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Abstract
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Frequency Scales for Current Statistics of Mesoscopic Conductors

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We calculate the third cumulant of current in a chaotic cavity with contacts of arbitrary transparency as a function of frequency. Its frequency dependence drastically differs from that of the conventional noise. In addition to a dispersion at the inverse RC time characteristic of charge relaxation, it has a low-frequency dispersion at the inverse dwell time of electrons in the cavity. This effect is suppressed if both contacts have either large or small transparencies.

What is the characteristic time scale of the dynamics of electrical transport in normal-metal mesoscopic systems if their size is larger than the screening length? The most reasonable answer is that this dynamics is governed by the RC time of the system, which describes the relaxation of piled up charge. It is this time that characterizes the admittance of a mesoscopic capacitor, the impedance of a diffusive contact, current noise in diffusive contacts, and charge noise in quantum point contacts and chaotic cavities. The free motion of electrons is superseded by the effects of charge screening and therefore the dwell time of a free electron is not seen in these quantities. The only known exception is the frequency dispersion of the weak-localization correction to the conductance. Because typical charge-relaxation times are very short for good conductors, the experimental observation of the dispersion of current noises is difficult.

Very recently, the third cumulant of current has been measured for tunnel junctions. In this case, the frequency dispersion is due only to the measurement circuit. In contrast to tunnel junctions, chaotic cavities have internal dynamics. It is the purpose of this paper to show that mesoscopic systems with such dynamics may exhibit an additional low-frequency dispersion in the third cumulant of current, which is absent for the first and second cumulants. Physically, this dispersion is due to slow, charge-neutral fluctuations of the distribution function in the interior of the system. Such charge-neutral fluctuations are similar to fluctuations of the effective electron temperature at a constant chemical potential. They do not contribute directly to the measurable current, but they modulate the intensity of noise and hence contribute to higher cumulants of current.

The zero-frequency electric noise in open chaotic cavities has been calculated by Jalabert et al. using random-matrix theory. More recently, these expressions were derived semiclassically by Blanter and Sukhorukov. Shot-noise measurements on chaotic cavities were performed by Oberholzer et al. The third and fourth cumulants of current were obtained by Blanter et al. for open cavities and in Ref. for cavities with contacts of arbitrary transparency. Recently, the statistics of charge in a cavity was analyzed by two of us.

The system we investigate is shown in Fig. 1. A chaotic cavity consists of a metallic mesoscopic conductor of irregular shape connected to leads $L, R$ via quantum point contacts with conductances $G_{L,R}$ and transparencies $\Gamma_{L,R}$. We assume that $G_{L,R} \gg e^2/h$ and the bias is much larger than the temperature and frequencies. The dwell time of electrons in the cavity is given by $\tau_D = e^2 Q_{N_F}$, where $R_Q = (G_L + G_R)^{-1}$ is the charge-relaxation resistance and $N_F$ is the density of states in the cavity, which is assumed to be continuous and constant. We assume that the dwell time is short as compared to inelastic scattering times. Then the mean occupation function may be written as a weighted average $f_0(\epsilon) = R_Q(G_L f_L(\epsilon) + G_R f_R(\epsilon))$ of the Fermi occupation factors $f_{L,R}$ of the leads which are step functions at zero temperature. In the absence of electrostatic interaction, fluctuations around $f_0$ would relax on the time scale $\tau_D$. However, due to screening there is a second shorter time scale $\tau_Q$, which describes fluctuations of the charge in the cavity. It is given by $\tau_Q = R_Q C_\mu$, where $C_\mu = C^{-1} + (e^2 N_F)^{-1}$ is the electrochemical capacitance and $C$ is the geometric capacitance.

Our semiclassical calculations are based on a large sep-
aration between the time scales describing the fast fluctuations of current in isolated contacts and slow fluctuations of the electron distribution in the cavity. This allows us to consider the contacts as independent generators of white noise, whose intensity is determined by the instantaneous distribution function of electrons in the cavity. Based on this time-scale separation, a recursive expansion of higher cumulants of semiclas-
sical quantities in terms of their lower-order cumulants was developed. Very recently, such recursive relations were obtained as a saddle-point expansion of a stochastic path integral. Here we follow the approach of Ref. 18.

Our aim is to obtain the statistics of time-dependent fluctuations of the current \( I_L(t) \) flowing through the left contact of the cavity. In general, the probability \( \mathcal{P}[A(t)] \) of a time-dependent stochastic variable \( A(t) \) is described by the characteristic functional \( \mathcal{S}[\chi_A] \) evaluated with an imaginary field

\[
\mathcal{P}[A(t)] = \int D\chi_A e^{-i\int_0^T dt \mathcal{A}_\chi + \mathcal{S}[\chi_A - i\chi_A]}. \tag{1}
\]

Functional derivatives \( \delta^n/\delta \chi_A(t_1) \cdots \delta \chi_A(t_n) \) of the characteristic functional \( \mathcal{S}[\chi_A] \) yield irreducible correlation functions \( \mathcal{A}(t_1) \cdots \mathcal{A}(t_n) \).

To obtain the characteristic functional \( \mathcal{S}[\chi_L] \) describing the fluctuations of \( I_L(t) \) we proceed in two steps. First, we consider the point contacts \((i = L, R)\) as sources of white noise that depend on two common time-dependent parameters, the electron occupation function of the cavity \( f(\epsilon - eU) \) and the electrostatic potential of the cavity \( U \). Their characteristic functionals are given by \( \mathcal{S}_i[\chi_{i,e}] = \int dt \int d\epsilon \mathcal{H}_i(\chi_{i,e}, \epsilon) \) with

\[
\mathcal{H}_i = \frac{1}{2}(\chi_{i,e}^2 \chi_{i,e}^2 + \frac{1}{6}(\chi_{i,e}^3 \chi_{i,e}^3 + \ldots). \tag{2}
\]

The correlators \( \langle \tilde{I}_i^n \rangle_e \) must be taken from a quantum-mechanical calculation

\[
\langle \tilde{I}_i^n \rangle_e = \frac{G_i}{\Gamma_i} \frac{\partial^n}{\partial \chi_i^n} \ln\{1 + \Gamma_i f_i(\epsilon)[1 - f(\epsilon - eU)](e^{-e\chi} - 1)
+ \Gamma_i f(\epsilon - eU)[1 - f_i(\epsilon)][e^{-e\chi} - 1]\} \mid_{\chi = 0}. \tag{3}
\]

In a second step, we take into account that the occupation function \( f(\epsilon - eU) \) and the potential \( U \) are not free parameters but are fixed by the kinetic equation

\[
\left( \frac{d}{dt} + \frac{1}{\tau_D} \right) \delta f(\epsilon) = \frac{dU}{dt} \frac{\partial f}{\partial U} - \frac{1}{N_F} (\tilde{I}_{L,e} + \tilde{I}_{R,e}) \tag{4}
\]

and the charge-conservation law

\[
C_0 \frac{dU}{dt} = - \int d\epsilon \left[ \frac{G}{e} \delta f(\epsilon) + \tilde{I}_{L,e} + \tilde{I}_{R,e} \right].
\]

We introduced the fluctuating part of the occupation function \( \delta f(\epsilon) = f(\epsilon - eU) - f_0(\epsilon) \). The two conservation laws are expressed by path integrals over Lagrange multipliers \( \lambda_e, \xi \) and integrated over the fluctuations of occupation function and potential to obtain the following result for the generating functional

\[
e^{\mathcal{S}[\chi_L]} = \int \mathcal{D}\lambda_e \mathcal{D}\xi \mathcal{D}f \times \exp \left\{ S_L[i\lambda_e + i\xi - i\chi_L] + S_R[i\lambda_e + i\xi] + S_C \right\}, \tag{5}
\]

where the conservation laws are expressed by the following dynamical action

\[
S_C = -i \int_0^T dt \left\{ \xi \left[ C \dot{U} + \int d\epsilon \frac{G}{e} \delta f(\epsilon) \right] + \int d\epsilon \lambda_e \left[ N_F \left( \frac{d}{dt} + \frac{1}{\tau_D} \right) \delta f - N_F \frac{dU}{dt} \frac{\partial f}{\partial U} \right] - G_L \chi_L \int d\epsilon \left[ f(\epsilon - eU) - f_L(\epsilon) \right] \right\}. \tag{6}
\]

In the semiclassical regime, this path integral may be evaluated in the saddle-point approximation. The saddle point equations are nonlinear differential equations for the four fields \( f(\epsilon - eU), \delta U, \lambda_e, \xi \) that contain the external statistical field \( \chi_L \) in inhomogeneous source terms. They describe the non-linear response of internal fields to these sources. In this publication, we are interested in the second and third order correlation functions of the current \( I_L \). For this purpose it is sufficient to expand the sum \( \eta = \lambda_e + \xi = \eta_1 + \eta_2/2 + \ldots \) up to second order in the external field \( (\eta_1 \propto \chi_L^2) \). The result can then be substituted into the action (5) to obtain the frequency dispersion of the correlation functions.

We first briefly discuss the frequency dependence of current noise to connect our theory to earlier results. To this end we collect the second order contributions of Eq. (5) and eliminate time derivatives using the saddle point equations

\[
S_2[\chi_L] = \frac{1}{2} \int dt d\epsilon \left[ (\tilde{I}_{L,e}^2)(\eta_1 - \chi_L)^2 + (\tilde{I}_{R,e})^2 \eta_1^2 \right]. \tag{7}
\]

The zero subscript indicates that the correlators \( \langle \tilde{I}_i^n \rangle_e \) are evaluated at \( f(\epsilon - eU) = f_0(\epsilon) \). We choose a large time interval \([0, T]\) and neglect any transient effects. It is then most convenient to work completely in Fourier space. The linear response of the internal fields \((1 + i\omega T_Q)\eta_1 = G_L R_Q \chi_L \) is entirely governed by the RC time, therefore the correlation function \( \langle I_L(\omega) I_L(\omega') \rangle = 2\pi \delta(\omega - \omega') P_2 \) takes a simple form

\[
P_2 = |Z_Q(\omega)|^2 \int d\epsilon \left[ (\tilde{I}_{L,e}^2)(\overline{G}_R(\omega))^2 + (\tilde{I}_{R,e})^2 \right]. \tag{8}
\]

where we introduced \( Z_Q(\omega) = (G_L + G_R + i\omega C_\mu)^{-1} \) and \( \overline{G}_R(\omega) = G_R + i\omega C_\mu \). At low frequencies, the current

\[
\begin{align*}
\langle I_L(\omega) I_L(\omega') \rangle & = 2\pi \delta(\omega - \omega') \left| Z_Q(\omega) \right|^2 \int d\epsilon \left[ (\tilde{I}_{L,e}^2)(\overline{G}_R(\omega))^2 + (\tilde{I}_{R,e})^2 \right] \\
& = 2\pi \delta(\omega - \omega') \left| Z_Q(\omega) \right|^2 \left[ (\tilde{I}_{L,e}^2)(\overline{G}_R(\omega))^2 + (\tilde{I}_{R,e})^2 \right].
\end{align*}
\]
noise shows correlations between left and right point contact. At high frequencies, these correlations disappear and we observe the bare noise of the left point contact. The transition frequency is given by the RC time. To complete this result, it remains to evaluate the bare noise correlators using Eq. (3).

We now turn to the much more complicated frequency dependence of the third order correlator. For the action (6) we find

\[
S_3[\chi L] = \frac{1}{6} \int dtd\epsilon \left\{ \langle \bar{f}_L^3 \rangle_0 (\eta_1 - \chi L)^3 + \langle \bar{f}_R^3 \rangle_0 \eta_1^3 \right. \\
+ \left. 3 \left[ \langle \bar{f}_L^2 \rangle_0 (\eta_1 - \chi L) + \langle \bar{f}_R^2 \rangle_0 \eta_1 \right] \eta_2 \right\},
\]

This correlator contains two contributions presented by diagrams in Fig. 2. The first one represented by diagram \(b\) is the minimal correlation result that depends only on the RC time. The second is represented by diagram \(c\) and gives the cascade correction,\(^{11,16}\) which contains the low-frequency dispersion. The equations of motion are of the form

\[
(1 + i\omega\tau_D)\lambda_2 = R_Q A_L (\eta_1 - \chi L)^2 + R_Q A_R \eta_1^2 - \xi_2,
\]

\[
(1 + i\omega\tau_Q)\xi_2 = R_Q B_L (\eta_1 - \chi L)^2 + R_Q B_R \eta_1^2
\]

(10)

and depend on both time constants \(\tau_Q\) and \(\tau_D\) that describe the decay of charged and charge neutral fluctuations of the occupation function \(f(\epsilon - eU)\). In turn, these fluctuations act back on the current noise.

We used here a notation \(A_i = \partial \langle \bar{f}_L^2 \rangle_0 / \partial f_i\) and \(B_i = (C_i / C) \int d\epsilon (\partial f_0 / \partial \epsilon) A_i\). It is now straightforward to substitute the second order response (10) into the action (9). Functional derivatives with respect to the external field \(\chi L\) then yield the irreducible part of the third order correlation function \(\langle I_L(\omega_1) I_L(\omega_2) I_L(\omega_3) \rangle = 2\pi \delta(\omega_1 + \omega_2 + \omega_3) P_3\). The same results are obtained by using a recursive diagrammatic expansion of the third cumulant.\(^{11,16}\)

In what follows, we will be interested in the most typical case where the dwell time \(\tau_D\) is much larger than the charge relaxation time \(\tau_Q = C_i R_Q\). Even for this case the general expression for \(P_3\) is too long and we present here only its limiting values. In the low-frequency limit \(\omega_i \ll (C_i R_Q)^{-1}\), \(P_3\) is of the form\(^{21}\)

\[
P_3(\omega_1, \omega_2) = e^2 i \left\{ 3 G_L G_R \frac{[C_L(1 - \Gamma_R) G_L^2 - (1 - \Gamma_L)G_R^2]^2}{(G_L + G_R)^6} \right. \\
- \frac{2}{(G_L + G_R)^5} \left[ 3 G_R G_L^4(1 - \Gamma_R) G_L^2 + (1 - \Gamma_L) G_R^3 \left( 1 - \frac{G_L}{G_R} \right) \right] \\
\left. \times \left[ \frac{1}{1 + i\omega_1 \tau_D} + \frac{1}{1 + i\omega_2 \tau_D} + \frac{1}{1 - (\omega_1 + \omega_2) \tau_D} \right] \right\}.
\]

(11)

which in general suggests a strong dispersion of the third cumulant of noise at \(\omega_{1,2} \sim 1/\tau_D\). This dispersion vanishes for symmetric cavities and cavities with two tunnel or two ballistic contacts. For a cavity with two tunnel contacts, the white-noise sources (3) are linear functionals of the distribution function and hence are not affected by charge-neutral fluctuations. For the case of two ballistic contacts, Eq. (2) depends only on \(f\) and not on \(f_i\). Then the low-frequency dispersion does not show up either because fluctuations of current and distribution function are uncorrelated due to the symmetry of \(H_c\). Note that Eq. (11) is symmetric with respect to indices \(L\) and \(R\) because pile-up of charge in the cavity is forbidden at low frequencies.

In general, \(P_3(\omega_1, \omega_2)\) exhibits a complicated behavior (see Fig. 3). It has also a dispersion at the inverse RC
relaxation time of the system. This effect has a purely
dispersion at frequencies much smaller than the charge-
time is of the same order or larger. Hence our results are
semiclassical systems like diffusive wires, where the dwell
time of the cavity can be measured as fluctuations of
voltage across a small resistor attached to it. Based
on the parameters of a chaotic cavity used in shot-noise
experiments, we also believe that similar low-frequency dis-
peral time and additional peculiarities at the scale $\tau_D^{-1}$ if one
of the frequencies or their sum tends to zero. The shape
of $P_3(\omega_1, \omega_2)$ essentially depends on the parameters of
the contacts. In particular, for a cavity with one tun-
nel and one ballistic contact with equal conductances
$G_L = G_R = G$ it exhibits a non-monotonic behavior
as one goes from $\omega_1 = \omega_2 = 0$ to high frequencies. A relatively simple analytical expression for this case may be
obtained if $\tau_D \gg \tau_Q$ and one of the frequencies is zero:

$$P_3(\omega, 0) = -\frac{1}{32} e^2 I \frac{1 + 2 \tau_D^2 \omega^2 + \tau_D^2 \tau_Q^2 \omega^4}{(1 + \omega^2 \tau_D^2)(1 + \omega^2 \tau_Q^2)}.$$  \hspace{1cm}(12)$$

The $P_3(\omega, 0)$ curve shows a clear minimum at $\omega \sim (\tau_D \tau_Q)^{-1/2}$ and the amplitude of its variation tends to
$P_3(0, 0)$ as $\tau_Q/\tau_D \to 0$ (see Fig. 4).

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The solid, dashed, and dash-dot curves correspond to $\tau_Q = 1/2$, $\tau_Q = 3$, and to the case of weak electrostatic coupling $\tau_Q = \tau_D$ ($C = \infty$).

FIG. 4. $P_3(\omega, 0)$ as a function of $\omega$ for $G_L/G_R = 1$, $\Gamma_L = 1$, $\Gamma_R = 0$, and $\tau_D = 10$ (dimensionless units). The

dispersion of the third cumulant may be observed in other
experiments, on the parameters of a chaotic cavity used in shot-noise

In summary, we have shown that the third cumulant of current in mesoscopic systems may exhibit a strong dispersion at frequencies much smaller than the charge-
relaxation time of the system. This effect has a purely
classical origin and the variations of the cumulant may
be of the order of its zero-frequency value even if the
number of quantum channels in the system is large.

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21. $P_3(\omega_1, \omega_2)$ is complex if neither of the frequencies $\omega_1$, $\omega_2$, and $\omega_1 + \omega_2$ is zero.