisotropic model (blue dashed line) remains slightly above the isotropic one (red solid curve), because of the lower angular momentum loss. Moreover, it reaches the critical rotation about 1 Myr before the isotropic model. The lower panel shows the total stellar content in angular momentum (including the envelope of the star). As expected, the anisotropic model keeps more angular momentum. However, we have to keep in mind that we have only considered the part of the stellar evolution before it reaches the critical velocity. After that, the star has to lose mass at the equator in order to maintain its surface at a sub-critical rotation parameter. This mechanical mass loss dominates over the radiative mass loss, and thus the angular momentum loss. The small difference in the angular momentum content established during the first phase of the MS is erased by this much stronger angular momentum loss.

Amazingly enough, the surface angular velocity of both models are quite similar (top panel of Figure 5.4 on page 73). The largest differences we find in $\omega$ are a combination between the variations in $\Omega$ and in the HRD tracks, which implies slightly different values for the stellar surface characteristics. We can thus obtain with the same angular velocity a different surface shape. The medium panel of the Figure 5.4 shows how the changes discussed above reflect on the equatorial velocity, while the bottom panel represents the first critical velocity as a function of time. Looking at these two last panels, we can see, as already noted by Ekström et al. (2008b) and Ekström (2008), that reaching the critical velocity is less a consequence of the increase of the equatorial velocity during the MS than a consequence of the decrease of the first critical velocity, due to the
5.1. Anisotropic radiative winds and stellar evolution

Preliminary conclusions

Even though the problem is considerably simplified in this first step, we can nevertheless draw some preliminary conclusions. Even for an extreme rotator, the effects of anisotropic winds during the phase of the stellar evolution between the ZAMS and the point where the star reaches the critical velocity remain extremely small. This result was predicted by the semi-analytical study of the problem. The following points are noticeable:

- The tracks in the HRD are only marginally modified by the action of the anisotropic winds.
- The angular momentum content of the star remains higher if the anisotropic winds are accounted for, if the star evolves towards very high rotation parameters. However, in that case, it is much likely that the star reaches the critical rotation, and that the loss of angular momentum will be dominated by the equatorial mass loss in both cases.
- The anisotropic model reaches the critical rotation slightly before the isotropic one.

These first indications suggest that the account for anisotropic winds in stellar evolution calculations is not necessary if one is only interested in the evolution of physical parameters of the star, particularly the evolution of its surface velocity. However, strong anisotropic winds may let an imprint on the circumstellar medium. The study of this aspect is an on-going work, which is presented in Chapter 10 on page 141.

On the other hand, the behaviour of our models is in agreement with the expected results obtained in the semi-analytical developed in Section 3.2. This validates the modifications implemented in the Geneva stellar evolution code in order to account for the anisotropic winds, and for the conservation of the angular momentum during stellar evolution.
CHAPTER 5. STELLAR EVOLUTION WITH ANISOTROPIC WINDS

5.1.2 Evolution towards the critical limit

The next step is to consider more complete stellar models. We computed a new set of 9 $M_\odot$ models similar as previously. However, we consider now the increase of the global mass loss rate induced by rotation (see Section 2.4 on page 21) and the possible variations of the force multiplier parameters $\alpha$ and $k$ on the surface. The mass loss prescription used here is the prescription by de Jager et al. (1988) (see Section 3.4.1 on page 53).

Evolution in the HRD

In Figure 5.5 on next page, we show, as in the previous case, the evolution through the HRD. Starting from the bottom-left corner, the evolution proceeds towards the top-right corner. The red solid line is the isotropic model, and the blue dashed line the anisotropic one. The initial rotation parameter is $\omega = 0.8$ for both models, and the points where the star reaches $\omega = 0.9$ and 0.95 are indicated. The behaviour of the tracks is quite different than in the constant mass loss rate case, with a crossing at the middle of the diagram. It is however difficult to determine a simple cause of that feature, as many parameters may vary between the two models. Nevertheless, both tracks look very similar. This confirms that, even with a much more complete treatment of the physics of the stellar surface, the effects of anisotropic winds on the evolutionary paths of the stars are negligible, even for extreme rotators.

Surface velocity and angular momentum content

The computation of the models discussed here was done with quite different time steps. This produces some differences in the burning reaction rates at the centre of these stars, leading to slightly different main-sequence lifetimes. In order to compare the variation of various surface parameters (as the mass loss rate, the surface angular velocity, etc...), it is more convenient to use the central hydrogen abundance $X_C$ instead of the time. Figure 5.6 on next page illustrates the
5.1. Anisotropic radiative winds and stellar evolution

Figure 5.5: Same as Figure 5.1, but with realistic mass loss rates and accounting for the variation of the force multiplier parameters.

Figure 5.6: Same models as Figure 5.5. Evolution as a function of the central hydrogen mass fraction $X_C$ of: the rotation parameter $\omega$ (upper panel) and the surface angular velocity $\Omega_S$ (lower panel).
rotation parameter $\omega$ (top panel) and the surface angular velocity $\Omega_S$ (bottom panel) as a function of the parameter $X_C$, for both our models. As in the case of constant mass loss rates, the anisotropic model shows a larger rotation parameter, but this difference cannot be related to a difference in the angular velocity, which is the same for both models. Rather, it seems to be mainly due to the differences in the tracks the models follow in the HRD.

The evolution of the mass loss rates for our models is presented in the top panel of Figure 5.7. The differences between the anisotropic and the isotropic models can be understood in terms of differences in the rotation parameter, and in surface characteristics ($L$, $T_{\text{eff}}$, ...), which intervene in the mass loss prescription. The bottom panel shows the total angular momentum content of the star. During the first part of the stellar life, the behaviour is the same as in the case of constant mass loss rate seen above: the anisotropic model keeps (very) slightly more angular momentum than the isotropic one. A peculiar feature appears when the central hydrogen abundance is $\sim 0.4$. From that point, the effective temperature of the star becomes low enough to produce a change in the force multiplier parameters (bistability jump)! This induces an enhancement of the equatorial mass flux, and at the same time, produces the abrupt change in the slope of the angular momentum content: the anisotropic model loses more angular momentum than the model where these effects are neglected!

![Figure 5.7: Same models as Figure 5.5. Mass loss rate $\log(\dot{M})$ (upper panel) and total angular momentum content $L_{\text{star}}$ (lower panel) as a function of the central hydrogen mass fraction.](image)

Conclusions

To end this section, let us make a few concluding remarks:

- For the first time, we checked that the system “star + stellar winds” is preserving its total angular momentum content. This validates the various modifications we implemented in the Geneva stellar evolution code to ensure this conservation.

- We found that even when realistic mass loss rates and variation of the opacities over the surface are fully accounted for, the effects of anisotropic winds remain small. For the general purpose of stellar evolution, they are negligible: the results obtained without accounting for them are very similar to the more refined model where they are considered.
• This result lies in sharp contrast with previous studies (see Maeder 2002; Meynet & Maeder 2007), where the account for anisotropic winds could lead to large effects during the stellar evolution. We can attribute these discrepancies to a more accurate treatment of the angular momentum kept in the envelope, and a different way to apply the correction induced by the anisotropic stellar wind to the angular momentum lost during a time step.

• Due to the bistability jumps, it is even possible that a star accounting for the anisotropic effects loses more angular momentum than an equivalent isotropic model.

• There is no easy way to determine which one of the isotropic or anisotropic model will keep the largest angular momentum content during its evolution. This is strongly dependent on the evolutionary tracks and the rotation parameter, which determine which part of the stellar surface crosses the bistability jump.

5.2 Equatorial mass loss: links with GRB progenitors and Be stars

In the previous section, we have discussed the effects of wind anisotropies on stellar models during the phase between the ZAMS and the moment where the star reaches the critical velocity. Here, we pursue these investigations, enlarging the computations to the whole stellar lifetime, including the phase where the star is critically rotating. As fast stellar rotation is thought to be directly linked with two astrophysical objects, the Gamma-Ray Bursts and the Be-stars, we begin this section by a few words about them. We then present the results obtained during this thesis related to that topic.

5.2.1 The Gamma-Ray Bursts: a short introduction

At the end of the 60s, the USA launched a fleet of satellites, the Vela spacecrafts, dedicated to the watch of the russian atomic space tests. After the declassification of the data, Klebesadel et al. (1973) published a paper announcing the existence of short duration gamma-ray flashes, randomly distributed all over the sky. This was the first announcement of the Gamma-ray burst (GRB) phenomenon. 20 years later, there was still no indication on the distance of these bursts, and their origin remained unknown. However, a study of the first BATSE catalogue (Fishman et al. 1993) and its 260 GRBs allowed Kouveliotou et al. (1993) to distinguish between two populations in that sample: a short-duration one, with typical duration of $0.3 \, \text{s}$ and a hard spectrum, and a long-duration one, with duration of $\sim 20 \, \text{s}$ and a softer spectrum.

On the theoretical side, the first attempts of explanation arose during the 80s and 90s. The GRBs were thought to be produced by the merger of two compact objects: either two neutron stars (Blinnikov & Rudzskii 1984), or, a neutron star and a black hole (Narayan et al. 1991, other references can be found in Woosley & Bloom 2006). Woosley (1993) developed the so-called collapsar model, which involves the collapse of a Wolf-Rayet star into a black hole, and the formation of an accretion disc. This could produce a relativistic jet along the rotation axis, generating the strong gamma-ray flux observed (see below for a more complete description of this phenomenon). He associated this model with the long-duration GRBs (LGRBs), the short-duration one being always explained by the merger model.

The first observational evidence that at least some LGRBs are related to the death of a massive star appears in 1998, with the observation of GRB 980425, and, simultaneously at the same location, the explosion of the supernova (SN) 1998bw, which was classified as a type Ic-BL (see Section 6.2 on page 97). To date, the co-occurrence of a SN explosion in direct link with a GRB is established for a dozen of cases, and some accurate spectroscopic data from the SN have been obtained for only 5 cases (Chornock et al. 2010, and references therein): GRB 980425 / SN 1998bw, GRB 030329 / SN 2003dh, GRB 031203 / SN 2003lw, GRB 060218 / SN 2006aj and GRB 100316 / SN 2010bh. In
all these cases, the associated SN was a type Ic-BL SN, showing strong broad-line features in their spectra, indicating a large velocity of the ejected matter.

The study of the nearby GRB host galaxies indicates that LGRBs occur preferentially in metal-poor, star forming galaxies (Prochaska et al. 2004; Sollerman et al. 2005; Modjaz et al. 2006; Stanek et al. 2006; Thöne et al. 2007; Wiersema et al. 2007; Margutti et al. 2007; Modjaz et al. 2008), with metallicities less than \( Z \sim (0.2 - 0.6) Z_\odot \). This trend is also apparent in more distant LGRB host studies (Fruchter et al. 2006; Svensson et al. 2010). This also indicates that LGRBs are related with the death of massive stars. It is however worth noting that some recent observations of “dark”-LGRBs seem to show that in some cases, GRB can occur in high-metallicity environment (Levesque et al. 2010).

The collapsar model

The collapsar model for LGRB (Woosley 1993) links the collapse of a massive rotating star to the GRB event. The subsequent physical processes are extremely complex, and parts of this scenario are still not well understood. We present here the general frame, without going into details. The interested reader will find further explanation in the following, not exhaustive list of paper: MacFadyen et al. (2001), Dessart et al. (2008) or Zhang & MacFadyen (2009).

At the end of the stellar life, the iron core collapses abruptly. In the collapsar scenario, a black hole (BH) must form. Actually, a neutron star would generate a jet too highly loaded in baryon, preventing it to reach the relativistic speed needed to generate the gamma-ray flux (Dessart, private communication). If the stellar material around the BH rotates fast enough, the matter does not fall into the BH, but can form an accretion disc around it. Once the disc is formed, some amount of energy is deposited near the rotation axis. Several mechanisms (neutrinos, magnetic instabilities in the disc or magneto-hydrodynamical extraction of the rotational energy of the BH) are liable to produce this deposition (see MacFadyen et al. 2001). The neutrinos should also play a role at some point. The energy released near the rotation axis initiates a collimated jet, which can pierce the star and reach the surface, where it becomes observable. The generation of the strong \( \gamma \)-flux is explained by the fireball model (Mészáros 2002), where the \( \gamma \)-photons are produced by \( e^+e^- \) interactions in the jet, and by shocks.

The constraints on the progenitors given by this model are thus:

- a massive enough core at the end of the evolution to produce a BH;
- a large enough specific angular momentum in the centre of the star at the pre-supernova stage, in order to obtain an accretion disc around the BH. We have typically to compare the specific angular momentum at the edge of the core with the specific angular momentum of the last stable orbit around the BH (using the Schwartzschild or the Kerr metric) (see e.g. Shapiro & Teukolsky 1983);
- the short time-scale of the burst implies that the phenomenon occurs through a relatively small region, smaller than the red supergiant stars. We thus need a star which has lost its external layer during its evolution. This is coherent with the observational fact that the supernovae associated with a good enough spectrum to determine their type are all of type Ic.

Alternative models

The millisecond magnetar model

In this model, the central engine is a milli-second highly magnetized neutron star. The energy is extracted from the rotation of the neutron star by the combined effect of energy neutrino deposition and magnetic fields (Thompson et al. 2004). With an initial rotation period of \( \sim 1 \) ms and a magnetic field of \( 10^{15} \) G, an energy of \( \sim 10^{51} \) erg can be extracted in
a few seconds. This value is comparable to the typical energy of a GRB and of the accompanying SN. However, Woosley & Bloom (2006) pointed out that this model is not accounting for the effect of the accretion on the neutron star. This could affect the physical process invoked here, and create some difficulties to this scenario.

Supranovae  This scenario assumes that the massive star first explodes as a normal SN. However, the fast rotation of the core permits the formation of a super-massive neutron star (sustained by centrifugal force), more massive than a non-rotating one (Vietri & Stella 1998). Since the neutron star progressively spins down, the sustaining force decreases, and at some point, the neutron star collapses in a BH. During this event, a jet can be ignited, creating the GRB. As the delay between the SN and the GRB can be large in this model, it cannot explain the cases were a SN occurs simultaneously with a GRB. However, it could explain some GRBs which were not accompanied by a SN.

5.2.2 The Be star phenomenon and fast rotating stars: observations

Be stars are a class of various hot objects exhibiting emission lines in their spectrum. They were first discovered by Father Secchi (1866). Classical Be stars are hot non-supergiant stars. In addition to their emission lines, they present an IR excess. The most generally accepted explanation for these features is that these stars are surrounded by an equatorial dense disc (Porter & Rivinius 2003), and by a radiative dilute polar wind. The formation of the disc is caused, or at least helped, by the fast rotation of the Be star. However, the formation process of the disc, its structure and its dynamics are to date still not well understood.

Table 5.1: Be disc masses and mass loss rates in the disc. The first column gives the name of the star, the second column the disc mass from the SIMECA code (Stee 2003), the third one the disc mass from Rinehart et al. (1999), the fourth one the mass loss rate in the disc (Stee 2003), and the fifth one, also the mass loss rate in the disc (Stee 2003, using data from Rinehart et al. 1999).

<table>
<thead>
<tr>
<th>Star</th>
<th>$M_{\text{disc}}$ [$10^{-10} \text{M}_\odot$]</th>
<th>$\log(M_{\text{disc}}$ [M$_\odot$ yr$^{-1}$])</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$ Per</td>
<td>10.0</td>
<td>9.3</td>
</tr>
<tr>
<td>$\eta$ Tau</td>
<td>22.9</td>
<td>–</td>
</tr>
<tr>
<td>$\zeta$ Tau</td>
<td>23.4</td>
<td>8.3</td>
</tr>
<tr>
<td>48 Per</td>
<td>11.2</td>
<td>3.7</td>
</tr>
<tr>
<td>$\phi$ Per</td>
<td>18.1</td>
<td>17.6</td>
</tr>
<tr>
<td>$\gamma$ Cas</td>
<td>43.6</td>
<td>81.4</td>
</tr>
<tr>
<td>$\alpha$ Aqr</td>
<td>4.2</td>
<td>0.25</td>
</tr>
<tr>
<td>EW Lac</td>
<td>2.6</td>
<td>3.2</td>
</tr>
<tr>
<td>$\kappa$ Dra</td>
<td>11.4</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Fast rotation is supported by several observations. Martayan et al. (2006) and Martayan et al. (2007) show that Be stars are generally fast rotating stars, and that they are faster rotators than classical B stars already on the ZAMS. Some direct measurement of the shape of the star was performed by interferometric technics, on the brightest Be star of the sky, Achernar ($\alpha$ Eri). The first one (Domiciano de Souza et al. 2003) highlighted an equatorial to polar ratio of $1.56$, which is in disagreement with the Roche model, described in Section 5.2.5 on page 51. However, a new estimation of this ratio, accounting for a small equatorial disc, leads to a value of $R_E/R_P = 1.4 - 1.5$ (Carciofi et al. 2008), complying with the Roche theory.

Chesneau et al. (2005) and Meilland et al. (2007) perform an interesting study of the Be star $\alpha$ Arae. They found the following properties for that star:
• a polar mass flux of $1.7 \times 10^{-9} \text{M}_\odot \text{yr}^{-1} \text{sr}^{-1}$,

• a disc mass of $2.3 \times 10^{-10} \text{M}_\odot$,

• a mass loss rate in the disc of $6 \times 10^{-7} \text{M}_\odot \text{yr}^{-1}$.

This directly supports the idea of a fast rotating star, with an enhanced radiative polar wind, as expected for such a star, and with an equatorial disc, indicating that this star could be close to the critical rotation. Other mass loss rates in the disc and disc masses have been measured by different groups (see Table 5.1 on previous page).

5.2.3 A $20 \text{M}_\odot$ model at very low metallicity

To evaluate the impact of anisotropic radiative winds coupled with mechanical equatorial mass loss on the evolution of massive stars, we have to focus on cases which are able to reach the critical velocity during their evolution. The most unfavourable cases are models which encounter very strong stellar winds. Such winds indeed remove a large amount of angular momentum from the surface of the star, and thus prevent it from reaching one of the critical velocities. In order to avoid strong mass loss during the stellar lifetime (or at least during the MS), there are mainly to possibilities:

• a star of moderate mass, where the radiative stellar winds are relatively inefficient, and where the mass loss rate is small,

• a more massive star, but at very low metallicity. As the winds scale with the metallicity, this ensures a modest mass loss, allowing the star to keep most of its angular momentum content.

In this section we study the second point, while the case of smaller mass stars will be discussed in the next section. Note also that we have performed several tests to check the validity of the whole new treatment of mass loss in the Geneva code, including the conservation of the angular momentum, anisotropic radiative winds and equatorial mechanical mass loss. These tests will be presented in Section 5.3 on page 89.

According to Yoon et al. (2006), we choose a model at very low metallicity which is liable to produce a GRB at the end of its lifetime. We computed a set of two $20 \text{M}_\odot$ models, at a metallicity of $Z = 5 \times 10^{-5}$, one with an isotropic mass loss, and the other accounting for the anisotropies of the stellar radiative winds (see Section 3.2.3 on page 39). In order to reach quickly the critical rotation, we set a relatively high rotation parameter $\omega = 0.75$ on the ZAMS. This corresponds roughly to $v_{\text{eq}}/v_{\text{Kep}} \sim 0.45$ (where $v_{\text{Kep}}$ is the keplerian velocity of the stellar equator, and does not exactly correspond to $v_{\text{crit},1}$). Looking at Figure 3 of Yoon et al. (2006), we see that such a model is at the edge of the GRB progenitor area. It is thus an interesting model to compute in the frame of GRB progenitor properties, particularly to study whether anisotropic winds and the reaching of the critical velocity have an impact on such models. We also included the effect of internal magnetic fields on the angular momentum and chemical species transport. Both models were followed up to the end of the central Silicon burning phase. The maximal allowed rotation parameter is set at $R_{\Omega,\text{max}} = 0.95$ for both models (see point 3 of the Section 3.2.3 on page 43).

Evolution in the HRD

Figure 5.8 on next page shows the evolution in the HRD of both our models. The red curve depicts the model computed with isotropic stellar winds, and the blue track the anisotropic one. Some key points for the evolution are labelled along the curves. The first one is the point where the models reach the (first) critical velocity for the first time. It occurs at the same time for both models (see below) and produces a small inflexion in the tracks. The end of the MS is not surprising, and
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Figure 5.8: HRD for the $20 \, M_\odot$ at $Z = 5 \cdot 10^{-5}$ (isotropic model: red solid line; anisotropic model: blue dashed line). Some important stages in the evolution of these models are labelled along the tracks.

behaves as usual. There is no noticeable difference between the isotropic and anisotropic model until roughly the end of the He-burning. From that point, the two curves are slightly different. Note however that the anisotropic model suffers from a rougher discretisation of its inner layers, due to a flaw in the automatic change of parameter during the late burning phases. This rougher discretisation in the core may produce the final differences in the tracks in the HRD.

Mass loss rates and final mass

In Figure 5.9 on next page, we see how the mass loss rates of the two models evolve as a function of the time (left panel). The solid (or dashed) curves represent the radiative mass loss rates. Owing to the very low metallicity we have chosen for these models, they are very small, and increase slightly during the MS. At around $8 \, \text{Myr}$, both models reach the critical velocity, beginning then to lose mass mechanically at the equator. The “instantaneous” mass loss rate they encounter is represented by the dots in the plot. To obtain this mass loss rate, we divided the amount of mass loss during a time step where the evolution of the star brings it to the critical velocity, according to relation (3.43), by the value of the time step. Note that this value is not very accurate. However, the order of magnitude is probably correct, and as it is computed only by considerations on the conservation of the angular momentum, the total amount of mass lost mechanically is certainly a good approximation (see Section 5.3.3 on page 92).

Qualitatively, there are no differences between both models. During the critical-rotation phase, the mechanical mass loss rate is 2 to 3 orders of magnitude above the radiative one. This produces a sharp decrease in the total mass of the star (right panel). In spite of the mechanical mass loss rate not occurring exactly in the same way for both models, the evolution of the total mass is absolutely similar, and the difference in the final mass is very small.
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Figure 5.9: Same models as Figure 5.8. Left panel: Mass loss rates evolution as a function of time. The curves represent the radiative mass loss rates; the points correspond to the mechanical mass loss rates encountered by the star once it reaches the critical velocity. Right panel: Time evolution of the total mass of each model. The moment when the critical velocity is reached is indicated along the curves.

Angular momentum loss

As this model was the first one to be computed with the equatorial mechanical mass loss treatment, it is also an important test for the conservation of the angular momentum during this critical phase. The upper panel of figure 5.10 on next page shows the total angular momentum content for both models (the colour and line coding is the same as previously). The small window is a zoom on the non-critically rotating phase. As expected from the case previously studied, the anisotropic model conserves slightly more angular momentum than the isotropic one during that phase. However, most of the angular momentum loss occurs in the equatorial disc, when the star is at the critical rotation. During that phase, both models lose the same amount of mass, as well as the same amount of angular momentum. The small difference accumulated during the first phase of the evolution between both models fades away, and the final angular momentum content of both models is the same.

The lower panel shows the sum of the angular momentum content of the star and the angular momentum brought away by the stellar winds during the past history of the star. We see that this value remains constant during all the stellar life, validating the modifications brought in the Geneva stellar evolution code in order to compute the mechanical mass loss, and to ensure the conservation of the angular momentum.

Final chemical and specific angular momentum structure

As already mentioned above, due to a flaw in the automatic adaptation of the parameters setting the discretisation of the stellar structure, the model computed with the account for the anisotropic radiative mass loss is not well described near the core at the end of the stellar evolution. However, the model computed with isotropic radiative wind does not suffer this problem. As the HRD tracks, the evolution of the mass loss and the final angular momentum are very similar between those models, we do not expect a large difference in term of final chemical structure and
Figure 5.10: Same models as Figure 5.8. **Upper panel:** Total angular momentum kept in the star as a function of time. The small window is a zoom on the first phase of the stellar evolution, between the ZAMS and the reaching of the critical rotation. **Bottom panel:** At each time step, sum of the total angular momentum in the star and of the total angular momentum brought away in the stellar winds during the past history of the star.

Specific angular momentum profile. The conclusions drawn below for the “isotropic” model are thus most probably valid for the other one.

In Figure 5.11 on next page, we can see the chemical abundance profile (left panel) for the most relevant chemical species, and the internal profile of specific angular momentum \( j = \frac{2}{3} \Omega r^2 \), compared with the corresponding specific angular momentum of the last stable orbit around a Schwartzschild black hole (\( j_{\text{Sch}} = \sqrt{\frac{2GM_c}{r}} \)) and a maximally rotating Kerr black hole (\( j_{\text{Kerr, max}} = \frac{GM_c}{c} \)).

Starting from the centre (on the left), the chemical structure of the star first shows a small core, composed essentially of \( ^{56}\text{Ni} \). Just above that core, we find a very thin shell of Si burning, surrounded itself by a larger O-burning shell. The C-burning region is very confined above the previous one. The last burning regions are the larger He- and H-burning shells, roughly between \( M_r/M_{\text{tot}} = 0.25 – 0.45 \). As the mass loss during the whole life of this star is very small, the surface abundances are only modified by the action of the mixing, which brings to the surface the products of the CNO-cycle: mainly, an increase of the He surface abundance, a modification of the relative CNO abundances (increase of the N abundance, decrease of the C and O abundances), and, of course, a decrease of the H surface mass fraction.

An important point to mention is the size of the CO-core at the end of the evolution. From the chemical abundance profile, we see that for this model, we have (approximately) \( M_{\text{CO}} \sim 4.9M_\odot \) (with a final mass \( M_{\text{fin}} = 19.58M_\odot \)). From the CO core mass, we can estimate the baryonic mass of the remnant using the relation given in Maeder (1992). We find a baryonic mass \( M_b = 2.14M_\odot \), which allows us to compute the gravitational mass of the remnant, following Hirschi et al. (2005). This leads to \( M_{\text{rem}} = 1.85M_\odot \). This mass is too low to allow the formation of a black hole during the collapse of the stellar core. The remnant left behind this model is a neutron star.

In the right panel of Figure 5.11 on next page, we can compare the specific angular momentum of our model with the specific angular momentum required for the last stable orbit around a Schwartzschild (or Kerr) black hole. Yoon et al. (2006) used the following criterion to determine if a model has enough specific angular momentum to produce an accretion disc around the (possible) central black hole: the disc forms whether any part of the CO core has a specific angular momen-
Figure 5.11: Isotropic model of $20 \, M_\odot$ at $Z = 10^{-5}$. Left panel: Profile of the main chemical abundances, in Lagrangian coordinates normalised to the total final mass (the centre is on the left, the surface on the right). The $y$-axis is the logarithm of the mass fraction. Each curve corresponds to a chemical species, labelled near the corresponding curve. Right panel: Profile of the specific angular momentum through the star (black solid curve). The red dotted curve is the specific angular momentum of the last stable orbit around a Schwartzschild Black-Hole, with a mass corresponding to the total mass $M_r$, $j_{Sch} = \sqrt{\frac{2GM_r}{c}}$. The blue dashed curve is the specific angular momentum of the last stable orbit around a maximally rotating Kerr Black Hole $j_{Kerr, max} = \frac{GM_r}{c}$ (in the most favourable case, see (Shapiro & Teukolsky 1983) for a detailed discussion on the last stable orbit around black holes).

According to the collapsar scenario for GRBs, we conclude that this model fulfils none of the required conditions:

- no BH production during the collapse event;
- not enough angular momentum in the core;
- the star keeps its H-rich envelope during its whole life, and is thus likely to explode as a type II SN.

This result does not agree with the corresponding model presented in Yoon et al. (2006), who found that a model of $20 \, M_\odot$ at $Z = 10^{-5}$ with an initial velocity of $v_{eq}/v_{Kep} = 0.45$ is able to produce a GRB. Their equivalent model follows a homogeneous evolution, while our model does not. As the computation methods are relatively different between the two codes, there could be many origins to this contradiction. However, this model is the limit case for homogeneous evolution in their approach, and it may be that only small differences in the physics considered in the codes is sufficient to produce large differences in the evolution of a model. Let us mention that in the code used by Yoon et al. (2006), the rotation is not treated as in the Geneva code (see Petrovic et al. 2005). There are also differences in the treatment of the internal magnetic field, and of the chemical
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species mixing. Finally, the mass loss rates used are also different (their model loses several $M_\odot$ during its whole lifetime, as our only roughly 0.5 $M_\odot$).

We can however conclude from our study that radiative wind anisotropies do not sufficiently modify the angular momentum content of the star to produce a completely different behaviour between anisotropic and isotropic models. Indeed, the reaching of the critical velocity and the consequent equatorial mechanical mass loss remove too much angular momentum from the surface of the model, extracting large amount from the core, and thus spinning it down.

In the future, it would be interesting to compute models with a still higher initial rotation velocity, in order to study the conditions leading to homogeneous evolution with the Geneva code physics. It is a key point for the evolution towards GRB progenitors, and the comprehension of the stars leading to such events would be improved with the computations of several additional models.

5.2.4 Models of Be stars

The Geneva code, in its 2006 version, has been already used to study the Be-star populations in the frame of single rotating star models (Ekström et al. 2008b). In this work, the authors computed a set of 112 stellar models, from 3 and 60 $M_\odot$ at four metallicities and with initial rotation parameters between $\omega = 0.1$ and 0.99. These models were followed up to the end of the MS, or, for those reaching the critical velocity, up to that point. Their conclusion was that, assuming that a star could eject some amount of matter at the equator when it rotates above $0.7v/v_{\text{crit,1}}$, and that the initial velocity distribution at low metallicity is shifted towards higher rotation parameter, these models are able to well reproduce the observed ratio of Be-stars/(Be-stars + B stars) at various metallicities.

With the rotation prescriptions chosen for the computation of the new stellar grids (see Chapter 8 on page 129), we have undertaken to pursue this work, computing a new sub-grid of stars likely to reach the critical velocity, and following them up to the end of the MS, including the phase where they lose mass at the equator. The computed masses are 3, 5, 7, 9, 12 and 15 $M_\odot$, and, to date, only solar metallicity was adopted (however, we plan to extend this work at sub-solar metallicity). The initial rotation parameters are $\omega = 0.8, 0.9$ and 0.95. As the effects of the radiative wind anisotropies are small, we neglected them (allowing computations with larger time steps). The parameter $R_{\Omega,\text{max}}$ is set to 0.99 (see Point 3 on page 43).

In Figure 5.12 on next page (left panel), we show the HRD for all our models of Be stars. Each group of three tracks corresponds to a different initial mass (which is indicated along the curves). For each initial mass, there is one track for the three different initial rotation parameters that we have computed. The track with the highest $L$ and $T_{\text{eff}}$ of each group corresponds to the model with $\omega_{\text{ini}} = 0.8$, and the track with the lowest ones to the model with $\omega_{\text{ini}} = 0.95$. The intermediate curve represents the models with $\omega = 0.9$. On the ZAMS and very beginning of the MS, this shift for the same initial mass is due to the fact that the centrifugal force adds a supplementary support against gravity, and the star behaves as if it had a lower mass (see e.g. Ekström 2008). Later in the evolution, the fastest rotating models develop a larger core, and their luminosity thus increases more than the slowest rotating one.

In the right panel of the same figure, we can see the evolution of the rotation parameter $\omega$, plotted as a function of time, and normalised by the total duration of the MS. This allows an easier comparison of different models, which have very different lifetimes. Each panel shows two different masses (for each mass, the three initial rotation parameters are represented, the colour code being the same as in the left panel.)

The reaching of the critical velocity is governed by three processes:

- the internal redistribution of the angular momentum, through advection and diffusion;
Figure 5.12: Left panel: HRD for our models of Be stars. For each mass, the curve the most to the left on the ZAMS corresponds to the model with an initial rotation parameter $\omega = 0.8$, the medium curve to $\omega = 0.9$, and the curve the most to the right to the model with $\omega = 0.95$. Right panel: Time evolution of the rotation parameter. Each panel shows the tracks for the two masses indicated. The colour and line styles are the same as in left panel. The $x$-axis it the time, normalised by the total duration of the MS (to allow an easier comparison between the different models).

- the mass loss, which tends to spin down the surface, and thus prevents the star from reaching the critical velocity;
- the time evolution of $\Omega_{\text{crit}}$ (see Figure 5.12). As evolution proceeds, the star loses mass and increases its radius. This induces a decrease in $\Omega_{\text{crit}}$.

Looking at Figure 5.12, a preliminary conclusion is that the minimal initial rotation parameter required to reach the critical velocity during the MS is $\omega \sim 0.8 - 0.9$. The models initially rotating slower than this value never reach the critical velocity, and models with $\omega = 0.8$ reach it at the very end of the MS. For the models without (or with moderate) radiative mass loss (i.e. the models between 3 and 12$M_\odot$), the radiative mass loss is insufficient to prevent the star from reaching the critical velocity. On the contrary, for models with an initial mass higher than $M_{\text{ini}} = 15M_\odot$, the radiative mass loss becomes so important that the critical velocity is never reached. We also see that the moment at which the star reaches the critical velocity (if it ever reaches it) is roughly independent of the mass: the models with an initial rotation parameter $\omega = 0.95$ becomes critically rotating after $\sim 60 - 70\%$ of the MS, and the models with $\omega_{\text{ini}} = 0.9$ after $\sim 80 - 90\%$ of the MS. An important point here is that single star models produce only Be stars at the end of the main sequence! They must thus show surface enrichments, as for example, an increase of the nitrogen abundance compared to the carbon one, which is produced by the mixing during the MS (see Table 5.2 on page 88 for typical values expected from our models).

An interesting quantity which is now accessible with the origin 2009 version of the Geneva code is the “instantaneous” mechanical, equatorial mass loss rate. By “instantaneous”, we mean the amount of mass which is lost during one time step (according to relation 3.43), divided by the time step. The equatorial mass loss occurs during some bursts (whose duration is purely numerical and
is not representative of the duration of the active/quiescent phases of observed Be stars), which bring the star below the critical velocity. Consequently, it takes a few time steps for the star to reach again the critical velocity, with no equatorial mass loss occurring during this time. We are fully aware of the incapability of the Geneva code, even with the new features, to precisely describe the whole complexity of the mass loss during the active phase of Be stars (which, at least, would need an hydrodynamic treatment of the star, and a complete model of the disc). However, we think that these instantaneous mass loss rates should at least give an idea of the amount of mass which has to be lost to maintain the star slightly below the critical velocity, even if they are very approximate. We discuss this point in more details in Section 5.3 on page 89.

In Figure 5.15 on page 91, we plot the logarithm of the instantaneous mechanical mass loss rate \( \log(\dot{M}_{\text{disc}}) \) of our models, as a function of \( \log(t) \), where \( t \) is the time in Myr. The initial mass is indicated below each cloud of points. The different colours in each cloud represent models with different initial rotation parameters \( \Omega/\Omega_{\text{crit}} \) (the colour code is the same as in Figure 5.12 on previous page). The most extended clouds along the time axis correspond to the initially fastest rotating models, which reach the critical velocity first. The dispersion of the value of \( \log(\dot{M}_{\text{disc}}) \) is very large. However, we can guess a mass trend, whereby the most massive stars tend to have a slightly higher mechanical mass loss rate. This point will be discussed below.

Even if our instantaneous mass loss rates are not very reliable, it is nevertheless interesting to compare them with observational data. Looking at the measured equatorial mass loss rates of Be stars (see Table 5.1 on page 78), which range between \(-8.8 < \log(\dot{M}_{\text{disc}}) < -6.6\), we can see that our mechanical mass loss rates are in reasonably good agreement (however, keep in mind that these mass loss rates should be slightly overestimated, see Section 5.3 on page 89). To compare with more refinement, we can choose, in the list of Be stars with estimated mass and measured equatorial mass loss rates, stars that have a mass comparable with some of our models. This is shown in Figure 5.14 on next page. We compare our \( 5 M_\odot \) models with \( \alpha \) Aqr (4.6 \( M_\odot \)) and \( \kappa \) Dra (5.3 \( M_\odot \)), and our \( 9 M_\odot \) models with \( \zeta \) Tau (8.6 \( M_\odot \)) and EW Lac (9.2 \( M_\odot \)) (the masses are taken from Zorec et al. 2005). The red dots correspond to the instantaneous mechanical mass loss rate of our \( 9 M_\odot \) model, and the blue ones the mechanical mass loss rate of the \( 5 M_\odot \) model, as a function
of time. The red and blue areas are the zones including the observed equatorial mass loss rates (the extension along the $x$-axis is arbitrary). We see that our models are (marginally) compatible with the observations, even if we generally have an underestimation of the mass loss rate. Note however that the observed mass loss rate depends also on models to interpret the interferometric measurements, and are probably not firm.

Figure 5.14: Comparison between the instantaneous mechanical mass loss rates for our 5 (blue points) and 9 $M_{\odot}$ (red points) models with an initial rotation parameter $\omega = 0.95$, and measurements of the mass loss rate in the discs of four Be stars with masses around 5 $M_{\odot}$ (blue area) and 9 $M_{\odot}$ (red area). The observed mass loss rates are from Rinehart et al. (1999) and Stee (2003). The estimated masses are from Zorec et al. (2005).

What is probably more robust is the mean mechanical mass loss rate of the model. By “mean mass loss rate”, we consider simply the total amount of mass lost mechanically by the model, divided by the whole time spent at the critical velocity. The main characteristics of our models are summarised in Table 5.2 on next page. During the critical-rotation phase, the models do not lose mass at each time step, but undergo mass loss episodes, which brings the star below the critical rotation, and it takes a few time for the star to reach again the critical rotation. This explain that the mean mass loss rate is smaller than the instantaneous one.

The mean equatorial mass loss rate is shown in the left panel of Figure 5.15 on page 91, as a function of the initial mass of the model, and for the three following initial rotation parameters: $\omega_{\text{ini}} = 0.8$ (blue), 0.9 (red) and 0.95 (green). The general trend is an increase of the mean equatorial mass loss rate with the initial mass, up to 9 $M_{\odot}$. By contrast, the 12 $M_{\odot}$ model has a mechanical mass loss rate smaller than the 9 $M_{\odot}$, and, of course, there is a quick drop towards the 15 $M_{\odot}$ model, which never reaches the critical velocity whatever the initial rotation velocity. This mass trend can be understood by the interplay of two phenomena:

- The meridional circulation, which brings angular momentum from the centre to the surface, becomes more and more efficient for the most massive stars (see Ekström 2008). This induces a more efficient input of angular momentum at the surface. Thus, to maintain those stars at the critical velocity, the mechanical equatorial mass loss has to carry away more angular momentum.

- The radiative mass loss, which also becomes more important for the most massive stars. It is particularly true for models above $\sim 12 M_{\odot}$ (see right panel). As radiative mass loss takes
5.2. Equatorial mass loss: links with GRB progenitors and Be stars

Table 5.2: Main characteristics of the Be star models. $M_{\text{ini}}$ is the initial mass of the model; $\Omega/\Omega_{\text{crit}}$ is the initial rotation parameter; $\Delta M_{\text{tot}}$ is the total amount of mass lost by the star during the MS; $\Delta M_{\text{disc}}$ is the amount of mass lost mechanically in the equatorial disc during the critical-rotation phase; $<M_{\text{disc}}>$ is the mean equatorial mass loss rate during the phase of critical rotation (see text); $\tau_{\text{cr}}/\tau_{\text{MS}}$ is the fraction of the MS spent at the critical velocity; $X_{\text{C,cr}}$ is the fraction of hydrogen in the burning core when the star reaches the critical velocity for the first time; $<v_{\text{eq}}>$ is the mean equatorial velocity during the MS; finally, $\Delta$(N/C) is the relative enrichment in nitrogen during the MS ($\Delta$(N/C) = $\frac{N/C}{N/C_{\text{cm}}}$).

<table>
<thead>
<tr>
<th>$M_{\odot}$</th>
<th>$\Omega/\Omega_{\text{crit}}$</th>
<th>$\Delta M_{\text{tot}}$</th>
<th>$\Delta M_{\text{disc}}$</th>
<th>$&lt;M_{\text{disc}}&gt;$</th>
<th>$\tau_{\text{cr}}/\tau_{\text{MS}}$</th>
<th>$X_{\text{C,cr}}$</th>
<th>$&lt;v_{\text{eq}}&gt;$</th>
<th>$\Delta$(N/C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>219.7</td>
<td>2.3</td>
</tr>
<tr>
<td>0.9</td>
<td>2.36 · 10^{-3}</td>
<td>2.36 · 10^{-3} (100%)</td>
<td>5.6 · 10^{-11}</td>
<td>0.10</td>
<td>0.16</td>
<td>265.8</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>5.49 · 10^{-3}</td>
<td>5.49 · 10^{-3} (100%)</td>
<td>4.7 · 10^{-11}</td>
<td>0.27</td>
<td>0.33</td>
<td>290.8</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>248.3</td>
<td>3.7</td>
</tr>
<tr>
<td>0.9</td>
<td>6.34 · 10^{-3}</td>
<td>6.34 · 10^{-3} (100%)</td>
<td>5.5 · 10^{-10}</td>
<td>0.10</td>
<td>0.16</td>
<td>300.5</td>
<td>5.3</td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>1.91 · 10^{-2}</td>
<td>1.91 · 10^{-2} (100%)</td>
<td>4.0 · 10^{-10}</td>
<td>0.38</td>
<td>0.40</td>
<td>349.4</td>
<td>8.8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.8</td>
<td>1.55 · 10^{-3}</td>
<td>4.93 · 10^{-4} (3%)</td>
<td>1.1 · 10^{-9}</td>
<td>0.001</td>
<td>0.0</td>
<td>268.3</td>
<td>5.3</td>
</tr>
<tr>
<td>0.9</td>
<td>1.33 · 10^{-2}</td>
<td>1.13 · 10^{-2} (85%)</td>
<td>1.5 · 10^{-9}</td>
<td>0.14</td>
<td>0.20</td>
<td>323.6</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>2.57 · 10^{-2}</td>
<td>2.34 · 10^{-2} (91%)</td>
<td>1.6 · 10^{-9}</td>
<td>0.27</td>
<td>0.31</td>
<td>356.0</td>
<td>10.4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
<td>1.53 · 10^{-3}</td>
<td>7.82 · 10^{-4} (5%)</td>
<td>6.2 · 10^{-9}</td>
<td>0.004</td>
<td>0.01</td>
<td>286.5</td>
<td>7.7</td>
</tr>
<tr>
<td>0.9</td>
<td>3.75 · 10^{-2}</td>
<td>1.98 · 10^{-2} (53%)</td>
<td>3.5 · 10^{-9}</td>
<td>0.17</td>
<td>0.22</td>
<td>347.7</td>
<td>10.6</td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>5.48 · 10^{-2}</td>
<td>3.45 · 10^{-2} (63%)</td>
<td>3.3 · 10^{-9}</td>
<td>0.31</td>
<td>0.33</td>
<td>377.0</td>
<td>11.7</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.8</td>
<td>1.15 · 10^{-1}</td>
<td>0.0 (0%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>300.3</td>
<td>7.5</td>
</tr>
<tr>
<td>0.9</td>
<td>1.47 · 10^{-1}</td>
<td>9.08 · 10^{-4} (1%)</td>
<td>3.2 · 10^{-10}</td>
<td>0.14</td>
<td>0.18</td>
<td>363.2</td>
<td>9.0</td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>1.73 · 10^{-1}</td>
<td>8.56 · 10^{-3} (5%)</td>
<td>1.5 · 10^{-9}</td>
<td>0.27</td>
<td>0.29</td>
<td>394.3</td>
<td>10.3</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.8</td>
<td>3.57 · 10^{-1}</td>
<td>0.0 (0%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>302.5</td>
<td>6.5</td>
</tr>
<tr>
<td>0.9</td>
<td>3.96 · 10^{-1}</td>
<td>0.0 (0%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>353.6</td>
<td>7.9</td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>4.29 · 10^{-1}</td>
<td>0.0 (0%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>384.4</td>
<td>9.0</td>
<td></td>
</tr>
</tbody>
</table>
care of removing angular momentum from the stellar surface, there is less and less need for an additional mechanical mass loss.

The combination of these two processes explains the behaviour of the mean mechanical mass loss rate during the critical rotation phase. This interplay appears when looking at the right panel of Figure 5.15 on page 91. Here, we represent the total amount of mass lost during the MS for each initial mass and initial rotation velocity, as well as the contribution of the mechanical mass loss (red) and radiative winds (blue).

The first interesting point is that the total mass lost by the star is increasing with the initial mass and the initial rotation parameter. Second, the total amount of mass lost mechanically is also increasing, from 3 to 9 $M_\odot$. However, as the radiative winds becomes very efficient for masses above 9 $M_\odot$, the relative contribution of the mechanical mass loss decreases as a function of the initial mass: from 100% for the 3 and 5 $M_\odot$ models, it becomes $\sim 50\%$ for the 9 $M_\odot$ star, and progressively decreases to 0 for the 15 $M_\odot$ model, which does not reach the critical velocity.

As the total mass lost in the equatorial disc is directly linked to evolutive processes (which bring the star to the critical velocity, and keep it close to it), it is almost independent of the way this mass is lost. It does not matter how this mass is lost (strong mass loss episodes punctuated by long quiescent phases, of weaker mass loss phases with shorter quiescent phases). Comparing our results (see Table 5.2 on previous page) with the observed mass of Be star disc (see Table 5.1 on page 78), we see that the mass lost in the disc obtained with our models is several orders of magnitude above the observed one. However, let us mentioned that the disc around a Be star should lose mass (for example, through photoevaporation). There are also observational evidences that the discs around Be stars have a limited lifetime (Wisniewski et al. 2010). Thus, the total mechanical mass loss of our models is not representative of the mass which is observed around Be stars.

To conclude this study of Be stars, let us recall the main points discussed in this chapter:

- On the basis of single stellar models, the Be star phenomenon is expected only at the end of the MS at solar metallicity.
- In the mass range considered, only initially very fast rotators are able to reach the critical limits ($\omega_{ini} > 0.8$). If the Be phenomenon occurs at a lower rotation rate than $\omega = 1$, this value of $\omega_{ini}$ could be lower.
- From $\sim 15$ $M_\odot$, the strong stellar winds prevent the star from reaching the critical velocity. We thus do not expect single Be (or Oe) stars with much higher mass, at solar metallicity. As the radiative mass loss rates are metallicity-dependent, this conclusion may not be valid at lower metallicity.
- The instantaneous and mean mechanical mass loss rates are dependent on the initial mass, but roughly independent of the initial rotation parameter. It is maximal for stars around 9 $M_\odot$, where the meridional circulation is efficient, and the radiative winds still not too large.
- We recall here that a precise description of the mechanical mass loss in the equatorial disc is not possible with the Geneva stellar evolution code. The instantaneous mass loss rates are thus to be considered with caution.

5.3 Critical tests of the numerical method

In this section, we would like to address the following (difficult) question: how reliable is the method developed during this PhD thesis? It is never simple to decide whether an algorithm provides good results or not. Below, we discuss three tests that have been conducted:
5.3. Critical tests of the numerical method

- A comparison between two relatively low mass star models, with a moderate initial rotation velocity. In this case, the radiative mass loss remains very small, and the effects of rotation are not as extreme as for models reaching very high rotation parameters. One model accounts for the modifications described in Section 3.2 on page 31, while the other not. We expect no, or very small, differences between those models.

- A comparison between two models of high mass star, with more important stellar winds. Some explainable differences are expected, in a quite predictable way.

- To check the robustness of the equatorial mechanical mass loss rates, and of the total amount lost in that way, we need some tests varying the time step during the critically rotating phase. These three points are discussed below.

5.3.1 A slowly rotating $7 \, M_\odot$ model

In order to verify the validity of the modifications brought to the Geneva stellar evolution code, we begin to perform a comparison between a set of two models of $7 \, M_\odot$ star, with an initial rotation parameter of $\Omega/\Omega_{\text{crit}} = 0.3$, at solar metallicity. The computation is done up to the end of the MS. One model is computed accounting for the modifications implemented to ensure the angular momentum conservation, while the other not. As the radiative winds for such a star remain modest during the MS, only very little angular momentum is removed from the star. Thus, we do not expect large differences between these two models.

As we can see in Figure 5.16 on next page, there are basically no differences between both models. The evolutive tracks in the HRD (top-left panel), the time evolution of the surface rotation parameter $\omega$ (top-right panel) and of the surface enrichment (bottom-right panel) are almost perfectly superposed.

In the bottom-left panel, we see the time evolution of the total angular momentum of the star plus the total angular momentum lost through the stellar winds in the past history of the star. We recall that if the model conserves angular momentum, this value should remain constant with time. We see that it is indeed the case for the model accounting for the last modifications of the Geneva code, but not for the model computed with the “old” 2006 version. In this model, the star keeps a too large amount of angular momentum at the end of the MS, compared to what we expect from the loss due to the stellar winds. This is due to the fact that in origin 2006, the loss of angular momentum was computed at the level of the first layer of the interior, and not at the stellar surface. Due to the smaller radius, the angular momentum loss was underestimate. Here, as the winds are very weak, the relative variation at the end of the MS compared with the initial value is very small: $\sim 6 \cdot 10^{-4}$.

However, because of the extreme weakness of the winds in this model, this has no implications for the stellar evolution; the differences do not appear in the other plots. This test reinforces our trust in the modifications implemented during this work (account for anisotropic winds and angular momentum conservation): they are effectively negligible when it is expected.

5.3.2 Expected differences for more massive stars

The next test concerns again two models of stars computed with the version 2006 and 2009 of the Geneva code. However, we choose this time to compute a more massive star, where the stellar winds are much stronger, and thus, where we expect to see some differences between the two models: two $25 \, M_\odot$ stars, at solar metallicity.

The same comparison as for the previous $7 \, M_\odot$ models is shown in Figure 5.17 on page 92. However, some quite large differences appear here. To begin, let us note that the “2006” model exhibits a very bad conservation of the angular momentum (bottom left panel), the variation during
Figure 5.15: Left panel: Mean mechanical mass loss rate in the disc for our Be models (see text), as a function of the initial mass. The models with an initial rotation parameter $\omega_{\text{ini}} = 0.8$ are represented by the blue curve, models with $\omega_{\text{ini}} = 0.9$ are in red, and models with $\omega_{\text{ini}} = 0.95$ are in green. Right panel: Histogram of the total mass loss during the MS for each initial mass and initial rotation parameter. The fraction lost mechanically at the equator is shown in red, and the fraction lost through radiative winds is shown in blue.

Figure 5.16: Comparison between two $7M_\odot$ models: one without accounting for the modifications implemented during this work (red solid curves) and the other accounting for them (blue dashed curves). Top-left panel: Evolutionary tracks in the HRD. Top-right panel: time evolution of the ratio $\Omega/\Omega_{\text{crit}} = \omega$. Bottom-left panel: time evolution of the quantity $L_{\text{star}} + \int \dot{L} \, dt$, where $L_{\text{star}}$ is the total amount of angular momentum in the star, and $\dot{L}$ is the rate of angular momentum removal by the stellar winds. Bottom-right panel: Surface enrichment of the ratio $^{14}\text{N}/^{12}\text{C}$ with respect to the initial ratio, as a function of time.
the MS of the total angular momentum of the system "star + wind" is 12%, several orders of magnitude larger than it was in the previous comparison. Accounting for the modification implemented in the "2009" version of the Geneva code, this quantity is now conserved. As an immediate implication, we see that the rotation parameter evolve towards lower values compared to the "2006" model (top-right panel). As the rotation is slower in the new version, the evolution in the HRD is also modified, being bluer and hotter. As the surface is more braked, the mixing is also modified (the stronger angular momentum gradient strengthens the mixing processes), and thus the surface abundance evolution is slightly different between both models.

For a star encountering strong mass loss during its evolution, the modifications brought in the 2009 version of the Geneva code produce some differences compared to the previous version. However, considering the bad conservation of the angular momentum in the old version, the differences between the two models presented here are expected, and follow the trend we can deduce from the slower rotating surface. This also confirms the validity of these modifications.

5.3.3 Effect of the time step on the equatorial mass loss rates

The last test we performed consists in varying the time step during the critical rotation phase, to check the consistency of both the total mechanical mass loss and the “instantaneous” mechanical mass loss rate. We use the $7\ M_\odot$ model with an initial rotation parameter $\omega = 0.95$ (see Section 5.2.4 on page 84) as the reference case (called hereafter the standard case).

We compute a set of 3 additional models, starting from the beginning of the critical rotation phase. One model has a time step corresponding to half the standard time step ($\Delta t_{\text{std}}$), one with $\Delta t_{\text{std}}/5$ and the last with $\Delta t_{\text{std}}/10$. All models were followed up to the end of the MS.

The results are illustrated in Figure 5.18 on page 94. On the left panel, we see the effect of the varying time step on the instantaneous mass loss rate. Note here that we did not plot all the points of our data in the figure, for readability considerations. We randomly chose 3000 points for each model. The time-axis range from the start of the critical rotation phase (on the left) to the end of the MS. We see that the distribution of the points seem to agree for all our models. To
probe the validity of this agreement, we computed the average and the standard deviation of the point cloud (of course, considering here all the points of the data). The results are not weighted by the time step, and are given in Table 5.3. The averages are in good agreement for all our models. There is a larger discrepancy in the standard deviation: models with short time step tend to have a larger dispersion than the standard model. We should keep in mind that the dispersion of the results presented in Section 5.2.4 on page 84 may be underestimated. For the smallest time step, the dispersion is almost 1 dex peak-to-peak. A simple statistical study as we did here is of course not a definitive conclusion. The comparison of the different point distributions would be treated with more refined statistical tools (e.g. Kolmogorov-Smirnov test).

Table 5.3: For each model with different time step, we give the average $\mu$ of the instantaneous mass loss rates, and its standard deviation $\sigma$. We also indicate the final mass of the model at the end of the MS (third column), and the total amount of mass lost mechanically in the equatorial plane (last column).

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$M_{\text{end MS}} [M_\odot]$</th>
<th>$\Delta M_{\text{mec}} [M_\odot]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard</td>
<td>−8.81</td>
<td>0.288</td>
<td>6.9743</td>
<td>0.0257</td>
</tr>
<tr>
<td>$\Delta t_{\text{std}}/2$</td>
<td>−8.84</td>
<td>0.365</td>
<td>6.9738</td>
<td>0.0262</td>
</tr>
<tr>
<td>$\Delta t_{\text{std}}/5$</td>
<td>−8.88</td>
<td>0.488</td>
<td>6.9732</td>
<td>0.0268</td>
</tr>
<tr>
<td>$\Delta t_{\text{std}}/10$</td>
<td>−8.83</td>
<td>0.416</td>
<td>6.9722</td>
<td>0.0278</td>
</tr>
</tbody>
</table>

The right panel of Figure 5.18 on next page shows the time evolution of the total stellar mass during the critical rotation phase. We note the following points:

- Models with small time step tends to lose more mass than the standard model. We thus probably underestimate the total mechanical mass loss for a few percents.

- However, the curves look very similar, and show the same general features.

- It is not clear how an even shorter time step than those we used here would affect these results.

- The discrepancy between the standard model and the model with the smallest time step is smaller than 8% (in terms of the total mass lost mechanically, see Table 5.3).

- The fact that the model with the shortest time steps lose more mass than the standard one seems in disagreement with the average mass loss rates presented in Table 5.3. Let us recall that the average has been computed without accounting for the time step. This indicates that our models provide reasonably uniform instantaneous mass loss rates comparing the different time step. The differences arise from the duration of the “quiescent” phase between two mass loss episodes, which are probably overestimated in the models with large time step.

To conclude, this test indicates that our models are indeed time-step dependent. However, the discrepancies are not so large. The values provided by our calculations are thus not firm quantities, however, the general agreement between our test models reinforce us about the ability of the new version of the Geneva code to provide at least good estimations of the instantaneous mass loss rates and of the total mechanical mass loss during critical rotation phases.

Some points remain nevertheless unclear:

- For which time step the results converges really? Note however that time steps still smaller than those we used here are too small to be used in our numerical simulations (as an example, the critical rotation phase of our standard model consists in 15279 stages, while the model
5.3. Critical tests of the numerical method

Figure 5.18: Left panel: Comparison of the logarithm of the instantaneous mechanical mass loss rates of our test models with various time steps, as a function of time. The corresponding colour code is indicated in the figure. Right panel: Time evolution of the total mass of our test models. The colours are the same than in the left panel.

- The distribution of the instantaneous mass loss rates (left panel of Figure 5.18) are not randomly distributed, but follow some kind of features. It would be interesting to understand why.

with the shortest time step needs more than 165450 stages). The computation time increases thus significantly.
Supernova and Gamma Ray Burst progenitors

In this chapter, we discuss the connexion between the type of a massive star at the end of its nuclear life (red- or blue-supergiant, Wolf-Rayet star), and the consequent core collapse supernova it will provide by exploding, in the frame of single star models. Binarity is probably involved in a fraction (which is probably significant) of all the SNe explosions (Podsiadlowski et al. 1992; Fryer et al. 2007; Eldridge et al. 2008; Yoon et al. 2010). However, the aim of this study is to determine whether the single star models are able to reproduce the observed trend or not.

This kind of study can put strong constraints on the stellar models. Since a couple of years, the number of observed SNe has considerably increased, allowing ever more accurate statistics about the relative number of each subtype of SNe. Several large surveys are now ongoing, for example, the Lick Observatoy SN Search (LOSS) (Leaman et al. 2010; Li et al. 2010b,a; Smith et al. 2010), or the Palomar Transient Factory (Arcavi et al. 2010), each with several tens of observed SNe. Other recent studies also link the SN event with the measured metallicity of the environment, or of the host galaxy, allowing the determination of the variation of the number of each type of SN as a function of the metallicity (Cappellaro et al. 1999; Prantzos & Boissier 2003; Prieto et al. 2008; Boissier & Prantzos 2009). This provides very good tests for stellar models, and this point is extensively discussed in this chapter.

After a short introduction to the classification of the Wolf-Rayet (WR) type stars, and on the SN classification, we present our results, which were published in the following paper (see on page 161):

THE DIFFERENT PROGENITORS OF TYPE IB, IC SNE, AND OF GRB
C. Georgy, G. Meynet, R, Walder, D. Folini & A. Maeder
2009 A&A 502, 611

6.1 The Wolf-Rayet stars

Wolf-Rayet stars (WR) were discovered at the end of the 19th century by Charles Wolf and Georges Rayet, who observed spectra with broad emission lines in three stars in Cygnus (Wolf & Rayet 1867). The comprehension of the origin of these emission spectra dates back to Beals (1929), who put forward the idea that the typical spectrum can be interpreted as a strong loss of matter originated from the star. It became rapidly clear that WR stars can be divided into two
6.1. The Wolf-Rayet stars

categories: the WN, which exhibits a spectrum dominated by He and N lines, and WC, with a spectrum showing strong He, C and O lines. With the increasing number of WR stars discovered, observers found out that the spectra of WR stars form a continuum from the O-type star spectra. The comprehension of the origin of the WR stars was progressively improved in the 60s and 70s, when the idea emerged that WR stars are O type stars which have lost their outer layers through strong stellar winds (Rublev 1965; Conti 1976).

6.1.1 The observational classification scheme for WR stars

The current classification scheme is based on the relative strength of the emission lines, and distinguishes between the following subtypes (see Crowther 2007, and references therein for more details).

- **WN subtypes**, based on the ratios of N\(_{\text{III-V}}\) and He\(_{\text{I-II}}\) lines. This class is divided into 10 subtypes, ranging from WN2 to WN11. The WN2 to WN5 stars are grouped under the generic name “early” WN stars (WNE), and WN7 to WN9 as “late” WN stars (WNL). WN6 are at the transition between the WNE and WNL types. WN10 and WN11 classes include the WR stars with spectrum very similar to the spectrum of O-type stars.

- **WC subtypes**, distinguished on the basis of line ratios (C\(_{\text{III}}\) and C\(_{\text{IV}}\)), and the appearance of the O\(_{\text{III-V}}\) lines. The subtypes range from WC4 to WC9, with WC4-WC6 the “early” (WCE) and WC7-WC9 the “late” types.

- **WO subtypes**, which is the subtypes for rare WR stars with strong O\(_{\text{VI}}\) lines. They range from WO1 to WO4, depending on the relative strength of the O\(_{\text{V-VI}}\) and C\(_{\text{IV}}\) lines.

- **WN/C subtype**. In the spectrum of some WN stars, we find unusually strong carbon lines. These stars are considered to be in an intermediate stage between WN and WC subtypes.

WR stars are rare objects. The last up-to-date catalogue of Galactic WR stars (van der Hucht 2001, 2006) includes 298 WR stars, among them we find 171 WN stars, 10 WN/WC stars, 113 WC stars and 4 WO stars. In the LMC, 134 WR stars are known (Breysacher et al. 1999), and 12 more are known in the SMC (Massey et al. 2003). In the Galactic neighbourhood, a few tens of WR stars are listed (see e.g. Bibby & Crowther 2010). Altogether, less than 500 WR are currently referenced. WO stars are the rarest objects in this category, with only 8 known representatives.

6.1.2 Classification scheme applicable to evolution codes

As the criterion to determine the WR star (sub-)type is based on the spectrum of the star, it is not easily applicable to stellar models (which have a simple treatment of the atmosphere, and do not allow the computation of the emitted spectrum). We have therefore to adapt the observational criteria to evolutionary criteria that can be applied to stellar models. The following criteria are chosen, based on the surface properties of the model (Smith & Maeder 1991; Meynet & Maeder 2005):

- A star with \(\log (T_{\text{eff}}) > 4\) and with a surface mass fraction of hydrogen \(X_S < 0.4\) is assumed to be a WR star. Otherwise, it is a standard O-type star (on the MS), or a blue- or red-supergiant.

- WR stars exhibiting hydrogen on their surface \((X_S > 10^{-5})\) are in the WNL phase.

- WR stars without hydrogen on their surface \((X_S < 10^{-5})\) and with a surface carbon abundance inferior to the nitrogen abundance are WNE stars.
• WR stars without hydrogen on their surface, with surface carbon abundance superior to the nitrogen one are in the WC or WO phase. To distinguish between both subtypes, we use the number ratio \( \frac{C}{He} \); if it is smaller than 1, we have a WC star, otherwise, we have a WO star.

These criteria allow an easy determination of the phase in which the star is, using only surface properties computed by the stellar code.

6.2 The supernova classification

Like the WR stars, supernovae are classified in various types and subtypes, according to their spectrum and light curve properties. The first distinction was definitely established by Minkowski (1941), between the type I and type II SNe. Type I SNe do not exhibit hydrogen lines in their spectrum, whereas type II SNe do.

![Figure 6.1: Typical SN spectra for the four main types. The main lines are indicated in the figure (from Filippenko 1997).](image)

Since that time, an increasing number of subtypes were proposed. The four main SN types are the following (see the review of Filippenko 1997):

• **Type I**: Its main characteristic is the fact that it does not exhibit hydrogen in its spectrum. There are three subtypes:
  - **Type Ia**: This subtype has a strong absorption near 615 nm, produced by blueshifted Si\(\text{II} \) lines.
  - **Type Ib**: This subtype exhibits He\(\text{I} \) lines.
  - **Type Ic**: This subtype has not this last He lines.

• **Type II**: This type groups the SNe with hydrogen in their spectrum.

Figure 6.1 shows typical spectra for these four SN types. In the current understanding of the mechanisms leading to the SN event, type Ia are thermonuclear supernovae, produced by the
6.3. The different progenitors of type Ib, Ic SNe, and of GRB

explosion of an accreting white dwarf, close to the Chandrasekhar mass (Hillebrandt & Niemeyer 2000). On the other hand, type II, Ib and Ic are produced by the collapse of the core of a massive star at the end of its nuclear lifetime. The type Ib and Ic SNe originate from stars which have expelled their external hydrogen-rich (or even helium-rich) layers, explaining the lack of hydrogen (or helium) in the spectrum. They are thus directly related to WR stars.

With the increasing number of observed SNe, this classification scheme has become more complex with the addition of a few new subtypes (Turatto 2003):

- **Type IIb:** This type has initially the spectrum of a type II SN, but progressively turns in type Ib. Type IIb SNe are the link between stars which have retained their external hydrogen-rich layers, and those which have lost it.

- **Type III and Type IIP:** for “linear” and “plateau”. This class gathers the bulk of the type II SNe. The distinction is made on the basis of the shape of the light curve: type III has a linearly decreasing light curve, while type IIP stops declining after the maximum, producing a plateau in the light curve.

- **Type IIn:** They exhibit narrow lines in their spectra, produced by the interaction between the ejecta and the circum-stellar medium.

6.3 The different progenitors of type Ib, Ic SNe, and of GRB

6.3.1 Physics of the models

In this work, we used stellar models previously computed (Meynet & Maeder 2003, 2005). The complete description of the physics used can be found in those papers. Let us briefly recall here the main aspects of these computations:

- We consider in this work rotating models. They are all calculated with an initial equatorial velocity $v_{eq} = 300 \text{ km s}^{-1}$ so as to ensure a mean equatorial velocity during the MS compatible with the measurements of Huang & Gies (2006) and Dufton et al. (2006).

- For non-WR stars, the mass loss rates of Vink et al. (2000, 2001) are applied. Outside the domain of validity of this prescription, the mass loss rates of de Jager et al. (1988) are used. For the WR phase, the mass loss rates of Nugis & Lamers (2000) are used (for a more complete description of these mass loss prescription, see Section 3.4.1 on page 48).

- The effect of rotation on the mass loss rate is taken into account (see Section 2.4 on page 21).

- During the non-WR phase, the mass loss rates are scaled with respect to the solar metallicity as $\dot{M}(Z) = \dot{M}(Z_{\odot}) \left( \frac{Z}{Z_{\odot}} \right)^{\frac{1}{2}}$ (Kudritzki & Puls 2000; Vink et al. 2001).

- Rotation is treated as described in Sections 2.2 on page 8 and 3.4.3 on page 59. The horizontal diffusion coefficient is given by Zahn (1992) (see relation (3.104) in this work), and the diffusion coefficient related to shear instability is given by Talon & Zahn (1997) (see relation(3.107) in this work).

Four metallicities were considered: $Z = 0.004$, $Z = 0.008$, $Z = 0.020$ (standard) and $Z = 0.040$. The different initial masses can be found in Table 6.1 on page 100. All the models were computed up to the end of the central He-burning. From this stage on, the evolution of the core and of the surface are decoupled, because of the very fast evolution of the core (governed by neutrino emission processes). We can thus assume that since that moment, the properties of the outer layer do not change until the end of the evolution. Moreover, the current mass of the star is very close to its real final value.
6.3.2 Nature of the supernova progenitor

In Table 6.1 on next page, we show the main characteristics of our models at the end of the He-burning phase, as a function of the initial mass $M_{\text{ini}}$ and the metallicity $Z$. For each model, we give the final mass $M_{\text{end, He}}$, the mass of the helium core $M_{\text{He}}$, the mass of the CO-core $M_{\text{CO}}$, the type of the SN progenitor (see below; if the star is a WR, the subtype is specified, otherwise, we mention only that it is a red- or blue supergiant by the label SG), the type of supernova (see Section 6.3.4 on page 102) and the type of the remnant, given by Hirschi et al. (2005) (see also Section 6.3.5 on page 106).

To determine the progenitor type, we use the classification scheme presented in Section 6.1.2 on page 96. Due to the stellar winds, a star may enter the WR phase as a WNL star. If the winds are strong enough, it may evolve through the different WR types: WNL $\rightarrow$ WNE $\rightarrow$ WC $\rightarrow$ WO. A star can end its evolution at any of these stages.

The next step is to find a way to determine, at each metallicity, what is the initial mass leading to a progenitor which is exactly at the edge of a WR star type. Let us illustrate our method by an example. Looking at Table 6.1 on next page, we see that, at a metallicity $Z = 0.020$, a star with an initial mass of $25 M_\odot$ ends its evolution as a WNL star. The next point in our grid is the $40 M_\odot$ model, which ends its life as a WC star. But we have no idea neither about the initial mass for which the transition occurs, nor whether there is an initial mass leading to a WNE star. The final structure of the $25 M_\odot$ model tells us that this star would lose still $0.35 M_\odot$ to become a WNE star. On the other hand, the evolution of the $40 M_\odot$ model tells us that it has lost $1.53 M_\odot$ since its entrance in the WNE phase. If we assign, by convention, a minus sign to this quantity (mass lost), and a positive sign to the former one (mass which has to be lost), we can plot it as a function of the initial mass. By connecting these two points, and looking for the point where this line cuts the horizontal axis, we can estimate the initial mass of a star which would enter in the WNE phase exactly at the end of its lifetime.

Proceeding in the same way for all the limits allows us to construct Figure 6.2 on page 101, which shows the type of the progenitor, as a function of the metallicity and the initial mass: the red area corresponds to the red- or blue-supergiants, the dark blue area on the left to the WNL stars, the intermediate blue one to the WNE stars, the light blue zone to the WC stars, while the dark blue island surrounded by the WC stars represents the area where we find WO stars. To complete this figure at zero metallicity, we used the models by Ekström et al. (2008a). The mass ranges for the SG and all the subtypes of WR star progenitors are also summarised in columns 2 to 6 of Table 6.4 on page 105.

Looking at this figure, we can do the following remarks:

- As expected from the metallicity dependence of the stellar winds and from the less efficient mixing at low metallicity, the lower mass limit to have a WR star progenitor increases with decreasing metallicity. This trend is much stronger at very low $Z$.

- The mass range of stars ending their life as a WN star is relatively narrow at all metallicities. Below this mass range, the stars do not succeed in entering the WR phase. Above it, the mass loss rates are efficient enough, and allows the star to evolve further into the WC or WO phase.

- WNE progenitors do not appear at low metallicity. Actually, WNE stars can be produced only when the star has evolved beyond the core H-burning phase: it is a necessary condition to have some regions of the star deprived hydrogen. At high metallicity, the strong mass loss rates allow to uncover the outer part of the H-burning shell in an early stage, when the He-burning core is still small, and has not yet transformed a large part of the former H-burning

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1 Defined as the mass interior to which the mass fraction of helium (or its fusion products) is higher than 0.75.
2 Defined in the same way as He-core, but with the abundances of C and O.
6.3. The different progenitors of type Ib, Ic SNe, and of GRB

used computed in the same way as in Hirschi et al. (2005), except for the homogeneous model (see text).

Note: The SN type is given for $\frac{M_{\text{ini}}}{M_{\odot}} = 0.6$ and supports that a SN occurs even when a BH is formed (see text). The remnant masses

<table>
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<th>Homogeneous model at $Z = 0.002$</th>
<th>$Z = 0.004$</th>
</tr>
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<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 6.1: Main characteristics of our models. First column indicates the initial mass $M_{\text{ini}}$, second one the mass of the core $M_{\text{He}}$ and of the CO core $M_{\text{CO}}$. The remnant estimated mass $M_{\text{rem}}$ figures in column

$M_{\text{rem}}$, $M_{\text{CO}}$, and $M_{\text{He}}$. All masses are given in $M_{\odot}$.
core into carbon and oxygen. The intermediate region between He-burning core and H-burning shell has thus a significant extension, increasing the probability that the star ends its lifetime as a WNE star (Meynet & Maeder 2005). At lower metallicity, the weak stellar winds uncover the H-burning core in a much later phase, when the He-burning core has grown in mass. The intermediate region between the He-burning core and the H-burning shell is small, and this reduces considerably the duration of the WNE phase, but also the possibility for a star to end its life as a WNE.

- The lower mass limit to produce a WC progenitor increases slightly with decreasing metallicity down to $Z \sim 0.01$. Below that metallicity, this limit increases much more rapidly.

- The WO progenitor are found only in an island at low metallicity. As already mentioned by Smith & Maeder (1991), this is due to the fact that, in order to be sufficiently rich in oxygen to produce a WO star, the He-burning core must be uncovered in a very late stage. This is more easily realised at low metallicity, because of the weaker stellar winds. We also note that WO progenitors arise neither from the most massive, nor the least massive stars that have their He-burning core uncovered. This reflects the delicate interplay between two counteracting effects: on the one hand, the star must lose enough mass to become a WO star, but on the other hand, the mass loss rates do not have to be too strong, so as to uncover the He-burning core in a late stage.

### 6.3.3 Chemical composition of the ejecta

As discussed in Section 6.2 on page 97, the determination of the SN type is dependent on the presence of hydrogen and helium in the emitted spectrum. It is thus important to determine, for our models, the chemical composition of the ejecta. Knowing the mass of the remnant (see Section 6.3.5 on page 106), we can integrate the chemical composition from the surface down to the
remnant edge for the most relevant chemical species, and therefore deduce the chemical composition of the ejecta. The results are presented in Table 6.2 on next page: we indicate the total released mass $M_{rel}$ (column 2), the ejected mass of He, C, N, O, and of heavy element (columns 3 to 8). We also indicate the corresponding fraction of the total ejected mass in parentheses. Heavy elements correspond to the sum of all the chemical species heavier than helium. To determine the mass of carbon and oxygen, we use the relation by Maeder (1992) between the mass of the CO core and the ejected mass of those elements.

The following trends are deduced:

- Supergiant progenitors usually eject more than 70% of their ejecta in the form of hydrogen and helium. At all metallicities, the proportion of the ejecta composed of hydrogen and helium is greater in lower initial mass stars.

- Note the special case of the $25 M_{\odot}$ model at $Z = 0.040$. The ejected quantity of hydrogen and helium is unusually small for a supergiant progenitor. Actually, this model is a blue supergiant ($\log (T_{\text{eff}}) = 4.34$) at the end of its nuclear lifetime, and is on the verge to become a WR star. This kind of model should probably explode as a type IIb SN, with a spectrum changing from a type II spectrum to a type Ib one, due to the very small amount of hydrogen present in the envelope of this star. It is interesting to mention the case of SN 2008ak, which is classified as a type Ibb SN, whose progenitor has an estimated mass of $25 - 30 M_{\odot}$ (Crockett et al. 2008; Pastorello et al. 2008), in agreement with this model of $25 M_{\odot}$ star.

- Our computed WNL progenitors eject less than 1 $M_{\odot}$ of hydrogen, but more than 1 $M_{\odot}$ of helium. They eject more carbon and oxygen than supergiant progenitors, and about 3/4 of the total ejected mass is in form of heavy elements.

- WC and WO progenitors eject more than 90% of their ejecta in the form of heavy elements. The mass of ejected helium is between 0.31 and 0.54 $M_{\odot}$.

- The chemical composition of the ejecta of WC and WO stars are identical.

- Eventually, the most important point is that the minimum helium content in the ejecta of a core collapse SN is about 0.3 $M_{\odot}$. In particular, type Ic SNe should contain some amount of helium. This conclusion is in line with results based on non-rotating models by Eldridge & Tout (2004), and with the binary models by Eldridge (private communication). To date, there is no clear measurement of the mass of helium ejected during a type Ic SN event (however see Elmhamdi et al. 2006).

### 6.3.4 The supernova type

As already pointed out in previous sections, the SN type is determined by the presence/absence of hydrogen or helium lines in the spectrum. For our models, we adopt a criterion based on the amount of hydrogen and helium in the computed ejecta. Let us call $m_{\text{H}}^{\text{min}}$ the minimum quantity of hydrogen that should be present in the ejecta for the spectrum of the SN to present H lines. In the same way, we define $m_{\text{He}}^{\text{min}}$ the minimum mass of helium needed in the ejecta for the spectrum to have He lines. We also write $m_{\text{H}}(M)$ the hydrogen mass ejected by a star with an initial mass $M$, and $m_{\text{He}}(M)$ the mass of helium released by the same star. In that case, the SN type produced by the star will be:

- a type II SN if $m_{\text{H}}(M) > m_{\text{H}}^{\text{min}}$;
- a type Ib SN if $m_{\text{H}}(M) < m_{\text{H}}^{\text{min}}$ and $m_{\text{He}}(M) > m_{\text{He}}^{\text{min}}$;
- a type Ic SN if $m_{\text{H}}(M) < m_{\text{H}}^{\text{min}}$ and $m_{\text{He}}(M) < m_{\text{He}}^{\text{min}}$.  

Table 6.2: Initial mass $M_{\text{ini}}$, total released mass $M_{\text{rel}}$, mass and fraction of ejected H ($M_{\text{H}}$), He ($M_{\text{He}}$), C ($M_{\text{C}}$), N ($M_{\text{N}}$), O ($M_{\text{O}}$) and of heavy elements ($M_{\text{heavy}}$), with all masses in $M_{\odot}$. In parentheses is indicated the corresponding fraction of the total mass of the ejecta.

<table>
<thead>
<tr>
<th>$M_{\text{ini}}$</th>
<th>$M_{\text{rel}}$</th>
<th>$M_{\text{H}}$</th>
<th>$M_{\text{He}}$</th>
<th>$M_{\text{C}}$</th>
<th>$M_{\text{N}}$</th>
<th>$M_{\text{O}}$</th>
<th>$M_{\text{heavy}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>10.4</td>
<td>5.52 (53%)</td>
<td>4.30 (41%)</td>
<td>0.15 (1.4%)</td>
<td>1.7 · 10^{-2} (0.16%)</td>
<td>8.9 · 10^{-2} (0.86%)</td>
<td>0.58 (5.6%)</td>
</tr>
<tr>
<td>15</td>
<td>12.4</td>
<td>6.11 (49%)</td>
<td>4.94 (40%)</td>
<td>0.25 (2.0%)</td>
<td>1.5 · 10^{-2} (0.12%)</td>
<td>0.42 (3.4%)</td>
<td>1.35 (11%)</td>
</tr>
<tr>
<td>20</td>
<td>16.0</td>
<td>6.57 (41%)</td>
<td>6.35 (40%)</td>
<td>0.49 (3.1%)</td>
<td>1.8 · 10^{-2} (0.11%)</td>
<td>1.41 (8.8%)</td>
<td>3.08 (19%)</td>
</tr>
<tr>
<td>25</td>
<td>17.5</td>
<td>5.48 (31%)</td>
<td>7.09 (41%)</td>
<td>0.88 (5.0%)</td>
<td>2.1 · 10^{-2} (0.12%)</td>
<td>2.64 (15%)</td>
<td>4.93 (28%)</td>
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<tr>
<td>30</td>
<td>16.0</td>
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<td>5.75 (36%)</td>
<td>1.45 (9.0%)</td>
<td>2.1 · 10^{-2} (0.13%)</td>
<td>5.07 (32%)</td>
<td>8.67 (54%)</td>
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<tr>
<td>40</td>
<td>18.8</td>
<td>0.45 (2.4%)</td>
<td>4.35 (23%)</td>
<td>2.11 (11%)</td>
<td>1.4 · 10^{-2} (0.07%)</td>
<td>9.93 (53%)</td>
<td>14.0 (75%)</td>
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<tr>
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<td>24.1</td>
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<td>0.41 (1.7%)</td>
<td>2.16 (9.0%)</td>
<td>1.4 · 10^{-6} (0.0%)</td>
<td>14.44 (60%)</td>
<td>23.69 (98%)</td>
</tr>
<tr>
<td>120</td>
<td>13.8</td>
<td>0.00 (0.0%)</td>
<td>0.48 (3.5%)</td>
<td>1.73 (13%)</td>
<td>0.00 (0.0%)</td>
<td>6.92 (50%)</td>
<td>13.32 (97%)</td>
</tr>
</tbody>
</table>

| $Z = 0.008$      |                  |                |                |                |                |                |                |
| 20               | 13.4             | 9.70 (73%)     | 3.55 (27%)     | 2.4 · 10^{-2} (0.18%) | 7.5 · 10^{-3} (0.06%) | 5.6 · 10^{-2} (0.42%) | 0.11 (0.8%) |
| 25               | 11.3             | 7.83 (69%)     | 3.40 (30%)     | 1.6 · 10^{-2} (0.14%) | 1.5 · 10^{-2} (0.14%) | 4.3 · 10^{-2} (0.38%) | 0.91 (0.8%) |
| 30               | 9.2              | 0.00 (0.0%)    | 0.54 (5.9%)    | 1.06 (12%)     | 1.8 · 10^{-3} (0.02%) | 3.30 (36%) | 8.66 (94%) |
| 40               | 13.9             | 0.00 (0.0%)    | 0.31 (2.2%)    | 1.75 (13%)     | 3.0 · 10^{-4} (0.0%) | 6.98 (50%) | 13.59 (98%) |
| 60               | 13.1             | 0.00 (0.0%)    | 0.49 (3.7%)    | 1.68 (13%)     | 0.00 (0.0%)     | 6.48 (49%) | 12.61 (96%) |
| 120              | 10.4             | 0.00 (0.0%)    | 0.44 (4.2%)    | 1.36 (13%)     | 0.00 (0.0%)     | 4.55 (44%) | 9.96 (96%) |

| $Z = 0.020$      |                  |                |                |                |                |                |                |
| 12               | 9.1              | 4.13 (45%)     | 4.20 (46%)     | 0.18 (2.0%)    | 5.4 · 10^{-2} (0.59%) | 0.18 (1.9%) | 0.77 (8.5%) |
| 15               | 8.5              | 2.53 (30%)     | 3.83 (45%)     | 0.32 (3.7%)    | 5.0 · 10^{-2} (0.59%) | 0.84 (9.9%) | 2.14 (25%) |
| 20               | 9.8              | 1.40 (14%)     | 3.51 (36%)     | 0.74 (7.6%)    | 4.7 · 10^{-2} (0.48%) | 2.22 (23%) | 4.89 (50%) |
| 25               | 8.8              | 3.78 · 10^{-3} (0.0%) | 2.04 (23%) | 1.13 (13%)    | 2.6 · 10^{-2} (0.29%) | 3.50 (40%) | 6.76 (77%) |
| 40               | 9.7              | 0.00 (0.0%)    | 0.53 (5.5%)    | 1.21 (12%)     | 3.8 · 10^{-6} (0.0%) | 3.77 (39%) | 9.17 (95%) |
| 60               | 11.0             | 0.00 (0.0%)    | 0.54 (4.9%)    | 1.43 (13%)     | 4.4 · 10^{-6} (0.0%) | 4.86 (44%) | 10.46 (95%) |
| 85               | 9.3              | 0.00 (0.0%)    | 0.53 (5.7%)    | 1.11 (12%)     | 3.7 · 10^{-6} (0.0%) | 3.43 (37%) | 8.77 (94%) |
| 120              | 8.5              | 0.00 (0.0%)    | 0.49 (5.8%)    | 0.96 (11%)     | 3.4 · 10^{-6} (0.0%) | 2.94 (35%) | 8.01 (94%) |

| $Z = 0.040$      |                  |                |                |                |                |                |                |
| 20               | 7.8              | 7.1 · 10^{-2} (0.91%) | 1.96 (25%) | 0.77 (9.9%)    | 4.62 · 10^{-2} (0.59%) | 2.29 (29%) | 5.77 (74%) |
| 25               | 7.9              | 0.00 (0.0%)    | 0.51 (6.5%)    | 0.65 (8.2%)    | 0.00 (0.0%)     | 1.88 (24%) | 7.39 (94%) |
| 40               | 8.6              | 0.00 (0.0%)    | 0.41 (8.3%)    | 1.00 (12%)     | 0.00 (0.0%)     | 3.07 (36%) | 7.89 (92%) |
| 85               | 5.0              | 0.00 (0.0%)    | 0.48 (9.6%)    | 0.33 (6.6%)    | 0.00 (0.0%)     | 0.89 (18%) | 4.52 (90%) |
| 120              | 4.9              | 0.00 (0.0%)    | 0.44 (9.0%)    | 0.33 (6.8%)    | 0.00 (0.0%)     | 0.90 (18%) | 4.46 (91%) |

Homogeneous model at $Z = 0.002$

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<th>$M_{\text{ini}}$</th>
<th>$M_{\text{rel}}$</th>
<th>$M_{\text{H}}$</th>
<th>$M_{\text{He}}$</th>
<th>$M_{\text{C}}$</th>
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<td>0.64 (2.7%)</td>
<td>2.33 (9.8%)</td>
<td>3.66 · 10^{-7} (0.0%)</td>
<td>15.56 (65%)</td>
<td>23.12 (97%)</td>
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(1) See note (1) in Table 6.1.
(2) Model computed up to the end of silicon burning.
Table 6.3: Mass limits between type II and type Ib SNe ($M_{\text{lim}}^{\text{II/Ib}}$), and between type Ib SNe and Ic SNe ($M_{\text{lim}}^{\text{Ib/Ic}}$) in M$_\odot$ (see text).

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<tr>
<td>0.008</td>
<td>30.0</td>
<td>29.9</td>
<td>29.7</td>
<td>35.7 - 49.1</td>
<td>31.6</td>
<td>30.0</td>
</tr>
<tr>
<td>0.020</td>
<td>25.1</td>
<td>24.2</td>
<td>23.1</td>
<td>-</td>
<td>110.2</td>
<td>39.1</td>
</tr>
<tr>
<td>0.040</td>
<td>20.5</td>
<td>20.5</td>
<td>20.5</td>
<td>-</td>
<td>26.2</td>
<td>24.7</td>
</tr>
</tbody>
</table>

The quantities $m_{\text{H}}^{\text{min}}$ and $m_{\text{He}}^{\text{min}}$ are a priori not known. If we choose $m_{\text{H}}^{\text{min}} = 0$ M$_\odot$, we obtain the limiting masses $M_{\text{lim}}^{\text{II/Ib}}$ indicated in the second column of Table 6.3. Stars with an initial mass below $M_{\text{lim}}^{\text{II/Ib}}$ will explode as a type II SN, and stars with an initial mass above $M_{\text{lim}}^{\text{II/Ib}}$ will end as a type Ib or Ic SN. The third and forth columns show the same result, but for values of $m_{\text{H}}^{\text{min}} = 0.25$ M$_\odot$ or 0.5 M$_\odot$. We see that adopting higher values for $m_{\text{H}}^{\text{min}}$ decreases $M_{\text{lim}}^{\text{II/Ib}}$, thereby increasing the range of mass leading to a type Ib SN and decreasing the mass range leading to type II SNe. Therefore, choosing $m_{\text{H}}^{\text{min}} = 0$ M$_\odot$ sets a lower limit for the number fraction of type Ibc to type II SNe. However, we see that, except at $Z = 0.004$, the changes remain modest. In the following, we choose $m_{\text{H}}^{\text{min}} = 0$ M$_\odot$ as our reference value, keeping in mind that we may underestimate the number fraction of type Ibc to type II SNe. Independently of the choice of $m_{\text{H}}^{\text{min}}$, we also note that, as expected due to the metallicity dependence of the stellar winds, $M_{\text{lim}}^{\text{II/Ib}}$ decreases as a function of $Z$.

The choice of $m_{\text{He}}^{\text{min}}$ is more delicate. Helium is a difficult element to observe, and a non-detection does not mean that this element is absent in the observed ejecta. How much helium could be hidden in a helium-free spectrum is still an opened question. As already mentioned, all the models still have some amount of helium in the ejecta. The minimum He content of the ejecta is about 0.31 M$_\odot$. Looking at the columns 5-7 of Table 6.3, we see that passing from $m_{\text{He}}^{\text{min}} = 0.4$ M$_\odot$ to $m_{\text{He}}^{\text{min}} = 0.5$ M$_\odot$ largely modifies the minimum initial mass $M_{\text{lim}}^{\text{Ib/Ic}}$ required to obtain a type Ic SN event. When one passes from $m_{\text{He}}^{\text{min}} = 0.5$ M$_\odot$ to 0.6 M$_\odot$, some smaller changes in $M_{\text{lim}}^{\text{Ib/Ic}}$ still appear. On the other hand, choosing a still higher $m_{\text{He}}^{\text{min}}$ would not change the results a lot. It thus appears that $m_{\text{He}}^{\text{min}} = 0.6$ M$_\odot$ is a kind of limiting value, because below that value, $M_{\text{lim}}^{\text{Ib/Ic}}$ is very sensitive to the exact value of $m_{\text{He}}^{\text{min}}$, while above that value, the results are no more sensitive to that choice. This brings us to the choice of $m_{\text{He}}^{\text{min}} = 0.6$ M$_\odot$ as our reference criterion to distinguish between the type Ib and type Ic SNe.

Choosing $m_{\text{H}}^{\text{min}}$ and $m_{\text{He}}^{\text{min}}$ enables us to determine the initial mass ranges leading to type II, type Ib and type Ic SNe for each initial mass and each metallicity. These values are summarised in Table 6.4 on next page (columns 7-9). Figure 6.3 shows the same data in the same kind of plot as Figure 6.2 on page 101. As for the previous figure, we used the models by Ekström et al. (2008a) for $Z = 0$. Comparing the mass ranges of the various progenitors and the mass ranges of the SN types shows that all the supergiant progenitors lead to a type II SN. The WNL stars usually produce also a type II SN, except for the most massive ones, which are on the verge to become a WNE star. WNE stars, which, according to our definition, contain no hydrogen, produce a type Ib SN, while they contain an important amount of helium. Among WC stars, the most massive ones end their life as a type Ic SN, while the least massive one still eject enough helium to explode as a typical Ib SN. All our WO star models lead to type Ic SNe, according to our criteria.
Table 6.4: Mass ranges for various supernova progenitors, supernova types, and compact remnants at different metallicities (in \( M_\odot \)).

<table>
<thead>
<tr>
<th>Z</th>
<th>SG</th>
<th>WNL</th>
<th>WNE</th>
<th>WC</th>
<th>WO</th>
<th>Type II</th>
<th>Type Ib</th>
<th>Type Ic</th>
<th>NS</th>
<th>BH</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>8 - 32</td>
<td>32 - 54</td>
<td>-</td>
<td>54 - 59; 114 - 120</td>
<td>59 - 114</td>
<td>8 - 54</td>
<td>54 - 59</td>
<td>59 - 120</td>
<td>8 - 29</td>
<td>29 - 120</td>
</tr>
<tr>
<td>0.008</td>
<td>8 - 25</td>
<td>25 - 30</td>
<td>-</td>
<td>30 - 41; 113 - 120</td>
<td>41 - 113</td>
<td>8 - 30</td>
<td>-</td>
<td>30 - 120</td>
<td>8 - 29</td>
<td>29 - 120</td>
</tr>
<tr>
<td>0.020</td>
<td>8 - 23</td>
<td>23 - 29</td>
<td>26 - 29</td>
<td>29 - 120</td>
<td>-</td>
<td>8 - 25</td>
<td>25 - 39</td>
<td>39 - 120</td>
<td>8 - 35</td>
<td>35 - 120</td>
</tr>
<tr>
<td>0.040</td>
<td>8 - 20</td>
<td>20 - 20.5</td>
<td>20.5 - 22</td>
<td>22 - 120</td>
<td>-</td>
<td>8 - 20.5</td>
<td>20.5 - 25</td>
<td>25 - 120</td>
<td>8 - 120</td>
<td>-</td>
</tr>
</tbody>
</table>

(1) At this metallicity, only the 40 \( M_\odot \) model produces a BH.

Note: The SN type is given for \( m_{\text{H}}^{\text{min}} = 0 \), \( m_{\text{He}}^{\text{min}} = 0.6 \) and assuming that an SN occurs even when a BH is formed (see text for more details).
6.3. The different progenitors of type Ib, Ic SNe, and of GRB

Figure 6.3: Supernova type as a function of the initial mass and metallicity. For each area, the SN type is indicated.

6.3.5 Remnant type

A still unanswered issue is whether a bright SN may occur when a black hole (BH) forms during the stellar core collapse. This question is to date a much debated issue, and there are arguments favouring both cases. Some authors claim that the formation of a BH by fall-back leads to a faint SN, and that direct BH formation even prevents the occurrence of an observable SN (Fryer 1999; Heger et al. 2003; Fryer et al. 2007). Another argument in that direction is the lack of observed massive WR progenitors for the known SNe, suggesting that these stars collapse without SN (Smartt et al. 2009). Rotation plays however a key role during the core collapse, and could change dramatically these conclusions. BH formation from fast rotating stellar core is an essential ingredient in the collapsar model for GRBs, which are accompanied by a SN, at least in some cases (Woosley 1993; Nomoto et al. 2005; Tominaga et al. 2007). Some observed BHs (in binary systems) have very unusual proper motions. This suggests that they underwent a kick during a “normal” SN explosion (Mirabel et al. 2001; Mirabel & Rodrigues 2003).

It is thus important to determine the status of the remnant of our models after the SN explosion. From the CO-core mass, we determined the remnant mass following the relation by Hirschi et al. (2005) (see Table 6.1 on page 100). To distinguish between a BH and a neutron star (NS), we assume that the maximal mass for a NS of $2.7 \, M_\odot$. This mass is in agreement with the maximal NS mass given in Shapiro & Teukolsky (1983). The recent discovery of a massive $2.1 \, M_\odot$ NS supports also this choice (Freire et al. 2008).

In Table 6.1 on page 100, we indicate the remnant type according to its mass. This allows us to determine the mass ranges where we expect a NS or a BH remnant (see columns 10 and 11). We see that the minimal initial mass required to produce a BH increases with the metallicity. Moreover, at twice-solar metallicity, the only model leading to the formation of a BH is the $40 \, M_\odot$ model, indicating that, from solar metallicity and above, we also have a maximal mass to form a BH! This has to be related to the very strong stellar winds encountered by the more massive stars, which
considerably reduce the mass of the star, and thus, the mass of the stellar core (for example, the 120 \( M_\odot \) model at \( Z = 0.040 \) ends its life with only 7.1 \( M_\odot \), which is lower than the final mass of the models with \( M_{\text{ini}} = 20 \)).

In Figure 6.4, we superimpose the region where a BH remnant is produced during the SN event over plots already presented in Figures 6.2 on page 101 and 6.3 on previous page. Up to a metallicity of \( Z \approx 0.005 \), all the WR stars should produce a BH. Above \( Z \approx 0.01 \), all the supergiants, and all the WNL and WNE stars end as a NS. In the same metallicity range, WC can finish their life either as a NS or as a BH. At low metallicity, all WC stars should lead to a BH. WO stars all produce a BH at the end of their life. Note that above twice-solar metallicity, no more BH formation is expected, and only NS remnant are produced.

Concerning the SNe, we see that below \( Z \approx 0.01 \), all the SN Ibc are in the BH area. This means that in cases where BH formation prevents the apparition of a bright SN, our models predict no observable type Ib nor type Ic SNe at low metallicity. Note that a few type Ic SNe with measured metallicity arise down to \( Z \approx 0.004 \), clearly in the BH zone of plots in Figure 6.4. Either the formation of a BH does not impede the occurrence of an observable SN, or another channel than the single star one (e.g. binary channel) is needed to explain those events. Above that metallicity, type Ib SNe progressively leave the BH zone. The type Ic SNe are out of this area only above \( Z \approx Z_\odot \).

Finally, taking a lower maximal mass for NS increases the size of the shaded area in these figures. If the creation of a BH at the end of the stellar lifetime prevents a “normal” SN event, we conclude that SNe of type Ib and Ic are expected only at quite high metallicity for single star models.

6.3.6 Ratio of supernova subtypes, and comparison with observations

In order to compute various ratios of supernova types, we have to weigh our results by an Initial Mass Function (IMF). In this work, we choose a Salpeter IMF (Salpeter 1955). The following
6.3. The different progenitors of type Ib, Ic SNe, and of GRB

Figure 6.5: For each metallicity considered in this work, the left column represents the ratio type Ib/type II SNe, and the right one the ratio type Ic/type II. The colour code of the columns indicates the progenitor type. To construct this plot, we used our reference cases for $m^\text{min}_H$ and $m^\text{min}_\text{He}$ (see Section 6.3.4).

relation is used (we give here the case of type Ib to type II SNe ratio, the other ones being similar):

$$\frac{\text{SN Ib}}{\text{SN II}} = \frac{\int_{M^\text{min,b}}^{M^\text{max,b}} M^{-2.35} dM}{\int_{M^\text{max,II}}^{8 M\odot} M^{-2.35} dM}, \quad (6.1)$$

where $M^\text{min,b}$ is the minimal initial mass leading to a type Ib SN, $M^\text{max,b}$ the maximal one, and $M^\text{max,II}$ the maximal initial mass leading to a type II SN. In a first place, we use the mass limits found in Table 6.4 on page 105, and we assume that a SN event occurs even if a BH is formed.

The results are shown in Figure 6.5, where we used our reference cases for $m^\text{min}_H$ and $m^\text{min}_\text{He}$. We see that at each metallicity, type Ic SNe are more frequent than type Ib (except at metallicity about solar, where their frequencies are comparable). WC and WO progenitors are the most usual progenitors for both types of SNe. WNL or WNE seem able to produce a type Ib or type Ic only marginally at solar and above-solar metallicity.

In Figure 6.6 on next page, we compare the ratio of type Ibc / type II SNe with some observations (Cappellaro et al. 1999; Prantzos & Boissier 2003; Smartt et al. 2009; Prieto et al. 2008; Boissier & Prantzos 2009) as well as with binary models by Eldridge et al. (2008) and Fryer et al. (2007). The grey areas indicate how our results change varying $m^\text{min}_H$ between 0 and 0.5 $M\odot$. Up to a metallicity around solar, our models fit well the observed data. For higher metallicity, we predict a too small number of type Ibc SNe compared to type II SNe. However, as the observations error bars are quite large, it is difficult to ascertain that this is a real deficit. Moreover, the most recent measurements (Boissier & Prantzos 2009, upside-down triangles) seem to be slightly below the measurements of previous studies. The increase of the ratio of type Ibc SNe to type II SNe directly comes from the metallicity dependence of the stellar winds. At high metallicity, they are so strong that it is easier to produce a star striped of its external hydrogen-reach layers, which eventually explodes as a type Ibc SN.

The green dots in the figure indicate the ratio we obtain using non-rotating stellar models. We see that rotation is a key ingredient for single star evolution, since it changes considerably
the computed ratio type Ibc / type II SNe. Not taking it into account leads to a worst fit, as the estimated ratio is largely below the observational ones.

It is interesting to see that binary models provide also a reasonable fit of the observed data. It is thus not possible to determine the relative importance of the single star versus binary channels on the base of the study of the metallicity variation of the type Ibc to type II SNe ratio. Actually, both channels probably have a significant importance, but it could be interesting to determine more precisely this importance, and whether it varies with the metallicity. It might be that both scenarios predict a different behaviour considering other ratios, as the metallicity dependence of the type Ib / type II ratio, or type Ic / type II ratio. Here, we computed those ratios in the frame of our single star models, and compare it with the observations.

To date, the only available observations of type Ib and type Ic SNe with metallicity measurements of the host galaxy are those by Prieto et al. (2008), who do not directly provide the needed ratios (however, they are extractable from their data), and Boissier & Prantzos (2009), who provide observational ratios of type Ib / type Ic SNe as a function of the metallicity. These measurements are compared to our predicted ratios in Figure 6.7 on next page. We see that at high metallicity, the number of type Ic SNe is about twice the number of type Ib SNe. The general trend from our single star models agrees well with the observed data from both Prieto et al. (2008) (left panel), and Boissier & Prantzos (2009) (right panel).

How do these results vary if we change our initial assumptions? In the left panel of Figure 6.8 on page 111, the same plot as in Figure 6.7 is reproduced, but with a minimum helium mass in the ejecta of a type Ib SN $m_{\text{He}}^{\text{min}} = 0.4 M_\odot$ instead of $0.6 M_\odot$, which was our reference case. This modification changes the results dramatically. Actually, only few models at $Z \sim 0.4 Z_\odot$ lead to a type Ic SN. Most of the type Ibc SNe are of type Ib at every metallicities, which is contrary to the observations. This illustrates the strong dependence of our models on the choice of $m_{\text{He}}^{\text{min}}$. If it is as low as $0.4 M_\odot$, then rotating single stellar models are unable to reproduce the observed data, while increasing it to $0.55 - 0.6 M_\odot$ provides a good fit.

The other major assumption we made is that all the core-collapse events produce an observable SN. If a BH formation during the collapse prevents the occurrence of a bright SN, our results are also strongly modified. As already mentioned (see Section 6.3.5 on page 106), at low metallicity,
most of the stars likely to produce a type Ibc SN indeed also produce a BH during the collapse. In this case, no type Ibc SNe are expected at low metallicity. This situation is illustrated in the right panel of Figure 6.8 on next page. At very low metallicity, the ratio of type Ibc to type II SNe is zero. From $Z \sim 0.4 Z_\odot$, the least massive progenitors of type Ib SNe begin to generate a NS instead of a BH, increasing that ratio. The virtual progenitor of type Ic SNe are however always producing a BH. Thus, up to solar metallicity, all the type Ibc SNe are of type Ib. Above solar metallicity, the situation tends towards the reference situation showed in Figure 6.7, because at twice solar metallicity and above, no more BH can be created, and all the remnants are NS. In that case, the single star models have also problems to reproduce the observations at low metallicity. For solar and super-solar metallicity, the situation is similar to the reference case discussed before.

Finally, we may wonder what happens if we choose a smaller maximal mass for NS. In that case, and if BH formation prevents the occurrence of an observable SN, the situation is even worse. The mass range where BH are expected (the shaded area in Figure 6.4) would be larger, and would thus prevent the apparition of type Ibc SNe up to a higher metallicity.

6.3.7 GRB progenitors

In Section 5.2.1 on page 76, we mentioned the main observational and theoretical characteristics that a well-behaved GRB progenitor should have according to the collapsar scenario. Among them, we noted that GRB seem to occur preferentially in low metallicity environments. Many physical reasons are evoked to explain this fact (Woosley & Bloom 2006):

- The weaker stellar winds at low metallicity, which allow the star to keep a higher angular momentum content. Moreover, the stellar masses at the end of the evolution are also higher, favouring the formation of BHs.
• The transport of angular momentum is less efficient in low metallicity stars, because of a weaker meridional circulation. The coupling between the core and the surface is weak, and the core can thus retain a larger amount of angular momentum.

• The chemical mixing is more efficient. This facilitates homogeneous evolution, which allows massive stars to produce type Ic SNe without losing too much mass.

• The distribution of initial velocity might favour faster rotation at low metallicity (Martayan et al. 2007).

The first interesting candidates to produce a GRB are of course the type Ic SNe, as GRB events are usually linked to such a SN. In Figure 6.9 on next page, we compare the observed ratio of type Ic SNe to the total number of core collapse SNe (CCSNe; triangles with error bars: Prieto et al. 2008) for various metallicities, with the shaded area showing the region where GRBs are expected. We see that type Ic SNe are not very good progenitor candidates for GRBs, for two reasons: first, even at low metallicity, where we find GRBs, the observed ratio of type Ic SNe to CCSNe is only marginally compatible with the GRB ratio, and second, we have to explain why more metallic type Ic SNe are not linked to a GRB event.

Other potential progenitor candidates are the WO stars, which have the advantage to be found only at low metallicity. The ratio of the type Ic SNe originating from a WO star to CCSNe is represented by the dashed curve in Figure 6.9 on next page. One can see that it is only marginally compatible with the observed ratio of GRBs to CCSNe, provided that the the collimating angle of the GRB jet is very small (around 1°). This conclusion was already found by Hirschi et al. (2005). So, even restraining the progenitors of GRB to WO stars still does not match the observations.

The key ingredient of the collapsar model is fast rotation in the core at the collapse time. We may wonder what happens to a low-Z, initially fast rotating star following a homogeneous evolution. Are there peculiarities in the chemical composition of the ejecta, allowing them to be distinguished by the SN spectrum? To answer this question, we computed a model of 60 M⊙ star at
6.3. The different progenitors of type Ib, Ic SNe, and of GRB

Figure 6.9: Observed ratio of type Ic SNe (Prieto et al. 2008, triangles with error bars) and theoretically predicted ratio of type Ic SNe whose progenitor is a WO star (dashed line) with respect to the total number of core collapse SNe (CCSNe). The grey rectangle represents the extension in metallicity (Modjaz et al. 2008) and ratio (Podsiadlowski et al. 2004) of GRB events.

$Z = 0.002$, with an initial rotation $\Omega/\Omega_{\text{crit}} = 0.75$. This model also includes the treatment of the magnetic field (see Section 2.2.4 on page 11). The model follows a quasi-homogeneous evolution during the MS, and contains enough angular momentum in the core at the collapse time to produce a GRB. Its main characteristics are presented in Tables 6.1 on page 100 and 6.2 on page 103. At the end of its evolution, this model is a WC star. The helium content in the ejecta ($0.64 M_\odot$) is slightly above the reference $m_{\text{He}}^{\text{min}}$ chosen in this work as the criterion to distinguish between type Ib and type Ic SN. It should thus explode as a type Ib SN. Let us note that the mass limit $m_{\text{He}}^{\text{min}}$ is not very firm, and that the explosion itself could be modified by the strong rotation of this model, which could change its spectral properties. It is thus not completely excluded that this model should lead to a type Ic SN.

The chemical composition of the ejecta of this model is very similar to the chemical composition of the equivalent non-homogeneous model (the $60 M_\odot$ at $Z = 0.004$). It is thus unlikely that we can observationally distinguish a model having followed an homogeneous evolution from a non-homogeneous model under the basis of the chemical composition of the ejecta at the explosion time.

6.3.8 Conclusions

In a previous study, Meynet & Maeder (2005) had shown that single star models can explain the increase of the ratio (SN Ib + SN Ic)/SN II as a function of the metallicity. In this work, with a more detailed method, we obtain several additional results.

- The first important point is that the single star models indicate that there is a minimum helium mass in the ejecta of a type Ic SN of around $0.3 M_\odot$. This result is also found with non-rotating models (Eldridge & Tout 2004). As helium is difficult to observe, there is to date no clear measurements of that quantity from the spectrum of type Ic SNe. Such a measurement may give a constraint on the mass limit $m_{\text{He}}^{\text{min}}$. 

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• Type Ib SNe are produced by the most massive WNL, WNE and the least massive WC stars. Type Ic SNe are produced only by WC or WO stars.

• Considering that the ejecta of type Ic SNe may contain up to $0.6 \, M_\odot$ of helium, and that the formation of a BH during the core-collapse does not prevent the occurrence of a bright SN, our single star models are able to reproduce simultaneously the cumulative ratio SN Ibc / SN II, and the peculiar ratio SN Ib / SN II, SN Ic / SN II and SN Ib / SN Ic. However, the observational measurements of those ratios are marred by large error bars. In the future, large SNe surveys should improve the accuracy of these results and provide strong constraints for the evolutionary models.

• If the formation of a BH prevents an observable SN, all type Ibc SNe at low metallicity should come from the binary channel. However, at solar metallicity and above, single star models can explain more than half of the total number of type Ibc SNe.

• Our models predict a too high GRB rate, even restraining the sample to type Ic SNe only, or WO stars progenitors only.

• On the basis of ejecta composition, it is not possible to distinguish a model having followed an homogeneous evolution from a similar non-homogeneous one.

Finally, we note that the study of the ratio (type Ib + type Ic) / type II SNe does not provide constraints on the relative importance of the single star channel and the binary channel. Looking at Figure 6.6 on page 109, we see that both binary models are also able to reproduce the observational trend. The separate ratio type Ib / type II or type Ic / type II, or the ratio of the subtypes type Ib / type Ic could provide stronger constraints. This study has to date never been performed for models accounting for the close interacting binary evolution.
6.3. The different progenitors of type Ib, Ic SNe, and of GRB
Chapter 7

Evolution near the $\mu^2 M$-limit

In this chapter, we discuss the implications of the mass loss rate recipe deduced in Section 3.3 on massive star evolution. After a short description of the models we computed, we present our results (mass loss rates, evolution of the rotation, WR phase, ...). Some consequences of this mass loss prescription on the chemical enrichment, the mechanical output in the interstellar medium are discussed. Finally, the implications on the fate of the star, particularly the possibility that it could explode as a GRB are presented.

This work has been submitted for publication in Astronomy & Astrophysics (see the current submitted version on page 193):

HOMOGENEOUS EVOLUTION NEAR THE $\mu^2 M$ LIMIT
C. Georgy, A. Maeder & G. Meynet
Submitted to A&A

7.1 Description of the models

In this work, we compare two models of $60 M_\odot$ at the metallicity of the Small Magellanic Cloud computed with the mass loss rate obtained with the $\mu^2 M$-limit (relation 3.64) and a model of the same mass and metallicity, computed with the standard mass loss rates found in the literature. We choose a high initial rotational velocity of $\omega = 0.9$ (which corresponds to an equatorial velocity of $690 \text{ km s}^{-1}$). This ensures an important mixing in the star, leading to a quasi-homogeneous evolution, required to fulfil the assumptions of the $\mu^2 M$-limit. All three models are computed up to the end of the core He-burning.

The initial chemical composition of the star is the following: $X = 0.747$, $Y = 0.251$ and $Z = 0.002$, and the distribution of the heavy elements is scaled as in the Sun (Asplund et al. 2005). For the CNO elements, we obtain the following initial abundances: $X \left(^{12}\text{C}\right) = 3.24 \cdot 10^{-4}$, $X \left(^{14}\text{N}\right) = 9.38 \cdot 10^{-5}$ and $X \left(^{16}\text{O}\right) = 8.13 \cdot 10^{-4}$.

These models take into account the treatment of the internal magnetic fields (see Section 2.2.4 on page 11). The induced strong coupling between the core and the surface produces a quasi solid-body rotation. The initial fast rotation of the models generates a very efficient mixing of the chemical species, by both the meridional circulation and the internal magnetic field. This makes the star to evolve quasi-homogeneously, allowing the use of the mass-loss recipe obtained from the $\mu^2 M$-limit.

We checked that our models remain nearly homogeneous during the evolution. On Figure 7.1 on next page, we show the most relevant chemical abundances profile (top panels) and the an-
7.1. Description of the models

Angular velocity profile (bottom panels) at two different stages of the star evolution (mid-MS, and beginning of the He-burning phase). We see that during the MS (left panels), the star is very close to being homogeneous, because of both internal mixing and a large convective core. During He-burning, some larger differences appear. Nevertheless, a great fraction of the star (almost 2/3) remains homogeneous.

Figure 7.1: Check of the validity of the homogeneity assumption for model C (see Table 7.1), for two different times (mid-MS, left panels, and beginning of helium-burning phase, right panels). Top panels: main chemical species abundance profiles (the centre of the star is on the left, the surface on the right). The concerned element is indicated along each curve. Bottom panels: angular velocity profile.

7.1.1 The mass loss recipe

The three models discussed here differ only by the mass loss prescription used to compute them. They are summarised in Table 7.1 on next page. Our reference model (model A) is computed with the standard mass loss rates. We use the prescription of Vink et al. (2000, 2001) on the MS as long as the star is an O type star. When it enters the WR phase, we use the Nugis & Lamers (2000) prescription scaled with metallicity (Eldridge & Vink 2006). These mass loss rate prescriptions are described in Section 3.4.1 on page 48. The criteria used to determine whether the star is a WR star are presented in Section 6.1.2 on page 96. Whatever the mass loss rate used for this model, the increase of the mass loss rate induced by rotation is accounted for (see Section 2.4 on page 21).

Models B and C are computed according to the theory developed in Section 3.3 on page 45. Model B assumes an Eddington constant $C_{\text{Edd}} = 44 M_\odot$, and model C a smaller value of $22 M_\odot$. Instead of computing directly the mass loss rate using relation (3.64), which would require the evaluation of the derivative of the mean molecular weight $\mu$, we rather compute the mean molecular weight through the whole star, which allows us to determine the maximum mass of the star with $M_{\text{max}} = \frac{C_{\text{Edd}}}{\mu}$. We then compare that value with the actual mass of the star $M$. If $M$ is larger than $M_{\text{max}}$, we remove the mass left over, and we evaluate the mass loss rate by dividing the amount of removed mass by the time step.
On the ZAMS, our models have a mean molecular weight $^{1} \mu \simeq 0.594$, leading to $\mu^2 M \simeq 21.16$. Both model B and model C are thus below their maximal mass. We let them evolve until they reach their respective maximal mass. Since that moment, we simultaneously evaluate the standard mass loss rate $\dot{M}_{\text{std}}$ and the mass loss rate related to the reaching of the $\mu^2 M$-limit $\dot{M}_{\text{Edd}}$. The effective mass loss rate of the star is then given by $\dot{M} = \max(\dot{M}_{\text{std}}, \dot{M}_{\text{Edd}})$.

7.2 Main results of the computations

7.2.1 Mass loss rates

The mass loss rates obtained for our models are presented in Figure 7.2 on next page. The three models show very similar features:

- a smoothly increasing part, starting from the ZAMS, corresponding to the standard mass loss prescription;
- a sharp jump (at $\sim 0.35$ Myr for model C, and at $\sim 3.6$ Myr for models A and B). This jump corresponds to the entrance into the WR phase for model A, and to the reaching of the maximal mass for models B and C;
- a second quick variation at the end of the MS (at $\sim 4.6$ Myr for model A, $\sim 4.7$ Myr for model B and at $\sim 5$ Myr for model C). During this phase, the standard mass loss rates are higher than $\dot{M}_{\text{Edd}}$ for models B and C. The increase of the mass loss rates are due to the quick evolution towards a higher luminosity and the redder part of the HRD (see Figure 7.4 on page 120).

Model C reaches the $\mu^2 M$-limit already during its O-type star phase. If such a star would be observable, it would exhibit the typical strong wind signature of WR stars, but with a different wind chemical composition, more typical of an O-type star: more hydrogen and less helium than a WR star.

Interestingly, models B and C show that once the star has reached its $\mu^2 M$-limit, its mass loss rate remains relatively constant over time. Writing the characteristic time scale of the mean molecular weight variation $\tau_\mu \equiv \frac{\mu}{\dot{\mu}}$, this constancy implies, with relation (3.64):

$$\frac{M}{\tau_\mu} \simeq \text{cst.} \quad (7.1)$$

As the stellar mass decreases when the evolution proceeds, it means that $\tau_\mu$ must thus decrease over time. This reflects the fact that the change of $\mu$ accelerates progressively, because the increase of the central temperature.

---

$^{1}$As the models evolve almost homogeneously, the average molecular weight is very close to the local mean molecular weight through the whole star. In the following, we note only $\mu$ instead of $\overline{\mu}$ for clarity reasons. However, the mass loss of our models are computed with the average value computed through the star.
7.2. Main results of the computations

Figure 7.2: Mass loss rates as a function of time (Model A: black solid line; model B: red dotted line; model C: blue dashed line).

The constancy of the $\mu^2M$-limit mass loss rates has another consequence. During the MS, the change of $\mu$ can be related to the change of the central hydrogen mass fraction $X$. Actually, we have:

$$\frac{1}{\mu} = \sum_i \frac{X_i}{A_i} (1 + E_i), \quad (7.2)$$

where $X_i$ is the mass fraction of element $i$, $A_i$ its atomic number, and $E_i$ the number of free electron provided by this element (depending on its ionisation level). Assuming that the medium is fully ionised, this relation becomes:

$$\frac{1}{\mu} = 2X + \frac{3Y}{4} + \sum_{i>2} \frac{X_i}{A_i} (1 + E_i), \quad (7.3)$$

with $X$ the hydrogen mass fraction and $Y$ the helium mass fraction. During the MS, only $X$ and $Y$ change (at least at the first order). Deriving the last relation with respect to time leads to:

$$-\frac{\dot{\mu}}{\mu^2} = 2\dot{X} + \frac{3\dot{Y}}{4}. \quad (7.4)$$

On the other hand, we must have $X + Y + Z = 1$ (where $Z$ is the mass fraction of all the elements heavier than helium). This implies immediately that $\dot{X} = -\dot{Y}$ on the MS. Introducing this result in relation (7.4), we obtain:

$$\frac{\dot{\mu}}{\mu} = -\frac{5}{4} \mu \dot{X}. \quad (7.5)$$

Thus, $\frac{\dot{\mu}}{\mu} \sim \mu \dot{X}$.

$\dot{X}$ is also related to the luminosity of the star through the relation $L = M \dot{X} \epsilon_{H\rightarrow He} c^2$, where we note $\epsilon_{H\rightarrow He}$ the efficiency of the burning of hydrogen into helium to produce energy ($\epsilon_{H\rightarrow He} \simeq 0.007$). Moreover, we assume here that the whole star contributes to the nuclear burning process (which is generally false, but in the homogeneous case, this approximation is roughly correct). We
have thus $\dot{X} \sim \frac{L}{M}$. Finally, introducing these results in the expression of the mass loss rates (3.64), we obtain:

$$\dot{M} \sim M \frac{\mu}{\mu} \sim M \mu \frac{L}{M} \sim \mu L.$$  \hfill (7.6)

During stellar evolution, $\mu$ increases because of the nuclear reactions in the stellar core. The constancy of $\dot{M}$ implies thus a diminution of the luminosity as a function of time, during the phase were the $\mu^2 M$-limit is reached (see Figure 7.4 on next page).

### 7.2.2 Total masses and surface velocities

Figure 7.3 (left panel) shows the time-evolution of the total mass of models A, B and C. For models B and C, we see that once the $\mu^2 M$-limit is reached, the strong and nearly constant mass loss rate encountered by these models produces a quick and linear decrease of the total mass.

Model C has a smaller mass loss rate than model B. However, as it enters very early in the $\mu^2 M$-limit phase, and remains at that limit during a large fraction of the MS and its whole He-burning phase, it ends its lifetime with the smallest mass of our models: $7.6 M_\odot$.

The strongest mass loss rate is encountered by model B, but the duration of the $\mu^2 M$-limit phase is much shorter than for model C. The final mass is higher: $14.6 M_\odot$.

![Figure 7.3: Same models as in Figure 7.2. Left panel: Total mass as a function of time. Right panel: Surface angular velocities as a function of time.](image)

Our reference model (model A) enters the WR phase at the same time model B enters the $\mu^2 M$-limit phase. During that time, it has generally a smaller mass loss rate. The final mass is thus still larger than model B, with $29.3 M_\odot$. The total mass re-injected in the interstellar medium is thus very dependent on the model: from around $30 M_\odot$ for model A to more than $50 M_\odot$ for model C. This could have important implications on the chemical enrichment of the interstellar medium.

The mass loss has a direct influence on the surface velocity, through the angular momentum removed by the stellar winds. The time evolution of the surface angular velocity $\Omega_S$ is presented in Figure 7.3 (right panel). The general trend is a decrease during the whole stellar lifetime. When the mass loss rates sharply increase (when entering the WR phase for model A, or reaching the $\mu^2 M$-limit for models B and C), the decrease of $\Omega_S$ becomes stronger. At the end of its lifetime, the stellar surface is almost non-rotating.
7.2. Main results of the computations

7.2.3 Evolution is the HRD and stellar lifetimes

The HR diagram for the three models is shown in Figure 7.4. The models begin on the ZAMS, and the evolution proceeds leftwards. Model C, which is very close to its $\mu^2 M$-limit, leaves the reference track very early. Model B reaches its own $\mu^2 M$-limit shortly before the star becomes a WR, and leaves the reference track.

Figure 7.4: Same models as in Figure 7.2. HRD with some key points of the evolution indicated: the position of the ZAMS, the beginning of the WR phase, the exhaustion of the hydrogen core (label $X_c = 0$), the ignition of the central He-burning (label He-b), and the point where the surface hydrogen abundance becomes zero (label $X_S = 0$). The effective temperature plotted here is the photospheric temperature.

Although we used different mass loss prescriptions for the computation of our three models, the tracks in the HRD show the same quantitative behaviour. The first striking point along the track of the reference model is the sharp change towards the blue side of the HRD. It is produced by the entrance in the WR phase. The strong increase of the mass loss rate during this phase allows for uncovering some hotter layers, increasing the effective temperature of the star. For models B and C, the same change occurs; it is however not produced by the entrance in the WR phase, but by the reaching of the $\mu^2 M$-limit. As models B and C lose more mass (see Figure 7.3 on previous page), they evolve at lower luminosity.

At central hydrogen exhaustion (label “$X_C = 0$”), the three models evolve temporarily redwards up to the ignition of the central helium burning. During this short phase, the star is out of thermal equilibrium. The core contracts, and produces a quick energy output. The thin envelope is not able to absorb this energy, which is consequently evacuated through an increase of the luminosity. The star finds again the thermal equilibrium at the ignition of the central He-burning. From that point, the luminosity continuously decreases, because of the decrease of the mass. The star contracts, producing an increase of the effective temperature.

The duration of the MS and of the He-burning phase are indicated in Table 7.2 on next page. The difference between models A and C are of the order of 10%. This is due to the difference in luminosity: model C is roughly one order of magnitude less luminous than the reference model.
Table 7.2: Time spent in the MS and He-burning sequence and total lifetime, and in the O-star, WNL, WNE, WC phases for the 3 models (times given in [Myr]).

<table>
<thead>
<tr>
<th>Model</th>
<th>MS lifetime</th>
<th>He-b lifetime</th>
<th>O-star</th>
<th>WNL</th>
<th>WNE</th>
<th>WC</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4.71</td>
<td>0.33</td>
<td>5.04</td>
<td>3.63</td>
<td>1.26</td>
<td>0.02</td>
</tr>
<tr>
<td>B</td>
<td>4.78</td>
<td>0.39</td>
<td>5.17</td>
<td>3.58</td>
<td>1.30</td>
<td>0.04</td>
</tr>
<tr>
<td>C</td>
<td>5.05</td>
<td>0.55</td>
<td>5.60</td>
<td>3.97</td>
<td>1.26</td>
<td>0.22</td>
</tr>
</tbody>
</table>

7.2.4 Surface abundances and WR phases

The time variation of the most relevant chemical species are plotted in Figure 7.5 (we represent here only the species involved in the hydrogen or helium burning). As the mass loss operates, layers displaying the result of hydrogen burning through the CNO cycle are progressively uncovered. The hydrogen abundance decreases, while the helium abundance increases simultaneously. The relative abundances of C, N and O are also modified, reaching the equilibrium value of the CNO cycle. N is produced, at the expense of C and O.

At the very end of the evolution, the products of helium burning appear at the surface. Helium decreases, while C and O abundances increase quickly. We note some differences in terms of He, C and O abundances at the end of the stellar lifetime. This is explained by the stage at which the helium-burning core is uncovered, because of the different mass loss history.

Using the criteria described in Section 6.1.2 on page 96, we can determine the type of star that we have at any times. Because of the strong mass loss, our models follow the evolution scheme O-star → WNL → WNE → WC, and end their life as WC stars. None of the three models finishes as a WO star. The duration of each phase is indicated in Table 7.2.
7.2. Main results of the computations

We note that the O-type star and WNE durations are significantly increased for model C. This model encounters a strong mass loss already during the MS. This reduces the mass and the luminosity of the star, increasing the MS and the O-type star duration. It has also another effect: it decreases the size of the helium-burning core. The part of the star which is depleted in hydrogen, but not enriched in He-burning products is thus larger, making the duration of the WNE phase longer. The other durations are similar in all three models.

7.2.5 The mass-luminosity diagram

In a mass-luminosity diagram (Figure 7.6), the three models show the same features (from right to left):

- first, an increase of the luminosity at relatively constant mass, because of the smaller mass loss rate when the star is an O-type star (for the reference model), or when the star is below its critical mass (for models B and C);
- then, an evolution along a line of constant slope;
- a peak, occurring at the helium core ignition.

Figure 7.6: Same models as in Figure 7.2. Mass-luminosity diagram, with the entrance in the WNL and WNE phases labelled along the tracks. The black dotted lines represent the position of stars at the Eddington luminosity (computed with the electron scattering opacity, or some multiples). The shaded area is the zone where we find WNE/WC stars.

We have overplotted in Figure 7.6 the position of stars which would be at the Eddington luminosity. To draw the line, we take the electron scattering opacity for an hydrogen-free star. As in the spectral lines, the opacity is higher, we also plot the lines corresponding to the Eddington limit with higher opacities (a few times the electron scattering opacity, the exact value being indicated along the curve). We see that the evolutionary tracks of our models lie in the region crossed by the Eddington limit corresponding to 2-5 times the electron scattering opacity. This close proximity of the Eddington limit should favour mass loss in such massive and luminous stars.

Many previous studies showed that WNE and WC stars follow a simple linear mass-luminosity relation (Maeder 1983; Maeder & Meynet 1987; Langer 1989; Schaerer & Maeder 1992). The shaded
area in Figure 7.6 indicates the region where our models are in the WNE or WC phase. A linear fit leads to:

$$\log \left( \frac{L}{L_\odot} \right) = 1.5360 \log \left( \frac{M}{M_\odot} \right) + 3.7574, \quad (7.7)$$

in good agreement with those previous works.

### 7.3 A few consequences of the $\mu^2M$-limit

In this section, we discuss the impact of the strong mass loss rates induced by the proximity of the $\mu^2M$-limit on the interstellar medium (chemical enrichment, energy output). We also examine the consequences of this strong mass loss on the fate of massive stars. Particularly, one question to be answered is whether those models are liable to end their life as a GRB?

#### 7.3.1 Chemical enrichment of the interstellar medium

In Table 7.3 on next page, we give for each model the total mass injected in the interstellar medium (i.e. the total mass lost during the whole stellar lifetime, without taking into account the matter released during the SN event). We also indicate the chemical composition of the ejected matter, particularly the masses of H, He, C, N and O (the mass and the corresponding fraction of the total ejected matter).

Our three models show mainly the products of hydrogen and helium burning: a poor hydrogen content, a large amount of helium and an enrichment in carbon and oxygen. Models A and B have very similar time-integrated wind compositions. Model C is quite different, with a much larger fraction of hydrogen, and less carbon and oxygen. Actually, this model encounters a strong mass loss very early during the MS, and loses an important fraction of its total mass loss during that phase, where the surface is still not enriched in helium-burning products.

We also note that our models do not produce primary nitrogen. Such a production indeed requires simultaneously a core helium burning with a shell hydrogen burning. Homogeneous evolution forbids such a configuration (even if our models are not perfectly homogeneous, the phase where both hydrogen and helium burning occur at the same time is too short to produce primary nitrogen). These stars are thus not an important source of primary nitrogen in the early universe, while this production seems necessary to explain the chemical evolution of galaxies (Chiappini et al. 2006).

#### 7.3.2 Mechanical energy outputs

We can define the power of the stellar winds, generally called the wind luminosity, as:

$$L_{\text{wind}} = \frac{1}{2} \dot{M} v_\infty^2, \quad (7.8)$$

where $\dot{M}$ is the stellar mass loss rate and $v_\infty$ is the terminal wind velocity, very far from the stellar surface. $v_\infty$ can be estimated using the relation given by Kudritzki & Puls (2000):

$$v_\infty = C(T_{\text{eff}}) v_{\text{esc}} \quad \text{with} \quad C(T_{\text{eff}}) = \begin{cases} 2.65 & T_{\text{eff}} \geq 21'000 \text{ K} \\ 1.40 & 10'000 \text{ K} < T_{\text{eff}} < 21'000 \text{ K} \\ 1.00 & T_{\text{eff}} \leq 10'000 \text{ K} \end{cases}, \quad (7.9)$$

with $v_{\text{esc}}$ the escape velocity at the stellar surface. Lamers & Cassinelli (1996) propose:

$$v_{\text{esc}} = \sqrt{\frac{2GM_{\text{eff}}}{R}}, \quad (7.10)$$
### Table 7.3: Total H, He, C, N and O masses ejected in the winds during the star lifetime.

<table>
<thead>
<tr>
<th>Model</th>
<th>Total mass loss</th>
<th>[%] M⊙</th>
<th>[%] M⊙</th>
<th>[%] M⊙</th>
<th>[%] M⊙</th>
<th>[%] M⊙</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.7 ± 1.0</td>
<td>0.0%</td>
<td>5.2 ± 1.0</td>
<td>0.1%</td>
<td>3.2 ± 0.1</td>
<td>0.1%</td>
</tr>
<tr>
<td>B</td>
<td>24.4 ± 2.3</td>
<td>27.0%</td>
<td>1.9 ± 1.3</td>
<td>3.7%</td>
<td>18.3 ± 2.1</td>
<td>5.2%</td>
</tr>
<tr>
<td>C</td>
<td>30.5 ± 1.7</td>
<td>4.9%</td>
<td>8.9 ± 1.4</td>
<td>1.0%</td>
<td>32.3 ± 2.0</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

Note: The values are given in units of solar masses (M⊙).
where \( R \) is the stellar radius and \( M_{\text{eff}} \) is the effective mass of the star, accounting for the effect of the radiation pressure (see equations 14-16 in Lamers & Cassinelli 1996). The effects of rotation are here neglected.

In the left panel of Figure 7.7, we show the time evolution of the wind luminosity for the three models. During the first phase of the evolution, when the star is in its “normal” mass loss rate phase, the wind luminosity is very low compared to the stellar luminosity: \( \log \left( \frac{L_{\text{wind}}}{L} \right) \sim -3 \). For models B and C, when the \( \mu^2 M \)-limit is reached, this ratio increases sharply up to around \(-1.6\). During the WR phase, the standard model reaches smaller values: the wind luminosity is roughly \( 1/100 \) of the star luminosity.

![Figure 7.7: Same models as in Figure 7.2. Time evolution of the wind luminosity (left panel), and time integration of the energy injected in the interstellar medium by the stellar winds (right panel).](image)

Integrating the wind luminosity over time, we obtain the mechanical energy released in the interstellar medium by the stellar winds:

\[
E_{\text{wind}}(t) = \int_0^t L_{\text{wind}}(\tau) d\tau.
\] (7.11)

The results are presented in the right panel of Figure 7.7. Models A and B are very close. The small difference is produced by the different wind luminosity during the WR phase (model A) and \( \mu^2 M \)-limit phase (model B). Model C enters the \( \mu^2 M \)-limit phase very early. During that period, it encounters a strong mass loss, and has a large wind luminosity, producing the quick increase in the mechanical energy deposition at around \( 0.4 \) Myr.

We also note that the final values of the mechanical energy released in the interstellar medium are roughly the same for all our models: model A releases \( 3.52 \cdot 10^{51} \) erg, model B \( 5.40 \cdot 10^{51} \) erg and model C \( 5.50 \cdot 10^{51} \) erg. These values are comparable with other studies (Freyer et al. 2003, 2006).

### 7.3.3 Evolution towards collapsar

As already mentioned before (see Chapters 5 and 6), fast rotating massive stars are good candidates for producing GRBs. We will not recall here the conditions assumed by the collapsar scenario. They can be found in Sections 5.2.1 on page 76 and 6.3.7 on page 110. Here, we discuss
7.3. A few consequences of the $\mu^2M$-limit

Table 7.4: For our three models presented in Table 7.1: CO-core mass ($M_{\text{CO}}$), remnant mass ($M_{\text{rem}}$), ejected masses of H ($m_{\text{H}}$) and He ($m_{\text{He}}$) during the SN event (all masses are given in $M_\odot$), and SN type.

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_{\text{CO}}$</th>
<th>$M_{\text{rem}}$</th>
<th>$m_{\text{H}}$</th>
<th>$m_{\text{He}}$</th>
<th>SN type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>27.13</td>
<td>5.28</td>
<td>0.0</td>
<td>1.04</td>
<td>Ib</td>
</tr>
<tr>
<td>B</td>
<td>13.74</td>
<td>5.04</td>
<td>0.0</td>
<td>0.57</td>
<td>Ic</td>
</tr>
<tr>
<td>C</td>
<td>6.26</td>
<td>3.84</td>
<td>0.0</td>
<td>0.67</td>
<td>Ib</td>
</tr>
</tbody>
</table>

whether stellar models encountering the $\mu^2M$-limit mass loss rate are able to fulfil these conditions or not.

The strong mixing inside the models leads to a very massive CO core at the end of the stellar lifetime. Using the same method as in Section 6.3.5 on page 106, we determined the mass of the remnant for the three models (see Table 7.4). For all the models, the remnant mass is large enough to allow the creation of a black hole.

Table 7.4 gives the amount of ejected hydrogen and helium during the SN event. Using the same criteria as in Section 6.3.4 on page 102, we have determined the SN type of each model. Model A leads clearly to a type Ib SN. Model B probably explodes as a type Ic SN. According to our criteria, Model C ends as a type Ib. However, it is very close to produce a type Ic SN as well.

Figure 7.8: Specific angular momentum profile of each model (black solid line), compared with the specific angular momentum of the last stable orbit around a Schwartzschild black hole (red dotted line) and a Kerr maximally rotating black hole (blue dashed line). Upper panel: model A; medium panel: model B; lower panel: model C.

The most critical quantity for the collapsar scenario is the internal specific angular momentum in the stellar core at the end of the lifetime. Here, the models were computed only up to the end of the helium-burning phase. However, the last nuclear burning phases are very short, much shorter than the characteristic time scale for the transport of angular momentum through advection and diffusion. We thus do not expect a huge modification of the specific angular momentum content between the end of the core He burning and the end of the nuclear lifetime. In Figure 7.8, we compare the specific angular momentum profile of our models with the specific angular momentum
of the last stable orbit around a Schwartzschild black hole and around a maximally rotating Kerr black hole. We see that our models have a too small specific angular momentum everywhere from the centre to the surface. They are thus unable to produce an accretion disc around the black hole during the core collapse. This is an insurmountable obstacle for the production of a GRB.

7.4 Conclusions

In this work, we show that models computed with a simple mass loss prescription based on the proximity of the $\mu^2M$-limit reproduce all the features of homogeneous models computed with standard mass loss prescriptions. We obtain the following main results:

- The mass loss driven by the $\mu^2M$-limit produces mass loss rates which are well in line with those obtained with the line driven wind theory. This physical process appears to be a reasonable mechanism for hot homogeneous stars (or, at least, a complement to the lines driven mass loss).

- If the $\mu^2M$-limit mass loss mechanism occurs already during the MS, we expect objects presenting the same characteristics of strong winds as WR stars, but with a chemical composition similar to O-type star winds.

- The models computed with the mass loss induced by the $\mu^2M$-limit lose significantly more mass than the standard model. This has important implications on the chemical enrichment of the interstellar medium.

- Models evolving near the $\mu^2M$-limit are also close to the $\Omega\Gamma$-limit.

- The angular momentum loss is larger when the $\mu^2M$-limit mass loss is accounted for. This restrains the domain of initial velocities required to produce a GRB at the end of the stellar evolution.

Of course, the simple treatment presented here does not intend to describe the complexity of the mass loss processes at the surface of stars near the $\mu^2M$-limit and the Eddington limit. However, it produces reasonable results, compared to what we know about WR stars.
7.4. Conclusions
Chapter 8

New stellar grids

A few years ago, the Geneva stellar evolution group has undertaken an important project: the computation of several grids of stellar models, at various metallicities, with and without rotation. A first set of models was computed between 2006 and 2008 (see Ekström 2008). However, the preliminary results of this work showed some problems, mainly the absence of blue loops for the stars around $9 \, M_\odot$, and a too small number of red supergiants at the metallicity of the SMC.

It was thus decided to compute the grids again, with a careful study of the rotation prescriptions, in order to fit as well as possible a number of observational facts. Among them, four are crucial tests for the setting of the free parameters of the code, and of the rotation prescription:

- the width of the main sequence,
- the N/C enrichment at the end of the MS,
- the number of blue- and red-supergiants,
- the presence of blue loops, allowing the existence of Cepheids.

The computation of the grids is still an on-going work. The results are to date very preliminary. We present them in this chapter, as well as the tests which were performed in order to set the rotation prescription and the free parameters.

8.1 Parametrisation and rotation prescription

The Geneva code allows for various rotation prescription and has some free parameters. Since the introduction of rotation in the code, the fit of the observed data was considerably improved (see the whole series of papers “Stellar evolution with rotation” and “Stellar evolution with rotation and magnetic field”\(^1\)). However, the treatment of rotation has considerably evolved during this period. The first version of the Geneva code with rotation was thus very different to the present one. We may wonder whether all the observational features, which were reproduced with different versions of the code, can be fitted with a unique set of parameters and prescription. Or, if this is not possible, what choice of such parameters allows the fit of the widest set of observations.

We briefly discuss here the results obtained with the best set of parameters we chose for the computation of the new stellar grids.

\(^1\)Meynet & Maeder (1997); Maeder (1997); Maeder & Zahn (1998); Maeder (1999); Meynet & Maeder (2000); Maeder & Meynet (2000, 2001); Meynet & Maeder (2002); Maeder (2002); Meynet & Maeder (2003, 2005); Hirschi et al. (2004, 2005); Maeder & Meynet (2003, 2004, 2005); Eggenberger et al. (2005).
8.1 Parametrisation and rotation prescription

8.1.1 Main sequence width

The calibration of the MS width was already done for the previous version of the grids (Ekström 2008), and allows the determination of the overshoot parameter $d_{\text{over}}$. This calibration was performed on low mass, weakly rotating stars, which are not much affected by rotation. We have not done this calibration again, as the main changes in the code only concern rotation. The parametrisation adopted is:

- $d_{\text{over}} = 0.1 H_P$ for stars more massive than $1.7 M_\odot$;
- $d_{\text{over}} = 0.05 H_P$ for stars between $1.25$ and $1.7 M_\odot$;
- $d_{\text{over}} = 0$ for stars less massive than $1.25 M_\odot$,

where $H_P$ is the pressure height scale.

Note also that the mixing length for convection (see Section 3.1 on page 27) was calibrated to fit the surface solar abundances. The value used in the code is $l = 1.6 H_P$. In Ekström (2008), the check of the position of the red giant branch theoretically obtained with the models compared to the observed one is also presented, and found consistent.

8.1.2 Rotation prescription and initial velocity chosen

After performing a consequent number of tests, varying the prescription for the shear mixing diffusion coefficient $D_{\text{shear}}$ and for the horizontal diffusion coefficient $D_{\text{h}}$ (see Section 3.4.3 on page 59). The best set of parameters appears to be:

- the $D_{\text{shear}}$ of (Talon & Zahn 1997) (note however that the critical Richardson number is taken as 1, and not its standard value of $1/4$ (see Maeder 2009), increasing the mixing),
- the $D_{\text{h}}$ of Zahn (1992).

As shown below, this set of rotation prescription allows the fit of various observational data.

We have also to determine the initial rotation velocity of our models. The idea is to choose a velocity representative of most of the rotating stars. Observationally, Huang & Gies (2006) obtain a peak at $200 \text{ km s}^{-1}$ in the velocity distribution for massive stars. Dufton et al. (2006) find a peak at $250 \text{ km s}^{-1}$, with a full width half maximum of $180 \text{ km s}^{-1}$ (both measurements at solar metallicity).

We determined that those typical velocities are well reproduced for models with an initial velocity set to $v_{\text{ini}}/v_{\text{crit,1}} = 0.4$. This ensures to obtain an average equatorial velocity of $\sim 200 \text{ km s}^{-1}$ during the whole MS.

8.1.3 C/N ratio at the end of the main sequence

Various observations seem to indicate that already during the MS, stars undergo mixing. The observed $^{14}\text{N}/^{12}\text{C}/^{14}\text{N}/^{12}\text{C}_\text{ini}$ range from $\sim 1.25$ to $\sim 28$ at solar metallicity (Gies & Lambert 1992; Lennon 1993; Przybilla et al. 2010). We have thus to check that our rotating models with the parametrisation chosen for the computation of the new stellar grids are able to reproduce such enrichment during the MS.

To check that point, we computed a set of $15 M_\odot$ models, with and without rotation, with the rotation prescription and initial rotation velocity discussed above. The left panel of Figure 8.1 on next page shows the HRD for those models. The effect of the metallicity is clearly visible, with models hotter and more luminous at low metallicity (mainly because of lower opacities). We see also the effect of rotation: the rotating models, initially at lower $T_{\text{eff}}$ and $L$, evolve towards higher
effective temperature and luminosity than the non-rotating ones, because of the increase of the convective core size produced by the rotational mixing.

On the right panel of the same figure, we present the ratio \( \frac{^{14}\text{N}/^{12}\text{C}}{^{14}\text{N}/^{12}\text{C}}_{\text{ini}} \) as a function of the central helium abundance \( Y_C \) for the same models. The ZAMS is located near \( Y_C = 0.23 \). First, evolution proceeds rightwards (the central helium abundance increases because of the hydrogen burning), up to \( Y_C \approx 1 \). Then, it proceeds leftwards, as the central helium abundance decreases during He-burning. Non-rotating models keep \( \frac{^{14}\text{N}/^{12}\text{C}}{^{14}\text{N}/^{12}\text{C}}_{\text{ini}} = 1 \) during the whole MS. During He-burning, as the models progressively cool down, a deep external convective zone appears, mixing the surface with regions enriched by H-burning products, which increases \( \frac{^{14}\text{N}/^{12}\text{C}}{^{14}\text{N}/^{12}\text{C}}_{\text{ini}} \). On the contrary, rotating models show already a mixing during the MS. The values of \( \frac{^{14}\text{N}/^{12}\text{C}}{^{14}\text{N}/^{12}\text{C}}_{\text{ini}} \) obtained at the end of the MS are indicated along the curves. We see that they are well in line with the observational values.

![Figure 8.1: 15 M☉ models at various metallicities, with and without rotation. For rotating models, the rotation prescriptions are those chosen for the computation of the new grids. The model corresponding to each curve is labelled in the figure. Left panel: HR diagram. Right panel: \( \frac{^{14}\text{N}/^{12}\text{C}}{^{14}\text{N}/^{12}\text{C}}_{\text{ini}} \) as a function of the central helium abundance \( Y_C \). The ZAMS is located at \( Y_C \sim 0.23 \), and the evolution proceeds first rightwards, up to \( Y_C = 1 \), and then leftwards up to \( Y_C = 0 \). The number along the rotating curves indicate the value of \( \frac{^{14}\text{N}/^{12}\text{C}}{^{14}\text{N}/^{12}\text{C}}_{\text{ini}} \) reached at the end of the MS.](image)

**8.1.4 Number of blue and red supergiants**

Observations seem to indicate that the ratio of the number of red to blue supergiant is larger at lower metallicities. Figure 8.2 on next page presents the behaviour of \( T_{\text{eff}} \) as a function of the central helium abundance for the same models as the previous section. We see that non-rotating models at low metallicity cross the HRD only at the end of the He-burning phase. We thus expect a very small number of red supergiants at low metallicity from non rotating stars, contrarily to what is observed! The minimum requirement for rotating models is thus to allow the existence of red supergiants at low metallicity.

Figure 8.2 indicates that the behaviour of the rotating models computed with the rotation prescriptions chosen for the grids is very different from the non-rotating ones. They cross the HRD...
very early after the central helium ignition, and allow the existence of red supergiants even at lower metallicities. Of course, we do not pretend here to reproduce the observed number ratio of blue- to red-supergiants, which would need a more complete and careful study of the models (and would need the use of complete stellar grids, which presently are still not computed). However, it shows that the presence of red supergiants at low $Z$ is possible with the chosen rotation prescriptions.

Figure 8.2: Same models as in Figure 8.1. $\log(T_{\text{eff}})$ as a function of the central helium abundance $Y_C$. The evolution proceeds from top to bottom.

8.1.5 Blue loops

The very fast crossing of the HRD at the beginning of the central helium-burning is not able to explain the observed number of Cepheid stars. These stars are expected during some peculiar event of the evolution of stars between $\sim 5 M_\odot$ and $10 M_\odot$, called blue loops. During those loops, the stars cross the Cepheid instability strip. As we see in Figure 8.3 on next page, a non-rotating $7 M_\odot$ model encounters a large blue loop. With the choice of the rotation prescriptions we made, we see that the rotating model exhibits also a blue loop, even if its extent is smaller than the one of the non-rotating model.

8.2 Preliminary results

Since the non-rotating grids published by Schaller et al. (1992), no other complete grids with uniform physical inputs and prescription was computed. The aim of this work is to fill this lack, by the computation of grids of stellar models at various metallicities, with and without rotation. The covered mass range spans from $0.8 M_\odot$ to $120 M_\odot$. To date, at least three metallicities are planed: solar metallicity and Magellanic Clouds metallicities. The physical inputs are those described in this work (see Sections 3.2.3 on page 39, 3.4 on page 48 and this chapter), and in Ekström (2008). We plan to perform the computations up to the end of central carbon burning for massive stars.
To date, the computation of the solar metallicity grids is running. Without rotation, all the models up to 25 $M_\odot$ are finished. The most massive ones are all in the He-burning phase. All the MS are thus computed. With rotation, all the models up to 15 $M_\odot$ are achieved. The stars between 20 $M_\odot$ and 40 $M_\odot$ have finished the MS, and are now in the He-burning phase. The most massive models are still on the MS.

To illustrate the state of these computations, Figures 8.4 on next page and 8.5 on page 135 show the HRD for the grids of non-rotating models (respectively rotating ones) at solar metallicity. The remaining computations are on-going, and the first complete results should be presented in a few months.
8.2. Preliminary results

Figure 8.4: Grid of non-rotating stellar models at solar metallicity.

Z = 0.014 without rotation
Figure 8.5: Grid of rotating stellar models at solar metallicity.

$Z = 0.014$

with rotation
8.2. Preliminary results
Other works

In this chapter, I shortly discuss a set of articles, to which I only marginally contributed, or which are not in the scope of this thesis. For each of them, I give a short summary of the main results. The interested reader will find the published article at the end of this work (see Appendix B on page 153).

9.1 Bulk composition of the transiting hot Neptune around GJ 436

The results of this work were published in the following paper (see on page 173):

**BULK COMPOSITION OF THE TRANSITING HOT NEPTUNE AROUND GJ 436**
P. Figueira, F. Pont, C. Mordasini, Y. Alibert, C. Georgy & W. Benz
2009 A&A 493, 671

Most of the exoplanets are found by the *radial velocity method* (see the review by Udry & Santos 2007). For a couple of years, another method, the *transit method*, becomes ever more used. Both of these methods are complementary. The first one allows the measurement of the planet mass, while the second leads to the planetary radius. The combination of these parameters makes possible the study of the internal composition of exoplanets.

In collaboration with the planet group of the Geneva observatory, I developed a planetary structure code. It uses very simple equations-of-state (EOS) for the four main ingredients assumed for the bulk composition of exoplanets in the present formation paradigm: iron-nickel, silicates, ices and H/He. For iron-nickel, silicates and ices, we use the EOS given by Fortney et al. (2007), and for the H/He layer, the EOS by Saumon et al. (1995) for an adiabat. We neglect the effects of temperature modifications on these EOS, since they are only marginally dependent on this parameter. Neglecting the equation of energy transport, the problem reduces to the simple case of the integration of the mass conservation equation, and of the hydrostatic equilibrium equation:

\[
\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^2 \rho} \tag{9.1}
\]

\[
\frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4} \tag{9.2}
\]

We first checked that this simple model provides reasonable results for the well-known objects of the Solar system.
We then applied this model to the case of planet GJ 436b, whose mass is estimated to 22.6 ± 1.9 M$_\oplus$ (Butler et al. 2004; Maness et al. 2007), and whose radius is measured to $4.19^{+0.21}_{-0.16}$ R$_\oplus$ (Gillon et al. 2007). Without additional constraint, the results are highly degenerated. However, a few cases can be excluded:

- the cases with more than 30% of H/He;
- the cases with more than 90% of iron/nickel;
- the case of an “ocean-planet”, i.e. without external layers of H/He, are also excluded.

However, it remains a wide variety of possible cases.

To bring a supplementary constraint, we coupled our structure model with the prediction of the formation model of the Bern group (Mordasini et al. 2009a,b), accounting for the effects of gas and solids accretion, the migration of the planet (type I and type II), and the progressive disappearance of the protoplanetary disc. Assuming a ratio 1:2 iron-nickel and silicates content, this model provides the mass and the composition of a large sample of synthetic planets. Considering only the cases in agreement with the measured mass of GJ 436b, we obtain the probability that this planet has a given composition.

![Figure 9.1: Possible composition of GJ 436b according to our structure models and the Bern model for planetary formation. Two possible metallicities for the host star are shown. The small rectangle (dotted line) represents the composition of Neptune.](image)

We finally obtained the following possible composition for GJ 436b (see Figure 9.1):

- 47 – 68% of iron-nickel and silicates (in the ratio 1:2);
- 17 – 40% of ices;
- 8 – 20% of H/He.

This composition is quite different from the composition Neptune (which has a similar mass and radius), with more silicates and less ice.

### 9.2 CNO enrichment by rotating AGB stars in globular cluster

The results of this work were published in the following paper (see on page 154):
In this paper, the chemical peculiarities observed in the Globular Clusters (GC) are studied within the frame of rotating models. Actually, giant and turnoff stars in GC show anomalies in their surface abundances of light elements (Li, C, N, O, Ba, Mg and Al). These anomalies are not observed at the surface of field stars. The chemical peculiarities are thought to be already present in the gas that formed the second generation of stars in the GC, and they originate from the dilution of hydrogen burning products ejected by a first generation of more massive stars with the pristine gas (see Prantzos et al. 2007).

Rotation induces an additional mixing in the star, strongly modifying the chemical structure of the star. During the helium-burning phase, some carbon produced in the core diffuses in the hydrogen-burning shell, leading to a net production of $^{14}$N through the CNO cycle. During the second dredge-up, the convective zone can dive deeply enough (for stars more massive than 4 $M_\odot$) to reach this region, producing an increase of the surface metallicity, and a strong enrichment of helium at the stellar surface. The strong enrichment of the surface in CNO-elements (see Figure 9.2) will affect the intra-cluster gas. If the second generation of stars would form from non-diluted gas from those first generation of stars, the difference in the CNO content between the first and second generation of stars would be at least 1.6 dex. If the gas is more diluted (to a factor of 10), it would still lead to differences of $\sim$ 0.6 dex. Such differences should be observable.

Nevertheless, it appears that the CNO content in GCs is constant, within the observational uncertainties. This discards massive rotating AGB stars as candidates to explain the chemical peculiarities of GCs.

**Figure 9.2:** Surface CNO enrichment after the second dredge-up ($\Delta [C + N + O] = [C + N + O] - [C + N + O]_{ini}$), for various metallicities (labelled along the curves). Models computed with the Geneva code (blue lines) and STAREVOL (red lines).
9.2. CNO enrichment by rotating AGB stars in globular cluster
Chapter 10

Conclusions and Perspectives

10.1 Improvement of the Geneva stellar evolution code

The quality of the models provided by the group to the astronomical community directly depends on the code we use. During this PhD thesis, the Geneva stellar evolution code has been considerably improved in several aspects. From a physical point of view, two points have mainly been modified:

- The coupling between rotation and mass loss process was modified, in order to be treated in a coherent way. The surface deformation and the wind anisotropies are properly accounted for. Eventually, even in case of “simple” computation without accounting for the anisotropic winds, the angular momentum conservation is now ensured for the system star+stellar winds.

- In link with the modifications produced by rotation on the stellar surface, I also modified the way the envelope of the star is integrating, allowing the account for the Solberg-Høiland criterion for convection. The effect of rotation on the radiative gradient is also included in this treatment.

Finally, from a purely programming point of view, and in close collaboration with Sylvia Ekström, a general rejuvenation of the Geneva code was performed:

- Adaptation of the Geneva code to the Linux operating system, ensuring its usage on more powerful machines, including the computing cluster recently installed at the Geneva Observatory;

- modernising of the writing of the numerous subroutines contained in the code;

- improvement and update of some parts of the code (automatic change of parameters, automatic change of the fitting mass, among others).

- correction of a few bugs.

10.2 Convection in rotating hot stars

After implementing some needed physics in the code, the effect of rotation on the convective envelope of massive, hot OB stars was studied. Rotation strongly modifies the structure of the external thin convective zones of such stars. The main effect is the modification of the radiative
10.3 Stellar evolution with anisotropic winds

The account for the Solberg-Høiland criterion plays only a minor role. In a star rotating close to the critical velocity, the extent in radius of the external convective zones is tripled at the equator. The mass content of these zones is also increased.

Sub-surface convective zones could also have an impact on the way the mass is lost by these stars. The large mass loss rates that they encounter produce probably a complex interaction between the stellar winds and the external convective zones. Actually, the star loses each year several times the mass contained in the convective zones. The convective motions might favour the escape of matter from the surface, contributing significantly to the vertical impulse, or generating acoustic waves amplified in the winds.

10.3 Stellar evolution with anisotropic winds

The modifications of the Geneva code, ensuring the conservation of the angular momentum, and allowing computations which take into account wind anisotropies, were applied to various cases. We first study the impact of anisotropic winds and equatorial mass loss during the critical rotation phase to a model which was initially thought to be a candidate for becoming a GRB progenitor. Eventually, the computed models did not fulfil the necessary conditions to produce a GRB in the frame of the collapsar scenario. It appeared that the account for anisotropic winds does not change significantly the final content in angular momentum of massive stars.

A systematic study of models of fast rotating Be stars has also been undertaken, with the computation of a grid of models between $3 \, M_\odot$ and $15 \, M_\odot$ at $Z_\odot$. For the first time, an estimation of the amount of mass lost mechanically in the equatorial disc during the critical rotation phase is given, as well as the mean equatorial mass loss rates during the same phase. If the Be phenomenon is produced by stars reaching the critical velocity, single star models show that it is expected only for initially fast rotating stars ($\Omega / \Omega_{\text{crit}}$,ini > 0.8), and in a mass range from $3 \, M_\odot$ to $12 \, M_\odot$. The strong stellar winds encountered by more massive stars prevent them to reach the critical velocity. The total mass lost in the equatorial disc is typically a few thousandths or a few hundredths of solar mass, and the mean mechanical mass loss rate ranges from $\sim 10^{-11} \, M_\odot \, \text{yr}^{-1}$ to $\sim 10^{-8} \, M_\odot \, \text{yr}^{-1}$. This result is below the estimated mass loss rates by the observations. However, it does not account for the alternation of burst and quiescent phases that observed Be stars undergo. Instantaneous mass loss rates are also deduced from the models. Note that the Geneva code does not provide a tool to obtain a precise determination of this instantaneous mass loss rates, and this result is only, in the best case, a rough estimation.

10.4 Supernova progenitors at various metallicity

During this thesis, a reanalyse of previously computed models of rotating massive stars was performed, to determine the existing link between a given type of progenitor and the subsequent SN type produced at the end of its lifetime. An important result is that the ejecta of all the models contain a minimal amount of helium, which does not appear in the spectrum of the SN. We set as a criterion that a maximal helium mass of $0.6 \, M_\odot$ is allowed in the ejecta of a type Ic SN and that a type Ibn SN must contain no hydrogen at all. With these criteria, we determined the SN type linked with each progenitor type obtained with our models. Type Ic SNe are found to originate only from WC or WO stars, while type Ib SNe comes from WNE, the most massive WNL or the less massive WC stars. Type II SNe mainly originate from supergiant progenitor.

Comparing these results with the observed ratios of type Ibn / type II, type Ib / type II or type Ic / type II shows that the models fit well the data. However, it was assumed here that all massive stars ($> 8 \, M_\odot$) end their life as a bright, observable SN, independently of the remnant left behind. If this very debated assumption is not realised, the model predictions at low metallicity show
discrepancies with the observed ratios of type Ibc to type II SNe, and another channel leading to this kind of event is needed, which could be binary stars.

10.5 Evolution near the $\mu^2 M$-limit

On the basis of considerations on the mass-luminosity relation and the Eddington luminosity, it is possible to determine a critical mass for homogeneous stars: the $\mu^2 M$-limit. Stars encountering this limit should lose mass according to a simple mass loss recipe, keeping the star close to this limit.

The computation of a set of models with this new mass loss prescription, and the comparison with a model computed with the standard mass loss recipe show amazing similarities. The tracks in the HRD and the mass-luminosity diagram for these models have the same features. Moreover, the homogeneous models evolve very close to the Eddington luminosity during their WR phase. The mass loss rates deduced with the new prescription are also very similar to the observed values from WR stars. This indicates that the $\mu^2 M$-limit should play an important role in the comprehension of the mass loss from WR stars.

According to the chosen value of the parameter $C_{Edd}$ entering in this prescription, the chemical enrichment of the interstellar medium by the stellar winds can be relatively different compared to the chemical enrichment produced by the standard model. The strong mass loss encountered during the $\mu^2 M$-limit phase quickly spins down the star. This can affect the fate of the star at the time of the collapse, preventing it to trigger a GRB.

10.6 Future perspectives

10.6.1 Grids of stellar models

The computation of new stellar grids is a priority project within the Geneva stellar evolution group. To date, a solar metallicity grid is in its way of achievement. However, the remaining models are also the most difficult to compute, and some difficulties have to be solved in the next weeks to finish this first grid. The computation of a SMC metallicity grid is scheduled immediately after the solar metallicity one, and a LMC one at a longer term.

In the future, we also look to provide, for a subset of those grids, a sample of models with various initial rotation velocities. Sub-grids for Be stars will also be computed (the solar metallicity one being largely undertaken, thanks to Anahí Granada). The inclusion of the effects of internal magnetic fields is also planed.

10.6.2 Effect of wind anisotropies on the circumstellar medium

During this thesis, we have shown that the anisotropic winds have only a minor impact on the evolution of fast rotating massive stars. However, they should affect in a peculiar way the circumstellar medium. To explore this direction, we began, in collaboration with Rolf Walder (ENS Lyon) the hydrodynamical study of the interaction of the strongly anisotropic winds of the $20 M_\odot$ star discussed in Section 5.2.3 on page 79 with the A-Maze code. We undertook in parallel the computation of a similar model without rotation, so as to provide a comparison and highlight the effects of anisotropic winds.

This kind of simulations need very long time to compute, so currently they are still running. One simulation is performed in three dimensions, providing better precision and follow-up of the turbulence. However, the complexity and the CPU requirements of this simulation do not allow for following the stellar evolution during its whole lifetime. We thus also perform the same
10.6. Future perspectives

![Figure 10.1: Effect of a strongly anisotropic stellar wind on the circumstellar medium. The rotation axis of the star is along the x-axis. The extension of the cavity along this axis is ∼ 3 pc. Left panel: density of the matter surrounding the star after 0.5 Myr. Right panel: same as left panel, but at t = 2 Myr. The extension of the cavity along this axis is ∼ 10 pc.](image)

Simulation in two dimensions, which is less time-consuming. To date, roughly 1/4 of the total stellar lifetime is already computed.

Preliminary results are presented in Figure 10.1. The anisotropic winds progressively dig a cavity in the circumstellar environment. After 0.5 Myr, the cavity is still more or less spherical. As the rotation parameter ω of the star progressively increases during the MS, the winds become ever more polar. The cavity around the central star becomes clearly aspherical, with a much longer extension along the rotation axis of the star.

One question we would like to address with this study is whether the anisotropic winds can leave an imprint in the circumstellar medium that could be observable, either directly, or by modifying the properties of the SN explosion.

10.6.3 Pre-supernova simulations

3-dimensional hydrodynamical simulations offer also a powerful tool to study in details the final stages of massive stars evolution. The Geneva code can provide some detailed structures of the star at the end of its nuclear life, including the effects rotation. These structures could be used as input data for 3D hydrodynamical simulations of the core collapse.

With Rolf Walder, we plan to perform such simulations, which are to date never computed with refined final stellar structures, but with simplified polytropic ones, and without accounting for the effects of rotation. In a first time, such simulations are realisable with the present structures. However, some peculiar points of the physics of the Geneva code should be improved to allow an ever more realistic computation of the final stages.

First, a modification of the equation-of-state. We currently use an equation-of-state based on the idealised case of perfect gases. If this assumption is good for most of the stellar evolution, the high densities reached during the final burning phases and the pre-collapse would need an improved equation-of-state. Moreover, the use of a common equation-of-state between the stellar evolution code and the hydrodynamical code would considerably facilitate the mapping of the stellar structure on the hydrodynamical code mesh, avoiding some conversion difficulties. Gilles Chabrier (ENS Lyon) is currently working on such an equation-of-state, and it is planed to implement it in the Geneva code.

Second, the Geneva code is presently built on the hypothesis of hydrostatic equilibrium. This assumption is acceptable for most of the phases of stellar evolution. However, there are some
periods during which this condition is not fulfilled, particularly during the HRD crossing (when
the star evolves quickly from the blue to the red side of the HRD), and during the last phases
of nuclear burning. It thus intended to add the acceleration term in the structure equations of
the Geneva code, which is rather a long-term work because of the complexity of the task. The
knowledge of the velocity field and of the acceleration of the internal layers of the stellar core
would still improve the quality of the final stellar structures.

The hydrodynamical study of the core collapse gives some important indications on the rem-
nant type left behind the star after it explodes, and its properties. The formation of the central
neutron star or black hole is still not well understood. The link between the angular momentum
contained in the stellar core and that retained in the remnant is also to date not clear.

On a longer term, these simulations could be extended, to include the bounce of the exter-
nal layers falling on the proto-remnant. This is a more difficult problem, as it should include some
complex neutrino physics which intervene in this process. It makes no doubt that rotation is an im-
portant ingredient during the SN explosion. Starting hydrodynamical simulations with accurate
stellar rotating structures will allow to launch the computations with the best initial conditions.
10.6. Future perspectives
Numerical aspects

In this appendix, we give more technical informations about the modifications implemented during this thesis work. For more details, the interested reader should refer to the documentation of the code itself.

A.1 Anisotropic correction

Routine modified

• aniso

List of modifications

Modifications in aniso:

• From the 2009 version of origin, the code goes through aniso whatever the value of ianiso.

• The anisotropic angular momentum loss is saved in the variable bdotan, the isotropic one in bdotis.

• The anisotropic correction $\Delta L_{\text{ani}} / \Delta L_{\text{iso}} - 1$ is saved in xlexces.

• The equatorial radius accounting for the stellar deformation is saved in raequat. It is used in xldote to compute the mechanical equatorial mass loss.

• In case of no- (or very slow) rotation, the same quantities are computed in the spherical case.

A.2 Estimated $\Omega_{\text{new}}$ and mechanical mass loss

Routines modified

• main

• xldote, which were completely rewritten.
A.3 Internal angular momentum conservation during advection/diffusion

List of modifications

Modification in main:

- The maximal allowed ratio of the angular velocity to the critical one $R\Omega_{\text{max}}$ is added in the parameter card under the name RAPCRILIM. If its value is 0, the modifications added to ensure the angular momentum conservation are not active, and the code runs in its origin2006 version.

Modifications in xldote:

- Computation of the variable $d\ell_{\text{lex}}$, which is the angular momentum lost during the time-step. If $\text{ianiso} = 1$, $d\ell_{\text{lex}} = \Delta L_{\text{ani}}$, otherwise, $d\ell_{\text{lex}} = \Delta L_{\text{iso}}$.
- For the envelope and each shell between the surface and $i = N_{\text{corr}}$, computation of the angular momentum.
- Computation of the correction factor $q_{\text{corr}}$, saved in the variable $q_{\text{corr}}$.
- Computation of the estimated new angular velocity of the surface according to (3.36).
- If that velocity is larger than RAPCRILIM, the equatorial mass loss is computed according to (3.43).
- The equatorial angular momentum loss is computed.
- The time derivative of the total angular momentum loss is computed. It is used in $\text{coedif}$ to compute the boundary conditions of the advection (see Maeder 2002).

A.3 Internal angular momentum conservation during advection/diffusion

Routines modified

- omenew
- advect

List of modifications

Modifications in omenew:

- In omenew, which is called in henyey to estimate the new angular velocity profile only based on local conservation of angular momentum, a save of the total internal angular momentum of the star before all the mixing processes is done. The variable is $x_{\text{lstarbefHen}}$.

Modifications in advect:

- After the diffusion (or advection) computation, $x_{\text{lstarbefHen}}$ is used to compute the corrective factor $C_{\text{tran}}$ (3.45).

A.4 Correction of the rotation profile

Routines modified

- main
- omescale, which is a new routine.
List of modifications

Modifications in main:

• The initial angular momentum of the previous model is read from binary in variable \texttt{dlelexprev}.
• The rotation profile correction $\Delta \Omega_{\text{corr}}$ is read from binary in variable \texttt{CorrOmega}.
• If the current model is a model with diffusion, the correction is applied to the rotation profile.
• The total angular momentum loss of the current model is computed in variable \texttt{dlelex}. If \texttt{dlelexprev} exists, it is add to \texttt{dlelex}.
• The new routine \texttt{omescale} is called once the current model has converged, to compute the correction $\Delta \Omega_{\text{corr}}$.
• The correction, and eventually the initial angular momentum of the model are stored in the binary file.

Modifications in \texttt{omescale}:

• From the angular momentum and the momentum of inertia of each shell, the angular velocity is computed.
• The rotation profile is homogenised in the convective zones.
• The factor $q_{\text{corr}}$ is computed according to (3.51).
• The correction $\Delta \Omega_{\text{corr}}$ is computed and stored in \texttt{CorrOmega}, for the $N_{\text{corr}}$ layers where the correction will be applied, or in the whole external convective zone if it has more than 100 layers width.
• The corrected rotation profile is homogenised in case of convective zones.

A.5 Opacity table choice

Routines modified

• \texttt{main}
• \texttt{readco}, located in the \texttt{xztrin_af2009} file

List of modifications

Modifications in \texttt{main}:

• A new parameter \texttt{IOPAC} is added in the parameter card. Its value can be:
  1 for the opacities corresponding to the solar values of Grevesse & Noels (1993);
  2 for the opacities corresponding to the $\alpha$-enhanced values of Grevesse & Noels (1993);
  3 for the opacities corresponding to the solar values of Asplund et al. (2005) and Cunha et al. (2006);
  4 for the opacities corresponding to the $\alpha$-enhanced values of Asplund et al. (2005) and Cunha et al. (2006).

Modifications in \texttt{readco};

• The reading of the corresponding file is done according to the value of \texttt{IOPAX}. 

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A.6 Rotation prescriptions

Routines modified

- main
- coedif

List of modifications

Modifications in main:

- A new parameter ICOEFF is added in the parameter card. Its values are summarised in Table A.1.

Table A.1: Values of ICOEFF corresponding to each rotation prescription implemented in the Geneva evolution code.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Zahn (1992)</td>
<td>11</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Maeder (2003)</td>
<td>12</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>Mathis et al. (2004)</td>
<td>13</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

Modifications in coedif:

- According to the value of ICOEFF, the choice of Dh and Dshear is made. The prescription rotation chosen is written in the .l file.
- Each time Dh or Dshear is computed, the right one is automatically computed.

A.7 FITM shifting

Routines modified

- main
- fitmshift, which is a new routine.

List of modifications

Modifications in main:

- Instead of the simple renumbering which was applied in origin 2006 when FITM was changed, the routine fitmshift is now called.

Modifications in fitmshift:

- Computation of the total angular momentum contained is the shells which will be removed.
- Shifting of the numbering.
- Computation of the new angular velocity of the first shell and of the envelope, conserving the initial angular momentum.
- The angular velocity is homogenised in the convective layers.
A.8 Automatic FITM change

Routine modified

• main

List of modifications

Modifications in main:

• A new parameter is added in the parameter card: IFITM. According to its value, the behaviour will be the following:

  0 IFITM is not automatically modified. Similar to the previous versions of the code.
  1 The fitting mass follows the evolution of the external convective zone, and is set at 3/4 of the extensions of this zone.
  2 the fitting mass depth is computed as a mix between the extension of the external convective zone, and the depth of the fully ionised medium.
  3 “hand-like” method, which mimics what was done before by hand, and applied at each time-step.
  4 “hand-like” method, which mimics what was done before by hand, and applied once by sequence of computation.

• At the end of a model computation, some tests on the variation of FITM during the time-step, and on the variation of the effective temperature, are done.

• If the required conditions are fulfilled, a new FITM is computed for the next time-step. The way it is done is directed by the parameter IFITM.

A.9 Automatic parameter change

Routine modified

• main

• dreck

List of modifications

Modifications in main:

• A new parameter is added in the parameter card: IAUTO. According to its value, the behaviour is those given in Section 3.4.4 on page 60.

• At several points in main, and according to the value of IAUTO, some parameters can be changed. They are listed above:

  – IAUTO=1: IMLOSS, FMLOS, DGRP, DGRL, DGRT, IADVEC, IDIALU, IDIALO
  – IAUTO=2: GKORM, FAKTOR, ALPH

Modification in dreck:

• If IAUTO = 2, and if the convergence in the envelope is not realised, the values of the triangle are changed to try another solution.
A.10. $\Omega$- and $\mu$-profile smoothing

A.10 $\Omega$- and $\mu$-profile smoothing

Routine modified

• dlonew

• grapmui

List of modifications

Modifications in dlonew:

• The parameters ncouom (setting the size of the fitted zone) and ordre (setting the order of the polynomial fit) are set to various values depending on the evolutive status, and the size of the radiative zones.

• Once the polynomial fit is performed, the $\chi^2$ value and the maximal distance between the fit and the fitted point are checked. If one of them is too large, the parameters are changed.

• If the new values of the parameters do not provide an acceptable fit, the execution is stopped.

Modification in grapmui:

• The parameters ncoumu (setting the size of the fitted zone) and ordre (setting the order of the polynomial fit) are initialized.

• Once the polynomial fit is performed, the $\chi^2$ value and the maximal distance between the fit and the fitted point are checked. If one of them is too large, the parameters are changed.

• If the new values of the parameters do not provide an acceptable fit, we allow up to four consecutive changes.

• After the fifth iteration, if the fit is not acceptable, the execution is stopped.
Appendix B

Publications related to this thesis

B.1 List of papers

Refereed articles

- **CNO ENRICHMENT BY ROTATING AGB STARS IN GLOBULAR CLUSTERS**
  T. Decressin, C. Charbonnel, L. Siess, A. Palacios, G. Meynet & C. Georgy
  2009 A&A 505, 727

- **THE DIFFERENT PROGENITORS OF TYPE Ib, Ic SNe, AND OF GRB**
  C. Georgy, G. Meynet, R. Walder, D. Folini & A. Maeder
  2009 A&A 502, 611

- **BULK COMPOSITION OF THE TRANSITING HOT NEPTUNE AROUND GJ 436**
  P. Figueira, F. Pont, C. Mordasini, Y. Alibert, C. Georgy & W. Benz
  2009 A&A 493, 671

- **CONVECTIVE ENVELOPES IN ROTATING OB STARS**
  A. Maeder, C. Georgy & G. Meynet
  2008 A&A 479, L37

Submitted articles in refereed reviews

- **EFFECTS OF ANISOTROPIC WINDS ON MASSIVE STARS EVOLUTION**
  C. Georgy, G. Meynet & A. Maeder
  submitted to A&A

- **HOMOGENEOUS EVOLUTION NEAR THE $\mu^2 M$ LIMIT**
  C. Georgy, A. Maeder & G. Meynet
  submitted to A&A

Publications in conference proceedings

- **WIND ANISOTROPY AND STELLAR EVOLUTION**
  C. Georgy, G. Meynet & A. Maeder
  in IAU Symposium 255: Low-Metallicity Star Formation: From the First Stars to Dwarf Galaxies,
  Eds L. K. Hunt, S Madden & R. Schneider
CNO enrichment by rotating AGB stars in globular clusters

T. Decressin\textsuperscript{1,2}, C. Charbonnel\textsuperscript{1,3}, L. Siess\textsuperscript{4,5}, A. Palacios\textsuperscript{6}, G. Meynet\textsuperscript{1}, and C. Georgy\textsuperscript{1}

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ABSTRACT

Context. AGB stars have long been held responsible for the important star-to-star variations in light elements observed in Galactic globular clusters (GCs).

Aims. We analyse the main impacts of a first generation of rotating intermediate-mass stars on the chemical properties of second-generation GC stars. The rotating models were computed without magnetic fields and without the effects of internal gravity waves.

Methods. We computed the evolution of both standard and rotating stellar models with initial masses between 2.5 and 8 M\textsubscript{\odot} within the metallicity range covered by Galactic GCs.

Results. During central He-burning, rotational mixing transports fresh CO-rich material from the core towards the hydrogen-burning shell, leading to the production of primary 14N. In stars more massive than \( M \geq 4 \, M\textsubscript{\odot} \), the convective envelope reaches this reservoir during the second dredge-up episode, resulting in a large increase in the total C+N+O content at the stellar surface and in the stellar wind. The corresponding pollution depends on the initial metallicity. At low- and intermediate-metallicity (i.e., [Fe/H] lower than or equal to \( -1.2 \), it is at odds with the constancy of C+N+O observed among GC low-mass stars.

Conclusions. With the given input physics, our models suggest that massive (i.e., \( \geq 4 \, M\textsubscript{\odot} \)) rotating AGB stars have not shaped the abundance patterns observed in low- and intermediate-metallicity GCs. Our non-rotating models, on the other hand, do not predict surface C+N+O enhancements, hence are in a better position as sources of the chemical anomalies in GCs showing the constancy of the C+N+O. However at the moment, there is no reason to think that intermediate-mass stars were not rotating. On the contrary there is observational evidence that stars in clusters have higher rotational velocities than in the field.

Key words. stars: AGB and post-AGB – star: rotation – star: abundances – globular clusters: general

1. Introduction

While the abundances of heavy elements (i.e., Fe-group and \( \alpha \)-elements) are fairly constant from star-to-star in any well-studied individual Galactic globular cluster (GC)\textsuperscript{1}, giant and turnoff stars present some striking anomalies in their light elements content (Li, F, C, N, O, Na, Mg and Al) that are not shared by their field counterparts (for a review see Gratton et al. 2007). Important is that the sum \( (C+N+O)/Fe \) for individual GC stars otherwise presenting very different chemical features is constant to within experimental errors in all the clusters studied so far, with the possible exception of NGC 1851 (for references see Sect. 5).

We now have compelling evidence that these peculiar chemical patterns were present in the intracluster gas from which second-generation (anomalous) low-mass stars formed, and that they resulted from the dilution of pristine material with the hydrogen-burning products ejected by a first generation of more massive and faster evolving GC stars (for a review see Prantzos et al. 2007).

Massive AGB stars that undergo efficient hot-bottom burning (HBB) during the thermal pulse phase (TP-AGB) have been proposed as the possible GC polluters in this so-called self-enrichment scenario (Ventura et al. 2001). The AGB hypothesis has been extensively discussed in the literature, first on a qualitative basis, and more recently with the help of custom-made standard (i.e., non-rotating) stellar models (Ventura et al. 2001, 2002; Denissenkov & Herwig 2003; Fenner et al. 2004; Herwig 2004a,b; Ventura & D’Antona 2005a,b,c, 2008a,b; Bekki et al. 2007). These studies point out several difficulties in building the observed chemical patterns in theoretical TP-AGB models. The main problem stems from the competition between the third dredge-up (3DUP) that contaminates the AGB envelope with the helium-burning ashes produced in the thermal pulse and HBB that modifies the envelope abundances via the CNO-cycle and the NeNa- and MgAI-chains. It is thus very difficult to obtain simultaneous O depletion and Na enrichment in the TP-AGB envelope, while keeping the C+N+O sum constant as required by the observations (see Charbonnel 2007, for more details and references).

To date, only AGB models in the mass range 5–6.5 M\textsubscript{\odot} have managed to simultaneously achieve an encouraging agreement

\textsuperscript{1} Except ω Cen (Norms & Da Costa 1995; Johnson et al. 2008).
with the observed O depletion and the Na enrichment (Ventura & D’Antona 2008a). These models include the FST formulation for convection (Canuto et al. 1996), which strongly affects the O depletion once the stars enter the TP-AGB (Ventura & D’Antona 2005b). Compared to the classical MLT treatment, FST indeed leads to higher temperatures at the base of the convective envelope (resulting in more advanced nucleosynthesis) and induces higher surface luminosities resulting in stronger mass loss and thus fewer thermal pulses and 3DUP events. As shown by Ventura & D’Antona (2005b), some O depletion can also be obtained with the MLT prescription, but only when the free parameter α is arbitrarily and strongly modified with respect to the value calibrated on the Sun. These models are also able to produce a slight decrease in Mg accompanied with a large increase in Al.

The Na enrichment is more difficult to estimate and requires fine-tuning of the NeNa-cycle reaction rates. More precisely, an increase in sodium is achieved in the most oxygen-poor ejecta of the 5–6.5 $M_\odot$ models only when the maximum allowed values for the $^{22}$Ne(p,γ) rate are adopted. In summary, and as clearly stated by Ventura & D’Antona (2008a), the AGB scenario is viable from the nucleosynthesis point of view, provided that only massive AGB stars of 5–6.5 $M_\odot$ contribute to the GC self-enrichment, and under the physical assumptions described above.

However the previously quoted models have only focused on physical uncertainties related to the TP-AGB phase, and their predictions have not been tested in different astrophysical contexts. In particular, the impact of rotation in our models on the nucleosynthesis predictions for AGB stars has never been investigated in the context of the GC self-enrichment scenario, although rotation is often invoked to understand a wide variety of observations (e.g., Maeder & Meynet 2000, 2006; Chiappini et al. 2008, see Sect. 4). The present paper addresses this question for the first time, using up-to-date treatment for rotation-induced processes. As we shall see, the main signature of rotation on the chemical composition of the stellar envelope and winds of intermediate-mass stars already show up during the second dredge-up event and cannot be erased during the TP-AGB phase.

2. Physical input of the stellar models

Although only massive AGB stars in a very narrow mass range between 5 and 6.5 $M_\odot$ are now suspected to play a role in the self-enrichment scenario (Ventura & D’Antona 2008a,b), we computed standard and rotating models of 2.5, 3, 4, 5, 7 and 8 $M_\odot$ stars with the code STAREVOL (V2.75) (Siess et al. 2000; Siess 2006). We present results for several metallicities, namely $Z = 4 \times 10^{-3}, 10^{-3}, 5 \times 10^{-4}, 10^{-4}$ and $10^{-5}$ (i.e., [Fe/H] = −0.66, −1.26, −1.56, −2.26 and −3.26, respectively). The composition is scaled solar according to the Grevesse & Sauval (1998) mixture and enhancement in α-elements ([α/Fe] = +0.3 dex) is accounted for. All models were evolved up to the completion of the 2DUP.

We used the OPAL opacity tables (Iglesias & Rogers 1996) above $T > 8000$ K that account for C and O enrichments, and

the Ferguson et al. (2005) data at lower temperatures. We followed the evolution of 53 chemical species from $^1$H to $^{14}$N using the NACRE nuclear reaction rates (Angulo et al. 1999) by default and those by Caughlan & Fowler (1988) otherwise (see Siess & Arnold 2008). The treatment of convection is based on the classical mixing length formalism with $\alpha_{MLT} = 1.75$, and no convective overshoot is included. The mass loss rates are computed with Reimers (1975) formula (with $\eta_R = 0.5$).

For the treatment of rotation-induced processes we proceed as follows. Solid-body rotation is assumed on the ZAMS and a typical initial surface velocity of 300 km s$^{-1}$ is chosen for all the models. (The impact of this choice is discussed in Sect. 3.3 where we present models computed with initial rotation velocities ranging from 50 to 500 km s$^{-1}$.) On the main sequence, the evolution of the internal angular momentum profile is accounted for with the complete formalism developed by Zahn (1992) and Maeder & Zahn (1998) (see Palacios et al. 2003, 2006; Decressin et al. 2009 for a description of the implementation in STAREVOL), which takes advection by meridional circulation and diffusion by shear turbulence into account. The initial solid-body profile relaxes on the main sequence on a short timescale of a few Myr and generates differential rotation. For our most massive stellar models ($M \geq 4 M_\odot$), the complete formalism for angular momentum transport is applied up to the 2DUP. For the lower mass models, however, the complete treatment is only applied up to the end of the main sequence, while a more crude approach is used in the more advanced evolution phases where angular momentum evolves only through local conservation (i.e., only the structural changes modify the angular momentum). This simplification is motivated by the evolution-ary timescale (i.e., the Kelvin-Helmholz timescale on the RGB) becoming shorter than the meridional circulation timescale. We test this assumption by running a 5 and 7 $M_\odot$ with and without the full treatment of rotational mixing and the results show little difference, validating our approximation. In all cases the transport of chemical species resulting from meridional circulation and both vertical and horizontal turbulence is computed as a diffusive process (Chaboyer & Zahn 1992) throughout the evolution.

3. Signatures of rotation-induced mixing up to the early-AGB phase

3.1. Models at [Fe/H] = −1.56

Up to the beginning of the TP-AGB phase, the surface composition of the standard models is only modified by the dredge-up event(s). Let us note that only the 2.5 and 3 $M_\odot$ models undergo deep enough first dredge-up so as to modify their surface abundances; however, all stars experience to various extents the second dredge-up (hereafter 2DUP) after central He-exhaustion. As described below, in the rotating models the effects of additional mixing become visible at the stellar surface during the quiescent central He-burning phase.

Figure 1 shows the abundance profiles (in mass fraction) of $^1$H, $^4$He, $^{12}$C, $^{14}$N and $^{16}$O, as well as the total sum C+N+O, at the end of central He-burning and before the 2DUP for standard (lower row) and rotating (upper row) 5 $M_\odot$ model.

In the standard case, $^{14}$N steadily increases as one moves inwards through the H-rich radiative layers down to the He-buffer (located between $M_\odot \sim 0.7$ and 1.2 $M_\odot$) as a result of CNO processing. Further inside, the stellar core has experienced complete He-burning and is essentially made of $^{16}$O and $^{12}$C.
Fig. 1. Chemical profiles (in mass fraction) at the end of central He-burning in rotating (top) and standard (bottom) 5 $M_\odot$ models at $Z = 0.001$, 0.0005, 0.0001 and 0.00001 (left to right). The elements shown are $^4$He (dotted-dashed lines), $^{12}$C (short-dashed lines), $^{14}$N (long-dashed lines), $^{16}$O (dotted lines), sum C+N+O (thick lines). The hatched area on top of each panel indicates the maximum extent of the convective envelope during the 2DUP.

In this 5 $M_\odot$ model, during the 2DUP, the convective envelope penetrates into the He-rich buffer (down to $M_r \approx 0.85 M_\odot$) as indicated by the hatched area. This produces an envelope enrichment in H-burning ashes: the surface abundances of $^{12}$C and $^{16}$O decrease, while that of $^{14}$N and $^4$He increase. The sum C+N+O, however, remains constant, since only the H-burning products are dredged up.

Rotation-induced mixing strongly modifies the internal chemical structure. As shown in the upper-row panels of Fig. 1, the abundance gradients are smoothed out in the radiative envelope (i.e., the region above the He-rich buffer up to the surface or the convective envelope) compared to the standard case: $^{14}$N produced in the internal H-burning layers diffuses outwards, while the $^{12}$C and $^{16}$O present in the envelope are transported inwards. As a consequence, during the whole central He-burning phase, rotational mixing produces a continuous surface increase in $^{14}$N concomitant to a decrease in $^{12}$C and $^{16}$O. At the same time, hydrogen from the envelope is also transported inwards and rapidly captured by $^{12}$C and $^{16}$O nuclei through CNO burning at high temperature. This leads to the production of a peak of primary $^{14}$N at the base of the HBS as seen in Fig. 1. The resulting chemical profiles at the end of central He-burning thus differ significantly from those obtained in the standard case where $^{14}$N is only produced in the HBS from the $^{12}$C and $^{16}$O originally present in the star and is therefore of secondary origin (Meynet & Maeder 2002). During the subsequent 2DUP, the convective envelope of the 5 $M_\odot$ rotating model reaches the polluted He-rich buffer (see Fig. 1) producing a large increase in $^4$He. Simultaneously the primary CNO and thus the overall metallicity increases in the envelope.

In the 2.5 and 3 $M_\odot$ rotating models, the convective envelope does not reach the contaminated He-buffer during the 2DUP. As a consequence in these low-mass models, only the H-burning products are dredged up to the surface: $^{14}$N strongly increases, while $^{12}$C and $^{16}$O decrease, but the sum C+N+O, as well as the total metallicity, remain unchanged (see Table 1). $^4$He also diffuses outwards into the radiative envelope leading to a surface enrichment.
Table 1. Surface abundance variations after the completion of the 2DUP with respect to the initial composition \( \delta [\text{X}/\text{Fe}]_{\text{2DUP}} - [\text{X}/\text{Fe}]_{\text{init}} \) for the models with initial value of \([\text{Fe}/\text{H}] = -1.56\).

<table>
<thead>
<tr>
<th>Mass (M⊙)</th>
<th>Standard models</th>
<th>Rotating models</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td></td>
<td></td>
</tr>
<tr>
<td>([\text{Fe}/\text{H}]<em>{\text{2DUP}} - [\text{Fe}/\text{H}]</em>{\text{init}})</td>
<td></td>
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</tr>
<tr>
<td>([\text{N}/\text{Fe}]<em>{\text{2DUP}} - [\text{N}/\text{Fe}]</em>{\text{init}})</td>
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</tr>
<tr>
<td>([\text{O}/\text{Fe}]<em>{\text{2DUP}} - [\text{O}/\text{Fe}]</em>{\text{init}})</td>
<td></td>
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</tr>
<tr>
<td>([\text{CNO}/\text{Fe}]<em>{\text{2DUP}} - [\text{CNO}/\text{Fe}]</em>{\text{init}})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>He</td>
<td></td>
<td></td>
</tr>
<tr>
<td>([\text{C}/\text{Fe}]<em>{\text{2DUP}} - [\text{C}/\text{Fe}]</em>{\text{init}})</td>
<td>0.29 0.29 0.32 0.34 0.35 0.36</td>
<td>0.29 0.29 0.32 0.34 0.35 0.36</td>
</tr>
<tr>
<td>([\text{N}/\text{Fe}]<em>{\text{2DUP}} - [\text{N}/\text{Fe}]</em>{\text{init}})</td>
<td>0.44 0.48 0.60 0.75 0.79 0.81</td>
<td>0.44 0.48 0.60 0.75 0.79 0.81</td>
</tr>
<tr>
<td>([\text{O}/\text{Fe}]<em>{\text{2DUP}} - [\text{O}/\text{Fe}]</em>{\text{init}})</td>
<td>-0.01 -0.01 -0.03 -0.06 -0.10 -0.11</td>
<td>-0.01 -0.01 -0.03 -0.06 -0.10 -0.11</td>
</tr>
<tr>
<td>([\text{CNO}/\text{Fe}]<em>{\text{2DUP}} - [\text{CNO}/\text{Fe}]</em>{\text{init}})</td>
<td>0.00 0.00 0.00 0.00 0.00 0.00</td>
<td>0.00 0.00 0.00 0.00 0.00 0.00</td>
</tr>
</tbody>
</table>

Fig. 2. Surface C+N+O increase index \( \delta [\text{C+N+O}]_{\text{2DUP}} = [\text{C+N+O}]_{\text{2DUP}} - [\text{C+N+O}]_{\text{init}} \) at the end of the 2DUP for rotating stars with various initial masses and metallicities. Squares and triangles indicate models computed with STAREVOL and with the Geneva code, respectively (see text for details).

Fig. 3. Top: enrichment in C+N+O after the 2DUP completion as a function of the mean main sequence velocity for a 5 M⊙, Z = 0.0005 for initial surface velocities of 0, 50, 150, 300, 500 km s⁻¹ as indicated. Bottom: distribution of surface velocities observed by Martayan et al. (2006) in the young LMC cluster NGC 2004 for B-type stars with mass from 2 to 10 M⊙ (v sin i measurements are multiplied by a factor 4/π).

3.2. Influence of metallicity

The impact of rotation on stellar properties and stellar yields is known to depend strongly on metallicity (see, e.g., Meynet & Maeder 2002). The metallicity dependence of our predictions is depicted in Fig. 1 showing the internal profiles of \(^{12}\text{C}, ^{13}\text{N}, ^{14}\text{O}\) at the end of central He-burning in standard and rotating 5 M⊙ models at four different metallicities \([\text{Fe}/\text{H}] = -1.26, -1.56, -2.26, -3.26\). As previously explained, the C+N+O profile outside the CO core is constant in the standard models, while it strongly increases in the He-rich layers below the HBS in the rotating models. This C+N+O step is higher in lower metallicity stars, which results in a stronger CNO surface enrichment after 2DUP in the most metal-poor stars as shown in Fig. 2. The maximal depth reached by the convective envelope for a given stellar mass hardly depends on metallicity; e.g., it reaches 1.04 and 1.07 M⊙ for the 7 M⊙ at Z = 10⁻³ and Z = 10⁻⁴, respectively.

At \([\text{Fe}/\text{H}] = -2.26\), the envelope (and thus the wind) of all the stars more massive than ~5 M⊙ undergo a C+N+O increase by 1 to 2 orders of magnitude, while an increase by a factor of 5 is obtained at \([\text{Fe}/\text{H}] = -1.26\). When metallicity becomes higher than \([\text{Fe}/\text{H}] \approx -1\), rotation-induced mixing increases the total C+N+O by less than a factor 2–3, which is undetectable with current observations (see Sect. 5). The effect is null at \([\text{Fe}/\text{H}] = -0.66\).

3.3. Influence of initial rotation velocity

We present several models for the 5 M⊙ star at Z = 0.0005 \([[\text{Fe}/\text{H}] = -1.56]\) with initial rotation velocities between 0 and 500 km s⁻¹ as part of investigating the dependence of our results on this parameter. As shown in Fig. 3, the C+N+O enhancement increases with increasing initial rotation. A higher initial velocity leads to a faster spinning core and to a larger differential rotation at the core during central He-burning. Since mixing stems mainly from shear turbulence and scales as \((\partial \Omega / \partial r)²\) (Talon & Zahn 1997), more CNO elements are stored in the...
He-rich buffer and then revealed at the surface after 2DUP. In the model with an initial velocity of 150 km s$^{-1}$, the C+N+O rises by a factor 3, while it increases by more than a factor 14 for an initial velocity of 500 km s$^{-1}$.

4. Discussion on the model uncertainties

4.1. Mean rotation velocity

As shown above, the total C+N+O enhancement depends on the assumed initial rotational velocity. We note that, for an initial (i.e., ZAMS) velocity of 300 km s$^{-1}$, the time-averaged velocity of our models on the main sequence ranges between 220 and 256 km s$^{-1}$ depending on the stellar mass and metallicity. What are the observational constraints on this ingredient?

Martayan et al. (2006, 2007a) find mean $v \sin i$ of 161 $\pm$ 20 km s$^{-1}$ and 155 $\pm$ 20 km s$^{-1}$ for SMC B-type stars of 2–5 $M_\odot$ (111 stars) and 5–10 $M_\odot$ (81 stars), respectively. Accounting in a statistical way for the $\sin i$ effect (i.e., multiplying the averaged $v \sin i$ by 4/$\pi$ supposing a random distribution of the rotational axis), we obtain averaged values for $v$ between 197 and 205 km s$^{-1}$ for B-type stars in the SMC. At first glance, the main-sequence time-averaged velocity of our models is slightly higher than the observed values for SMC stars. However, the following points have to be considered:

1. The mean observed values quoted above do not account for Be-type stars. Let us recall that Be stars are fast rotators that present emission lines originating in an outward equatorial expanding disk probably formed due to strong stellar rotation (Martayan et al. 2007b). Therefore Be stars belong to the upper part of the velocity distribution, and it is legitimate to incorporate them in the estimate of the averaged velocities of B-type stars. Taking them into account does enhance the observed average velocities. As an illustrative example, the mean $v \sin i$ for SMC Be stars in the mass ranges 2–5, 5–10 $M_\odot$ are 277$\pm$34 km s$^{-1}$ (14 stars) and 297$\pm$25 km s$^{-1}$ (81 stars), respectively. As can be seen in Fig. 3, our assumptions on the stellar rotation velocities are thus totally realistic.

2. Martayan et al. (2007a) (see also Hunter et al. 2008b) find that, for both B and Be stars, the lower the metallicity, the higher the rotational velocities. This agrees with the finding by Maeder et al. (1999) and Wisniewski & Bjorkman (2006) that the fraction of Be stars with respect to the total number of B+Be stars increases when the metallicity decreases. Since the metallicities considered in this work are lower than that of the SMC, we may expect that the averaged velocities of the stars would be somewhat higher than the one quoted above for the SMC.

3. Last but not least, there is evidence that the rotation rates of stars are higher in clusters than in the field (Keller 2004; Huang & Gies 2005; Strom et al. 2005; Dufton et al. 2006; Wolff et al. 2007).

Thus, in view of these remarks, it does appear that our adopted rotation velocities are probably very close to the averaged velocities of stars in clusters for the range of masses and metallicities considered. If the previous extrapolations are correct, the values given in Fig. 2 should be considered as lower limits.

4.2. Treatment of rotation

Of course, rotating models are not free from uncertainties and their predictions should be carefully compared with well-observed features. The physics included in the present models is the same as the one that provides a good fit to the following observed features:

- the surface enrichments in nitrogen in main-sequence B-type stars (Maeder et al. 2009), even if invoking binarity or magnetic fields is required to explain the whole observational pattern (Hunter et al. 2008a);
- the observed number ratio of blue to red supergiants in the SMC (Maeder & Meynet 2003);
- the variation with metallicity of the number fraction of WR to O-type stars (Meynet & Maeder 2003, 2005);
- the lithium abundance patterns in A-type and early F-type dwarf stars, as well as in their subgiant descendants (Charbonnel & Talon 1999; Palacios et al. 2003).

Moreover, they provide a reasonable explanation for the origin of the high N/O observed at the surface of metal-poor halo stars, as well as for the C/O upturn (Chiappini et al. 2006, 2008). Therefore, while the present models are not free of uncertainties, they have the nice property of accounting for the above observed features.

4.3. Rotational rate of remnants

The present models have some difficulty, however, in accounting for the observed rotation rates of young pulsars and white dwarfs (see e.g., Kawaler 1988; Heger et al. 2005; Suijs et al. 2008). Moreover, they predict too fast rotation of the stellar cores in the advanced phases. This maybe stem from different causes that could lead to additional angular momentum loss from the central regions at different evolutionary phases:

- already during the nuclear lifetime;
- at the time of the supernova explosion in the case of neutron stars or during the TP-AGB phase at the time of the superwind episode in the case of white dwarfs;
- by the neutron stars or the white dwarfs themselves shortly after their formation.

In the second and third cases mentioned above, the physics and the predictions of the present models would not need to be revised since the loss of angular momentum would occur after the evolutionary stages covered by the present computations. However, if the loss of angular momentum occurs before second dredge-up, another mechanism should be included at earlier phases in rotating models.

The study of the s-process nucleosynthesis in low-mass TP-AGB stars could provide some hints to the rotational evolution of the stellar core. At the moment the properties and the behaviour of rotating TP-AGB stars are poorly known. Calculations using a diffusive treatment for the transport of angular momentum by Langer et al. (1999), Herwig et al. (2003) and Siess et al. (2004) have shown that rotationally induced instabilities provide enough mixing to trigger the 3DUPs. However, a shea layer at the base of the convective envelope leads to an efficient pollution of the $^{13}$C pocket (the neutron source) by $^{14}$N with the result of strongly inhibiting the s-process nucleosynthesis.

\footnote{In this framework, the neutrons needed for the s-process are released in the $^{13}$C($\alpha$,n) reaction.}
This is at odds with the observations of s-stars and tends to indicate that the modelling of rotation used in these models needs to be improved and that angular momentum must have been removed by the time the thermal pulses start. Therefore, if the extraction of angular momentum occurs before core He-burning, no CNO enrichment will be expected because it relies on shear due to a fast spinning core. On the other hand, if the extraction occurs during the early-AGB phase, it could allow both the CNO enrichment of the He-buffer and the s-process during the TP-AGB phase.

The mechanism frequently pointed to for removing angular momentum from the core is the magnetic field. For instance, Heger et al. (2005) have shown that magnetic coupling between the core and envelope can account for the rotation rates of young neutron stars. Suja et al. (2008) also find that magnetic torques may be required to understand the slow spin rate of white dwarfs. However the dynamo model (Spruit 2002) on which the current rotating models with magnetic fields are based has recently been tested through hydrodynamical computations by Zahn et al. (2007), who do not find the amplification of the magnetic field as expected from the theory, casting some doubt on its validity.

In view of the remaining theoretical uncertainties associated with the treatment of magnetic fields, it seems reasonable to stick to the present models whose predictions account for a broad variety of observations as described in Sect. 4.3.

Finally let us note that Talon & Charbonnel (2008) predict that angular momentum transport by internal gravity waves should be efficient in intermediate-mass stars during the early-AGB phase. If these waves were the culprit, then the predictions of the present models would be valid as far as the CNO enrichment is concerned since it builds up earlier in the life of the star.

5. Consequences for the self-enrichment scenario

5.1. Summary of the theoretical predictions

During central He-burning, rotational mixing efficiently transports primary $^{12}$C and $^{16}$O outside the convective core in the H-burning region where these elements are processed by the CNO-cycle, resulting in an important production of primary $^{14}$N. In the massive ($M \geq 4 \ M_\odot$) rotating models, after central He-exhaustion, the convective envelope penetrates into the layers affected by rotation-induced mixing. In contrast to standard models, the 2DUP produces a large surface enrichment in total C+N+O that cannot be erased by hot-bottom (hydrogen) burning during the subsequent TP-AGB evolution. 3DUP episodes further increase the total C+N+O mass fraction as they bring the products of He-burning to the surface.

As a consequence, if rotating massive AGB stars were responsible for the abundance patterns observed in GCs, one would expect large C+N+O differences between (O-rich and Na-poor) first-generation stars and (O-poor and Na-rich) second-generation stars (see e.g., Prantzos & Charbonnel 2006). Such differences would easily amount to 1.6 dex in GCs with $[\text{Fe}/\text{H}] \sim -1.5$, and to 2.2 dex in the most metal-poor clusters with $[\text{Fe}/\text{H}] \sim -2.2$ for the cases where the ejecta of the first-generation would not have been diluted with pristine material. Diluting these ejecta with 10 times more pristine material would still give differences of 0.6 to 1.2 dex, well above the observed dispersion (see Sect. 5.2). Actually, to maintain C+N+O constant would require such a high dilution of the ejecta with the intracluster matter that the O-Na and Mg-Al anticorrelations would be erased and, as far as abundances are concerned, first- and second-generation stars would be indistinguishable.

5.2. Comparison with the observations

C, N and O abundances have been determined simultaneously in stars of several GCs. Up to now, no significant star-to-star variation of the total C+N+O has been detected, except in NGC 1851.

In the case of the most metal-rich GCs such as 47 Tuc (Carretta et al. 2005) and NGC 6712 (Yong et al. 2008), this C+N+O constancy is actually compatible with pollution by both standard and rotating AGB stars since the 2DUP CNO enrichment is found to be negligible in the most metal-rich rotating models. This is of course without considering possible C+N+O increase due to the 3DUP of He-processed material during the TP-AGB phase, which is beyond the scope of the present study.

In the case of intermediate-metallicity ([Fe/H] $\sim -1.2$) GCs, such as NGC 288 and NGC 362 (Dickens et al. 1991) and M4 (Ivans et al. 1999; Smith et al. 2005), the total C+N+O is found constant from star-to-star, within observational uncertainties that amount to ±0.25–0.3 dex. At this metallicity, the most massive intermediate-mass rotating models ($M \geq 5$) predict a C+N+O increase between 0.5 and 0.7 dex at odds with the observations. The only exception is NGC 1851 (Yong et al. 2009, [Fe/H] $\sim -1.2$), where variations in the total CNO by 0.57 ± 0.15 dex are found, which is compatible with the CNO enrichment produced by our rotating models as shown in Fig. 2. We note, however, that the stars observed by Yong and collaborators in NGC 1851 are very bright objects. In view of its position in the colour-magnitude diagram, the only star with C+N+O exceeding the typical error bar could actually be an AGB star. In that case, the high value of the total CNO may not have been inherited at birth by the star, but rather from processes within the star itself. Being an AGB would also explain why this star is the only one among the sample to exhibit some enhancement in s-process elements. We note that in this cluster a double subgiant branch has been detected by photometric measurements (Milone et al. 2008) that can be interpreted as caused by an age difference among cluster stars of about 1 Gyr$^4$

However, Cassisi et al. (2008) propose to fit the double sequence assuming two coeval cluster stellar populations with a C+N+O difference of a factor of 2. CNO measurements in NGC 1851 unveeled stars (as done in most of the other clusters studied so far) are thus crucial for settling the problem.

Finally, the CNO predictions for the rotating most massive intermediate mass models lead to increasing factors up to 10 (5 $M_\odot$) and 100 (7 $M_\odot$) at low-metallicity ([Fe/H] $\sim -2.2$), in clear contradiction with the observations in the metal-poor clusters studied so far, i.e., M3 and M13 (Smith et al. 1996; Cohen & Meléndez 2005), NGC 6752, and NGC 6397 (Carretta et al. 2005)$^5$. CNO measurements in other very metal-poor GCs like M15 or M92 would be extremely valuable in this context.

6. Conclusions

In this paper we have investigated the effects of rotation in low-metallicity intermediate-mass stars during their evolution up to the completion of the 2DUP and arrival on the AGB. Our rotating stellar models include the complete formalism developed

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$^4$ The hypothesis of two populations with similar age and [Fe/H] differing beyond the errorbars has been ruled out by the spectroscopic study of RGB stars by Yong et al. (2009) and by the photometric study of RR Lyrae by Walker (1998).

$^5$ NGC 6397 ([Fe/H] $\sim -1.95$) does not show a very extended O-Na anticorrelation, a more modest C+N+O increase is expected in that case.
by Zahn (1992) and Maeder & Zahn (1998) and accounts for the transport of chemicals and angular momentum by meridional circulation and shear turbulence. With respect to standard models, the most important change concerning the nucleosynthesis is the large $^{14}\text{N}$ production resulting from the diffusion of protons below the HBS in a region enriched with primary C and O during central He-burning. During the subsequent 2DUP, the convective envelope of massive AGB stars deepens in this region, producing a large surface increase in the total C+N+O, which irreversibly imprints the yields.

This behavior is in sharp contrast to what is observed in low- and intermediate-metallicity GCs where the sum C+N+O is constant within the experimental errors. Our rotating models based on the Zahn (1992) formalism, which neglect the effects of magnetic fields and internal gravity waves, suggest that massive rotating AGB stars can be discarded as potential polluters in the self-enrichment scenario in GCs, unless the crowded environment prevented intermediate-mass stars from rotating. This latest hypothesis is highly improbable in view of the observations finding higher stellar rotation velocities in clusters than in the field.

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The different progenitors of type Ib, Ic SNe, and of GRB

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ABSTRACT

\textbf{Aims.} We discuss the properties of the progenitors of core collapse supernovae of type Ib and Ic and of long soft gamma ray bursts, as they can be deduced from rotating stellar models of single stars at various metallicities. \textbf{Methods.} The type of the supernova progenitor was determined from the surface abundances at the pre-supernova stage. The type of the supernova event was obtained from the masses of hydrogen and helium ejected at the time of the core-collapse supernova event. \textbf{Results.} We find that the minimum amount of helium ejected by a core-collapse supernova (of whatever type) is around 0.3 \(M_\odot\). There is no difference between the WC and WO stars in the ejected masses of helium, CNO elements, and heavy elements. Also, no difference is expected between the chemical composition of a WC star resulting from a normal or a homogeneous evolution. The progenitors of type Ib supernovae are WNL, WNE, or less massive WC stars. Those of type Ic are WC and WO stars. WO stars are produced in a limited mass range (around 60 \(M_\odot\)) and only at low metallicity (for \(Z \lesssim 0.010\)) as already found. The WC and WO stars are the progenitors of only a small fraction of type Ic. Present stellar models indicate that, at solar metallicity, there is about 1 type Ib supernova for 1 type Ic, and this ratio rises to 3 type Ic for 1 type Ib SN at twice solar metallicity. At this metallicity, type Ic’s are more frequent than type Ib’s because most massive stars that go through a WNE stage evolve further into a WC/WO phase. Current models can account for the observed number ratios SN Ib/SN II and SN Ic/SN II and for their observed variation with the metallicity. In case no supernova occurs when a black hole is formed, single-star models can still account for more than half of the observed (SN Ib+SNIc)/SN II ratio for \(Z \geq Z_\odot\). For the gamma ray burst rate, our models produce too large a number for such an event, even if we restrict the progenitor to the WC stars. This confirms that only a fraction of the WC/WO stars evolve toward gamma ray burst event, most likely those arising from stars that were initially very rapid rotators.

\textbf{Key words.} stars: Wolf-Rayet – stars: supernovae: general – gamma rays: bursts

1. Introduction

Type Ib supernovae are core-collapse supernovae whose spectrum shows no hydrogen lines. The spectra of type Ic show no hydrogen and helium lines (see e.g. Wheeler et al. 1987; Nomoto et al. 1994). The progenitors of these core collapse supernovae are thus believed to be stars stripped of their original H-rich envelope for type Ib’s and also of their He-rich envelope for type Ic’s. Progenitors are therefore Wolf-Rayet stars. The observed frequency at solar metallicity of type Ibc supernovae is about 20\% the frequency of type II supernovae (see e.g. Cappellaro et al. 1999), which represents a significant fraction of all core collapse supernovae. In four cases, the typical spectrum of a type Ic supernova has been observed associated with a long soft gamma ray burst (GRB) event (Woosley & Bloom 2006), indicating a privileged link between type Ic’s and the most powerful supernova explosions observed in the Universe. That type Ibc represents a significant proportion of core collapse supernovae and the link between type Ic’s and the long soft GRBs justify the study of the evolution leading to such events.

In this context, a very interesting feature observed by Prantzos & Boissier (2003) is the increase with the metallicity in the number fraction of type Ibc to type II supernovae. Such a trend is reproduced well by rotating single-star models accounting for metallicity-dependent stellar winds (Meynet & Maeder 2003). On the other hand, a well known scenario explaining this trend comes from close binary-star evolution (see for example Podsiadlowski et al. 1992; Vanbeveren et al. 2007; Eldridge et al. 2008). In this scenario, the hydrogen-rich envelope is removed either through a Roche lobe overflow process or during a common envelope phase, producing a WR star.

In the present work, we take the opportunity of recent observations by Prieto et al. (2008) from which separated frequencies for type Ib and Ic can be deduced to get a step further. The questions we want to address are: how many type Ib and Ic supernovae normalized to the number of type II supernovae can we expect from single-star models at different metallicities, how the numbers derived from single-star models compare with the observed values, and how the results change no supernova event occurs if every time a black hole forms. Answers to those questions will provide some hints as to the importance of the single star scenario in explaining the evolution leading to the type Ibc supernovae events and will provide useful quantities for more comparisons with similar outputs obtained from close binary evolution scenarios.

Other questions will be addressed, for instance, the absence of H and He lines does not necessarily imply that the associated elements are completely absent from the ejecta. It only tells us that the physical conditions (the chemical composition being only one of them) are such that no lines of these elements are formed at the time of the supernova event. Actually, the quantities of hydrogen and helium ejected by these two types of
supernovae are not known. In this paper, guided by stellar models for single stars, we estimate the minimum quantities of hydrogen and of helium, we may expect in type Ib and type Ic supernovae. We also indicate the relations of filiation between type Ib, Ic supernovae and the WNL, WNE, WC, and WO stars.

The second section of this paper briefly recalls the main physical ingredients of the stellar models used in this study. Based on these models, we discuss in Sect. 3 the nature of the supernova progenitors, and in Sect. 4 the chemical composition of the supernova ejecta. In Sect. 5, we discuss the link between the chemical composition of the ejecta and the nature of the supernova. The nature of the stellar remnant is discussed in Sect. 6. Theoretical rates for different supernova types are estimated in Sect. 7 and compared with observations. Section 8 discusses the implications for the long soft GRB progenitors and finally conclusions are presented in Sect. 9.

2. Stellar models
All the rotating stellar models we use here come from Meynet & Maeder (2003, 2005) (hereafter Papers X and XI). A complete description of the physics used can be found in these two papers.

- all the models considered in this work are computed with an initial equatorial velocity of 300 km s$^{-1}$. This initial velocity implies time-averaged equatorial velocities between 160 and 250 km s$^{-1}$ depending on the initial mass and metallicity (see Tables 1 in Papers X and XI). These values are in the range of observed values for OB stars; for instance, Huang & Gies (2006) present projected rotational velocities for 496 OB stars belonging to 19 young galactic clusters with estimated ages between 6 and 73 Myr. Mean $v$ sin $i$ values of 139, 154, and 151 km s$^{-1}$ were obtained for groups of O9.5-B1.5, B1.5-B5.0, and B5.0-B9.0 type stars. These authors derived the underlying probability distribution for the equatorial velocities $v$ and obtained a peak at 200 km s$^{-1}$, Dufour et al. (2006) obtain a peak of $v$ at 250 km s$^{-1}$ with a full width half maximum of approximately 180 km s$^{-1}$ for the unevolved targets in the galactic clusters NGC 3293 and 4755;
- mass loss rates are those of Vink et al. (2000, 2001), which take wind bistability into account. The rates of de Jager et al. (1988) are used outside the domain of application of the rates of Vink et al. (2000, 2001). During the WR phase, the mass loss rates are those of Nugis & Lamers (2000). These mass loss rates, which account for the clumping effects in the winds, are lower by a factor of 2-3 than the ones used in our previous non-rotating stellar grids (Meynet et al. 1994);
- the dependence of mass loss on rotation (Maeder & Meynet 2000) was taken into account, as is the dependence on the metallicity. During the non–WR phases of the present models, we assumed that the mass loss rates depend on the initial metallicity as $M(Z) = (Z/Z_\odot)^{1/2} M(Z_\odot)$ (Kudritzki & Puls 2000; Vink et al. 2001).

Models were computed for four metallicities: $Z = 0.004, Z = 0.008, Z = 0.020$ (standard) and $Z = 0.040$. The initial masses considered are indicated in Table 1. All the models were computed up to the end of the core helium-burning phase. The very short durations of the advanced phase imply that the star at the end of the core He-burning phase has already reached its final mass. Moreover, the decoupling between the rapid evolution of the core governed by the neutrino emission and the slow evolution of the envelope does not allow the envelope to change a lot during the advanced phases. Thus we consider that the properties of the outer layers of our models obtained at the end of the core He-burning phase are very near the one that would have obtained had we pursued the computation until the presupernova stage.

In previous papers (see e.g. Meynet & Maeder 2000; Maeder & Meynet 2001; Maeder et al. 2008, Paper X; Paper XI), we compared the outputs of the rotating models to many observed features of massive stars and showed that the models accounting for the effects of rotation gave much better agreement for fitting observed features such as:

- surface enrichments;
- the blue to red supergiant ratio in the SMC;
- the variation with metallicity in the number ratio of WR to O-type stars;
- the observed number ratio of WN to WC stars in the LMC and SMC;
- the variations with metallicity in the number ratio of type Ib to type Ic supernovae;
- the existence of Wolf-Rayet stars showing both at their surface H- and He-burning products.

In Papers X and XI we discussed the last four points above in details. Here, as indicated in the introduction, we focus on the nature of the progenitors for the type Ib and type Ic SNe, taken separately.

3. Nature of supernova progenitors
The nature of the supernova progenitors was determined by the surface composition of the star at the presupernova stage. The following classification scheme was adopted:

- all the stars not ending their lifetime as a Wolf-Rayet star end their nuclear life as a blue or a red supergiant. They are hereafter called supergiant (SG);
- the star is considered to be a WR star when log T$_{\text{eff}}$ > 4.0 and the mass fraction of hydrogen at the surface X$_H$ is inferior to 0.4 (same criterion as in Meynet & Maeder 2005);
- Wolf-Rayet stars with hydrogen at their surface (X$_H$ > 10$^{-5}$) are WN stars;
- Wolf-Rayet stars without hydrogen on their surface (X$_H$ < 10$^{-5}$) and with surface carbon abundance inferior to nitrogen abundance are WNE stars;
- Wolf-Rayet stars with carbon abundance on the surface superior to nitrogen abundance are WC or WO stars. To distinguish between WC and WO, we follow Smith & Maeder (1991): if the ratio of $C/He$ (in number) is less than 1, we have a WC, otherwise a WO.

A star enters the WR phase as a WN, then may evolve through the sequence WN$\rightarrow$WC$\rightarrow$WO. A star can end its evolution at any of these stages. We do not consider the WN/ WC phase here, which can occur as a very short transition phase between the WNE and WC phases. 

Table 1 gives for the 4 metallicities considered here the initial mass of the star $M_{\text{ini}}$, the mass at the end of the helium-burning phase $M_{\text{endH}}$, the mass of the He-core, $M_{\text{He}}$, defined as the mass interior to which the mass fraction of helium (or of its products after fusion) is higher than 0.75, the mass of the carbon–oxygen core, $M_{\text{CO}}$, defined as the mass interior to which the mass fraction of carbon plus oxygen is higher than 0.75, the mass of the
remnant $M_{\text{rem}}$ (given by Hirschi et al. 2005), the type of the progenitor, the SN type produced (see Sect. 5), and the remnant type (see Sect. 6).

In Fig. 1, we show how the initial mass ranges of various supernova progenitors vary as a function of metallicity. To build this figure we had to adopt a method to determine the mass limit of the intersection of the line connecting these two points with a plane "mass to be lost" versus "initial mass", the abscissa of this last quantity (mass lost) and a positive sign to the former one (mass which has to be lost), plotting these two quantities in a plane "mass-to-be-or-lost" versus "initial mass", the abscissa of the intersection of the line connecting these two points with the horizontal line at ordinate 0 gives the initial mass for which the endpoint of the evolution just corresponds to the stage where "no mass" has to be lost to enter into the WNE phase and where no mass has been lost since the entry into that phase. Thus this is the minimum initial mass to end its evolution as a WNE star. Using similar procedures for all the limits, we obtain the regions shown in Fig. 1. Note also that we used the results of Ekström et al. (2008) for complementing the figure with Pop III stellar models.

Looking at that figure, we can make the following remarks.

- as expected, the lower mass limit for having a WR progenitor increases when the metallicity increases. We see that the dependence on the metallicity of this limit is much stronger at low $Z$ than at high $Z$;
- the mass range of stars ending their life as a WN star is relatively narrow. Below this mass range, the stars do not succeed in entering the WR phase; above it, mass loss rates are efficient enough for allowing the star to evolve further into the WC or WO phase;
- we note that at metallicities higher than about 0.02, the WN progenitors are more or less equally distributed among WNL and WNE stars, while for metallicities below 0.01 all WN progenitors are WNL stars. Actually present stellar models predicts such a narrow range of masses for WNE progenitors below $Z = 0.008$ that we neglected it. This can be explained in the following way: first let us point out that WNE stars can be produced only when the star has evolved beyond the core H-burning phase. Indeed, regions without hydrogen are present in the star only after the main-sequence phase has ended. At high metallicity, because of strong stellar winds, the outer part of the H-burning core is uncovered at an early stage when the size of the He-burning core is still small and has not yet transformed a significant part of the former H-burning core into carbon and oxygen. This allows the intermediate region between the He-burning core and the H-burning shell to have a large extension in mass. Thus, the WNE phase has a significant duration (see Fig. 9 of Meynet & Maeder 2005), which allows a narrow range of initial masses for which the endpoint of the evolution occurs when the star is in the WNE phase. At low metallicity, the outer part of the H-burning core is uncovered at later evolutionary stages when the He-burning core has grown in mass, reducing the extent in mass of the region with chemical composition typical of the WNE phase. This reduces the duration of the WNE phase and also the initial mass range of stars ending their evolution in that stage;
- above 30 $M_{\odot}$ and for metallicities higher than 0.008, all stars end their life as WC/WO stars. For metallicities below 0.008, the minimum initial mass for stars ending their lifetime as WC stars rapidly increases when the metallicity becomes lower;

\begin{table}
\centering
\begin{tabular}{ccccccccccc}
\hline
$M_{\text{ini}}$ & $M_{\text{tot,He}}$ & $M_{\text{He}}$ & $M_{\text{He}}$ & $M_{\text{He}}$ & $M_{\text{He}}$ & $M_{\text{CN}}$ & $M_{\text{He}}$ & $M_{\text{He}}$ & $M_{\text{He}}$ & $M_{\text{He}}$ \\
\hline
12 & 11.8 & 3.74 & 1.78 & 1.4 & SG II & NS & 12 & 10.5 & 3.81 & 1.94 & 1.4 & SG II & NS \\
15 & 14.1 & 5.01 & 2.84 & 1.7 & SG II & NS & 15 & 10.2 & 5.97 & 3.61 & 1.7 & SG II & NS \\
20 & 18.0 & 7.45 & 4.79 & 2.0 & SG II & NS & 20 & 11.8 & 9.01 & 6.62 & 2.0 & SG II & NS \\
25 & 20.0 & 9.95 & 7.06 & 2.5 & SG II & NS & 25 & 11.3 & 11.33 & 8.98 & 2.5 & WNL II & NS \\
30 & 18.9 & 14.16 & 11.14 & 2.9 & SG II & BH & 30 & 12.7 & 12.70 & 11.75 & 3.0 & WC Ic & BH \\
40 & 22.3 & 21.59 & 17.16 & 3.5 & WNL II & BH & 40 & 14.6 & 14.60 & 13.67 & 3.6 & WC Ic & BH \\
60 & 28.5 & 28.50 & 28.46 & 4.4 & WC Ic & BH & 60 & 12.3 & 12.30 & 11.22 & 3.0 & WC Ic & BH \\
120 & 17.2 & 17.20 & 17.18 & 3.4 & WC Ic & BH & 120 & 11.3 & 11.30 & 10.40 & 2.8 & WC Ic & BH \\
\hline
\end{tabular}
\caption{For all our models, various quantities are given here (see text for details).}
\end{table}
the WO progenitors region appears as an “island” in the “WC” sea, located at low metallicity. As first explained by Smith & Maeder (1991), the low metallicity position of this island comes from the fact that, in order to form a WO star, the He-burning core must be uncovered at a very advanced stage of its evolution when a lot of helium has been converted into carbon and oxygen. This is more easily realized at low metallicity where the mass loss rates are lower, hence the core is revealed at a later stage. We also note that the WO progenitors do arise neither from the most massive nor from the less massive stars having their He-burning core uncovered. This reflects the delicate interplay between two countering effects. On one hand, in order for a star to become a WO star, strong enough stellar winds are necessary; on the other hand, as explained above, the stellar winds should not be too strong, otherwise no WO stars but instead WC stars are formed!

4. Chemical composition of the ejecta

The masses of hydrogen and helium ejected at the time of the explosive event are the key quantities for determining the type of the supernova event. These quantities are given for each of our stellar models in Table 2, which indicates for each metallicity and initial mass considered the total mass ejected at the time of the supernova explosion (Col. 2), the mass of hydrogen, of helium, of carbon, nitrogen and oxygen, and of heavy elements (Cols. 3 to 8). Heavy elements are the sum of the mass ejected under the form of elements heavier than hydrogen and helium. For carbon and oxygen, we used relations given by Maeder (1992) between the mass of the CO core and the ejected mass of these two elements at the time of the supernova event.

From the results shown in Table 2, we can deduce the following trends.

- most of supergaint progenitors eject more than 70% of their ejecta in the form of hydrogen and helium. At any given metallicity, the proportion of the ejecta under the form of hydrogen and helium is greater in lower initial mass stars;
- the 20 $M_\odot$ stellar model at $Z = 0.04$ is a special case of supergiant progenitor in the sense that it only ejects small amounts of hydrogen and helium. This model is a blue supergiant ($log T_{\text{eff}} = 4.338$) at the end of its stellar lifetime and is actually on the verge of becoming a Wolf-Rayet star. It should lose only 0.09 $M_\odot$ to uncover layers with a hydrogen mass fraction inferior to 0.4. Probably a slightly more massive star would show an evolutionary path of the type O–red SG–blue SG–WR. Let us mention here that the single-star evolutionary scenario predicts a strong anti-correlation in the number of RSG and WR stars in single-aged populations (Leitherer 1999). Close binary evolution may produce WR stars more easily from lower initial mass stars (Podsiadlowski et al. 1992) and thus explain the simultaneous presence of RSG and WR stars even in stellar populations with age spread inferior to a few Myr. It would be interesting to obtain from well-observed, single-aged stellar populations quantitative constraints on the occurrence of RSG and WR stars and to compare them with population synthesis models accounting for single and close binary evolution;
- our computed WNL progenitors eject less than 1 $M_\odot$ of hydrogen, but more than 1 $M_\odot$ of helium (more generally every time some hydrogen is present in the ejecta, the mass of ejected helium is above ~2 $M_\odot$). They eject greater quantities of carbon and oxygen than supergiant progenitors. About 3/4 of the mass of the ejecta is in the form of heavy elements;
- WC and WO progenitors eject more than 90% of their ejecta in the form of heavy elements. In these cases, the mass of ejected helium is between 0.31 and 0.54 $M_\odot$;
- we see that the minimum helium content in the ejecta of a core collapse supernova is about 0.3 $M_\odot$. This means in particular that type Ic SNe should at least contain this minimum amount of helium in their ejecta. This agrees with the results of non rotating models presented in Eldridge & Tout (2004), and with binary models of Eldridge (private communication). There is to date no clear measurement of the mass of helium contained in the ejecta of type Ic SNe (although see Elmhamdi et al. 2006). It would be very interesting to find a correlation or an anticorrelation between the mass of ejected helium and the mass of another element produced in the same region of the star that would be more easily detectable. Typically, such correlations exist between the ejected mass of He and of N, as can be seen in Fig. 3. We see that the correlation depends on the metallicity so that to use such theoretical guideline for the He-content, some measurement of the metallicity is needed (such correlations would allow an idea of the metallicity in case the amounts of helium and of nitrogen in the ejecta can be measured!). To advance on the question of the He-content in type Ic supernova ejecta, we would need similar correlations but of course with another element, since in type Ic’s the amount of nitrogen is zero. Probably $^{22}$Ne would be an interesting candidate, since it traces the He-rich regions of the star. The difficulty would then be to distinguish the $^{22}$Ne abundances from that of $^{20}$Ne in the spectrum analysis. In a future work, we shall study this question;
Table 2. Initial mass $M_{\text{ini}}$, total released mass $M_{\text{rel}}$, mass and fraction of ejected H ($M_{\text{He}}$), He ($M_{\text{He}}$), C ($M_{\text{C}}$), N ($M_{\text{N}}$), O ($M_{\text{O}}$) and of heavy elements ($M_{\text{heavy}}$), with all masses in $M_\odot$.

<table>
<thead>
<tr>
<th>$M_{\text{ini}}$ ($M_\odot$)</th>
<th>$M_{\text{rel}}$ ($M_\odot$)</th>
<th>$M_{\text{He}}$ ($M_\odot$)</th>
<th>$M_{\text{C}}$ ($M_\odot$)</th>
<th>$M_{\text{N}}$ ($M_\odot$)</th>
<th>$M_{\text{O}}$ ($M_\odot$)</th>
<th>$M_{\text{heavy}}$ ($M_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>10.4</td>
<td>4.30 (41%)</td>
<td>0.15 (14%)</td>
<td>1.7 $	imes$ 10^{-2} (0.16%)</td>
<td>8.9 $	imes$ 10^{-3} (0.86%)</td>
<td>0.58 (5.6%)</td>
</tr>
<tr>
<td>15</td>
<td>12.4</td>
<td>4.94 (40%)</td>
<td>0.25 (20%)</td>
<td>1.5 $	imes$ 10^{-2} (0.12%)</td>
<td>0.42 (3.4%)</td>
<td>1.35 (11%)</td>
</tr>
<tr>
<td>20</td>
<td>16.0</td>
<td>6.35 (40%)</td>
<td>0.49 (31%)</td>
<td>1.8 $	imes$ 10^{-2} (0.11%)</td>
<td>1.41 (8.8%)</td>
<td>3.08 (19%)</td>
</tr>
<tr>
<td>25</td>
<td>17.5</td>
<td>7.09 (41%)</td>
<td>0.88 (50%)</td>
<td>2.1 $	imes$ 10^{-2} (0.12%)</td>
<td>2.64 (15%)</td>
<td>4.93 (28%)</td>
</tr>
<tr>
<td>30</td>
<td>16.0</td>
<td>5.75 (36%)</td>
<td>1.45 (90%)</td>
<td>2.1 $	imes$ 10^{-2} (0.13%)</td>
<td>5.07 (32%)</td>
<td>8.67 (54%)</td>
</tr>
<tr>
<td>40</td>
<td>18.8</td>
<td>4.35 (23%)</td>
<td>2.11 (11%)</td>
<td>1.4 $	imes$ 10^{-2} (0.07%)</td>
<td>9.93 (53%)</td>
<td>14.0 (75%)</td>
</tr>
<tr>
<td>60</td>
<td>24.1</td>
<td>0.00 (0.0%)</td>
<td>0.41 (1.7%)</td>
<td>2.16 (90%)</td>
<td>1.4 $	imes$ 10^{-6} (0.0%)</td>
<td>14.44 (60%)</td>
</tr>
<tr>
<td>120</td>
<td>13.8</td>
<td>0.00 (0.0%)</td>
<td>0.48 (3.5%)</td>
<td>1.73 (13%)</td>
<td>0.00 (0.0%)</td>
<td>6.92 (50%)</td>
</tr>
</tbody>
</table>

Table 3. Mass limits between type II and type Ib SNe ($M_{\text{lin}}^{(\text{II})}$), and between type Ib SNe and Ic SNe ($M_{\text{lin}}^{(\text{Ib} / \text{Ic})}$) in $M_\odot$ (see text).

<table>
<thead>
<tr>
<th>Z</th>
<th>Mass limit between type II – type Ib SNe $m_{\text{lin}}^{(\text{II} / \text{Ib})}$</th>
<th>Mass limit between type II – type Ic SNe $m_{\text{lin}}^{(\text{II} / \text{Ic})}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>54.0 0.25 39.4</td>
<td>46.2 0.25 29.2</td>
</tr>
<tr>
<td>0.008</td>
<td>30.0 0.25 29.7</td>
<td>35.7 49.1 21.6</td>
</tr>
<tr>
<td>0.020</td>
<td>25.1 0.25 23.1</td>
<td>37.5 49.1 11.0</td>
</tr>
<tr>
<td>0.040</td>
<td>20.5 0.25 20.5</td>
<td>26.2 24.7</td>
</tr>
</tbody>
</table>

5. The supernovae types

As already emphasized in the introduction, the conditions for producing type Ia and type Ibc supernovae are based on the presence/absence of H, He lines in the supernova spectrum. Here we adopt a criterion based on the quantities of hydrogen and of helium ejected at the time of the supernova event to link a given chemical structure of the supernova progenitor to a given supernova type. Let us call $m_{\text{lin}}^{(\text{H})}$ the minimum quantity of hydrogen that should be present in the ejecta for the spectrum of the supernova to present H lines and $m_{\text{lin}}^{(\text{He})}$, the minimum quantity of helium that should be present in the ejecta for the spectrum of the supernova to present He lines. In that case a given initial mass star $M$ presenting at the end of its evolution a quantity $m_{\text{H}}(M)$ of hydrogen in its envelope, $m_{\text{He}}(M)$ of helium, will produce

- a type II supernova if $m_{\text{H}}(M) > m_{\text{lin}}^{(\text{H})}$;
- a type Ib supernova if $m_{\text{H}}(M) < m_{\text{lin}}^{(\text{H})}$ and $m_{\text{He}}(M) > m_{\text{lin}}^{(\text{He})}$;
- a type Ic supernova if $m_{\text{H}}(M) < m_{\text{lin}}^{(\text{H})}$ and $m_{\text{He}}(M) < m_{\text{lin}}^{(\text{He})}$.

The quantities $m_{\text{lin}}^{(\text{H})}$ and $m_{\text{lin}}^{(\text{He})}$ are not known a priori. In the case of $m_{\text{lin}}^{(\text{He})} = 0$, we obtain the limiting masses, $M_{\text{lim}}^{(\text{He})}$, indicated...
in the second column of Table 4, where stars with initial masses below $M_{\text{lim,He}}^{\text{min}}$ explode as type II supernovae and stars with masses above $M_{\text{lim,He}}^{\text{min}}$ explode as type Ib or type Ic supernovae. We see that, when the metallicity becomes higher, this mass limit decreases as expected. Interestingly also, we see that adopting $m_{\text{He}}^{\text{min}} = 0.25$, or 0.5 $M_\odot$, pushes $M_{\text{lim,He}}^{\text{min}}$ towards lower values, thereby decreasing the mass range for type II supernovae and increasing it for type Ib supernovae. Therefore $m_{\text{He}}^{\text{min}} = 0$ leads to a lower limit for the number fraction of type Ibc to type II supernovae. Except at the metallicity $Z = 0.004$, the changes remain quite modest. In the following we adopt the value $m_{\text{He}}^{\text{min}} = 0$ as our reference value keeping in mind that it may underestimate the number fraction of type Ibc to type II supernovae.

The choice of $m_{\text{He}}^{\text{min}}$ is more delicate. We first note that there are no models without any helium in the ejecta. For all the metallicities and masses considered here, the minimum value of the mass of helium in the ejecta is 0.31 $M_\odot$. Second, we see that passing from the value 0.4 to 0.5 changes the mass limit a lot between the type Ib and the type Ic. Still, some changes are brought when one passes from 0.5 to 0.6, but in general are much more modest except at solar metallicity. Choosing values for $m_{\text{He}}^{\text{min}}$ between 0.6 and 1–1.5 $M_\odot$ would not change the results significantly when adopting 0.6. From these considerations it appears that 0.6 is a kind of limiting value, because below that value, the mass limit between type Ib and type Ic presents an high sensitivity to the exact value adopted for $m_{\text{He}}^{\text{min}}$ (for instance, choosing 0.4 would make type Ic appearing only in a narrow interval of metallicities). Above it the sensitivity of the mass is much weaker and the mass limits are much less dependent on the exact value chosen. In that respect the value of 0.6 does appear to us as the most reasonable choice. Also this choice gives a monotonic decrease in the mass limit between type Ib and type Ic when the metallicity increases and allows all WC and WO stars to be type Ic progenitors. Thus, in the following we adopt the limiting values corresponding to $m_{\text{He}}^{\text{min}} = 0.6$.

We are now able to determine the type of the supernova it will give for each model (see Col. 7 of Table 1). Table 4 gives for each metallicity the initial mass ranges leading to different progenitors, SN events, and stellar remnants. A comparison between the mass ranges for the different types of progenitors and of SN types shows that all SG stars produce a type II SN. The WNL stars generally lead to type II SNe except may be those WN stars on the verge of becoming WNE stars. They present nearly no hydrogen on their surface (see the case of 25 $M_\odot$, model at $Z = 0.02$). WNE stars contain no hydrogen, but enough helium to become a type Ib SN. WC stars end their life as type Ic except the less massive ones that contain too much helium to be classified as a type Ic SN and thus would explode as a type Ib. Finally, all the WO stars lead to a type Ic SN event.

Using Table 4, we can build Fig. 2, showing the different supernova types expected for various initial masses and metallicities. As for Fig. 1, we use the models of Ekström et al. (2008) to complete the figure for $Z = 0.0$.

We see that type Ib SNe arise from a mass band recovering the WNL / WNE band (see Fig. 1) and the lower range of WC stars. Interestingly, the estimated initial mass of the progenitor of SN 2008ak, which exhibits a transition spectrum between type II and type Ib, is in the range of 25–30 $M_\odot$ for a metallicity around solar (Crockett et al. 2008; Pastorello et al. 2008). This agrees with the present stellar models. Type Ic SNe cover a much broader range of initial masses than type Ib. This trend reflects that most stars which, at a given stage, in their evolution have peeled off their H-rich envelope do not stop at that stage. Their evolution drives them beyond up to the stage where most of their He-rich layers have been peeled off.

One of the main uncertainties of our models is the mass loss rate. Some recent studies suggest that this rate could be overestimated by a factor of 2 (Vanbeveren et al. 2007). We briefly discuss here qualitatively how the results presented here would be modified if we had considered lower mass loss rates. The main differences are:

- the entrance in the WR phase is delayed, so this phase is shorter;
- at a given metallicity, the minimum initial mass to obtain a WR star is increased;
- 

<table>
<thead>
<tr>
<th>$Z$</th>
<th>SG</th>
<th>WNL</th>
<th>WNE</th>
<th>WC</th>
<th>WO</th>
<th>Type II</th>
<th>Type Ib</th>
<th>Type Ic</th>
<th>NS</th>
<th>BH</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>8–32</td>
<td>32–54</td>
<td>–</td>
<td>54–59; 114–120</td>
<td>59–114</td>
<td>8–54</td>
<td>54–59</td>
<td>59–120</td>
<td>8–29</td>
<td>29–120</td>
</tr>
<tr>
<td>0.040</td>
<td>8–20</td>
<td>20–20.5</td>
<td>20.5–22</td>
<td>22–120</td>
<td>–</td>
<td>8–20.5</td>
<td>20.5–25</td>
<td>25–120</td>
<td>8–120</td>
<td>–</td>
</tr>
</tbody>
</table>

(1) At this metallicity, only the 40 $M_\odot$ model produces a BH. Note: The SN type is given for $m_{\text{He}}^{\text{min}} = 0$, $m_{\text{He}}^{\text{min}} = 0.6$ and supposing that an SN occurs even when a BH is formed (see text for more details).
stead of a neutron star (NS) (see also Table 7). For metallicities equal to or above solar, only BH are expected to result from a type Ic event.

6. Nature of the stellar remnants

Superposed to Figs. 1 and 2, we have indicated, as a light grey zone, the regions where a black hole (BH) might be formed instead of a neutron star (NS) (see also Table 4). For drawing this zone, we assumed that 2.7 \( M_\odot \) is the maximum mass of a NS. This value is compatible with the one given by Shapiro & Teukolsky (1983) and also with the recent discovery of a massive NS of 2.1 \( M_\odot \) (Freire et al. 2008). By adopting this mass limit and the relation \( M_{\text{NS}} \) versus \( M_{\text{CO}} \) deduced from the models of Hirschi et al. (2005), we can estimate the mass ranges of stars producing a NS for each metallicity, or a BH. For \( Z \leq 0.01 \), all stars with an initial mass higher than 30 \( M_\odot \) finish their life as a BH, that is, all WR stars and the most massive red– or blue– supergiants. The minimum initial mass to become a BH is higher at higher metallicities. There is also a maximum initial mass to become a BH at high metallicity. This comes from the intense stellar winds that peel off the most massive stars and prevent them from having sufficient massive cores to produce a BH.

As a result, the mass range of BH progenitors decreases when more and more metal-rich environments are considered, and for \( Z \geq 0.040 \), no BH is expected from the single-star scenario (see Fig. 1).

We note that, except at low metallicity (\( Z \leq 0.01 \)), the majority of WNL and WNE stars produce a NS. WC stars may produce either NS or BH for metallicities higher than about 0.01. Below that metallicity they would only produce BH. WO stars produce only BH. In the same manner looking at Fig. 2, we see that most of type Ib’s produce a NS. Type Ic’s produce either a BH or a NS for metallicities above 0.02. For lower metallicities, only BH are expected to result from a type Ic event.

7. Frequency of type Ib, Ic SNe, and comparison with observations

In Fig. 4 we show the variation with metallicity in the number ratio of type Ib, Ic SNe for each type of progenitor with respect to type II SNe. To obtain these results, we used a Salpeter initial mass function (IMF), and apply the relation (in case of the Ib/II ratio):

\[
\frac{\text{SN Ib}}{\text{SN II}} = \frac{\int_{M_{\text{min},b}}^{M_{\text{max},b}} M^{-2.35} \, dM}{\int_{M_{\text{min},b}}^{M_{\text{max},b}} M^{-2.35} \, dM}
\]

where \( M_{\text{min},b} \) is the minimum initial mass giving a type Ib SN, \( M_{\text{max},b} \) the maximum initial mass giving a type Ib SN, and \( M_{\text{max},II} \) the maximum initial mass giving a type II SN. We used the mass limits given in Table 4, and we supposed that an SN event occurs, even when a BH forms. Similar expressions are used to compute the other ratios with respect to type II SNe.

The first striking point when looking at Fig. 4 is that, for most of the metallicities studied here, type Ic are more frequent than type Ib. This was already apparent in Fig. 2, but here it is confirmed after the mass ranges involved have been weighted by the IMF. Only at solar metallicities would present models predict a similar frequency between Ib and Ic. A second point deserving to be mention is that the vast majority of the type Ibc supernovae come from WC or WO stars. WN stars contribute only to a relatively small proportion of the type Ib’s and only at metallicities equal to or above solar.
These frequencies are compared with recent observed ratios in Fig. 5, and our models with the observed rates given by Cappellaro et al. (1999); Prantzos & Boissier (2003); Smartt et al. (2009) and Prieto et al. (2008).2 We also add the prediction of binary star models of Eldridge et al. (2008) and of Fryer et al. (2007). The gray areas indicate the variation of our computed ratio taking the mass limits \( M_{\text{lim}}^{\text{SN}} \) obtained varying \( m_{\text{He}}^{\text{min}} \) between 0 and 0.5.

Our models reproduce the observations up to \( Z = Z_\odot \). At metallicities higher than about 0.02, present models predict too few type Ib supernovae, although in view of the big error bars, it is difficult to ascertain that this is a real deficit. The increase in the ratio of type Ib/type II SNe with the metallicity naturally arises from the metallicity dependence of the stellar winds. At high metallicity stellar winds are stronger, making the formation of stars without an H-rich envelope that explode as a types Ib or Ic supernova easier.

The results obtained including close binary evolutionary scenarios are also able to reproduce the general increase in the SN Ib/SN II ratio with respect to the metallicity. In these models most of the type Ib supernovae occur as a result of mass transfer in close binary systems. Thus we face here the situation where two very different models (single stars rotating/close binary evolution with mass transfer) are both able to give a reasonable fit to the data. Actually both scenarios probably contribute to the observed populations of type Ib supernovae. However, it would be interesting to know their relative importance and how their relative importance changes with the metallicity. It might be that both scenarios predict different behaviors for the way the frequencies of the type Ib and type Ic SNe vary as a function of the metallicity. Below we discuss the predictions of single-star rotating models and compare them with the available observations.

From the data of Prieto et al. (2008), we extract separate observed rates for type Ib and type Ic SNe. They are represented for the SN Ib/SN II rate, and for SN Ic/SN II in Fig. 6. We note that, in the high metallicity range, the observed rate of type Ib SNe is lower than the rate of type Ic by about a factor two. The trend deduced from the present theoretical models is in good agreement with the observed one.

If we adopt \( m_{\text{He}}^{\text{min}} = 0.4 \) instead of 0.6 (see Fig. 7), we see that our models never produce type Ic SNe (except a small number at \( Z = 0.4 Z_\odot \)). In that case, models would predict that most type Ibc supernovae would be type Ib, which is contrary to what is observed. This numerical experiment illustrates the strong dependence of the present results on the maximum quantity of helium in the ejecta that can be accommodated in a type Ic supernova event. In case it were confirmed that this value is as low as 0.4 \( M_\odot \), then present models would not provide a good fit to the observed data. Enhancing this quantity to values superior or equal to 0.55–0.6 suffices to provide a good fit.

How do the results change when one makes the hypothesis that when a BH is formed no SN event occurs? The situation for the variation with metallicity in the number ratio of type Ibc to type II SNe is shown in Fig. 5. In Fig. 8, we show what would be the predictions of the present models for the frequencies of type Ib and type Ic supernovae and adopting

2 In their recent paper, Prieto et al. (2008) analyze a sample of 77 core collapse SNe (and 38 type Ia SNe) in the redshift range 0.01 < \( z < 0.04 \). This sample is composed of 38 SN II, 13 SN Ic, 3 SN Ib, and 3 SN Ibc, all with a measurement of log(O/H) + 12 for the host galaxy.

3 In Meynet & Maeder (2005), we already made this comparison, although with a less sophisticated method to link the progenitor to the supernova type it produces.
$m_{\text{He}}^{\text{min}} = 0.6$. At low metallicity, no type Ibc SNe are expected. At $Z \sim 0.010 = 0.5 Z_\odot$, type Ib SNe begin to come out of the BH domain, making the SN Ib/SN II ratio begins to increase. For $Z \geq 0.02$, single-star models might still account for more than half of the type Ibc supernovae. However, at $Z = 0.02$, all the SNe coming from single-star evolution would have type Ib, which is not consistent with the observations. At twice solar metallicity, all our models lead to a neutron star, and the ratios are identical to the one obtained in Fig. 6. These comparisons show that, considering that the hypothesis “when a BH is formed no SN event occurs” makes a big difference for metallicities below about 0.02. Above that metallicity, the results are only weakly affected.

If we take a lower maximal mass for neutron stars, this situation becomes more extreme: almost all the WR stars produce a BH, thus the rate of SN Ibc/SN II is zero or very low at every metallicity. This illustrates the high sensitivity of the above results on either the mass limit for forming BH and/or the possibility or impossibility to have a SN event when a BH is formed.

Probably, the hypothesis according to which no supernova event appears when a BH is formed is too restrictive. Until no clear understanding of the physics involved in core collapse supernova explosion is reached, it is difficult to draw firm conclusions at that point, especially when one considers the possible effects of rotation. In that context, the collapsar scenario for GRBs (Woosley 1993) needs the formation of a BH (Dessart et al. 2008), and this formation is at least accompanied in some cases by a type Ic supernova event. Also, the observation of the binary system GRO J1655-40 containing a BH (Israelian et al. 1999) suggests that a few stellar masses have been ejected when the black hole formed and therefore that an SN event occurred. This is suggested by the important chemical anomalies observed at the surface of the companion whose origin is attributed the companion being accreted a part of the SN ejecta. These examples show that there are at least some circumstances where the formation of a BH does not prevent an SN event to occur. Probably, reality is somewhere between the two extreme cases discussed before: “all massive stars produce an SN, independently of the remnant type”, and “only the stars leading to a neutron star produce an SN”.

As in Sect. 5, it is interesting here to infer how a decrease in the mass loss rate modifies the previous results. At first order, a decrease in the mass loss rates is equivalent to a shift in the curves presented in Figs. 5 to 8 to the right along the metallicity (horizontal) axis. In general, in that case, the theoretical ratios will be lower than the observed values.

8. Gamma ray burst progenitors

It is now admitted that there is a connection between so-called long-soft GRB events and type Ic supernovae (Woosley & Bloom 2006). As seen in the previous section, this kind of supernova can be produced by both WC or WO stars. In this way, WC and WO stars are natural candidates to be GRB progenitors. However, GRB are primarily found in metal–poor environments (Modjaz et al. 2008), while type Ic’s SNe appear mainly at high metallicity. Many physical reasons have been invoked to explain why GRBs seem to occur only at low metallicities. Among them are the following (see the review by Woosley & Bloom 2006):

- at low metallicity, stellar winds (even during the WR phases) are weaker, thus bringing away small quantities of angular momentum. Black holes are also more easily formed since higher final masses are obtained;
- at low metallicity the transport of angular momentum between the core and the envelope is less efficient than at high metallicities, because of slower meridional currents in metal poor stars;
- since the chemical mixing due to rotation is more efficient at low Z, homogeneous evolution is more easily obtained in metal-poor regions. Homogeneous evolution allows massive stars to produce a type Ic SN event without having to lose large amounts of mass. Indeed, a perfectly homogeneous
evolution (actually never realized) would allow the formation of a pure CO core (and then lead to a type Ic SN event) at the end of the core He-burning phase without the need for the star to lose any mass!

– the distribution of initial velocities at low metallicity might contain more fast rotators than at high metallicities (see Fig. 9 in Martayan et al. 2007).

At the moment, these arguments remain quite speculative.

What can be done presently is to compare the observed GRB frequency with the observed frequency of potential candidates. First, as mentioned above, type Ic supernovae at low metallicity do appear interesting candidates. From Fig. 9, one can see that the observed rate of type Ic supernovae from single star models is still above the estimated number ratio GRB/core collapse supernovae (CCSNe) even when one only considers low-metallicity, type Ic SNe. Here we supposed that the formation of a BH does not prevent an SN event. In case of course only rare circumstances would allow an SN to occur when a black hole is formed, then the situation might be very different (see below).

Other interesting candidates are the WO stars that primarily occur at low metallicity. Table 5 lists all the known WO stars. If we take the value log(O/H) + 12 = 8.66 (Grevesse et al. 2007) for the solar oxygen abundance, we see that 6 out of a total of 8 occur in regions with a metallicity under Z/Z⊙ < 0.9. These stars will explode as a type Ic SN. The frequency of type Ic’s SNe with WO star progenitors is shown in Fig. 9. The expected rate is only marginally compatible with the observed GRB rate (assuming that the aperture angle of the bipolar jet is very small, typically around 1°). This conclusion has already been obtained by Hirschi et al. (2005), so even restraining the progenitors of GRB to WO stars would still not match the observed frequency of GRBs.

From the above discussion, it can be deduced that exploding as a type Ic SN in metal-poor regions (or having a WO progenitor) is not a sufficient condition for a GRB. Physical characteristics shared by a subsample of the metal poor type Ic events exists that are needed to obtain a GRB event. Probably this physical characteristic is the high angular momentum in the core (Yoon et al. 2006; Woosley & Heger 2006).

If GRB would only occur for initially very fast-rotating stars, rotating so fast that these stars would follow a homogeneous evolution (Maeder 1987; Yoon et al. 2006; Woosley & Heger 2006; Meynet & Maeder 2007), can we expect to find any peculiar feature in the chemical composition of the ejecta testifying this previous homogeneous evolution? Or in other words, is there any difference in the chemical composition of the ejecta between a type Ic having “normal WC or WO” progenitors and those arising from a model that followed a homogeneous evolution during the MS phase? To that purpose we computed the evolution of a 60 M⊙ model followed up to the end of central Si-burning, with an initial metallicity of Z = 0.002 and an initial rotation rate of Ω/Ωcrit = 0.75. The model was computed including magnetic field, following Spruit (2002) and Maeder & Meynet (2005). This model follows a nearly homogeneous evolutionary track during the MS phase. The angular momentum content at the time of the presupernova is sufficient for producing a collapsar. In Tables 1 and 2, we present various characteristics of that model.

Table 1 shows that the model is in the WC phase at the end of its evolution. The mixing induced by the magnetic field produces larger cores than in the models without magnetic field. Thus, due to mass loss, the core is uncovered at an early stage of He-burning, leading to a WC star. The helium content of the ejecta (0.64 M⊙) is also slightly greater than in our other models of similar mass and is also just above the value of 0.6 M⊙ below which a type Ic is supposed to occur in our classification scheme. This implies that our homogenous progenitors lead to a type Ib SN. On the other hand, as mentioned above, the limit of 0.6 has not to be taken as a firm limit, and stars with higher He-content might also explode in a type Ic event. Moreover, the models having followed a homogeneous evolution will end their evolution with more angular momentum in their central regions. This may have a significant impact on the way the stars explode, as well as on the spectral features arising from the SN explosion!

It is also interesting to compare the chemical composition of the ejecta as they are obtained in the present models when they follow a homogeneous evolution or not. Compare for instance the 60 M⊙ at Z = 0.004 (heterogeneous case) with the 60 M⊙ at Z = 0.002 (homogeneous case). The differences in the masses of He, CNO elements, and Z are very small. Thus there is little chance from the observations of the composition of the ejecta in CNO elements to be able to distinguish between a normal and a homogeneous evolution.

A point that would be interesting to check in a later work is the following: since the star rotate very fast during a great part of its evolution, it will produce anisotropic stellar winds (see Fig. 10), typically bipolar winds and probably equatorial mass loss when the critical limit is reached (Maeder 1999). These anisotropic winds will shape the circumstellar environment of the star (van Marle et al. 2008, and see also Fig. 10) and it might be that some traces of the resulting particular morphology will still be present at the time of the SN event (for instance, equatorial mass loss probably gives rise to a slow equatorial expanding disk whose traces might still be present when the star explodes). In that case the circumstellar environment of GRB may be peculiar. This can in turn have an impact on some features in the
The maximal extension of the wind is has evolved during 21.552 yr since it has left the ZAMS. The past fast rotation is not too remote.

---

Table 5. WO stars catalog.

<table>
<thead>
<tr>
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</table>

Note: 1st column is the name, 2nd one is the host galaxy, 3rd one is the galactocentric radius (only for Galactic WO), 4th one is the oxygen abundance, 5th one is the number in the circumstellar environment of some stars, now slowly rotating, traces of their very rapid rotation in a previous phase of their evolution.

9. Conclusion

In Meynet & Maeder (2005) we showed that single-star models can account for the increase with the metallicity in the ratio (SN Ib+SN Ic)/SNII. In the present work we obtained the following additional results:

- Type Ib supernovae may be produced by WNL, WNE, or less massive WC stars. Type Ic SNe are only produced by WC or WO stars;
- If we consider that the ejecta of a type Ic supernova may contain up to 0.6 $M_\odot$ of He and that an SN event occurs even when a BH is formed, single-star models are able to reproduce the observational cumulative ratio of SN Ibc/SN II well along with both detailed ratios SN Ib/SN II and SN Ic/SN II. Because of the small number of observed events with known log(O/H), the actual estimations of the ratios SN Ib/SN II and of SN Ic/SN II have large uncertainties. Additional measurements will permit better constrained stellar models;
- In the framework of the “BH-no supernova event” hypothesis, all type Ibc supernovae at low metallicity should come from close binary evolution. At solar and higher-than-solar metallicities, more than half of the type Ibc might still be explained by single-star models;
- Our models predict too high a rate of GRB, even considering that only WO stars lead to this rare event. Probably GRB only originate in the most rapid rotators;
- There is no way to distinguish on the basis of the chemical composition of the ejecta a type Ic supernova arising from a normal heterogeneous evolution from a type Ic supernova arising from a homogeneous evolution. The past fast rotation of the GRB progenitor may have left some imprint on the circumstellar matter distribution;
- For $Z \geq 0.04$, all massive stars produce neutron stars. When the metallicity decreases, the range of initial mass producing neutron stars decreases. Below a metallicity of about 0.01, all stars more massive than about 30 $M_\odot$ produce a black hole.

It does appear difficult on the basis of the variation in the ratio (SN Ib + SN Ic)/SN II with the metallicity to differentiate the relative importance of the single and binary channels. As can be seen from Fig. 5, scenarios with both a significant proportion of close binaries and with a modest one are able to reproduce the observational trend. To make progress it would be interesting to study the predictions of the close binary channel for the relative frequency of type Ib and type Ic supernovae and to compare them with the observations. Also it would be useful to have predictions similar to those presented here for what concerns the chemical composition of the ejecta.

One can also note that, in case the hypothesis “no SN occurs when a BH forms” is correct, the binary scenario is responsible for most, if not all, type Ic and type Ib events at low metallicities, while at high metallicities single stars would still account for more than one half of the type Ibc supernovae. In the...
framework of that hypothesis, the long-soft GRB (at least those associated to type Ic events) would originate in binary systems. Let us note however that some observations show that at least in some circumstances, a SN occurs when a BH is formed.

A promising way to study the respective importance of the single and close binary channel for the formation of type Ib/c SN progenitors is to look for single-aged stellar populations with masses at the turn off between 8 and 25 $M_{\odot}$. If such clusters very rarely show both RSG and WR stars, then this can be interpreted as these two types of stars coming from very different mass ranges just as would be expected in single-star scenarios. If on the contrary, the simultaneous occurrence of both RSG and WR stars is frequent, then it may indicate that there is an overlap of the initial mass ranges of progenitors of RSG and WR stars. In that case, close binary evolution is probably required (except maybe at high metallicity where the overlap also exists for single-star scenarios).

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Bulk composition of the transiting hot Neptune around GJ 436

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ABSTRACT

The hot Neptune orbiting around GJ 436 is a unique example of an intermediate mass planet. Its close-in orbit suggests that the planet has undergone migration and its study is fundamental to understand planet formation and evolution. As it transits its parent star, it is the only Neptune-mass extrasolar planet of known mass and radius, being slightly larger and more massive than Neptune ($M \approx 22.6 M_\oplus$, $R \approx 4.19 R_\oplus$). In this regime, several bulk compositions are possible: from an Earth-like core with a thick hydrogen envelope to a water-rich planet with a thin hydrogen envelope, and comprising a Neptune-like structure. We combine planet-structure modelling with an advanced planet-formation model to assess the likelihood of the different possible bulk compositions of GJ 436 b. We find that both an envelope-free water planet (“Ocean planet”) as well as a diminute version of a gaseous giant planet are excluded. Consisting of a rocky core with a thick hydrogen/helium envelope, a “dry” composition produces not only too small a radius but is also a very unlikely outcome of planet formation around such a low-mass star. We conclude that GJ 436 b is probably of much higher rock content than Neptune (more than 45% in mass), with a small H-He envelope (10–20% in mass). This is the expected outcome of the gathering of materials during the migration process in the inner disk, creating a population of which the hot Neptune is representative.

Key words. planetary systems – planetary systems: formation – stars: individual: GJ 436

1. Introduction

The 22-Earth-mass planet orbiting around the late M dwarf GJ 436 (Butler et al. 2004; Maness et al. 2007) transits its parent star (Gillon et al. 2007a). Photometric monitoring of the transits from the ground and with the Spitzer Space Telescope inferred a planetary radius of 4.19$^{\pm0.12}$–0.21 $R_\oplus$ (Gillon et al. 2007b). It is the first intermediate-mass planet for which we have a radius measurement and is found at an orbital distance that implies that it has been affected by migration. It provides us with a unique testbed to study the formation, evolution and migration of planets. For the first time we are able to analyze the different characteristics of an exoplanet of this class.

In the current paradigm of planet formation and composition – still heavily based on the single case of the Solar System – planets consist of four main categories of compounds: H/He, ices, silicates and iron/nickel1. Planets consisted almost entirely of only one of these four materials have very different radii due to the wide range in densities. Since different materials are mixed in the same planet, a certain amount of degeneracy is unavoidable due to the fact that a mass-radius pair can be formed by different arrangements of elements. Nevertheless, the observed mass and radius of a planet can provide valuable constraints on its bulk composition.

The mass-radius combination for GJ 436 b suggests that the planet consists primarily of heavy elements (iron/rocks/ices), surrounded by a lighter envelope – just like Uranus and Neptune. Given its tight orbit, it is unlikely to have formed close to its present location and is therefore supposed to have undergone type I migration within the protoplanetary disc. Its proximity to its parent star implies that it may also have been affected by evaporation. We explore all the possible composition scenarios and address three key questions requiring a finer analysis than presented so far:

– how likely is the planet to be a “dry” neptune, i.e. to have a core of refractory elements that accreted a H/He envelope inside the snow line, without accumulating ices in significant amounts?
– could it be an “ocean planet”, i.e. a water-dominated planet, whose H/He envelope has been lost by evaporation?
– could its composition be significantly different from that of Neptune due to its peculiar formation process?

To attempt to answer these questions, we combine simple but realistic planetary structure models with the formation models of Alibert et al. (2005) and Mordasini et al. (2008a,b). In Sect. 2 we present our structure model and in Sect. 3 we review the characteristics of the formation model employed. Section 4 describes the results obtained using the two models, first separately and then by using the models together. We discuss our results in Sect. 5 and present our conclusions in Sect. 6.

2. Structure model

The discovery by radial velocity surveys of close-in planets in the 4–20 Earth-mass range (e.g. Lovis et al. 2004; Udry et al. 2007; Mayor et al. 2009) has motivated the development of several sets of structure models of these planets

1 Kuchner et al. (2003) also consider the case of “carbon planets”, planets with a carbon-to-oxygen ratio higher than unity. Until the observations indicate otherwise, we can assume that the C/O ratio around GJ 436 is normal, since the object is an otherwise unremarkable M-dwarf in the solar neighbourhood.
As emphasized by these authors, the mass-radius relation is a robust prediction of the structure models, far less sensitive to details and model-input uncertainties than, for instance, predictions about the thermal history or atmospheric spectrum of exoplanets. Sasselov et al. (2007) discussed the effects of the different sources of uncertainties on the radius predictions, and found that the most significant effect—the equation of state (see review by Guillot 2005)—produced an error of 2% in the radius. This is smaller than the observational uncertainties in most cases (the radius uncertainty is 4% for GJ 436 b). Therefore, a relatively simple model is sufficient to interpret the position of GJ 436 b in the mass-radius diagram in terms of composition. The relative insensitivity of radius predictions to the details of the structure models is due to the large spread in density between the different raw materials (spanning almost two orders of magnitudes from hydrogen to iron), and the cubic relation between the mean density and the radius. An error of 10% in the equation of state in a certain regime will produce only a 3.3% difference in the radius of the layer concerned.

The authors mentioned above have concentrated generally on the $M < 10 M_\oplus$ regime, because these masses are too small for the planet to accrete a significant gaseous envelope. Therefore, the correspondence between planetary radius and composition is more direct. Nevertheless, even if an unequivocal determination of the composition is impossible for GJ 436 b, educated guesses can be made from the same type of structure models.

We developed a structure model for Neptune-type planets close to their parent star (“hot neptunes”), and present an application to the case of GJ 436 b. Our approach closely follows that of Fortney et al. (2007). We use four layers in our model: iron/nickel, silicates, ices, and hydrogen/helium. We use the equations of states from Fortney et al. (2007) for iron and silicates at zero temperature. As suggested by these authors, we neglect the small thermal expansion factor for iron and silicates. We use the “hot water” equation of state from Fortney et al. (2007) for the ices layer, including a first-order account of the thermal effect for a reasonable temperature profile. For the hydrogen/helium mixture, we use the equation of state of Saumon et al. (1995) of a mixture of 75% hydrogen and 25% helium, with a temperature profile taken along an adiabat. We define the outer limit of our structure models as being at a pressure of 1 kbar. The thickness of the outer layer, from 1 kbar outwards, is estimated with a different procedure (see below).

Depending on the exact temperature history of the formation of the initial planetary core, a certain amount of ammonia and methane can be mixed with the water. To estimate the importance of a potential enrichment in these non-water ices, we also consider a suite of models with the mixed water+ammonia+methane, using the density ratio of pure water ice to mixed ice from the Neptune/Uranus models of Hubbard et al. (1995). Since it is not known, even in Neptune and Uranus, whether the refractory elements are mixed with the water layer or separated in a central core, it is not useful in this mass regime to attempt to refine the equations of state beyond a certain point. Moreover, the pressure at the centre of a 22-Earth-mass planet (10–100 Mbars) exceeds the point reached by high-pressure experiments in laboratories considerably for the relevant compounds, so that a certain degree of uncertainty is unavoidable. The key point is that the uncertainties in the model radius remain smaller than the observational uncertainties in the radius of the planet.

The thickness of the outer layer, from 1 kbar to the pressure at which the optical depth becomes small, has a stronger dependence on the temperature profile. We define the “transit radius” as being that at which the pressure is 0.1 bar. There is some level of uncertainty in this value, and it is also wavelength-dependent. We compute the thickness of the outer layer by extending the measured pressure-density profile and scale height of the atmosphere and outer layers of Neptune (Podolak 1976) to the hotter temperature of GJ 436 b. Between 0.1 and 1 bar (“atmosphere”), we adjust the scale height according to the ideal gas law $H \sim T/\mu g$. We use the equilibrium temperature of GJ 436 b (∼600 K) as an indication of the temperature in its isothermal layer. Between 1 bar and 1 kbar (“outer layer”), we follow the analytical treatment of Arras & Bildsten (2006) to modify the temperature profile for a higher temperature in the isothermal layer.

This modelling is approximate and does not provide realistic predictions of the atmosphere’ thickness of the planet. The typical thickness of the 1kbar to 0.1 bar zone in our models with a H/He envelope is around $1 \times 10^3$ km, or 4% in terms of radius. Therefore, even if these estimates are uncertain, they will affect the total radius only at the percent level, which is acceptable for our purposes.

For models without an H/He envelope, we proceed as follows: for water (steam atmosphere), we use the same approach for $H/He$, scaling according to an ideal gas law for the difference in mean molecular weight. In an alternate model, we use a temperature of $1 \times 10^3$ K for the atmosphere, to account for the possibility of a strong greenhouse effect elevating the temperature of the isothermal layer several hundred degrees above the equilibrium temperature. If the outermost layer is silicates or iron, then we fix the density of the outer layer to the zero-pressure density of the solid state of the material (in that case the thickness of the outer layer has a negligible effect on the total radius).

After the equations of state have been chosen, the mass and density profiles can be calculated. The effect of the temperature profile in our models is integrated into the equations of state, and we do not solve the equation of energy transfer. We are left with two basic structure equations: the continuity condition

$$\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^4 \rho}$$

and the hydrostatic equilibrium

$$\frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4}$$

where $M_r$ is the included mass at the level $r$ and $P, \rho$ the pressure and density at that level.

Equations (1) and (2), in addition to the equation of state, form a full system in which all the variables are determined at each level. For the integration, we use a standard shooting method (e.g. Press et al. 1992), beginning at the center and going towards the surface. We start with an arbitrary central pressure, and compute the surface value. Depending on the result, we modify successively the central value, until the surface value reaches the required value. The mass fraction of the different layers is an input parameter, which enables the composition parameter space to be completely explored.

Our structure model was tested with Solar System objects. With accepted composition mixtures, we were able to reproduce the radius of Mars, Earth, the Moon, Titan, Uranus and Neptune to a closer margin that a few percents, usually about 1%. This
does not prove that the composition mixtures adopted were correct, since they were also based on models developed under the same assumptions. It demonstrates, however, that no critical property cannot be reproduced by our simplified approach. We compared our model results with the equivalent of Fortney et al. (2007) and Seager et al. (2007) for the mass range that we explored. The comparison with the former shows that the output radii always agree to within 4% and the differences most of the time are around 1%. When we apply the polytropic state equations of Seager et al. (2007), we recover their radii to within 1%. Once again, the numerical approach of the model is validated.

The structure models employed here are far too simple to take into account effects such as the radioactive decay of elements or excitation pumping. We decided against trying to include these particular effects because it would have increased the degeneracy of the solutions, due to the insufficient amount of current available data.

The inputs to our structure model are the mass fraction of iron, silicates, ices, and H-He, the total mass, and the surface pressure. The outputs are the mass, pressure and density profiles, and the total radius. We ran the model for a variety of composition mixtures, sampling the parameter space around the observed mass-radius position of GJ 436 b.

3. Formation model

As stated in the previous section, the measured mass and radius of GJ 436b provide constraints on its internal structure. On the other hand, the formation process itself establishes the final composition of the planet. The bulk internal composition must be consistent not only with the measured radius and mass of GJ 436 b but also with a plausible formation scenario.

We use here the latest version of the extended core-accretion models including protoplanetary disk structure and evolution, as well as migration of the forming planet. Details of the formation model itself can be found in Alibert et al. (2005), whereas information about the initial conditions that we consider here are described in Mordasini et al. (2008a). In short, the calculation of the formation includes, in a consistent way, three effects: 1) the accretion of solids and gas; 2) the migration of the planet; and 3) the progressive dispersal of the protoplanetary disk, due to viscosity and photoevaporation. The accretion of solids and gas is calculated as in standard core-accretion models (e.g. Pollack et al. 1996). The migration of the planet occurs within two regimes: low mass disks migrate within type I, whereas higher mass planets migrate within type II. The switch from type I to type II occurs when the planet’s hill radius exceeds the local disk thickness. However, for the low-mass planets considered here, type II migration is not important. The type I migration rate is calculated using the analytical work of Tanaka et al. (2002), reduced by a constant factor $f_1$, which stems from different effects that can in fact reduce the migration velocity compared with the value derived for the ideal case considered in Tanaka et al. (2002). These effects include in particular magnetic induced turbulence (Nelson & Papaloizou 2004), or non-isothermal effects (Menou & Goodman 2004; Paardekooper & Mellema 2006). To reproduce the bulk internal composition and atmospheric composition of Jupiter and Saturn, as well as the statistical properties of extrasolar planets, values of $f_1$ between 0.001 and 0.1 are required, as established by Alibert et al. (2005). The contemporary work of Mordasini et al. (2008b) pushed this approach further, and studied the effect of not only $f_1$ but also of the dust-to-gas ratio and the disk viscosity parameter, among others, on the final population properties. In the end, a coherent picture was attained in which the observable properties of extrasolar planet population, such as mass, semi-major axis and “metallicity effect” (increase of detection probability with metallicity) were reproduced. The models presented here stand on this parametrization and assumptions, and in particular $f_1$ takes the value of 0.001.

For the case of the GJ 436 system, we considered formation models around a 0.5 $M_\odot$ star with a solar or half-solar ([Fe/H] = $-0.3$) metallicity, reflecting the uncertainties in observations (Maness et al. 2006; Bonfils et al. 2005). We have calculated tens of thousands of formation models, each one assuming a different set of initial conditions, namely the mass and lifetime of the disk, and the initial location of the embryo that will (eventually) lead to a planet. The statistical distribution of the disk lifetime is taken from Haisch et al. (2001), whereas the distribution of disk mass is scaled down from observations around solar mass stars, using the relation $M_{\text{disk}} \propto M_\oplus^{\beta}$ with $\beta \sim 1.2$ (Beckwith & Sargent 1996). This latter relation allows us to calculate accretion rates in computed disks, which evolve with the square of the star mass, as indicated by observations of protoplanetary discs (Muzerolle et al. 2003; Natta et al. 2004).

Although a scaling of the mass of the disk with the host star mass is theoretically expected from the gravitational link between these quantities, observations do not support such a clear picture. For example, Andrews & Williams (2005) demonstrated that from submillimeter continuum slopes in Taurus Auriga one cannot infer a correlation between $M_{\text{disk}}$ and $M_{\text{star}}$ for the interval of $M_{\text{star}}$ of interest here. As a limiting case, we therefore considered the scenario in which the disk mass does not scale with $M_{\text{star}}$ between FGK and M stars i.e. $\beta = 0$.

3.1. Evaporation

Our formation models terminate when the disc evaporates, and do not follow the subsequent evolution of the planet. In particular, subsequent evaporation of part of the planet atmosphere could modify its radius and bulk composition. Mass loss through atmospheric evaporation is thought to affect close-in planets, and could even strip a planet of most of its mass (e.g. Baraffe et al. 2006).

However, comparison with other known close-in planets suggests that mass loss has not been a strong factor in the evolution of GJ 436b. Despite its proximity to the parent star, the planet receives far less incident flux than hot Jupiters such as HD 209458b, due to the low luminosity of the star. GJ 436 b occupies a position in the comparison of the potential energy at the top of its atmosphere with the incident stellar flux (see Lecavelier 2007), which implies that a significant part of its mass has not escaped through atmospheric evaporation.

4. Results

4.1. Structure

Several structure models can be excluded at the 2-$\sigma$ level. The observed radius of GJ 436 b is not consistent with being a planet with more than 30% in mass of light gases (small Jovian planet) or 90% of rock and iron (a “super Earth”).

In the absence of H/He, with a purely “Ocean planet” model, we were also unable to reproduce the observed radius. Models without a H/He envelope predict radii that are smaller than the observed radius by at least 20%. Even when replacing water ice entirely with methane ice, adding a hot steam atmosphere at $1 \times 10^{38}$ K, and using the upper end of the observed mass range,
the calculated radius for hydrogen-free planets with the mass of GJ 436 b does not reach the 1-$\sigma$ lower error bar of the observed radius. The minimum amount of H/He required is 5% in mass, with a purely water core.

Up to this point, our conclusions correspond to those already presented in Gillon et al. (2007a).

At the other extreme, a “dry” composition with a rocky core of 18.3 $M_{\oplus}$ and 16% H/He in mass (~50% in radius) is also consistent with the observations. A higher H/He proportion produces a larger radius than observed. Changing the silicate to iron ratio from 2:1 to 1:1 modifies these results only at the 1.5% level, corresponding to a ~350 km shrinkage in radius for the most likely bulk compositions.

Finally, many intermediate models, with a certain amount of ices compensated by lower amounts of light gases to retain the radius at a constant value, are compatible with the observed mass and radius.

Table 1 provides the composition, radius, and mass for five models with four fundamentally different bulk compositions: the “Jovian planet”, the “ocean planet”, “hot Neptune”, and “dry Neptune” options. Only the last three provide results that are compatible with the mass and radius observations. For the “hot neptune” variety, two different compositions are given, one with a majority of ices, the other with a majority of heavy elements.

These results are in agreement with those of Adams et al. (2007), as expected since our structure models are very similar.

### 4.2. Formation

We calculated different sets of formation models, differing in value of $\beta$ and the metallicity of the system (solar or half solar). In each case, we selected planets whose final location was <0.1 AU (close-in planets), and total mass in the range 17.6−27.6 $M_{\oplus}$. For these planets, the internal composition (fraction of ices, refractory elements and H/He) was given by the formation model. The formation models do not consider the evolution of planets after their formation, and in particular possible further evaporation (see Sect. 3.1). The effects of the presence of multiple embryos and giant impacts are also not considered by the model (for a discussion of the importance of these effects the reader is referred to Ikoma et al. 2006; and Thomas et al. 2008, respectively). We emphasize that in our models, the formation parameters are established to provide a good match for the extrasolar planets’ properties around solar mass stars (see Mordasini et al. 2008b). Figure 1 illustrates the composition of planets satisfying the mass and orbital distance criteria, for different values of the $\beta$ parameter, which was left as a free parameter for the reasons mentioned in Sect. 3.

We note that the 17.6−27.6 $M_{\oplus}$ mass range corresponds to the most common outcome of the models for a 0.5 $M_{\odot}$ star mass. Therefore, the formation of GJ 436 b can be explained by the models, which predict that it has a high probability of occurring, i.e. as the element of a typical population and not as a single event case.

### 4.3. Formation and structure

We now apply our structure model to calculate a radius for each of the planets produced by the formation model. Since the formation model does not distinguish between rocks and iron components, we use a 2:1 rock-to-iron ratio. As stated earlier, using other realistic ratios does not significantly affect the radius predictions.

To evaluate the likelihood that a given synthetic planet corresponds to GJ 436 b, we compare the model outputs with the measurements of planet mass and radius (Gillon et al. 2007b) simultaneously. This allows us to determine the parameter space that can reproduce both measurable quantities. However, when selecting the synthetic planets that correspond to the observations, one has to take into account not only the observational errors but also the uncertainties present in the modelling itself. We already estimated our radius modelling errors to be inferior to 4% (about 0.17 $R_{\oplus}$), which we can consider to be a conservative upper limit. In what concerns mass uncertainties we have

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**Table 1.** Bulk composition and computed radius for some key models. Note that the GJ 436 b measured mass and radius are $22.6^{+1.9}_{-1.8} M_{\oplus}$ and $4.19^{+0.02}_{-0.02} R_{\oplus}$ (~26 500−1340 km) according to Gillon et al. (2007b).

<table>
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<th></th>
<th>Jovian planet</th>
<th>Ocean planet</th>
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<th>Hot Neptune 2</th>
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<tr>
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</tr>
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<td>Radius [km]</td>
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<td>20 833</td>
<td>27 355</td>
<td>29 929</td>
<td>27 096</td>
</tr>
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</table>

**Fig. 1.** Internal composition of intermediate mass planets ($22.6 \pm 5 M_{\oplus}$) at low distance from the central star (below 0.1 AU). The two curves delimit the envelope of the corresponding planets in two cases: $\beta = 1.2$ in black, $\beta = 0.0$ in blue. For each cases, two metallicities are considered, namely solar and sub-solar metallicity ([Fe/H] = −0.3). The regions occupied in the two cases are however very similar, and are not distinguished in the figure for clarity. The lines in dash are lines of constant gas mass fraction (value indicated on the figure).
models for solar metallicity. The composition of synthetic planets is expressed in terms of rock to the planets created using [Fe/
the formation model does not include evaporation or subsequent mergers. In the context of mass uncertainties, we recall that
injection timescale which therefore determines an end to the evolu-
Besides that, the precise final mass depends on the disk evapo-
dynamical evolution of the system, including possible mergers.

Fig. 2. Internal composition of intermediate-mass planets (18.6−27.6 M⊕) at low distance from the central star (below 0.1 AU) in the formation models for solar metallicity. The composition of synthetic planets is expressed in terms of rock + iron and ices ratios. The left panel corresponds to the planets created using [Fe/H] = 0.0 and the right [Fe/H] = −0.3; in both simulations we assumed that β = 0, since the results do not depend strongly on this parameter (see Table 2 for details). The dotted box corresponds to the composition of Neptune if we assume a protosolar ice/rock ratio (as considered in Guillot 1999). The symbol types indicate the level of agreement of the computed radii and mass with those measured for GJ 436 b. Filled circles represent simulated planets within the estimated modelling errors from the observations and open circles within twice these errors. Crosses represent all other models located within 3 times the estimated errors for mass and radius.

to note that the formation model does not include evaporation or subseqent dynamical evolution of the system, including possible mergers. In the context of mass uncertainties, we recall that the formation model does not include evaporation or subsequent dynamical evolution of the system, including possible mergers. Besides that, the precise final mass depends on the disk evaporation timescale which therefore determines an end to the evolution of the planets. To take into account these uncertainties we introduce a conservative −10% margin, or 2 M⊕ in the comparison between models and observational determination of planet mass. We selected all of our synthetic planets within 0.17 R⊕ and 2 M⊕ from the measured radius and mass, as candidates to represent GJ 436 b internal composition. A second group within twice the above margins is also selected and analysed. We obtain approximately the same final composition, which demonstrates that the results do not depend on a precise estimation of the modelling uncertainties, as we can see in Table 2. The resulting planetary population is shown in Fig. 2, with the different groups represented.

The synthetic population is remarkably homogenous in composition, as the clumping of data points in Fig. 2 testifies. As reported in Table 2, the assumption of different β scaling factors produces minimal population changes. The robustness of the results to the different physical parameters variations allows us to draw some conclusions about the bulk composition of GJ 436 b.

First, no model reproduces the observed orbital distance and mass without including a significant amount of water in the composition. Thus, each and every embryo that evolved into the planets of the selected population was formed beyond the ice-line. Pure rock models either migrate too rapidly to accumulate sufficient mass before reaching the close neighborhood of the star, or migrate too slowly and reach runaway gas accretion creating a gas giant. Retaining a “dry neptune” in the 17−28 M⊕ mass range requires a fine-tuning that was never achieved in our simulations. Planets without H-He envelopes (“ocean planets”) also never occur in the models, because a significant amount of gas is always captured from the disc during the migration, even without runaway accretion. The composition of the model planets on close-in orbits and in the correct mass and radius range from 17% to 40% of their mass in the form of water, 10−20% in the hydrogen-helium envelope, and 45%−70% in the rock+iron core.

5. Discussion

The precise radius of GJ436 b measured by Gillon et al. (2007b) and Deming et al. (2007) suggested a Neptune-like bulk composition for this planet. This paradigm for the planet is not, however, confirmed by our work. Using our structure models we have demonstrated that, in the current planet composition
scenario, an envelope of light gases accounting to 10–20% in mass should indeed be present. An envelope-free planet would be substantially smaller, even assuming a high proportion of methane and ammonia combined with the water ice and a hot greenhouse atmosphere. With this quantity of light gases, the radius of GJ 436 b is consistent with all ratios of water to refractory elements, from a purely water core to a purely dry core.

The combination of the structure and formation models yield water contents of between 17% and 40% in mass for planets such as GJ 436 b. Rock and iron should therefore constitute more than half the bulk mass of the planet, unlike Neptune and Uranus which contain a higher fraction of ices: 25% rock+iron, 60–70% ices and 5–15% H-He (Guillot 1999; Podolak et al. 2000). This is because, although the planet begins its formation beyond the snow line, it accumulates a substantial fraction of its mass in regions of the disc closer to the star, where the fraction of refractory elements is higher when compared with the outer regions of the disk. As a result, we measure a composition for GJ 436 b that differs significantly from that of Neptune: the rock+iron ratio is probably between 45% and 70%, the ices ratio between 17%, and 40% and the H-He ratio between 10% and 20%.

The migration of a Neptune-mass planet and consequent accretion are constrained by the parameter $f_1$, and different values may correspond to a different final structure. Even though this variable was fixed by Mordasini et al. (2008b), we decided to test its impact on the final structure. We calculated additional models using $f_1 = 0.01$ and 0.1 (using solar metallicity and $\beta = 1.2$) and compared their results with those obtained previously. While the fraction of H-He remained approximately constant, the amount of rock+iron ranged from 40% to 80% for $f_1 = 0.01$ and from 40% to 95% for $f_1 = 0.1$. This was expected since the protoplanets can be drawn to the inner limit of the numerical disk far more rapidly, creating a wider range of rocky contents at the expense of a reduced ice content, which does not have enough time to develop. It is therefore clear that the conclusions reached about the richness of the rock and iron content in GJ 436 b are not the result of fine-tuning the $f_1$ parameter but a typical outcome of a migration process within the inner part of the disc.

It is important to note that we have assumed that separated layers of different materials in our composition exist, which may not be the case. Usually produces poor solutions when compared with models that allow a progressive mixing. Once again, this reinforces our belief in the high rocky content of the planet.

6. Conclusion

We have combined formation and structure models to narrow the composition parameter space that corresponds to the mass and radius measurements of a transiting planet. The strength of this method is its ability to reject composition scenarios providing predictions that remain plausible for structure models alone, independently of their sophistication. This problem is caused by the span in densities of different compositions and the cubic dependence of the radii on the density. By using a planetary formation model to provide realistic outputs from the formation process, this degeneracy can be significantly reduced.

Our study has demonstrated that GJ 436 b can be described within the current paradigm of core accretion planet formation, including type I migration and the dependence of planet formation on the properties of the parent star. We propose that this planet is not a Neptune analogue but instead a planet of higher rock and iron content (above 45%), due to the accumulation of these compounds during its migration within the inner disk.

For most of our study we used the nominal $f_1$ parameter value of 0.001, as established by Mordasini et al. (2008b). Since this parameter is poorly constrained theoretically, we tested that our results are not sensitive to its variation. As we experimented with other plausible values and the migration rate increased, the iron and rock content of GJ 436 b were also found to increase, reaching 95% in mass for some planets, which corresponds to a “dry Neptune” configuration. We are therefore convinced that the rocky nature of the hot-Neptune around GJ 436 is not an artifact of our model or a consequence of our parametrization but a natural outcome of the formation process depicted.

Further detections of low-mass transiting planets are of course required to establish robust statistical conclusions. With the results of several ongoing or planned programs, such as the space missions CoRoT and Kepler, and ground-based radial-velocity planet surveys, these kind of studies can be performed. By analysing a low-mass planet population, we will be able to constrain the floating parameters of the formation models and thus expand our understanding of the underlying physics.

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LETTER TO THE EDITOR

Convective envelopes in rotating OB stars

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ABSTRACT

Aims. We study the effects of rotation on the outer convective zones of massive stars.

Methods. We examine the effects of rotation on the thermal gradient and on the Solberg-Holmblad term by analytical developments and by numerical models.

Results. Writing the criterion for convection in rotating envelopes, we show that the effects of rotation on the thermal gradient are much larger and of opposite sign to the effect of the Solberg-Holmblad criterion. On the whole, rotation favors convection in stellar envelopes at the equator and to a smaller extent at the poles. In a rotating 20 $M_{\odot}$ star at 94% of the critical angular velocity, there are two convective envelopes, with the bigger one having a thickness of 13.2% of the equatorial radius. In the non-rotating model, the corresponding convective zone has a thickness of only 4.6% of the radius. The occurrence of outer convection in massive stars has many consequences.

Key words. stars: evolution – convection – stars: rotation

1. Introduction

It is generally considered that the Cowling model applies to massive OB stars: i.e., a convective core surrounded by a large radiative envelope. However, long since stellar models have shown that massive stars have an outer convective envelope encompassing several percent of the stellar radius (Maeder 1980).

Also, Langer (1997) has shown that an Eddington factor $\Gamma = \kappa L/(4\pi GM)$ tending toward 1.0 implies convection. Our aim is to show that fast rotation amplifies the size of the convective envelope in OB stars as well as to develop anisotropic convective envelopes.

Various limits can be considered about the effects of rotation and high luminosity on the stellar stability (Langer 1997; Maeder & Meynet 2000): the $\Gamma$-limit, which is the Eddington limit for $\Gamma \rightarrow 1$; the $\Omega$-limit, which is reached by stars at rotational break-up with a small or negligible effect of the Eddington factor $\Gamma$; the $\Omega\Gamma$-limit, which applies to stars where both luminosity and rotation play significant roles. We show that not only the stars at the $\Gamma$-limit, but also the stars at the $\Omega\Gamma$-limit and at the $\Omega$-limit, have amplified external convective zones.

The occurrence of outer convective envelopes in OB stars and their anisotropic structure lead to many astrophysical consequences:

- convection generates acoustic modes that may allow asteroseismic observations of OB stars;
- convective motions may play a role in driving mass loss by stellar winds;
- for stars close to the critical rotation, convective motions lower the effective break-up velocities;
- an outer convective envelope may make a dynamo and contribute to some chromospheric activity generating an X-ray emission from OB-stars;
- a convective envelope transports chemical elements and angular momentum;

- the occurrence of outer convection may modify the von Zeipel theorem (von Zeipel 1924).

A closer investigation is justified. We start by an analytical approach (Sect. 2), and finish by two-dimensional models of 20 $M_{\odot}$ rotating envelopes (Sect. 3).

2. Convection in rotating stars

In a rotating star of mass $M$, luminosity $L$, and angular velocity $\Omega$ (supposed to be shellular, i.e. constant on shells), the total gravity is the sum of the gravitational, centrifugal, and radiative accelerations:

$$g_{\text{tot}} = g_{\text{eff}} + g_{\text{rad}} = g_{\text{grav}} + g_{\text{rot}} + g_{\text{rad}}.$$  \hspace{1cm} (1)

The vector $g_{\text{eff}}$ has both radial and tangential components, the radial component at colatitude $\theta$ is

$$g_{\text{eff},r} = \frac{GM_r}{r^2} \left( 1 - \frac{\Omega^2 r^3}{GM_r} \sin^2 \theta \right).$$  \hspace{1cm} (2)

where $r$ is the radius at colatitude $\theta$. The radiative acceleration is directed outward

$$g_{\text{rad}} = -\frac{1}{\rho} \nabla P_{\text{rad}} = \frac{\kappa(\theta) F}{c},$$  \hspace{1cm} (3)

where $F$ is the flux. On an isobaric surface, $F$ is given by the von Zeipel theorem (von Zeipel 1924).

$$F = \frac{L(P)}{4\pi G M_\star} g_{\text{eff},r},$$  \hspace{1cm} (4)

with \( M_\star(r) = \frac{M}{1 - \frac{\Omega^2 r^3}{2GM_r}} \), \hspace{1cm} (5)

and $L(P)$ is the luminosity on the isobar, $\rho_0$ the internal average density. In baroclinic stars, there are other terms (Maeder 1999),
however they are small and neglected here. The flux is proportional to the effective gravity $g_{\text{eff}}$. The effective mass $M_\star$ is the mass reduced by the centrifugal force. Let us note that one has $Q^2/(2\pi^2\Omega\varpi) = (4/9)(v_{\text{esc}})^2 \approx (16/81)\varpi^2$, where $\omega = \Omega(\varpi)$, $r$ is the rotation velocity at the level considered and $v_{\text{esc}} = (2/3)(GM/R_{\text{rad}}(\theta = 0))^{1/2}$.

2.1. Effect of rotation on the thermal gradient

Formally the Solberg-Holmquist criterion is to be considered in a rotating star, as in Sect. 2.2. However, the radiative gradient $\nabla_{\text{rad}}$ is also modified by rotation, an effect generally not accounted for in Schwarzchild’s criterion. The local flux and the equation of hydrostatic equilibrium are

$$ F = -\chi \nabla T \quad \text{and} \quad \nabla P = \varrho \nabla g_{\text{eff}}, $$

with $\chi = 4\pi a T^3/(3k\varrho)$. Radiation pressure is included in $P$, the total pressure. The local radiative gradient becomes in a rotating star,

$$ \nabla_{\text{rad}} = \frac{dP}{d\varrho} \frac{\varrho}{T} = \frac{3}{16\pi a c G M_\star} \frac{kL(P)P}{\bar{\rho}^2}\frac{1}{\Omega(r)T^4}, $$

where the derivatives are computed along a direction $n$ perpendicular to the isobars. Except $L(P)$, the terms are local and thus have to be taken at a given $(r, \theta)$. We ignore the horizontal thermal gradient and take $L(P)$ as constant in the envelope. With (5) and the expression of the Eddington factor $\Gamma$, we get

$$ \nabla_{\text{rad}} = \frac{\Gamma}{4(1-\beta) \left(1 - \frac{\Omega(r)}{\Omega_0}\right)}, $$

where $\beta = P_{\text{ad}}/P$ is the ratio of gas pressure to the total pressure, thus $P/(aT^4) = 1/[3(1 - \beta)]$. The adiabatic gradient $\nabla_{\text{ad}}$ is

$$ \nabla_{\text{ad}} = \frac{8 - 6\beta}{32 - 24\beta - 3\beta^2}. $$

As $T$ varies with $\theta$, $\beta$ also varies with colatitude, and we write $\beta(\theta)$. That $\beta(\theta)$ is higher at the equator favors equatorial convection. The criterion for convective instability $\nabla_{\text{rad}} > \nabla_{\text{ad}}$ becomes

$$ \frac{\Gamma(\theta)}{1 - \frac{\Omega(r)}{\Omega_0}} > 4 \left[1 - \beta(\theta)\right] \nabla_{\text{ad}}, $$

where the $\theta$-dependence of $\beta$ comes only through $\kappa(\theta)$ (Maeder & Meynet 2000). Equation (10) has various interesting consequences:

- in the absence of rotation, expression (8) is equivalent to Langer’s result (Langer 1997). The right-hand side of Eq. (10) is always smaller than 1.0, thus if $\Gamma \to 1$, the criterion is satisfied. Convection is present in layers close to the Eddington limit;

- in a rotating star, inequality (10) is more easily satisfied. Thus, rotation favors convection in stellar envelopes;

- the occurrence of convection depends on both $\kappa(\theta)$ and $\beta(\theta)$. Equatorial ejection is always favored, even for electron scattering opacity caused by the higher $\beta$;

- when the centrifugal force can be derived from a potential (conservative case), the temperature and density are constant on isobars and so that $\Gamma$ and $\beta$ are also constant on isobars. In that case, rotation also favors convection as can be seen from Eq. (9).

One can wonder whether rotating stars that are neither at the critical nor at the Eddington limit may develop a convective envelope. The ratio $\beta$ decreases with mass (Fig. 1), while the Eddington factor $\Gamma$ increases, e.g. $\Gamma = 2.5 \times 10^{-3}$, 0.0047, 0.021, 0.098, 0.239, 0.343, 0.544 for 1, 5, 9, 20, 40, 60, and 120 $M_\odot$ stars on the ZAMS. The parameter $\beta$ is at a minimum in the stellar centers, reaches a maximum in the envelope, and is zero at the stellar surface. One has the following relation between the maximum $\beta$-values in the outer layers and $\Gamma$,

$$ \beta = 1.0 - s \Gamma \quad \text{or} \quad \Gamma = \frac{s}{1} (1 - \beta), $$

with $s = 0.72 \pm 0.01$ between 20 $M_\odot$ and 120 $M_\odot$. Using Eq. (11), one eliminates $\Gamma$ from criterion (10) and gets

$$ s \left(1 - \frac{\Omega(r)}{\Omega_0}\right) > \frac{4}{32 - 24\beta - 3\beta^2}. $$

This relation indicates above which value of $\omega$ there is convection for a given value of $\beta$ at the maximum (for lower $\beta$ in the envelope, the inequality is evidently satisfied more easily). For example, for $\beta \to 1.0$, the inequality becomes $\frac{\Omega(r)}{\Omega_0} > 0.132$ or $\langle v_{\text{rad}} \rangle > 0.54$. This is an approximation owing to the simplified relation adopted and because the various parameters also vary with depth. However, it shows that rotating massive stars not even at the critical limit may have enhanced convection.

2.2. The Solberg-Holmquist criterion

A fluid element displaced in a rotating star is also subject to the restoring effect of angular momentum conservation. This leads to the Solberg-Holmquist criterion for stability (Kippenhahn & Weigert 1990), which is (for constant mean molecular weight $\mu$)

$$ \nabla_{\text{ad}} - \nabla_{\text{rad}} + \nabla_{\text{ad}} \sin \theta > 0 $$

with

$$ \nabla_{\text{ad}} = \frac{H_P}{\varrho g_{\text{eff}}(\theta)} \frac{1}{\varpi^2} \frac{d(\Omega^2 \varpi^2)}{d\varpi}, $$

where $\varpi = r \sin \theta$ is the distance to the rotation axis and $\delta = -\left(\partial \ln \varrho / \partial \ln T\right)_P$. The quantity $\nabla_{\text{ad}}$ depends on the distribution
of the specific angular momentum \( j = \varpi^2 \Omega \), which results from transport processes. As \( j \) decreases outward, \( \nabla \Omega \) generally has a stabilizing effect. Let us consider the two extreme cases for \( \Omega(r) \):

1. **Constant specific angular momentum**: a distribution \( \Omega \propto r^{-2} \) may result from the Rayleigh-Taylor instability. This distribution is also sometimes considered in convective regions, with the argument that the plumes rapidly redistribute the angular momentum. If so, \( \nabla \Omega = 0 \), and one is brought back to Schwarzschild’s criterion.

2. **Constant angular velocity**: this assumption is also used in convective regions, with the argument that turbulent viscosity favors solid rotation. If so, \( \nabla \Omega \) simplifies to

\[
\nabla \Omega = \frac{4 \Omega^2}{g_{\text{grav}}} \frac{H_p}{\delta} = \frac{4 \Omega^2}{g_{\text{grav}}} \frac{P}{g_{\text{grav}} \delta}
\]

(15)

We can simplify this expression further. In the outer layers, as long as \( \kappa \approx \) const. and \( g_{\text{grav}} \approx \) const., at an optical depth \( \tau \) one has \( P \approx (g_{\text{grav}}/\kappa) \tau \). This gives

\[
\nabla \Omega \approx \frac{4 \Omega^2}{g_{\text{grav}} \kappa \delta} \tau \approx \frac{4 \left( \frac{\Omega R}{G M} \right)^4}{\rho_{\text{grav}}} \frac{\tau}{\rho_{\text{grav}} R^4}.
\]

(16)

The term in the first parenthesis is \( \omega^2 \), while that in square brackets is just the ratio \( (R - r)/R \) (assuming \( \vartheta = 1 \)), which is small in the envelope. The Solberg-Hoiland criterion becomes in this approximation,

\[
\Gamma \frac{(1 - \beta)}{1 - \frac{\Omega R}{G M}} > 4 (1 - \beta) \left( \frac{\rho_{\text{grav}} + \omega^2}{\rho_{\text{grav}}} \frac{(R - r)/R}{\sin \vartheta} \right),
\]

(17)

where as above the various quantities are local ones. At low rotation, the Solberg-Hoiland term \( \nabla \Omega \) is negligible with respect to the other terms. At high rotation for constant \( \Omega \), it is not negligible, but in general smaller than the other terms because the convective zone lies very close to the surface and the term \( (R - r)/R \) is small.

Since the actual rotation laws are likely between the two extreme cases \( \Omega(r) \approx \) const. and \( \Omega \propto r^{-2} \), we conclude that the main effect of rotation on convection in stellar envelopes is not the inhibiting effect due to the Solberg-Hoiland criterion, but the effect of rotation on the thermal gradient (Eq. (10)), which enhances convection.

### 3. Numerical models

We do some 2D models of the outer regions of a 20 \( M_\odot \) fast-rotating star with \( X = 0.70 \) and \( Z = 0.020 \) (Fig. 2). At each latitude we integrate the equations of the structure for the corresponding effective gravity and \( T_{\text{eff}} \) of the Roche model of the given rotation, also taking the effect of the reduced mass into account. In the envelope, we suppose that \( \Omega \) is a constant as a function of depth (the problem is conservative). We first consider only the effect of rotation on the thermal gradient and then the complete Solberg-Hoiland criterion to see the differences.

#### 3.1. Effects of rotation on the thermal gradient

Without rotation, a 20 \( M_\odot \) model at the end of the MS evolution has two outer convective zones. The first one is very close to the surface and is due to an increase of the opacity caused by partial He ionization. It extends from \( r/R = 0.992 \) to 0.999, i.e. only 0.7% of the radius, and contains a very small fraction of the total stellar mass (2.5 \( \times 10^{-8} \)). The second one is deeper, between \( r/R = 0.915 \) to 0.962 (4.7% of the total radius), and contains a fraction 7.4 \( \times 10^{-7} \) of the total mass. Both convective zones are associated with opacity enhancements.

For fast rotation, with a ratio \( \Omega / \Omega_{\text{crit}} \approx 0.94 \), where \( \Omega_{\text{crit}} \) is the critical angular velocity, the convective layers are shown in Fig. 2. They are more extended than without rotation. The thin upper convective zone extends from \( r/R = 0.987 \) to 0.999 at the pole, i.e., over 1.2% of the stellar radius, and it contains 1.3 \( \times 10^{-8} \) of the stellar mass. The deeper convective zone covers the region between \( r/R = 0.836 \) to 0.936 (i.e. 10.0% of the polar radius), and its mass fraction is 2.8 \( \times 10^{-8} \). At the equator, we have the following sizes for the two convective layers: between \( r/R = 0.958 \) and \( r/R = 0.988 \) for the first one (3.0% of the equatorial radius) and between \( r/R = 0.727 \) and \( r/R = 0.862 \) for the deeper one (13.5% of the radius). The included masses are the same as at the pole.

In agreement with Sect. 2.1, convection is more extended in the rotating model than in the non-rotating one. Figure 2 shows that contrarily to the classical Cowling model, massive rotating stars have a large-size convective envelope. Interesting is that, if we look at the structure of the envelope of the star as a function of the pressure, we see that this structure is independent of the colatitude. This is expected since we have supposed \( \Omega \) constant in the envelope. In that case, as recalled above, \( \Gamma \) and \( \beta \) are constant on isobars and the extensions of the convective zones, expressed in term of differences of pressure between the bottom and the top of the convective zone are the same at the pole and at the equator. The spatial extensions are, however, greater in the equatorial region than in polar ones. This comes from the variations in the spatial gradients of the pressure and temperature with the colatitude imposed by the hydrostatic equilibrium (gradients of pressure have to balance \( \rho g_{\text{grav}} \)). Another result of the constancy of pressure and temperature (and thus density) on an isobar is that the radiative gradient is also constant on this isobar, except for the change in the effective mass as given by \( M_e \), which is lower than \( M \) in rotating stars. Thus, for rotating stars the radiative gradient is larger and convection is favored not only at the equator, but at each colatitude, compared with the non-rotating model.

The mass loss rate in the considered model is 6.2 \( \times 10^{-7} \) \( M_\odot \, \text{yr}^{-1} \). This means that within one year about 4 times all
the matter in the thin convective zone is lost in the stellar winds! Thus, the matter carried by the winds is continuously passing through the superficial convective zones in a dynamical process.

3.2. Solberg-Hoiland criterion

We also computed the envelope structure with the Solberg-Hoiland criterion for convection. The values of \((\nabla_{\text{ad}} - \nabla_{\text{rad}})\) and \(\nabla_{\Omega} \sin \theta\) are shown in Fig. 3 for a 20 \(M_{\odot}\) model at the equator, i.e. where the effect of the Solberg-Hoiland criterion is the strongest. The solid line shows the difference between the adiabatic and the radiative gradient (identical to the values obtained in the model computed with the Schwarzschild criterion). The short-dashed line shows the Solberg-Hoiland term. The limits of the convective zones when the Solberg-Hoiland criterion is used are inside the regions where \((\nabla_{\text{ad}} - \nabla_{\text{rad}}) + \nabla_{\Omega} \sin \theta = 0\). Convective zones are thus smaller than those obtained with the Schwarzschild criterion. We find the following values for the extension of the convective zones at the equator when the Solberg-Hoiland criterion is used: between \(r/R = 0.960\) and \(r/R = 0.988\) for the superficial one (2.8% of the equatorial radius) and between \(r/R = 0.727\) and \(r/R = 0.859\) for the deeper one (13.2% of the total radius), i.e. values slightly smaller than but very similar to those obtained in the model computed with the Schwarzschild criterion. This numerical example confirms that the Solberg-Hoiland term \(\nabla_{\Omega} \sin \theta\) has a very limited influence in stellar envelopes, as discussed in Sect. 2.3.

4. Conclusions

In stellar envelope of rotating stars, the effects of rotation on the thermal gradient are stronger and with the opposite sign with respect to the Solberg-Hoiland criterion, so that rotation favors convection instead of inhibiting it. The increase of the convective zone occurs mainly at the equator and also a bit at the poles. In a fast-rotating 20 \(M_{\odot}\) Pop I star, there are two equatorial zones covering a total of 16% of the stellar radius at the equator.

There are several consequences of this result to be examined in future. The outer convective motions may lower the escape velocity as well as the critical rotation velocity. The matter accelerated in the winds continuously goes through the convective zone in a dynamical process, suggesting that convection plays a role in accelerating the stellar winds and in producing the clumps in the winds. The convective pistons generate acoustic waves of periods of several hours to a few days. The density is very low, and it is thus likely that convection injects oscillations into the wind rather than into the interior.

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Effects of anisotropic winds on massive stars evolution

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ABSTRACT

Context. Whenever stars are rotating very fast (Ω/Ω_{crit} > 0.7) radiative stellar winds are enhanced in polar regions. This theoretical prediction is now confirmed by interferometric observations of fast rotating stars.

Aims. Polar winds remove less angular momentum than spherical winds and thus allow the star to keep more angular momentum. We quantitatively assess the importance of this effect.

Methods. We first use a semi-analytical approach to estimate the variation of the angular momentum loss when the rotation rate increases. Then we compute complete 9 M_{⊙} stellar models at very high velocities with and without radiative wind anisotropies.

Results. When wind anisotropies are accounted for, the angular momentum loss rate is reduced by less than 4% for Ω/Ω_{crit} < 0.9 with respect to the case of spherical winds. The reduction amounts to at most 30% when the star is rotating near the critical velocity. These values result from two counteracting effects: on one side polar winds reduce the loss of angular momentum, on the other side, surface deformations imply that the mass which is lost at high colatitude is lost at a greater distance from the rotational axis and thus removes more angular momentum.

Conclusions. In contrast with previous studies, which neglected surface deformations, we show that the radiative wind anisotropies have a relatively modest effect on the evolution of the angular momentum content of fast rotating stars.

Key words. Stars: evolution, Stars: mass-loss, Stars: rotation, Stars: winds

1. Introduction

Over the last years, the Geneva evolutionary code has experienced several major improvements, as rotation, the inclusion of extended nuclear reaction network allowing to explore the advanced phases (neon, oxygen and silicon burning) of massive star evolution, and the inclusion of magnetic field.

The inclusion of rotation improved the agreement between the outputs of numerical models and observational results, as the surface enrichments, the blue to red supergiant ratio in the SMC, the ratio of WR to O-type stars and the ratio of type Ibc to type II supernovae (see for example Meynet & Maeder 2000; Maeder & Meynet 2001; Meynet & Maeder 2003; Georgy et al. 2009). The treatment of internal magnetic field in the models gives a better rate of Gamma Ray Bursts as a function of the metallicity (Yoon et al. 2006), produces a rotation rate for the young pulsars in much better agreement with the observations (Heger et al. 2005), and allows to explain the flat rotation profile of the Sun (Eggenberger et al. 2005).

However, the rotation acts not only in the interior of the stars, but has also several effects on the surface. Maeder & Meynet (2000) show that the global mass loss rate is increased by rotation. Another point is that it modifies the shape of the star, and consequently various surface quantities: effective gravity, effective temperature and radiative flux. Interestingly, Maeder (2002) shows that the mass loss in fast rotating massive stars does not remain isotropic, but becomes more and more anisotropic as the rotation approaches the critical rotation rate. This favours a bipolar stellar wind, and modifies the quantity of angular momentum removed from the star.

With the developments of the interferometric technics, some of the predicted effects have recently become observable. For example, Carciofi et al. (2008) have obtained for the ratio of the equatorial to polar radius a value of 1.5 for the very fast rotating star Achernar as is expected from theory. Monnier et al. (2007) provide a map of the effective temperature over the surface of Altair, showing the temperature gradient between the pole and the equator of this star, in good agreement with the von Zeipel theorem (von Zeipel 1924). Meilland et al. (2007) observed evidences of a disc and of a polar wind around the star μ Ara as is predicted for a star rotating at the critical limit.

At the moment the effects of wind anisotropies on stellar models have been quantitatively explored only in two previous publications (Meynet & Maeder 2003, 2007). In the present work we reexamine this issue after improving the numerical treatment with respect to these works. The improvements brought here are the following:

- First, we propose a semi-analytical approach to estimate the effect of fast rotation on the loss of angular momentum. The relative effects obtained in that way depend only on one parameter, the ratio Ω/Ω_{crit}, where Ω is the surface angular velocity and Ω_{crit}, the critical angular velocity, i.e. the angular velocity such that the centrifugal acceleration at the equator compensates for the gravity. The semi-analytical estimates are used to
check the validity of the numerical results and also to study in a clearer and simpler context the various effects intervening in the loss of angular momentum.

– For the first time, we account not only for the variation of the mass flux with the colatitude as was done in the previous work, but also account for the surface deformation of the star. As we will see this last effect cannot be neglected.

– We use an updated expression for the mass flux obtained by Maeder (2009), and a corrected expression for the total mass loss rate (see Sect. 2.1).

– We accounted for the variation of the force multiplier parameters over the surface of fast rotating stars (see below for more details on that point).

The paper is organized in the following way: in Sect. 2, we recall the theoretical aspects of wind anisotropy. The third section presents our semi-analytical approach. In Sect. 4, we discuss results based on complete numerical stellar models. Conclusions are presented in Sect. 5.

2. Rotation and wind anisotropy

2.1. Increase of the global mass loss rate induced by rotation

As shown by Maeder & Meynet (2000), the local radiative mass loss rate $\Delta \dot{M}$ by unit surface $\Delta \sigma$ can be written:

$$\Delta \dot{M} \sim A \left( \frac{\alpha c}{4} \right)^{\frac{2}{3}} \left[ \frac{L}{4 \pi G M} \right]^{\frac{2}{3}} \Gamma \frac{\delta_{\text{eff}}}{1 + \left\{ \left( 1 + \frac{\Omega}{\Omega_{\text{crit}}} \right)^{\frac{1}{2}} \right\}^{\frac{1}{2}}}. \tag{1}$$

In this expression $A = (k \alpha)^{\frac{2}{3}} \left( \frac{\Omega}{\Omega_{\text{crit}}} \right)^{\frac{1}{2}}$, where $\alpha$ and $k$ are the force multiplier parameters empirically determined (Lamers et al. 1995), $M = M_{\text{tot}} - M_{\text{star}}$ is the reduced mass, where $M_{\text{star}}$ is the internal average density, $L$ is the stellar luminosity, $\delta_{\text{eff}}$ is the effective gravity at the colatitude $\theta$ (i.e. the gravitational acceleration minus the centrifugal one) and $\Omega$ expresses the deviation from the von Zeipel theorem produced by shellular rotation (Maeder 1999, this term is generally negligible). $\Gamma_{\text{crit}}(\theta)$ is the local Eddington factor, taking into account the effect of rotation:

$$\Gamma_{\text{crit}}(\theta) = \frac{\kappa(\theta) L \Sigma}{4 \pi c G M \left[ 1 - \frac{\Omega^2}{2 \pi G \rho_{\text{star}}} \right]}, \tag{2}$$

with $\kappa(\theta)$ the opacity at the colatitude $\theta$. The term $1/8$ in the power of expression (1) was added by Maeder (2009, see his chapter 14.4) and does not appear in Maeder & Meynet (2000, a $\Gamma_{\text{eff}}^{-1}$ was absent in their expression 4.24).

Averaging expression (1) over the whole stellar surface $\Sigma$, one obtains the total mass loss of the star

$$\dot{M} \sim \frac{A L}{(4 \pi G M)^{\frac{2}{3}} \Sigma^{\frac{2}{3}}} \left[ 1 - \frac{\Omega^2}{2 \pi G \rho_{\text{star}}} \right]^{\frac{1}{2}} \left[ 1 - (1 - \Gamma_{\text{crit}})\Sigma^{\frac{1}{2}} \right]. \tag{3}$$

This allows us to compute the ratio of the mass lost by a rotating star to the mass lost by a non-rotating one lying at the same position in the Hertzsprung–Russel diagram:

$$\frac{\dot{M}(\Omega)}{\dot{M}(\Omega = 0)} = \frac{1 - \left( 1 - \Gamma^{1/2} \right)}{\left( 1 - \frac{\Omega^2}{2 \pi G \rho_{\text{star}}} \right)^{1/2} \left( 1 - \Gamma_{\text{crit}} \right)^{1/2}}. \tag{4}$$

where $\Gamma$ is the classical Eddington factor for a non-rotating star. As a result, we see that the faster the star rotates, the more mass will be lost.

2.2. Wind anisotropy

According to Maeder (1999), two main effects contribute to the development of anisotropies in the stellar winds. The first, called the $g_{\text{eff}}$-effect, is due to the variation of the effective gravity with the colatitude: $g_{\text{eff}}$ is smaller at the equator than at the poles, and thus, the mass loss, which is directly related to $g_{\text{eff}}$ (see eq. (1)), is favoured at the poles for a rotating star.

The second effect is called the $\kappa$-effect. Due to the so-called bistability in the stellar winds (see Lamers et al. 1995), the $A$ term in eq. (1) increases for lower values of the effective temperature, i.e. towards the equatorial regions (see Fig. 6 in Ekström et al. 2008). This favours an equatorial mass loss.

Looking at eq. (1), we also expect a contribution to the latitudinal variation of the mass loss due to the term $\kappa(\theta)$ in $\Gamma_{\text{crit}}(\theta)$. However, for hot enough stars, the opacity we should consider in eq. (2) is the electron scattering opacity $\kappa_{\text{esc}}$, which is latitude-independent.

2.3. Critical velocities

As the concept of critical velocity is treated in very different way through the literature, it is useful here to briefly recall some definitions. Following Maeder & Meynet (2000), we can define two critical velocities. The first one is the traditional keplerian velocity at the equator when the star rotates at the critical velocity defined by $g_{\text{eff}} = 0$:

$$v_{\text{crit, 1}} = \sqrt{\frac{GM}{R_{\text{eb}}}} = \sqrt{\frac{2GM}{3R_{\text{eb}}}}, \tag{5}$$

where $R_{\text{eb}}$ ($R_{\text{pb}}$) is the equatorial (polar) radius when the critical velocity is reached. The numerical factors $2/3$ comes from the polar to equatorial radius ratio when the star is at the critical velocity and the Roche approximation is valid (see e.g. Ekström et al. 2008). We also define the critical angular velocity $\Omega_{\text{crit}} = \frac{v_{\text{crit, 1}}}{R_{\text{eb}}}$, and the ratio $\omega = \frac{\Omega}{\Omega_{\text{crit}}}$.

The second critical velocity is reached when the star is at the so-called $\Omega$-limit, i.e. when the local Eddington factor (accounting for the effects of rotation) defined in eq. (2) equal 1. According to Maeder & Meynet (2000), this term is equal to:

$$v_{\text{crit, 2}} = \frac{81}{16} \frac{1 - \Gamma GM}{V'(\omega) R_{\text{pb}}^2}, \tag{6}$$

with $\Gamma$ the Eddington factor, $V'(\omega)$ is the ratio of the volume enclosed by the surface to the volume of a sphere with a radius equal to $R_{\text{pb}}$ (the polar radius when the star rotates at the break-up velocity, see below) and $R_{\text{pb}}$ is the equatorial radius.

3. Effects of fast rotation on angular momentum loss: a semi-analytical approach

In this section we derive the variations of

– the shape of the star,
3.1. Shape of the surface

In the frame of the Roche model and neglecting the variation of $R_p$ with $\omega$, the equation of the surface of the star can be given as a function of the rotation rate $\omega$ (Maeder 2002)

$$\frac{1}{x(\omega, \theta)} + \frac{4}{27} \omega^2 x^2 (\omega, \theta) \sin^2(\theta) = 1,$$

(7)

where $x(\omega, \theta) = \frac{R_p}{R}$ is the ratio of the radius at a given colatitude to the polar one. We can easily express $\theta$ as a function of the normalized radius $x$:

$$\theta(x) = \arcsin \left( \sqrt{\frac{27(x - 1)}{4\omega^2 x^2}} \right)$$

if $\omega \neq 0$

$$x(\theta) = 1$$

if $\omega = 0$. (8)

We see that the shape depends only on $\omega$. The range of satisfactory values for $x$ is a function of $\omega$. It starts from 1 (to have a positive value under the square root), and goes up to the first positive root of the equation $4\omega^2 x^3 - 27x + 27 = 0$. On Fig. 1, we show how the shape of the surface varies for various values of $\omega$, starting from $\omega = 0$ (non rotating case) to $\omega = 1$ (critically rotating case). If the rotation rate $\omega = 0$, the shape of the surface is of course spherical. As $\omega$ increases, the centrifugal force deforms more and more the star, and the equatorial radius increases. When the star is exactly at the critical angular velocity ($\omega = 1$), we see from eq. (7) that the equatorial radius is 1.5 times larger than the polar one.

It is interesting to remark here that in the expression of $\omega$ only the first traditional critical velocity appears. That means that stars reaching the second critical velocity will have a ratio $\omega$ inferior to 1. These stars, despite being at the critical velocity, will not show as strong deformations as stars reaching the first classical critical limit. Since, as we will see, it is the deformation of the star which triggers the wind anisotropies, it means that stars which would be at the $\Omega$-limit would present not as strong wind anisotropies as stars at the classical $\Omega$-limit. According to this line of reasoning, the fact that $\eta$-Carinae presents strong polar winds, implies that this star should rotate with velocities not far from the first classical critical rotation velocity.

3.2. Mass flux variations with the latitude

To study how the mass flux is modified by rotation as a function of the colatitude $\theta$, we use eq. (1) to compute the local mass loss rate per unit surface. As said above, we normalize the total mass loss to a value of $4\pi$, in order to have a local mass flux of 1 at every colatitude in the non rotating case.

The results of these calculations are shown in Fig. 2. Not surprisingly, the mass loss rate per unit surface is constant when there is no rotation. As the rotation rate increases, the mass flux increases towards the pole, and decreases towards the equator, due to the variation of the effective gravity, producing a strong anisotropy in the winds. In the extreme case, when the rotation parameter $\omega = 1$, the effective gravity at the equator is zero, and the radiative mass flux becomes also zero in this idealized representation. Obviously, our treatment is not detailed enough in the case of the exact critical rotation. In this critical regime, some other energy transport processes must develop, as convection, at least
near the equatorial region. Maeder et al. (2008) show that fast rotation favours convection at the equator. This convection can help equatorial mass loss, especially when the effective gravity becomes low. Another important point to mention here is that in our treatment, we have considered that the force multiplier parameters $\alpha$ and $k$ remain constant over the whole surface of the star. Near the critical velocity, when the effective temperature of the equatorial regions can become very low, this hypothesis is no more fulfilled, and the variation of these parameters enhances the mass loss near the equator (see Sect. 4.3).

From Fig. 2, we see that the faster the star rotates, the more mass will be lost in the polar region. Typically the mass flux at the pole is greater than two times the mass flux at the equator for $\omega \gtrsim 0.8$. Integrating the mass flux from the pole to a given colatitude $\theta$ and dividing by the total mass loss rate, we obtain the fraction of the total mass lost in a cone of semi-aperture $\theta$, for a given rotation factor $\omega$. The result is shown in Fig. 3. For a non-rotating star (lower curve), we see that 50% of the total mass is lost in a cone of semi-aperture 60°. When the star is at the first critical velocity, the aperture of the cone containing 50% of the total mass loss is slightly reduced: it becomes around 48°.

3.3. Latitude dependency of the angular momentum loss

Once we know the local mass loss flux and the shape of the surface, it is possible to compute the local loss of angular momentum induced by the stellar winds, for a given angular velocity of the surface (we suppose in this work that the angular velocity of the surface $\Omega$ is constant over the whole star).

The loss of angular momentum per surface unit and time is given by:

$$\frac{d\dot{L}}{d\sigma} = \frac{\Delta \dot{M}}{\Delta \sigma}(\theta)\Omega R^2(\theta),$$

where $R$ is the distance from the considered unit surface element to the rotation axis at the colatitude $\theta$. Introducing the surface element $d\sigma = r^2 \sin(\theta) d\phi d\theta$, where $\varepsilon$ is the angle between the local effective gravity and the radial direction, and integrating over $\phi$ to obtain only the colatitudinal variation of the angular momentum loss, we have:

$$\frac{d\dot{L}}{d\theta} = 2\pi \frac{\Delta \dot{M}}{\Delta \sigma}(\theta) r^4 \Omega \sin^3(\theta) \frac{\cos(\varepsilon)}{\cos(\varepsilon)},$$

which is the contribution to the total angular momentum loss of an infinitesimal ring at the colatitude $\theta$. In order to avoid the $\Omega$-dependency, and as in our model, the surface of the star rotates at a constant angular velocity, we consider further the moment of inertia $I = \mathcal{L}/\Omega$. This permits to easily compare models with various rotation rates. The distribution of the momentum of inertia brought away by the wind is shown on Fig. 4.

On this figure, we see, for various rotation rate $\omega$, how the angular momentum lost is distributed as a function of the colatitude $\theta$. Two effects are in concurrence: first, the increase of the equatorial radius (see Fig.1), which tends to increase the angular momentum lost near the equatorial regions, and second, the decrease of the local mass loss rate near the equator, which tends to decrease the angular momentum lost in the same area.

Without rotation, the mass loss rate per surface unit is constant over the whole surface of the star, and the angle $\varepsilon$
between the effective gravity direction and the radial direction is zero. Examining eq. (10), we see that $d\dot{L}/d\theta$ varies as $\sin^3(\theta)$ (since all other terms are constant). The corresponding curve is labelled $\omega = 0$ on Fig. 4. Progressively increasing the rotation rate, we see that the deformation of the stellar surface produces an increase of the angular momentum loss in the equatorial region. Once the rotation factor $\omega \approx 0.75$, the increase of the equatorial radius becomes counterbalanced by the progressive decrease of the local mass loss flux in the same region. The angular momentum loss becomes thus more and more reduced at the equator, and the maximum of momentum loss is shifted towards the pole, up to a colatitude of $\sim 70^\circ$ when the rotation becomes critical. At that moment, no more angular momentum is lost at the equator, since there is no equatorial mass loss through radiative winds in this regime in our model.

### 3.4. Total angular momentum lost

Once the distribution of the angular momentum loss is known, we can integrate it over the colatitude to obtain the total angular momentum brought away by the stellar winds:

$$L = 2\pi \int_0^{\pi} \frac{\Delta M}{\Delta \sigma} (\theta)^{-1}\Omega^2 \sin^3(\theta) \cos(\varepsilon) d\theta.$$  \hspace{1cm} (11)

To well understand the effect of the anisotropic winds on the total angular momentum loss, we distinguish the following cases:

- **case 1**: we determine a mean stellar radius $r_{\text{mean}}$ using the following relation:
  $$L = \Sigma \sigma (\langle T^4_{\text{eff}} \rangle) \equiv 4\pi r^2_{\text{mean}} \sigma (\langle T^4_{\text{eff}} \rangle)$$  \hspace{1cm} (12)
  where $L$ is the stellar luminosity and $\Sigma$ the total stellar surface. We neglect the stellar deformation and the wind anisotropy, and the loss of angular momentum is thus computed on a sphere of radius $r_{\text{mean}}$, $L = \frac{2}{9} \Omega r_{\text{mean}}^2$. This radius is the one we would find if we measure the luminosity and the effective temperature of the star, and suppose that it is perfectly spherical. It gives the angular momentum loss as computed in numerical models where the effects of rotation on the shape of the surface and the mass loss distribution is neglected;

- **case 2**: the deformation of the star is accounted for, but the mass loss is uniformly distributed over the stellar surface (i.e. the winds anisotropy is not taken into account). This case is academical, but is interesting in the sense that it allows to see the effect of the deformation only;

- **case 3**: both the deformation of the shape of the surface and the anisotropy are accounted for. The angular momentum loss is computed with the relation (11).

The results we obtain are shown on Fig. 5. The top panel shows the variation of the moment of inertia lost by the wind per unit time in the cases “neither deformation, nor anisotropy”, computed with the mean radius discussed above ($\dot{L}_{\text{sh}}$, case 1), “deformation only” ($\dot{L}_{\text{de}}$, case 2), and “deformation + wind anisotropy” ($\dot{L}_{\text{ani}}$, case 3).
Fig. 3. Fraction of the total mass loss contained in a cone with a semi-aperture $\theta$ given by the $x$-axis. The lower curve is the result for a non-rotating star losing its mass isotropically. The upper curve is for a critically rotating star ($\omega = 1$). The intermediate curves are the results of the same rotation factor omega as in Fig. 2, i.e. $\omega = 0.5, 0.75, 0.9, 0.96, 0.98, 0.99, 0.995$ respectively (from bottom to top).

The increase of $\dot{I}_{\text{sph}}$ in case 1 with $\omega$ is due entirely to the increase of the mean radius $r_{\text{mean}}$ when $\omega$ increases. When the deformation is accounted for, we see that more angular momentum is lost because most of the mass leaves the surface of the star at a greater distance from the rotational axis. We see that, at the critical limit, deformation would increase the angular momentum loss rate by around 20% with respect to case 1. For case 3, we see that the wind anisotropy largely compensates for the effect of the deformation and slows down the rate of angular momentum loss by 49% with respect to case 2 and by 25% with respect to case 1 when the star rotates at the critical velocity ($\omega = 1$). We see that a proper account of the effects of the winds anisotropy needs also a proper account for the star deformation.

The lower panel of Fig. 5 shows the ratio of $\dot{I}_{\text{sph}}$ over $\dot{I}_{\text{ani}}$, or, in an equivalent way, of the ratio $L_{\text{sph}}/L_{\text{ani}}$. This curve shows the real impact of the account for the anisotropy of winds and the deformation of the stellar shape, compared with a model where we consider an isotropic spherical wind on the surface, with a radius determined by the stellar luminosity and mean effective temperature (as most of the stellar evolution codes).

Interestingly enough, the error committed on the angular momentum loss when neglecting the effects of wind anisotropies is small in most of the cases. It is less than 4% if $\omega < 0.9$. At the critical velocity, the error is larger. Up to 25% more angular momentum can be kept in the star when the effects of wind anisotropies are accounted for. Therefore, the effects of the wind anisotropies become important only for the faster rotators. This indicates that for most of the cases studied in stellar evolution, the precise account for the anisotropies are not relevant, and the errors induced by neglecting it will remain small.

4. Effects of fast rotation on angular momentum loss: a numerical approach

The analytic relations above can provide some orders of magnitude estimates of the impact of the wind anisotropies on the loss of angular momentum assuming that $\omega$ remains more or less constant as a function of time. To obtain more realistic values, it is necessary to compute numerical stellar models.

An additional complication, let aside in the above estimate, comes from the fact that the global mass loss rate increases with faster rotation. We first consider a case where the mass loss is kept constant. This will allow to make a more direct comparison with the semi-analytical results obtained above and thus to check that the implementation of the process was done correctly in the stellar evolution code. In a second step (see Sect. 4.3), we shall consider the case of a model with all the usual prescriptions, in particular accounting for the evolution as a function of time of the mass loss rates.

B.3. Articles submitted to a refereed review

Georgy et al.: Effects of anisotropic winds on massive stars evolution
4.1. Models with constant mass loss rate

We examine the effects of the anisotropic stellar winds on the evolution towards the critical velocity of two sets of models of $9 \, M_\odot$ star. To see some anisotropic effects already on the zero age main sequence (ZAMS), we launch the star with a very high initial rotation rate of 80% of the critical velocity. The metallicity is taken equal to $Z = 0.002$, i.e. equivalent to that of the Small Magellanic Cloud. The rotation is treated as in Maeder & Meynet (2005), accounting for the internal magnetic field and its impact on the transport of angular momentum (Spruit 2002). The account for the magnetic field ensures a strong coupling between the centre and the surface of the star, and leads to higher surface velocity. We expect thus a more important effect of the anisotropic winds in this context. These models were computed with a constant mass loss rate of $10^{-9} \, M_\odot \, yr^{-1}$, independent of the stellar surface parameters, and independent of the rotation rate. One model was computed with the account for the anisotropic winds, and one assuming isotropic winds. In both cases, the deformation of the shape of the star is accounted for. The anisotropic model evolves slightly faster than the isotropic one. During the time between the ZAMS and the reaching of the critical rotation rate, the anisotropic model (red dashed line) with constant mass loss rate. The rotation rate is indicated for some points along the tracks.

On Fig. 6, we show the Hertzsprung-Russel diagram (HRD) of both models. The points where the models reach 80%, 90% and 95% of the critical rotational velocity are indicated on the tracks. The ZAMS is bottom-left, and the evolution proceeds towards the top-right corner. We see that the account for the anisotropic winds has only a minor impact on the evolutionary track. Some small effects begin to appear when the surface velocity is around 90% of the critical velocity. The anisotropic model evolves slightly more in the red side of the HRD. This is due to the fact that this model rotates faster than the isotropic one (see the top panel of Fig. 7). Its surface is slightly larger, and thus, for a given luminosity, the mean effective temperature will be lower.

Fig. 7 shows on the top panel the evolution of the ratio $\Omega/\Omega_{crit}$ as a function of time for both models. The bottom panel shows the total content of angular momentum of the star. As expected, the mass loss due to the stellar winds causes a decrease of the angular momentum kept in the star. The model with the account for the wind anisotropy loses less angular momentum than the isotropic model. As a consequence, the stellar surface of the anisotropic model rotates slightly faster than the isotropic one.

The mean $\Omega/\Omega_{crit}$ for the anisotropic model is $0.867$. According to Fig. 5, we expect that the anisotropic model keep 1.13 times more angular momentum than the isotropic one. During the time between the ZAMS and the reaching of the critical rotation rate, the anisotropic star loses an amount of angular momentum $\Delta L_{ani} = 3.56 \cdot 10^{50} \, g \, cm \, s^{-1}$. During the same time, the isotropic model loses $\Delta L_{iso} = 4.13 \cdot 10^{50} \, g \, cm \, s^{-1}$. The final ratio $\Delta L_{iso}/\Delta L_{ani} = 1.16$ is very close to the estimate based on the mean $\Omega/\Omega_{crit}$.

4.2. Total mass loss rate as a function of $\omega$ and $\Gamma$

Before discussing the results of models accounting for time dependent mass loss rates, let us briefly recall how rotation enhances the global mass loss rate. Examining eq. (4), we see that the mass loss rate of a rotating star is simply expressed as a function of the angular velocity $\Omega$, the classical (i.e. with no account for the rotation) Eddington factor $\Gamma$ and the rotating one $\Gamma_\Omega$. Let us introduce in this relation the rotation parameter $\omega$ as defined above, and the definition of $\Gamma_\Omega$ given by relation (2):

$$\frac{\dot{M}(\Omega)}{\dot{M}(\Omega = 0)} = \left(1 - \frac{\Gamma}{\Gamma_\Omega}\right)^{\frac{\omega^2}{1 - \omega^2}}$$

To obtain Eq. (13), we used eq (5), and replaced the mean density $\rho_m$ by $V/M$, with $V$ the volume enclosed by the stellar surface.

Using the expression of $v_{crit,2}$ in Eq. (6), the expression above can be written

$$\frac{\dot{M}(\Omega)}{\dot{M}(\Omega = 0)} = \left(1 - \frac{\Gamma}{\Gamma_\Omega}\right)^{\frac{\omega^2}{1 - \omega^2}} \frac{1}{1 - \frac{\omega^2}{v_{crit,2}^2}}.$$  

Thus, we see that the mass loss enhancement is governed by the ratio $\omega/v_{crit, z}$, while the deformation of the star is governed by the ratio $\omega/v_{crit, \Omega}$. In the expression of $v_{crit,2}$ intervenes the Eddington factor $\Gamma$, thus the global enhancement factor of the mass loss will depend on two parameters $\Omega$ and $\Gamma$.

Fig. 8 shows the variation of the ratio $\dot{M}(\omega)/\dot{M}(\omega = 0)$ as a function of these two parameters. For this plot, we took a value for $\alpha = 0.43$, which is adapted for effective temperatures $4.05 \leq T_{eff} \leq 4.3$ (Lamers 2004, private communication). The volume $V$ enclosed by the stellar surface is numerically computed using eq. (8) for the shape of the surface.

For Eddington factors greater than 0.639, $v_{crit,2} < v_{crit,1}$ (Maeder & Meynet 2000), thus the second limit is the one to be considered. At this limit, the mass loss rate becomes...
very high. The precise value of the enhancement cannot be given since, at that limit and beyond, the hypothesis made in deriving eq. (13) no long holds. For instance, near the Eddington limit, continuous radiation field contributes in pushing out the outer layers, while for obtaining the value is equal to one. In that domain, the continuous emission will participate in pushing out the matter and very important mass loss rates are expected (van Marle et al. 2008). Depending on the value of \( \Gamma \), the winds can be more or less anisotropic for \( \Gamma \)-values just above 0.639, \( v_{\text{crit},2} \) is near \( v_{\text{crit},1} \) and strong anisotropies are expected, when \( \Gamma \) is near 1, \( v_{\text{crit},2} \) is much lower than \( v_{\text{crit},1} \) and the winds are expected to be isotropic.

4.3. Models with realistic mass loss rate

Here we discuss 9 M\(_{\odot} \) models using a more realistic mass loss rate (Vink et al. 2001), and accounting for the increase of the mass loss rate induced by rotation (see above). One model takes the effect of anisotropic winds into account, and one has an isotropic mass loss. We also included in the anisotropic model the variation of the force multiplier parameters \( k \) and \( \alpha \) due to the variation of the local effective temperature as a function of the colatitude. Models were followed until they reached the critical velocity.

The variations of the force multiplier parameters are accounted for in the following way. In the expression of the increase of the global mass loss rate due to rotation (13), we take \( \alpha \) and \( k \) given by the mean effective temperature of the star. For the computation of the anisotropic effects (1), we use at each colatitude \( \alpha \) and \( k \) corresponding to the local effective temperature, allowing variations over the stellar surface.

Fig. 9 shows the HRD for this set of models. The black curve represents the model with an isotropic mass loss over the surface, and the red curve the anisotropic one. As in the previous case, the tracks in this diagram are very similar for both models, even if a more complete physics is considered. This confirms that even with full treatment of the wind anisotropy, including a realistic mass loss rate and the variation of the force multiplier parameters, the effect of the anisotropic mass loss for very fast rotors remains very small.

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**Table 1.** Maximum rotation parameter \( \omega_{\text{max}} \) and maximum increase of the mass loss rate as a function of the Eddington factor \( \Gamma \) (see text).

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( \omega_{\text{max}} )</th>
<th>( M(\omega)/M(0) )</th>
<th>( \Gamma )</th>
<th>( \omega_{\text{max}} )</th>
<th>( M(\omega)/M(0) )</th>
</tr>
</thead>
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<tr>
<td>0.0</td>
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<td>1.810</td>
<td>0.6</td>
<td>1.0</td>
<td>21.696</td>
</tr>
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<td>0.619</td>
<td>1.0</td>
<td>( \infty )</td>
</tr>
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<td>1.0</td>
<td>2.214</td>
<td>0.7</td>
<td>0.968</td>
<td>( \infty )</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0</td>
<td>2.612</td>
<td>0.8</td>
<td>0.861</td>
<td>( \infty )</td>
</tr>
<tr>
<td>0.4</td>
<td>1.0</td>
<td>3.383</td>
<td>0.9</td>
<td>0.659</td>
<td>( \infty )</td>
</tr>
<tr>
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<td>1.0</td>
<td>5.444</td>
<td>0.95</td>
<td>0.484</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

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**Fig. 7.** Top panel: Rotation rate as a function of time for the isotropic (black solid line) and anisotropic model (red dashed line) with constant mass loss rate. Bottom panel: Total angular momentum contained in the star in unit of \( 10^{53} \text{g cm}^2 \text{s}^{-1} \). The models are represented as in top panel.

**Fig. 8.** Logarithm of the ratio \( \dot{M}(\omega)/\dot{M}(\omega = 0) \) as a function of \( \omega = \omega_{\text{crit}} \) for various values of the Eddington factor (value indicated at the top of each curve). For \( \Gamma \geq 0.639 \) the curve tends towards infinity when \( \omega \) approaches \( \omega_{\text{max}} \) (see table 1). \( \alpha \) is set to 0.45 (see text).
On the top panel of Fig. 10, we see the evolution of the ratio $\omega$ as a function of the central hydrogen mass fraction for the isotropic model (black solid line) and the anisotropic one (red dashed line). The ZAMS is on the left, and the point where $\omega = 1$ is on the right. The anisotropic model rotates ever faster than the isotropic one, but the difference is limited. The rotation increase of the isotropic model near $X_c = 0.52$ produces the crossing of the tracks in the HRD (see Fig. 9). The inflation of the surface induced by the higher rotation rate decreases the mean effective temperature, pushing the track on the right.

The bottom panel of Fig. 10 shows an interesting feature. From the ZAMS until $X_c \sim 0.4$, the behaviour of the total angular momentum contained in the whole star is similar to the models with constant mass loss rate: the anisotropic model keeps more angular momentum due to the polar wind. However, from that point, the star reaches a high enough rotation rate to strongly decrease the equatorial effective temperature. The force multiplier parameters are different in this area, and generate a strong equatorial mass loss. This effect produces the change in the angular momentum loss rate: the anisotropic model loses more angular momentum than the isotropic one. When the star reaches the critical limit, the isotropic model finally has a larger angular momentum content than the anisotropic one!

The final angular momentum of the star strongly depends on the angular momentum removed by the mechanical mass loss that the star undergoes during the critically rotating phase. It is difficult to estimate which of the isotropic or anisotropic model will have the higher content at the end of the stellar evolution, and to quantify this difference without a model accounting for the mechanical mass loss. This question, and first estimates of the mass lost in the equatorial disk, will be addressed in a forthcoming paper.

5. Conclusion

The main result of this paper is the fact that radiative wind anisotropies will not deeply affect the angular momentum content of stars, in contrast with previous estimates. The different conclusion obtained here comes mainly from a precise account of the effect of the surface deformation in addition to the effects induced by the variation of the mass flux with the colatitude. Interestingly the account for the variation of the force multiplier parameters over the surface when the star is near the critical limit can favour an equatorial-enhanced mass loss rather than a polar mass loss. In that case, the angular momentum loss when the effects of wind anisotropies are accounted for can be higher than when they are neglected!

If the anisotropic winds do not seem to have a strong influence on the evolution of the star, the strongly enhanced polar flux has probably a big importance on the evolution of the circumstellar medium (see Georgy et al. 2009). The creation of a strongly asymmetric nebula around fast rotating stars is likely.

Another point however which appeared in that work is the importance of the equatorial mass loss triggered by the reaching of the first critical limit. Two effects can act to keep the star at the critical limit: the first one is the variation of the force multiplier parameters $\alpha$ and $k$ in the equatorial regions when the local effective temperature becomes low enough under the effect of rotation. It triggers strong equatorial radiative winds, and is already accounted for in

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**Fig. 9.** HR diagram for the isotropic model (black solid line) and anisotropic model (red dashed line) with realistic mass loss rate. The rotation rate is indicated for some points along the tracks.

**Fig. 10.** Top panel: Rotation rate as a function of the central hydrogen mass fraction for the isotropic (black solid line) and anisotropic model (red dashed line) with realistic mass loss rate. The evolution proceeds from left to right. Bottom panel: Total angular momentum contained in the star in unit of $10^{53} \text{g cm}^2 \text{s}^{-1}$. The models are represented as in top panel.
this study. The other is the mechanical mass loss in the equatorial plane, when the equatorial effective gravity vanishes. In that case, the important losses of material through an equatorial disk remove a lot of angular momentum. In a forthcoming paper we shall study in a quantitative way the impact of such an anisotropic mechanical mass loss on the evolution of fast rotating stars.

Finally let us stress that the presence of a strong enough magnetic field at the surface may keep a strong coupling between the wind nebula and the star and thus imply much stronger losses of angular momentum than accounted for in the present models.

References

Maeder, A. 2009, Physics, Formation and Evolution of Rotating Stars (Springer Verlag)
Homogeneous evolution near the $\mu^2M$ limit

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ABSTRACT

Aims. We derive from basic relations an upper limit for the quantity $\mu^2M$ for hot and homogeneous stars, $\mu$ being the mean molecular weight and $M$ the mass of the star. We study the effects of the mass loss obtained when a hot, homogeneous star evolves near that limit.

Methods. We computed very fast rotating 60 $M_\odot$ stellar models evolving homogeneously. When the evolution of the star reaches the critical limit $M_\mu^2 = C_{\text{EDD}}$, where $C_{\text{EDD}}$ is a constant, the star begins to lose mass at a rate such that the star remains at this limit.

Results. We show that the $\mu^2M$ limit for hot, homogeneous stars is a consequence of the mass–luminosity relation and of the Eddington limit. The main result of this paper is that all properties of WR stars in the WNE and WC stages as observed and given by usual models, can be reproduced by models where the condition $M_\mu^2 \leq C_{\text{EDD}}$ is used. In the WNE and WC stages the stars evolve keeping at the upper limit, where the mass loss rates are independent of the value of the constant. The agreement is verified for the mass loss rates, the actual WR masses and luminosities, the surface abundances in He, N, C and O and the surface rotational velocities.

As side remark, we suggest that the WNL stars may evolve at the $\Omega$–limit, where rotation plays a role. We also point out that collapsar progenitors evolve near the $\mu^2M$ limit and lose important amounts of angular momentum. This reduces the domain of initial conditions leading to long soft Gamma Ray Bursts.

Key words. stars: evolution, stars: mass-loss, stars: Wolf-Rayet

1. Introduction

The evolution of hot homogeneous stars is interesting for at least three purposes:

1. Some stars in their advanced phases can be considered as nearly homogeneous. These are the Wolf-Rayet stars of the types WNE, which are nearly pure He-core and the WC and WO stars, which are nearly pure carbon-oxygen cores (see e.g. the review by Crowther 2007).

2. Some authors have proposed homogeneous evolution of massive hot stars as the scenario leading to long–soft gamma ray bursts (GRB, Yoon & Lauger 2005; Woosley & Heger 2006). In that case, homogeneity is due to the fast axial rotation of the star.

3. Homogeneous stars can also be formed during merging of stars. This scenario has been recently reexamined by Glebbeek et al. (2009) as a possible scenario leading to intermediate mass black hole in globular clusters.

As is the case for all massive stars, mass loss is a key physical ingredient governing the evolution of hot homogeneous stars. Unfortunately this quantity is still not very well known. For instance, mass loss rates for Wolf-Rayet stars are still debated. Different prescriptions exist, giving the mass loss as a function of various stellar parameters (see e.g. Hamann et al. 1995; Schmutz 1997; Nugis & Lamers 2000; Eldridge & Vink 2006; Gräfener & Hamann 2008).

The mass loss prescriptions used in the computations of the progenitors of long soft GRB and of the results of stars mergers are those used in standard computations and thus present the same degree of uncertainties.

We propose here a mass loss rate prescription different from those used in those works. First, we show here that an upper limit for $\mu^2M$ exists for hot homogeneous stars (Sect. 2). Then, we deduce from this limit a prescription for the mass loss rate\(^1\) (Sect. 3). We apply such a prescription to the evolution of magnetic fast rotating stars. The ingredients of the models are briefly described in Sect. 4. In Sect. 5, we present the results of our computations and compare it with a standard model. Some consequences of the evolution near the $\mu^2M$ limit are discussed in Sect. 6. Finally, we present our conclusions in Sect. 7.

2. A $\mu^2M$ upper limit for hot homogeneous stars

An upper limit for $\mu^2M$, where $\mu$ is the mean molecular weight and $M$ the actual mass can be derived for hot and homogeneous stars using the mass-luminosity relation and the expression for the Eddington luminosity.

2.1. The mass-luminosity relation

The mass luminosity relation can be obtained from the three equations below: the equation of radiative equilibrium

$$\frac{L}{4\pi r^2} = -\frac{ac}{3\mu} \frac{dT}{dr}. \quad (1)$$

\(^1\) The present mass loss rate prescription is similar to that used in Maeder (1985), although its physical justification is somewhat different. Also we apply it in a different context.
the equation of hydrostatic equilibrium
\[ \frac{dP}{dr} = -\rho g, \]  
(2)

the equation of state, where we suppose here a mixture of perfect gas and of radiation,
\[ P = \frac{P_{\text{gas}}}{\beta} = \frac{k}{\beta \mu \mu_{\text{M}_{\odot}}} \rho T. \]  
(3)

The equation of hydrostatic equilibrium tells us that
\[ P \sim \frac{M^2}{R^4}, \]  
(4)

where the subscript c indicates that the quantity is evaluated at the centre. The equation of state implies that
\[ T_c \sim \frac{\mu \beta}{\rho_c} M \rho. \]  
(5)

The radiative equilibrium equation tells us that
\[ L \sim \frac{R}{\kappa \rho} T_c^4, \]  
(7)

where \( \kappa \) is the mean value of the opacity in the star.

Inserting the expression of \( T_c \) as a function of \( M \) and \( R \) inside the expression of \( L \), we obtain
\[ L \sim \mu^4 \beta^4 \rho_c^4 R^3 \kappa. \]  
(8)

We can write a similar equation for a reference homogeneous model (it is not correct here to use solar values as the reference model since the sun is not a hot homogeneous star and thus only very approximately follows the relations given above). Let us give a subscript \( \text{r} \) to this reference model. We can then write
\[ L_R \sim \mu_r^4 \beta_r^4 \rho_{c_r}^4 \frac{M_R^3}{\rho_{\text{c}_R}^4} \kappa_R. \]  
(9)

Finally, dividing \( L \) by \( L_R \), we obtain
\[ \frac{L}{L_R} = \mu^4 \beta^4 \rho_c^4 M^3 \rho_{\text{c}_R}^4 \kappa_R. \]  
(10)

Fig. 1. The continuous line shows the value \( C_{\text{EDD}} \) of as a function of the mass. For some points, we have indicated the mass-averaged values of the mean molecular weight, \( \mu \), of the central density, \( \rho_c \), of the mass-averaged density and of the central density, respectively, \( \rho \) and \( \rho_c \), in g cm\(^{-3} \).

The present homogeneous stellar models B were used to compute these quantities.

2.2. The Eddington limit

Near the Eddington limit, we have by definition that the radiative acceleration is equal to the gravity. This will occur only in the most outer layers. In these layers we have that
\[ \frac{1}{\rho_c} \frac{dP_{\text{rad}}}{dr} = \frac{\kappa_s L_{\text{EDD}}}{4 \pi c R^2} = \frac{GM}{R^2}, \]  
(11)

This gives the classical Eddington limit that we normalize with the luminosity \( L_R \).
\[ \frac{L}{L_R} \leq \frac{L_{\text{EDD}}}{L_R} = \frac{4 \pi c GM}{\kappa_s L_R}. \]  
(12)

At that point however we might wonder the following question: should we also account for the acceleration due to the perfect gas pressure gradient? We might indeed think that the matter will begin to be expelled before the Eddington limit is reached, since in addition to the radiative acceleration there will be also an acceleration induced by the perfect gas pressure gradient. But this line of reasoning is not correct. In a perfect gas star we have that at every level the acceleration induced by the perfect gas pressure is equal to the gravity but we have no instability linked to that! This is due to the fact that the perfect gas pressure gradient adjusts itself in order to compensate for the gravity everywhere inside the star.

In the case of radiation, the situation is different. In the outer layers, the luminosity is given by the condition of hydrostatic equilibrium, this means the radiative acceleration cannot be changed by local movement of the matter.
Let us imagine that we can increase gradually the radiative acceleration, what would happen? First the perfect gas pressure gradient will decrease, since there is less need of it to compensate for the gravity. At a given point the matter can be completely sustained by radiation making the perfect gas pressure gradient equals to 0 and if the radiative acceleration still increases then instabilities will set in. Thus we see that it is correct not to account for the acceleration of the perfect gas pressure in the Eddington limit.

Finally let us stress that in stellar models at the Eddington limit, supra-Eddington luminosity is reached only in the very outer layers. Typically, as shown in Maeder (1989) and in Maeder (1992, see Fig. 3), the mass above the supra-Eddington layers is only of the order of $10^{-2} - 10^{-3} M_\odot$. Thus, in situations, where the star is at the Eddington limit, the layers where $\beta$ tends to zero contain very little amount of mass. In most of the star, the value of $\beta$ is greater than 0.4-0.5!

2.3. An upper limit for $\mu^2 M$

The luminosity needs to be inferior to the Eddington limit, $L_{\text{Edd}}$, otherwise "... the radiation observed to be emitted ... would blow up the star" as written by Eddington (1926). The Eddington luminosity, $L_{\text{Edd}}$, is such that

$$ L/L_\odot = \frac{L_{\text{Edd}}}{L_\odot} = \frac{4 \pi c G M}{\kappa_L R} $$

which implies after some rearrangements that

$$ \mu^2 \frac{M}{M_\odot} \leq \left( \frac{\rho}{\kappa_r} \right) \left( \frac{R}{R_\odot} \right)^2 \frac{M}{M_\odot} \left( \frac{4 \pi G c}{\rho} \right)^2 \frac{c}{\rho c} \frac{c}{\rho}.$$

We have simplified the mean opacity $\kappa$ with the surface opacity $\kappa_s$ since, when the star is hot enough, the free electron opacity is the dominant effect and thus would be the same in the whole stellar structure in a homogeneous star.

As a reference model, we chose our 60 $M_\odot$ stellar model (model B in Table 1), when its actual mass is equal to 30 $M_\odot$. We obtain the following values for the different quantities involved in the equation above: $L_\odot = 684000 L_\odot$, $\rho/L_\odot = 0.33$, $\rho/L_\odot = 0.50$, $\kappa_r = 0.19$, $\mu_r = 1.21$. Inserting these numerical values we obtain

$$ \mu^2 \frac{M}{M_\odot} \leq C_{\text{Edd}} = 1.9 \left( \frac{\rho}{\kappa_r} \right)^2 \left( \frac{\rho}{\rho c} \right)^2. \quad (15) $$

For such a limit to have a sense, it should be a constant or at least only show a weak dependence with the mass. To check that point, in Fig. 1, we show estimates of $C_{\text{Edd}}$ using our homogeneous stellar models to compute $(\frac{\rho}{\kappa_r})^2 (\frac{\rho}{\rho c})^2$.

The continuous line has been obtained using our 60 $M_\odot$ model (model B in Table 1). We have considered only the part of the track where homogeneity is realized since this is a necessary condition for applying the above relations. This part covers the whole phase from the last part of the core hydrogen burning phase to the end of the core He-burning phase. We can see that, in the mass range from 15 to 55 $M_\odot$, $C_{\text{Edd}}$ remains around a value of 62.3 $M_\odot$, with variations less than 1.3%. Thus this limit is fairly well constant.

Let us recall that the $\mu^2 M$ limit as determined above applies only to homogeneous stars and for cases where the opacity is due to free electron scattering opacity (this last hypothesis makes possible the simplification between the averaged opacity appearing in the mass-luminosity relation and the surface opacity in the Eddington luminosity).

Before the strict $\mu^2 M$ limit is reached, instabilities may appear which can also help in driving some mass loss, like the $\varepsilon$-mechanism, the strange modes (Glatzel et al. 1993), and even the $\varepsilon$-mechanism. For all these mechanisms there are still great uncertainties on how they may drive mass loss and to which extent. Let us simply note here that, from a detailed analysis of the $\varepsilon$-process, Ledoux (1941, 1970) gives an upper limit for $\mu^2 M$ equal to 40, and Schwarzschild & Hars (1958) a value of around 22 in the following, in order to account for instabilities which may occur before the $\mu^2 M$ limit is reached, we have explored the cases when $C_{\text{Edd}}$ is equal to respectively 22 and 44.

3. Mass loss rates near the $\mu^2 M$ limit

When $\mu^2 M$ is equal to $C_{\text{Edd}}$, a mass loss is applied so that the star remains at this critical limit. The mass loss rate is given by

$$ \dot{M}_{\text{Edd}} = \frac{2 M}{\mu}. \quad (16) $$

We note that the mass loss rate does not depend directly on the value of $C_{\text{Edd}}$.

Let us estimate the order of magnitude of the mass loss rates that we can expect from this limit. Eq. (16) indicates that an increase of 2% of the mean molecular weight induces a decrease of 1% of the mass. During the main sequence (MS), the mean molecular weight increases from around 0.6 (a mixture of 75% of H and 25% of He) to 1.3 (pure He), that means by $\sim 120\%$. The mass should then decrease by about 60% during the MS. The lifetime of a fast rotating 60 $M_\odot$ star is around 5 Myr. Thus, a mean mass loss rate can be estimated. It is equal to $\sim 0.7 \times 10^{-3} M_\odot/yr$, slightly inferior to the mass loss rates of WR stars, but of the same order of magnitude. This great similarity in the mass loss rates is striking and indeed indicates that such stars are not far from the $\mu^2 M$ limit. As we shall see below this is indeed the case (see Fig. 8).

As emphasized above, the mass loss rates do not depend on the value of $C_{\text{Edd}}$. The value of $C_{\text{Edd}}$ determines when the mass loss rate given by eq. (16) switches on and thus how long the above process shapes the mass loss. Thus, its value somewhat encompasses the uncertainties bearing on what happens near the $\mu^2 M$ limit. Lower its value, earlier in the evolution the $\mu^2 M$-mechanism will set in and more mass will be lost due to this process. Ideally, of course, a stability analysis should be made for all evolutionary models with a detailed treatment of the running waves in the outer layers to get the mass loss rates at each time.

4. Description of the models

In this work, we computed a set of 3 models of a 60 $M_\odot$ star at the metallicity of the Small Magellanic Cloud, with a very high initial velocity rate of $\Omega_{\text{SMC}} = 0.9$ (corresponding to an equatorial velocity $v_{\text{eq}} = 690 \text{km s}^{-1}$). The initial
composition of the star was taken as follows: $X = 0.747$, $Y = 0.251$ and $Z = 0.002$. The distribution of the heavy elements is taken as in the Sun (Asplund et al. 2005), in particular, $X(^{12}\text{C}) = 3.24 \cdot 10^{-4}$, $X(^{14}\text{N}) = 9.38 \cdot 10^{-5}$ and $X(^{16}\text{O}) = 8.13 \cdot 10^{-4}$.

The high velocity chosen ensures an efficient transport of the angular momentum and chemical species, and allows the star to follow a quasi-homogeneous evolution. The three models were computed until the end of core helium-burning. The treatment of the internal coupling due to magnetic fields is computed following the Tayler-Spruit dynamo theory (Spruit 2002), as in Maeder & Meynet (2005). The internal magnetic fields generate a strong coupling of the rotation between the core and the surface, producing a quasi-solid body rotation. The rotation favors the internal mixing of the star, by both meridional circulation and magnetic fields. This favors an homogeneous evolution, needed to validate the mass loss rate prescription we used in this work. We checked that this hypothesis remains valid most of time. In Fig. 2, we show for two different stages of the evolution (mid-main sequence, and beginning of He-burning) the internal profiles of the main abundances and of the angular velocity. We see (left panel) that during the MS, the homogeneous assumption is well respected, with only small variations in the abundances near the surface. For the more advanced phases (right panel), some more important variations occurs near the surface, but around two third of the star remains homogeneous.

Fig. 2. For the model C (see Table 1), at two times : 2 Myr (on the left, main sequence) and 5 Myr (on the right, helium ignition). Upper panel: Abundance profile. H is in black (solid line), He in red (dotted line), C in blue (short-dashed line), N in green (long-dashed line), and O in violet (dot-dashed line). Lower panel: Angular velocity profile for the same model (colour figure available on-line).

Table 1. Main parameters of the models. First column gives the label, second one the initial mass, third one the metallicity and the last column gives the mass loss law (see text).

<table>
<thead>
<tr>
<th>Model</th>
<th>mass $[M_{\odot}]$</th>
<th>$Z$</th>
<th>mass loss law</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>60</td>
<td>0.002</td>
<td>standard</td>
</tr>
<tr>
<td>B</td>
<td>60</td>
<td>0.002</td>
<td>$\mu^2M = 44$</td>
</tr>
<tr>
<td>C</td>
<td>60</td>
<td>0.002</td>
<td>$\mu^2M = 22$</td>
</tr>
</tbody>
</table>

4.1. Mass loss rates

The three computed models A, B and C differ by the mass loss rates used (see Table 1): Model A is the reference model, computed without taking into account the mass loss due to the $\mu^2M$ limit. The mass loss rates adopted are the following:

- on the main sequence, if the star is not in the Wolf-Rayet (WR) phase, we used the mass loss prescription of Vink et al. (2000, 2001);
- as soon as the star enters in the WR phase (see below), we use the prescription of Nugis & Lamers (2000), scaled with metallicity (Eldridge & Vink 2006).

The criteria used to determine if a star is in the WR phase is the same as in Meynet & Maeder (2005): the star must have a $\log(L_{\text{Edd}}/L_{\text{normal}}) > 4$ and a surface hydrogen mass fraction $X_H < 0.4$.

The two other models take into account the mass loss due to the $\mu^2M$ limit. The first assumes a maximum mass for the star given by $M_{\mu^2M} = 44$ (model B), the second has a maximum mass given by $M_{\mu^2M} = 22$ (model C). On the ZAMS, the mean molecular weight in the star is $\mu = 0.594$, and we have $M_{\mu^2M} \approx 21.16$. Thus, models B and C are under their critical mass. We let them evolve until they rich their own critical masses. After this point, the mass loss rate adopted is given by $\dot{M} = \max(\dot{M}_{\text{edd}}, \dot{M}_{\text{normal}})$, where $\dot{M}_{\text{edd}}$ is given by eq. (16), and $\dot{M}_{\text{normal}}$ is the mass loss rate obtained using the standard recipes given above. To evaluate $\dot{M}_{\text{edd}}$, we compute at each time step the mean molecular weight $\mu$ across the whole star. This allows us to determine the maximum mass of the star given by $M_{\mu^2M}/\mu^2$, and compare it to the actual mass of the star. If this mass is too high according to the $\mu^2M$ limit, we remove the mass left over.

In all cases, the effect of rotation on the mass loss rate is taken into account (Maeder & Meynet 2000).

5. Results of the computations

5.1. Mass loss rates

The mass loss rates are represented in Fig.3. The first noticeable point is that the two models computed with the mass loss given by the $\mu^2M$ limit present a mass loss rate on average one order of magnitude higher than the standard mass loss rate used during the MS (the rate of Vink et al. 2000, 2001). The strong enhancements of mass loss shown by models C and B at respectively an age of 0.25 and 3.5 Myr on Fig. 3 are due to the reaching of the $\mu^2M$ limit, while the enhancement shown by model A at an age of 3.5 Myr is due to the entrance of the model into the WR phase.
Since model C has a strong wind, already during its O-star phase, such a star would show the typical signature of the strong winds of WR stars, but with the chemical composition of the winds of an O-star, i.e. a larger fraction of H and a lower content of He than in the WR winds.

Our numerical simulations confirm the simple order of magnitude estimates for the mass loss rates made in Sect. 2. They show also that depending on the value of the upper limit for $\mu^2M$, mass loss rates slightly above or below the usual WR mass loss rates are obtained. Indeed, the WR mass loss rates adopted in model A are well framed by the two models accounting for the $\mu^2M$ limit. This also supports the idea that the $\mu^2M$ limit plays an important role in WR mass loss rates.

We note also that once the mass loss is driven by the $\mu^2M$ limit, the mass loss rates remain constant over time. Following Eq. 16, this implies that $M\dot{\mu}/\mu$ remains constant, or that the inverse of the timescale for the variation of $\mu$ (i.e. $\dot{\mu}/\mu$) varies as the inverse of the mass. This means that when the mass decreases as a function of time, the $\mu$ variation timescale decreases also. This simply reflects the fact that, when evolution proceeds (and $M$ decreases), the change of $\mu$ accelerates, due to increasing central temperatures.

The constancy of the mass loss rates induced by the $\mu^2M$ limit implies that the luminosity roughly varies as the inverse of the mean molecular weight. Indeed it can be easily shown that $\dot{\mu}/\mu$ is proportional to about $\mu X$, where $X$ is the time derivative of the mass fraction of hydrogen. Now, $X$ is proportional to the luminosity through the relation $L = MX \cdot 0.007c^2$ (in the classical version of this equation, a parameter $q$ equal to the mass fraction occupied by the convective core appears. Since here the star is homogeneous, $q$ is close to one). Thus, $X$ is proportional to $L/M$. Inserting these relations into the expression for $\dot{M}$ leads to $\dot{M}$ proportional to $\mu (L/M) \cdot M = \mu L$. Then, the near constancy of $\dot{M}$ implies that $L$ varies as the inverse of $\mu$. This rule only applies when the star is in radiative equilibrium, i.e. when the luminosity is compensated by nuclear energy production in the centre. Thus, when the $\mu^2M$ limits enters into play, the luminosity decreases when $\mu$ increases (see Fig. 6).

At the end of the MS, the quick evolution towards higher luminosity and towards the red part of the HRD (see Fig. 6) produces the small peak in the mass loss rates for our 3 models (at around 4.6 Myr, 4.7 Myr and 5.0 Myr for models A, B and C respectively). During this short phase, the radiative mass loss computed following the standard recipe becomes larger than the mass loss rate induced by the $\mu^2M$ limit for our models B and C. This explains the variations in $\dot{M}$ in Fig. 3. Interestingly, during that phase the star approaches the $\Omega \mu^2$-limit as defined in Maeder (2002), i.e. the actual luminosity of the star is near the expression of the Eddington luminosity modified by the effects of rotation. It might be that this peculiar phase is associated with a much stronger mass loss episode than presently accounted for in the models, and be somewhat related to the apparition of ring nebulae around WR stars. This will be studied in a forthcoming paper. Interestingly, this behaviour at the end of the core H-burning phase is quite general and occurs also for non homogeneous evolution.

5.2. Evolution of the mass and of the surface velocities

The evolution of the mass of models A, B, and C is represented in Fig. 4. As expected from the mass loss rates (see Fig. 3), once the mass loss is ensured by the $\mu^2M$ limit, the mass decreases linearly with time. Model C undergoes an important mass loss during almost all the main sequence and all the helium-burning sequence. This implies that its final mass is the lower of the three models: $7.6 \, M_\odot$.

Model B has a more important mass loss rate than model C when it evolves along its critical mass, but this occurs only near the end of the main sequence. Integrated over the whole evolution, the total mass lost is smaller than for the model C, and the final mass of this model is $14.6 \, M_\odot$.

Our reference model (A) undergoes a smaller mass loss than model B almost all the time. The mass of this model at the end of the helium-burning is $29.3 \, M_\odot$. The amount of matter re-injected in the interstellar medium is thus very different for our three models: from around $30 \, M_\odot$ for the standard case, up to more than $50 \, M_\odot$ for the model which encounters the stronger mass loss.

We see that there is some convergence between the tracks of models B and C in Fig. 4: the one which enters later in the $\mu^2M$ unstable phase compensates its late entry by a stronger mass loss rate.

It is striking that the final masses and luminosities obtained with the simple recipe (16) are in excellent agreement with the estimates of the final masses and luminosities of WR stars.

Seeing the strong mass loss rates of our models when they encounter the $\mu^2M$ limit or when they are in their WR phase, we expect a strong influence of the mass lost, and thus, of the angular momentum removed, on the surface velocity. Our models were computed with a very high ini-
Fig. 4. Evolution of the mass of our 3 models as a function of time. The models are represented as in Fig. 3. For each track, we indicate the moment when the star enters the WNL, WNE and WC phase (colour figure available on-line).

Fig. 5. Time evolution of the surface angular velocity of our models. The models are represented as in Fig. 3 (colour figure available on-line).

tential velocity of $\Omega/\Omega_{\text{crit}} = 0.9$, to ensure a mixing as efficient as possible. The evolution of the surface angular velocity is shown in Fig. 5. Due to the mass loss, which removes angular momentum from the star, the angular velocity decreases with time during the main sequence. Models A and B show a very similar behaviour in the evolution of their surface velocity. For models B and C, the decrease in the surface velocity accelerates when the star enters in the $\mu^2M$ limit phase. For model A, the same phenomenon occurs, but it is produced by the entrance in the WR phase.

5.3. Hertzsprung-Russell diagram and lifetimes

The evolutionary tracks of our 3 models in the Hertzsprung-Russel Diagram (HRD) are shown on Fig. 6. Our reference model is in black (solid line), model B is in red (short-dashed line) and model C in blue (long-dashed line). All the models begin at the same point on the ZAMS. Model C, which is already very close to its critical mass, leaves the reference track very early after the ZAMS. Model B takes more time to reach the critical mass and remains on the reference track. It reaches its critical mass just before the star becomes a WR, and leave the reference track at this moment.

Even if the mass loss laws are different between these three models, we see from Fig. 6 that the evolutionary tracks show the same qualitative behaviour. For the reference model, the sharp change towards the blue side of the HRD when it reaches the WR phase is due to the stronger mass loss rate used during this phase. It allows to uncover some hotter layers, producing the bluwards evolution. For the models B and C, the increase of the mass loss rate is not produced by the entrance in the WR phase, but because they reach their respective critical mass, and thus, lose mass to remain at this limit. Models B and C lose more mass than model A (see Sect. 5.1) and thus evolve along lower luminosity tracks.

Once the central hydrogen is exhausted (label $X_C = 0$ in Fig. 6), all the models evolve temporarily towards the right of the HRD, up to the ignition of the central helium (label He-b). In this segment of the track, the models are out of thermal equilibrium, the core contracts, producing a rapid energy output. The very thin envelope of the star is not very efficient to absorb this energy, and it is evacuated through an increase in the luminosity. After He ignition, the star is again in thermal equilibrium. The luminosity decreases, mainly due to the decrease of the mass. The star progressivley contracts, producing an increase in the effective temperature. Interestingly, all our models make a quick “red loop” when the surface abundance of hydrogen becomes null (label $X_S = 0$). This is due to the modification of the opacities near the surface at this point.

In Table 2, the duration of the MS phase and of the He-burning phase, and the total lifetime of our models are indicated. These values vary by more than 10% between model A and model C. This is due to the difference of luminosity between the 3 models : model C is roughly 1 order of magnitude less luminous than model A (see Fig. 6), thus its lifetime is increased.

5.4. Surface abundances and WR phases

The time evolution of the surface abundances is shown in Fig 7. As the star loses its mass, layers with more and more H-burning products are uncovered. This implies an increase of the He content. In the same way, as the hydrogen is burned through CNO cycle, the relative abundances of this
Table 2. Time spend in the MS and He-burning sequence and total lifetime, and in the O-star, WNL, WNE, WC phases for our 3 models (times given in [Myr]).

<table>
<thead>
<tr>
<th>Model</th>
<th>MS lifetime</th>
<th>He-b lifetime</th>
<th>O-star</th>
<th>WNL</th>
<th>WNE</th>
<th>WC</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4.71</td>
<td>0.34</td>
<td>5.94</td>
<td>3.68</td>
<td>1.26</td>
<td>0.02</td>
</tr>
<tr>
<td>B</td>
<td>4.78</td>
<td>0.39</td>
<td>5.17</td>
<td>3.58</td>
<td>1.30</td>
<td>0.04</td>
</tr>
<tr>
<td>C</td>
<td>5.05</td>
<td>0.55</td>
<td>5.60</td>
<td>3.97</td>
<td>1.26</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table 3. Total H, He, C, N and O masses ejected in the winds during the star lifetime.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30.674</td>
<td>6.168 (20.1%)</td>
<td>23.325 (76.0%)</td>
<td>0.947 (3.1%)</td>
<td>3.2 · 10^{-2} (0.1%)</td>
<td>0.172 (0.6%)</td>
</tr>
<tr>
<td>B</td>
<td>45.436</td>
<td>8.972 (19.7%)</td>
<td>34.510 (76.0%)</td>
<td>1.659 (3.7%)</td>
<td>4.6 · 10^{-2} (0.1%)</td>
<td>0.241 (0.5%)</td>
</tr>
<tr>
<td>C</td>
<td>52.431</td>
<td>24.970 (47.0%)</td>
<td>27.106 (51.7%)</td>
<td>0.244 (0.5%)</td>
<td>5.2 · 10^{-2} (0.1%)</td>
<td>1.7 · 10^{-2} (0.0%)</td>
</tr>
</tbody>
</table>

Fig. 6. Hertzsprung-Russel diagram for our 3 models. Some key points of the evolution are indicated: the position of the ZAMS, the beginning of the WR phase, the point where the central abundance of hydrogen becomes zero (label \(X_C = 0\)), the beginning of the central helium-burning phase (label He-b) and the point where the surface abundance of hydrogen becomes zero (label \(X_S = 0\)). Here, the effective temperature \(T_{\text{eff}}\) is not the temperature in the optically thick winds, but the photospheric temperature of the WR star. The models are represented as in Fig. 3 (colour figure available on-line).

3 elements are modified, up to the equilibrium abundances of the CNO cycle. N is produced to the detriment of C and O.

At the very late stages of the evolution, the products of He-burning begin to appear on the surface, and the abundances of C and O increase, while the He abundance decreases quickly. The differences between the models A, B, and C in term of relative surface abundances of He, C and O at the end of He-burning reflects the fact that the core is uncovered at different stage of its evolution, depending of the mass loss rate history. Here again, we note the similarity between models A and B, which directly support the idea that WR stars evolve very close to the \(\mu^2 M\) limit.

With the surface abundances, it is possible to determine the type of star we have at any times. We use here the same criteria than Georgy et al. (2009). Due to the strong mass loss rates we used in this work, all our stars follow the path O-star → WNL → WNE → WC, and end their life as a WC star. No one of the 3 models finishes its evolution as a WO star. In Table 2, the time spend in each of the previous phases are indicated.

We see that the O-star phase is increased for model C, as well as the WNE phase. This model encounters a strong mass loss very soon in its evolution. This reduces significantly the mass of the star already during the MS.
The He-burning core will thus be much smaller than in a model which loses less mass. This means that the portion of the star depleted in helium, but not enriched in He-burning product extends over a relatively greater portion of the star, making the WNE phase longer. All the other durations are comparable for the 3 models.

In particular, we note that the ratio of the WR-lifetime to the O-type star lifetime remains around 0.4 for all three models. This comes from the fact that as long as we have homogeneous evolution (and of course reasonable mass loss rates), then the entrance into the WR phase is due to mixing, not to mass loss. The mass loss can differ (see Fig. 3), but their impact on the WR/O ratio is very limited.

5.5. The mass–luminosity diagram

Fig. 8 shows the evolution of our models in the \( \log(M/M_\odot) - \log(L/L_\odot) \) plane. The three curves show the same features: first, an increase in the luminosity at relatively constant mass (due to the smaller mass loss rate when the star is under its critical mass for models B and C, or when the star is in its O-star phase for model A), followed by an evolution along a line of constant slope. The peak in luminosity occurs when central helium ignition occurs. After, the luminosity decreases again linearly.

We have overplotted on Fig. 8, the positions of stars which would be at the Eddington limit. The curve labeled with \( L_{\text{Edd}}(\kappa_{\text{es}}) \) was computed assuming that the opacity is equal to the electron scattering opacity for H-free stars. Of course the actual opacity in the star may be higher, especially in the outer regions and in the wavelengths of the lines. In these domains, the luminosity is much closer to the maximum luminosity. This should favor the mass loss phenomenon in such stars. Thus, we show also curves corresponding to the Eddington limit for higher opacities equal to a few times the electron scattering one for H-free material. We see that our evolutionary tracks evolve in regions crossed by the Eddington limits corresponding to 2-5 times the electron scattering opacity.

Many previous studies show that WNE and WC stars follow a simple mass–luminosity relation (Maeder 1983; Maeder & Meynet 1987; Langer 1989; Schaerer & Maeder 1992). Schaerer & Maeder (1992) found:

\[
\log \left( \frac{L}{L_\odot} \right) = 1.7267 \log \left( \frac{M}{M_\odot} \right) + 3.4949. \tag{17}
\]

In Fig. 8, we show the region where our models are in the WNE/WC phase as a shaded area. This region is also well linear, whatever the mass loss prescription we used. The linear interpolation in this area gives:

\[
\log \left( \frac{L}{L_\odot} \right) = 1.5360 \log \left( \frac{M}{M_\odot} \right) + 3.7574. \tag{18}
\]

very similar to the previous one.

6. A few consequences of the \( \mu^2 M \) mass loss rates

6.1. Chemical enrichment

In Table 3, we show the time-integrated composition of the winds of our models. It represents the total mass injected in the interstellar medium during the life of the star, and do not take into account the material released during the SN event (note that our models will produce a black hole, see Sect. 6.3, and thus, most of the matter ejected during the SN event will be swallowed by the black hole, so the chemical enrichment is due mainly to the wind contribution). The released mass of hydrogen \( m_\text{H} \), helium \( m_\text{He} \), carbon \( m_\text{C} \), nitrogen \( m_\text{N} \) and oxygen \( m_\text{O} \) are indicated in solar masses and in fraction of the total mass lost by the wind. In Table 4, the same quantities are indicated in a slightly different way, with the ratios \( \text{[C/H]} \), \( \text{[N/H]} \) and \( \text{[O/H]} \), as well as the isotopic ratio \( \text{^{12}C/^{13}C} \).

In the three cases, the wind composition reflects the effects of H- and He-burnings. The wind ejecta of models A and B have a very similar composition. Model C is slightly different. Since it reaches quite early the \( \mu^2 M \) limit, it loses a lot of mass already during a large portion of the MS. This implies that the total composition of the wind is more rich in H burning products than models A and B.

Looking at Fig. 7, one notes that there is no primary nitrogen ejected by the winds. This is expected. A perfectly

![Fig. 8. Evolutionary tracks in the mass–luminosity diagram with overplotted (dotted lines) the Eddington limit computed for different values of the opacities expressed in units of the free electron opacity for stars without any hydrogen, i.e. with \( \kappa_{\text{es}} = 0.2 \text{ g cm}^{-2} \). The entrance in the WNL and WNE phases are indicated. The shaded area represents the region where we find WNE and WC stars. The evolution goes from right to left. Models are represented as in Fig. 3 (colour figure available online).](image-url)
homogeneous evolution would not lead to the production of primary nitrogen since one needs inhomogeneity to do that: H burning regions must be present simultaneously with He-burning ones! Even when we have not strict homogeneity as in the present models, the time during which these two nuclear phases are present at the same time in the star are too short for allowing primary nitrogen production. This means that such homogeneously evolving stars cannot be important sources of nitrogen in the early Universe. Indeed many observational facts as the high N/O ratio observed at the surface of metal poor halo field stars or the very high CNO abundances at the surfaces of carbon enhanced metal poor (CEMP) stars needs the production of important amount of primary nitrogen (Chiappini et al. 2006; Meynet 2010).

6.2. Mechanical energy outputs

Fig. 9 shows the evolution of the ratio of the luminosity of the stellar wind over the stellar luminosity. The wind luminosity is defined as:

$$L_{\text{Wind}} = \frac{1}{2} \dot{M} v_{\text{esc}}^2,$$

where $\dot{M}$ is the mass loss rate of the star, and $v_{\text{esc}}$ the velocity of the stellar wind very far from the surface. Here, to determine $v_{\text{esc}}^2$, we used the relation given by Kudritzki & Puls (2000):

$$v_{\text{esc}} = C(T_e) v_{\text{eff}},$$

$$C(T_e) = \begin{cases} 2.65 & T_e \geq 21'000 \text{ K} \\ 1.4 & 10'000 \text{ K} < T_e < 21'000 \text{ K} \\ 1.0 & T_e \leq 10'000 \text{ K}, \end{cases}$$

with $v_{\text{esc}}$ the escape velocity on the surface. To compute this velocity, we follow Lamers & Cassinelli (1996), who give $v_{\text{esc}} = \sqrt{\frac{GM_{\ast}}{R_{\ast}}}$, with $M_{\ast}$ the effective mass of the star, taking into account the radiation pressure (see their eqs. 14-16), and $R_{\ast}$ the radius of the star. We neglect here the effects of rotation, which introduces a latitudinal dependence in the escape velocity. But as the surface velocity decreases, the error is probably not very large.

From Fig. 9, we see that when the star is in its standard mass loss phase, the luminosity of the stellar wind remains small, at around $\log(L_{\text{Wind}}/L) \sim -3$. Once the star enters the phase where it loses mass through the $\mu^2 M$ limit scenario (for models B and C), the wind luminosity increases up to $\log(L_{\text{Wind}}/L) \sim -1.6 - 1.8$, slightly increasing with time. For model A, which does not undergo the mass loss through the $\mu^2 M$ limit process, but has the mass loss rate given by Nugis & Lamers (2000), this value is lower, at around $\log(L_{\text{Wind}}/L) \sim -2$.

We show in Fig. 10 the total energy released in the interstellar medium by the stellar winds at a given time, given by:

$$E_{\text{wind}}(t) = \int_0^t L_{\text{Wind}}(\tau) d\tau.$$  \hspace{1cm} (21)

If we concentrate only on the end of our simulation, we see that there is not a big difference between our three models. The final values for the energy injected in the interstellar medium are $3.52 \cdot 10^{51}$ erg, $5.40 \cdot 10^{51}$ erg, and $5.50 \cdot 10^{51}$ erg for models A, B, and C respectively. This values are compatible with other studies of the stellar winds of massive star (see e.g. Freyer et al. 2003, 2006).

The difference between models A and B comes from the slightly higher $\dot{M}$ for model B when it reaches its critical mass than for model A when the Wolf-Rayet mass loss rate is taken into account. However, these two models show globally the same behaviour. Model C released a larger amount of mechanical energy in the interstellar medium much earlier, as soon as it enters in its $\mu^2 M$ limit phase, after roughly 0.4 Myr.

6.3. The $\mu^2 M$ limit and the evolution towards collapsars

According to recent works, fast rotating models following homogeneous evolution are good candidates for producing gamma ray bursts (GRB) (according to the collapsar scenario of Woosley 1993) at the end of their evolution (see Yoon & Langer 2005; Yoon et al. 2006; Woosley & Heger 2006). It is thus interesting to check whether the models we computed in this work, accounting for the $\mu^2 M$ limit, are able or not to produce a GRB event.

For having a collapsar, a black hole needs to be produced during the SN event. To determine the mass of the remnant, we use the same method as in Hirschi et al. (2005). Due to the strong internal mixing, the CO-core of our models are very large (see Table 5): more than 80% of the total mass for all the models. This implies a very massive remnant. Even for the model C, which finished its He-burning with the smallest total mass, the deduced mass of the remnant is much more than 3.5 $M_{\odot}$. This is much more than the maximum mass possible for a neutron star. We conclude that the remnant resulting from the SN event is a black hole for our three models.
In at least three cases (see Woosley & Bloom 2006), a type Ic SN has been observed associated to the GRB. It is thus interesting to determine the SN type which will be produced by our models. To determine the SN type, we use the same criterion as in Georgy et al. (2009): we computed in the same way the ejected masses of hydrogen $m_{\text{H}}$ and of helium $m_{\text{He}}$ during the SN event (see Table 5). If $m_{\text{He}} < 0.6 \, M_\odot$, we have a type Ic SN, otherwise, we have a type Ib SN (obviously, the condition that $m_{\text{H}} = 0$ is fulfilled in our models, which end their lives as WC stars).

Under these assumptions, model A will produce a type Ib SN. The mass loss during the whole life of this star is not strong enough to uncover regions where a large fraction of He has been depleted. For model C, the mass loss is so strong that the CO-core is proportionally smaller, and not completely uncovered at the end of the evolution. Between this two cases, model B encounters a stronger mass loss than model A, but smaller than model C. This produces a large CO-core, with a sufficient mass loss rate to uncover it at the pre-SN stage and to produce a type Ic SN. We see by these numerical examples that the production of type Ic supernovae requires neither too small, nor too high mass loss rates (Georgy et al. 2009).

A key point of the collapsar scenario is the amount of angular momentum kept in the core at the end of the evolution. In order to produce an accretion disc around the black hole formed by the collapse, we need that the specific angular momentum of the centre of the star remains higher than the corresponding specific angular momentum of the last stable orbit around a Schwarzschild black hole $j_{\text{Schwa}}$.

In Fig. 11, we plot for our three models: the specific angular momentum of the model $j_{\text{mod}}$ (solid black line), the corresponding specific angular momentum of the last stable orbit around a Schwarzschild black hole $j_{\text{Schwa}}$ (red short-dashed line) and $j_{\text{Kerr}}$ for a maximally rotating Kerr black hole plotted as a function of the mass (blue long-dashed line) (colour figure available on-line).

We see that for all the three models, the specific angular momentum at the end of core He-burning is already too small everywhere in the interior of the star. Since the specific angular momentum is modified only by transport processes such the diffusion and the advection, and since the characteristic timescale of the evolution after the central helium-burning phase becomes very short comparing to the timescale of these processes, this result is a good approximation of what will be the situation at the pre-supernova stage.

Our models are thus too slow rotators at the end of their evolution to produce a GRB. This is not surprising in the sense that this model is comparable to the models discussed in Yoon et al. (2006), and their corresponding 60 $M_\odot$ model is not in the GRB area they predicted in the $M_\text{ini} - v_\text{ini}$ plane. Our models B and C lose much more mass than model A, and thus much more angular momentum. As we can see in Fig. 11, their final content in specific angular momentum is lower than for model A.

Thus, no one of our models fulfill all the conditions to produce a collapsar. Since present models have cores with lower specific angular momentum than models computed with usual mass loss rates, we conclude that adoption of mass loss induced by the $\mu^2 M$ limit reduces the initial mass–velocity domain of GRB.

7. Conclusions

In this work, we have studied the effects of mass loss driven by the $\mu^2 M$ limit on the evolution of fast rotating mas-
Homogeneous evolution at the $\mu^2 M$ limit

Table 5. CO-core mass, remnant mass, and ejected mass of H and He at the SN event for our three models (all masses are given in $M_\odot$).

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_{\text{CO}}$</th>
<th>$M_{\text{rem}}$</th>
<th>$m_\text{e,J}$</th>
<th>$m_\text{e,II}$</th>
<th>SN type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.73</td>
<td>5.28</td>
<td>0.0</td>
<td>1.01</td>
<td>Ib</td>
</tr>
<tr>
<td>B</td>
<td>13.74</td>
<td>5.04</td>
<td>0.0</td>
<td>0.57</td>
<td>Ic</td>
</tr>
<tr>
<td>C</td>
<td>6.26</td>
<td>3.84</td>
<td>0.0</td>
<td>0.67</td>
<td>Ib</td>
</tr>
</tbody>
</table>

We showed that the $\mu^2 M$ limit for hot homogeneous stars is a consequence of the mass-luminosity relation and of the Eddington limit.

The most striking results of this study is that the mass loss driven by the $\mu^2 M$ limit produces mass loss rates which are well in line with those obtained from the line driven theory. Therefore supposing that WR stars evolve very close to their $\mu^2 M$ limit produces very similar results as classical models based on line driven mass loss. This is well illustrated by the comparison of models A and B, which present similar evolution in many respects. Thus, this physical process appears as a reasonable mechanism for hot homogeneous stars which can be an alternative or at least a complement to the line driven-wind mass loss.

When applied to fast rotating stars, strong mass loss induced by the $\mu^2 M$ limit can set in very early during the evolution (see case C). Such objects would present broad emission lines in their spectra as shown by classical Wolf-Rayet stars, but their surface composition would be different. They would show material with H content than in classical WR stars.

Homogeneously evolving models computed with the present prescription lose significantly more mass than those computed with line driven mass loss. This has an impact on the way such stars contribute to the enrichment of the interstellar medium through their winds.

In addition to evolve near the $\mu^2 M$ limit, Wolf-Rayet stars may also approach the $\Omega^2$-limit, notably at the end of the core H-burning phase. This evolutionary stage may be characterized by stronger mass loss than presently accounted for.

Due to the above effect, more angular momentum is lost when the effects of the $\mu^2 M$ limit are accounted for. This restrains the domain of initial velocities required in order to produce a collapsar at the end of the evolution.

Of course, the very simple relation we use in this work to compute the mass loss rates cannot pretend to provide a detailed description of the complex processes occurring near the $\mu^2 M$ limit. However, it gives reasonable results which reasonably cannot be due to chance. Progresses will come from improvements of our understanding of the effects of the instabilities occurring in stars with a high $L/M$ ratio.

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Wind anisotropy and stellar evolution

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Abstract. Mass loss is a determinant factor which strongly affects the evolution and the fate of massive stars. At low metallicity, stars are supposed to rotate faster than at the solar one. This favors the existence of stars near the critical velocity. In this rotation regime, the deformation of the stellar surface becomes important, and wind anisotropy develops. Polar winds are expected to be dominant for fast rotating hot stars.

These polar winds allow the star to lose large quantities of mass and still retain a high angular momentum, and they modify the evolution of the surface velocity and the final angular momentum retained in the star’s core. We show here how these winds affect the final stages of massive stars, according to our knowledge about Gamma Ray Bursts. Computation of theoretical Gamma Ray Bursts rate indicates that our models have too fast rotating cores, and that we need to include an additional effect to spin them down. Magnetic fields in stars act in this direction, and we show how they modify the evolution of massive star up to the final stages.

Keywords. stars: evolution, stars: magnetic fields, stars: mass loss, gamma rays: bursts

1. Effects of rotation on the stellar surface

Rotation has a strong influence on the stellar surface. Indeed, it adds a centrifugal component to the gravity, which modifies the shape of the surface, and various quantities such as the effective temperature $T_{\text{eff}}$ and the mass loss flux. These effects can be derived from the von Zeipel theorem (von Zeipel 1924), which is originally valid for conservative cases of angular momentum distribution, and was treated in the more general case of the so-called “shellular rotation” by Maeder (1999). This theorem gives the relation between the local flux $F$ of the star and the local effective gravity:

$$F = -\frac{L(P)}{4\pi GM_\star(P)} g_{\text{eff}} (1 + \zeta(\theta))$$

where $L(P)$ is the luminosity on the isobar and $g_{\text{eff}}$ the local effective gravity. The two remaining terms are given by

$$\zeta(\theta) = \left[ (1 - \frac{\chi_T}{\delta}) \Theta + \frac{H_T}{\delta} \frac{d\Theta}{dr} \right] P_2(\cos(\theta))$$

$$M_\star = M \left( 1 - \frac{\Omega^2}{2\pi G \rho_m} \right)$$

Here, $\rho_m$ is the internal average density, $M_\star$ represents the effective mass, modified by the rotation velocity $\Omega$, $\chi = 4aT^3/(3\kappa \rho)$ is the thermal conductivity coefficient and $\chi_T$ is its partial derivative with respect to $T$, $\Theta = \frac{\delta}{\xi}$ is the ratio of the horizontal density fluctuation to the average density on the isobar (Zahn 1992). $\delta$ is the thermodynamic coefficient $\delta = -\left( \frac{\partial^2 \ln \rho}{\partial \ln T} \right)_{\rho, T}$, and $H_T$ is the temperature scale height. Generally the term $\zeta(\theta)$ is very small, and we can neglect it.
The total gravity at the surface of the star is given by $g_{\text{tot}} = g_{\text{eff}} + g_{\text{rad}}$ with $g_{\text{rad}} = \frac{\kappa F}{c}$ as the term due to radiative forces and $\kappa$ the total Rosseland mean opacity. The Eddington limit in a rotating star is defined by the vanishing of $g_{\text{tot}}$ and we find the limiting flux

$$F_{\text{lim}} = -\frac{c}{\kappa}g_{\text{eff}}.$$  

The Eddington factor at a given colatitude, which is given by the ratio of the local flux to the limiting flux, becomes (Maeder 1999)

$$\Gamma_{\Omega}(\theta) = \frac{L(P)}{L_{\text{max}}}$$

with $L_{\text{max}} = \frac{4\pi c GM}{\kappa(\theta)(1 + \zeta(\theta))} \left(1 - \frac{\Omega^2}{2\pi G \rho_m}\right)$.  

A first interesting consequence is that the maximum luminosity of a star, given by $\Gamma_{\Omega}(\theta)=1$, is lowered by rotation (compare $L_{\text{max}}$ given above with the non-rotating one $L_{\text{max,nonrot}} = \frac{4\pi c GM}{\kappa}$).

Rotation has also a strong impact on the mass loss rate of the star. Maeder and Meynet (2000) found the following relation between the mass loss rate at a given rotational velocity $\dot{M}(\Omega)$ and the non-rotating one:

$$\frac{\dot{M}(\Omega)}{\dot{M}(0)} = \left(1 - \frac{\Omega^2}{2\pi G \rho_m} - \Gamma\right)^{\alpha - 1}$$

where $\Gamma = \kappa L_{\text{max}}$ is the “non-rotating” Eddington factor. We see that rotation will increase the total mass loss rate of the star.

Rotation has another effect on the mass loss rate: it is no longer isotropic at the surface of a rotating star, but becomes colatitude-dependent. Maeder and Meynet (2000) give the following local mass loss rate $\Delta \dot{M}$ per unit surface $\Delta \sigma$:

$$\frac{\Delta \dot{M}}{\Delta \sigma} \simeq (k\alpha)^{\frac{1}{2}} \left[\frac{L(P)}{4\pi GM_{\odot}(P)}\right]^{\frac{1}{2}} \frac{g_{\text{eff}}}{(1 - \Gamma_{\Omega}(\theta))^{\frac{1}{2}}}$$

with $k$ and $\alpha$ the force multiplier parameters. We neglect here the small effect of $\zeta(\theta)$. Mass loss occurs preferentially where $g_{\text{eff}}$ is small, i.e. at the poles. Then mass loss rate is thus varying as a function of the colatitude, producing the anisotropic wind phenomenon.

Figure 1 shows various effects of the rotation at the surface of a 20 $M_{\odot}$ star at a metallicity of $10^{-5}$ and at 95% of the critical rotation velocity. We can first remark that the star becomes oblate, with an equatorial-to-polar radius ratio $R_{\text{eq}}/R_{\text{pol}} \approx 1.3$. Then we note the variation of the effective temperature with respect to colatitude: $T_{\text{eff}}$ at the pole is around 48000 K, while it is only around 34000 K at the equator. Finally, we see that the mass loss flux is larger at the pole by a factor of $\sim 3.8$. This allows the star to lose 10% less angular momentum than if the same amount of mass was lost isotropically.

2. Models without magnetic field and rate of GRB

Let us briefly recall here the main assumptions of the so-called ‘collapsar” model for long-soft Gamma Ray Bursts (Woosley 1993). In order to produce such an event, the following conditions must be fulfilled:

- formation of a black hole;
- enough angular momentum in the stellar core in order to form an accretion disk around the BH;
- formation of a type Ic supernova (see Woosley and Bloom 2006).
Wind anisotropy and stellar evolution

Figure 1. Effects of rotation on a 20 M\(_\odot\) star at Z = \(10^{-5}\) and with \(\Omega/\Omega_{\text{crit}} = 0.95\). The star is seen equator-on, and the axis are in R\(_\odot\). The latitudinal variation of \(T_{\text{eff}}\) is shown (gray scale), and the arrows represents the mass flux at a given colatitude.

Figure 2. SNIb/SNII (solid line), SNIb/SNII (dotted line) and SNIc/SNII (dashed line) ratios. The triangles are observed SNIb/SNII ratio at various metallicities and the upside down gray triangles observed SNIc/SNII ratio (extracted from Prieto et al. (2008)). These three points are consistent with a fast rotating massive star. It is thus interesting to study the evolution of the rate of type Ic supernovae. Figure 2 shows the relative rates for type Ib and type Ic SNe, computed with rotating models without a magnetic field. To distinguish between type Ib and type Ic, we use a criterion based on the amount of He ejected during the SN event: all models ejecting more than 0.55 M\(_\odot\) of helium and no hydrogen are considered as type Ib, the models ejecting less than 0.55 M\(_\odot\) of helium (and still no hydrogen) are considered to give birth to a type Ic SN event. The interesting curve for our purpose is the dashed one, representing the SN Ic / SN II ratio with respect to the metallicity. We see that the number fraction of type Ic supernova becomes higher at higher metallicity. This is well in agreement with the trend shown by the observed type Ic to type II SNe ratios given by Prieto et al. (2008). There is much observational
evidence indicating that long-soft GRBs occur preferentially in metal-poor regions. For instance, Modjaz et al. (2008) find that GRB events appear at low metallicity: between $0.2 < Z/Z_\odot < 0.7$. Thus only type Ic events in metal poor regions (or part of them) can occur simultaneously with a GRB event.

Moreover, if we consider our models at metallicities that are compatible with the observed GRB range of metallicity, we see that all the models producing a type Ic SN keep enough angular momentum in the core to fulfilled the collapsar model conditions (see Hirschi et al. (2005)). We can thus determine the ratio of GRB event to the total number of core collapse SNe; we found that this rate is around 15% for metallicities between 0.4 and 0.7 $Z_\odot$. In comparison, Podsiadlowski et al. (2004) found an observational rate of $0.04\% - 8\%$, depending of the aperture angle of the bipolar jets produced during the GRB event. Our theoretical rate is therefore much larger than the observational one. These two facts lead to the conclusion that not all type Ic SNe produce a GRB. We have thus to find a way to reduce the number of GRB progenitor candidates, in order to reproduce the observational rate. One possibility is to introduce new ingredients in our models to extract more angular momentum from the core during the stellar life.

3. Models with magnetic field and wind anisotropy

Following Spruit (2002), we have included in our models the effect of magnetic field amplified at the expense of the excess energy in the shear. As noted just above, this produces a strong coupling between the differentially rotating layers, and tend to build a solid–body rotation profile. Contrarily to the previous models, where the rotational velocity is only weakly coupled between the surface and the core, models with magnetic field have a strong coupling, and thus, the loss of angular momentum due to mass loss is quickly transmitted to the core. This implies a strong extraction of angular momentum during the evolution, particularly when the mass loss is strong at the surface (e.g. during the Wolf–Rayet phases).

To explore the combined effects of magnetic field and wind anisotropy, we computed two 60$M_\odot$ models at $Z = 0.002$ with $\Omega/\Omega_{\text{crit}} = 0.75$, with and without the treatment of the wind anisotropy. With respect to the work by Meynet and Maeder (2007), we have improved significantly the treatment of the wind anisotropy, allowing this treatment to apply even when the critical velocity is reached and checking very carefully that the sum of the angular momentum remaining in the star and the angular momentum lost in the wind remains constant all over the evolution (see Fig. 3, bottom panel). As we shall see below, these improvements lead to effects which although still important are less pronounced as in Meynet and Maeder (2007).

For both models, the strong mixing induced by the magnetic field leads to the so-called “quasi-chemically-homogenous” evolution (see Yoon and Langer 2005 and Woosley and Heger 2006). Figure 3 (top panel) shows the evolution of the total angular momentum of the stellar interior during its evolution. We see that the anisotropic model (dashed curve) is slightly higher than the isotropic one (solid curve). This is due to the effect of wind anisotropy, as shown in the medium panel: the anisotropic model loses less angular momentum (around 7%), while the rotational velocity is high (first part of the evolution). Then, the star becomes a WR, and the high mass loss rate implies a strong breaking of the rotation: the surface velocity is sufficiently below the critical velocity for removing any anisotropy in the wind, and there is no more difference between these two models.

We see that when magnetic field is accounted for, the WR phase has a crucial influence on the total angular momentum kept in the star (and thus in the core, through the coupling produced by the magnetic field). Our model does not keep enough angular
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Figure 3. Top panel: evolution of the total angular momentum kept in the star (in units of \(10^{45} \text{ g cm}^2 \text{s}^{-1}\)). The dashed curve is the model with anisotropic wind, the solid curve for the isotropic one. Middle panel: angular momentum removed by wind at each time step, in the same units: dashed curve for anisotropic model, solid for isotropic one. Bottom panel: Sum of the angular momentum of the star and the integrated angular momentum removed by wind.

momentum for the collapsar model, and is thus not a good GRB progenitor candidate. However, models with very high initial rotational velocity would most probably develop larger differences between the iso- and anisotropic treatment. The same would be true for models with lower mass loss rate (less massive models, lower metallicity). We will adress this point in a forthcoming paper.

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