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Abstract

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Interaction induced magnetic field asymmetry of nonlinear mesoscopic electrical transport

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We demonstrate that the nonlinear I–V characteristics of a two probe conductor is not an even function of magnetic field. While the conductance of a two-probe conductor is even in magnetic field, we find that already the contributions to the current which are second order in voltage, are not even in general. This implies a departure from the Onsager microreversibility principle already in the weakly nonlinear regime. Interestingly, the effect that we find is due to the Coulomb interaction. A measurement of the magnetic field asymmetry can be used to determine the effective interaction strength. As a generic example, we discuss the I–V characteristics of a chaotic quantum dot. The ensemble averaged I–V of such a cavity is linear: nonlinearities are due to quantum interference. Consequently, phase-breaking reduces the asymmetry. We support this statement with a calculation which treats inelastic scattering with the help of a voltage probe.

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I. INTRODUCTION

The linear transport regime is governed by a number of fundamental principles. It is interesting to ask whether these principles extend into the non-linear regime and if not what causes their breakdown. For instance, the Onsager-Casimir theory of irreversible processes\(^2,3\) applied to electrical conduction\(^2\) implies that the conductance of a two probe conductor is an even function of magnetic field \(G(B) = G(-B)\). We might expect that this symmetry is valid even in the non-linear regime and that quite generally the current is an even function of magnetic field \(I(B, V) = I(-B, V)\) for any voltage \(V\) applied to the conductor. However, a closer inspection shows that this is not true. In a recent work\(^6\), we have shown that already the weakly nonlinear current response quadratic in voltage is in general not an even function of magnetic field. This implies that the Onsager relations are strictly limited to the linear transport regime and that the I–V characteristics contains terms that are odd under magnetic field reversal. Consequently, it is sufficient to demonstrate that nonlinear transport even in a two probe geometry is sensitive to the Hall potential in order to generate an I–V characteristics that contains terms that are odd under magnetic field reversal. We have analyzed a model with a quasi-localized state coupled to chiral edge states\(^9\). We have shown that the I–V curve exhibits a magnetic field asymmetry if either the charges on the edge states couple has been focussed on the asymmetries shown in the rectification current of mesoscopic systems when a magnetic field is applied in addition to the external dc bias.

We have first investigated a simple example of a resonant impurity in the quantum Hall regime.\(^3\) The Hall potential is odd under magnetic field reversal. Consequently, it is sufficient to demonstrate that nonlinear transport even in a two-probe geometry is sensitive to the Hall potential in order to generate an I–V characteristics that contains terms that are odd under magnetic field reversal. We have analyzed a model with a quasi-localized state coupled to chiral edge states.\(^9\) We have shown that the I–V curve exhibits a magnetic field asymmetry if either the charges on the edge states couple

There has been a growing interest in the investigation of the nonlinear current through low-dimensional electronic structures in which electron transport is phase coherent. At the same time, high-field effects have found much interest in the rapidly developing field of molecular transport. We highlight the observation of rectification effects in solid-state mesoscopic junctions\(^5,6,7\) and molecular structures\(^10,11,12\) which, when the temperature is low enough, arise already at small bias voltages. Our work

FIG. 1: Magnetic field asymmetry of nonlinear two-terminal transport. A chaotic cavity is coupled via quantum point contacts with \(N_1\) and \(N_2\) transverse modes to reservoirs and via a capacitance \(C\) to a gate (left inset). \(P(\Phi)\) is the probability to find a cavity with a certain asymmetry \(\Phi\) [see Eq. (7)] in the presence of a magnetic field generating a flux of the order of \(h/e\). The variance of \(\Phi\) as a function of the total channel number \(N = N_1 + N_2\) is shown in the inset to the right. After Ref. \(^9\)
asymmetrically to the impurity state (electrical asymmetry) or if the impurity is coupled asymmetrically to the edge states simply due to asymmetric tunnel couplings (scattering asymmetry).

A second system that we have examined consists of a chaotic cavity coupled via quantum point contacts to metallic contacts (see left inset of Fig. 1). An ensemble of chaotic cavities, in which individual members differ only by small changes in shape, exhibits an ensemble averaged $I$–$V$ characteristics which is linear in voltage. In contrast an individual cavity exhibits a nonlinear $I$–$V$ characteristics which is odd under magnetic field is a consequence of the magnetic field asymmetry of the electrical potential landscape inside the cavity. For chaotic cavities, this effect is the stronger the better the local potential neutralizes excess charge which might arise due to the applied voltage. In our work the cavity is ballistic (the only scattering arises due to the potential walls of the cavity and possibly due to diffraction of the carriers at the contacts to the cavity). Alternatively, one can consider a small conductor which is metallic diffusive. Even though a ballistic cavity and a small metallic diffusive sample are very different conductors, the theory gives qualitatively similar predictions. Both for the ballistic cavity and the diffusive cavity the asymmetry becomes smaller very rapidly with increasing conductance. For the ballistic cavity we found detailed results in terms of the number of open channels of the quantum point contacts linking the sample to the metallic contacts. The effect we find is very small, at best of the order of the conductance quantum. Possibly, this is connected to the fact that the ensemble averaged $I$–$V$ characteristics of a chaotic cavity is linear. We can not exclude that there are special geometries that exhibit a magnetic field asymmetry which is much stronger than the effect reported here. Below, we investigate the effect of inelastic scattering with the help of a voltage probe and demonstrate that for the chaotic cavities it reduces further the magnetic field asymmetry.

The magnetic field symmetry of transport coefficients is an important probe of our understanding of transport phenomena. It is well known that the magnetic field symmetry of linear conductance played an important role in the development of a scattering theory that is applicable to a wide range of experimental configurations. The magnetic field symmetry of ac-transport conductances has similarly been discussed, but only a few experimental works are available. The self-consistent, nonlinear scattering approach to dc-transport which we will consider a small conductor which is metallic diffusive. Alternatively, one can consider a small conductor which is metallic diffusive. The magnetic field symmetry of pumped currents is the subject of Refs. 25, 26. The magnetic field symmetry of an adiabatic two-parameter pump in the presence of a dc-voltage is discussed by Moskalets and Büttiker.

The nonlinear conductance fluctuations asymmetric in $B$ have been measured in experiments in chaotic cavities. The experiment finds fluctuations which are smaller than predicted by the theoretical model. Since in a realistic situation dephasing time might be of the same order as the dwell time inside the cavity, we report below an investigation of the role of dephasing.

## II. SCATTERING THEORY OF WEAKLY NONLINEAR TRANSPORT

We consider a generic mesoscopic conductor connected to $\alpha = 1, \ldots, M$ reservoirs and gates. We model the transport with the scattering approach. In this picture, the probability amplitude for an electron to go from lead $\beta$ to lead $\alpha$ is given by the quantum-mechanical scattering matrix $s_{\alpha\beta}(E)$, which is a function of the carrier’s energy $E$. It describes elastic scattering within the conductor, assuming that inelastic processes take place only in the reservoirs far away from the conductor. The reservoirs are maintained at thermal equilibrium with Fermi distribution function $f(E - \mu_\alpha)$, where $\mu_\alpha$ is the electrochemical potential. Thus, due to microscopic reversibility the amplitude for an electron to be transmitted from lead $\beta$ to lead $\alpha$ at a given $B$ equals the amplitude from lead $\alpha$ to lead $\beta$ when the magnetic field is reversed, i.e., $s_{\alpha\beta}(E, B) = s_{\beta\alpha}(E, -B)$. It follows that the two-probe linear conductance

$$G = \frac{e^2}{h} \int dE \text{Tr}(s_{12}s_{12}^\dagger)(-\frac{\partial f}{\partial E}),$$

fulfills the Onsager’s symmetry $G(B) = G(-B)$.

For linear transport the scattering matrix is evaluated for electron motion in the equilibrium potential $U_{eq}(\vec{r})$. For a nonequilibrium situation, the scattering matrix is a function of the potential generated by the charges piled up in the mesoscopic conductor. The potential $U(\vec{r}, \{V_\gamma\})$ now depends on the set of electric potential shifts $\{V_\gamma\}$ applied to the external reservoirs and nearby gates. In what follows, we assume for simplicity that $U$ is uniform inside the sample. The full theory takes into account spatial inhomogeneities $U(\vec{r}, \{V_\gamma\})$. The electrostatic potential is essential to a charge-conserving transport theory since the resulting current–voltage characteristics must obey gauge invariance, i.e., the current expressions must depend only on voltage differences. Therefore, the Onsager’s symmetries of the scattering matrix $s_{\alpha\beta}(E, U)$ depend crucially on whether or not the screening potential is an even function of $B$. In linear response, $s_{12}$ is evaluated at the equilibrium potential $U_{eq}$, thereby the conductance is an even function of the magnetic field, as should be.
case, a generalized reciprocity theorem holds. In contrast, in the nonlinear regime the potential is not an even function of magnetic field as we will now discuss.

To ensure charge conservation the potential $U$ of the conductor floats up and down in response to the density of carriers injected through the leads. In addition, interaction with nearby gates causes a response of $U$. As a consequence, the electron density in the sample feels a potential which depends on the shifts $V_\gamma$ of both bias and gate voltages. To leading order in the voltage shifts we can write

$$U = U_\alpha + \sum \alpha u_\alpha V_\alpha + O(V^2),$$

where the characteristic potentials $u_\alpha = (\partial U/\partial V_\alpha)_{\alpha\alpha}$ relate the variation of the screening potential in the sample to a voltage shift in the contact $\gamma$.

To compute $u_\alpha$ we need the charge density in response to a variation of the voltage at the contact and need the charge response to a small variation of $U$. These response functions can be obtained from the partial density of states,

$$\frac{dn_{\alpha\beta}}{dE} = -\frac{1}{4\pi i} \text{Tr}\left( s_{\alpha\beta}^1 \frac{\partial s_{\alpha\beta}}{\partial U} - \frac{\partial s_{\alpha\beta}}{\partial U} s_{\alpha\beta} \right),$$

which is the portion of the density of states (DOS) associated with carriers that enter the conductor from contact $\beta$ and leave it through contact $\alpha$. (When the spatial dependence is needed, the partial derivative with respect to the potential $U$ should be replaced by a functional derivative). The injectivity of lead $\beta$ describes the DOS of those carriers which are injected from lead $\alpha$ regardless to which reservoir the carriers are finally scattered, i.e., $\overline{D}_\beta = \sum_\alpha dn_{\alpha\beta}/dE$. The emissivity of lead $\alpha$, $\overline{D}_\alpha = \sum_\beta dn_{\alpha\beta}/dE$, contains information only about the carriers which are leaving the sample through contact $\alpha$ irrespective of the injecting contact(s). Knowledge of these quantities is essential to find the charges which pile up in the conductor in response to a voltage shift. Since imposing charge conservation involves a balance for the internal potential and the scattering matrix depends itself on $U$, this problem needs to be solved self-consistently. We postpone the solution for the chaotic cavity to the next section.

An expansion of the current through lead $\alpha$ in powers of the applied voltages,

$$I_\alpha = \sum_{\beta} G_{\alpha\beta} V_\beta + \sum_{\beta\gamma} G_{\alpha\beta\gamma} V_\beta V_\gamma,$$

contains a linear term given by the linear conductance conductance matrix $G_{\alpha\beta}$, and a leading-order rectification term $G_{\alpha\beta\gamma}$ which reads, for spinless electrons,

$$G_{\alpha\beta\gamma} = \frac{e^2}{h} \int dE \left( \frac{\partial f}{\partial E} \frac{\partial A_{\alpha\beta}}{\partial U} [2u_\gamma - \delta_{\beta\gamma}] \right),$$

where $A_{\alpha\beta} = \text{Tr}[1_{\alpha\beta} \delta_{\alpha\beta} - s_{\alpha\beta}^1 s_{\alpha\beta}]$ with 1 the identity matrix. Although Eq. 5 gives only the initial departure from linear behavior, the expansion has the advantage that all quantities (like the linear conductance) are evaluated in the equilibrium state. Therefore, from a statistical mechanical point of view, we are on safe grounds. We now specialize to the two-terminal case: $I \equiv I_{12} = -I_{21}$. The conductance matrix is then given by $G_{11} = G_{22} = -G_{12} = -G_{21} = G$ with $G$ given by Eq. 4. The nonlinear $I$–$V$ characteristics depends not only on the voltage shifts of the contacts which permit particle exchange but also on how we shift the voltage on the nearby gates. To be definite, we assume here $V_1 = V$ and $V_2 = V_\delta = 0$. Then, to second-order in the bias voltage $V$ the differential conductance takes the following form:

$$G = \frac{dI}{dV} = G_{11} + 2G_{111}V.$$

Hence, the magneto-asymmetry of $G$, defined as

$$\Phi_G = \frac{1}{2} [G(B) - G(-B)],$$

depends only on the asymmetry of the rectification coefficient $G_{111}$. Moreover, we define the magneto-asymmetry of the screening potential:

$$\Phi_U = \frac{1}{2} [U(B) - U(-B)].$$

In turn, $\Phi_U$ depends on the symmetry properties of the characteristic potentials. It is important to stress that injectivity $\overline{D}_\alpha(B)$ and the emissivity $\overline{D}_\alpha(-B)$ are evaluated in the equilibrium potential and therefore satisfy the reciprocity relation $\overline{D}_\alpha(B) = \overline{D}_\alpha(-B)$. The injectivities and the emissivities are in general not even functions, in contrast to the total density of states (the sum of all injectivities or the sum of all emissivities). As a consequence, the characteristic potentials (and thereby the potential landscape) which, as we will show, depend in an essential way on the injectivity are in general not even functions. Therefore, the transport potential is in general not an even function of magnetic field. Below, we prove that for a generic conductor (a chaotic cavity) the fluctuations of the potential possess a magnetic-field asymmetry and that, as a result, the fluctuations of the nonlinear current–voltage characteristics are not an even function of $B$.

### III. Nonlinear Fluctuations of a Quantum Dot

We consider a ballistic quantum dot in which in the classical limit the electron trajectories are chaotic (see left inset of Fig. 1). This defines a chaotic cavity which we connect to external reservoirs via two quantum point contacts with adjustable number $N_1$ and $N_2$ of propagating modes. Scattering in such open cavities is successfully described by random matrix theory.20,30.
We assume that the cavity is in a perpendicular magnetic field with a total magnetic flux through the cavity of the order of one flux quantum, $h/e$. This is sufficient to break time-reversal symmetry. The probability distribution of the scattering matrix is uniform over the unitary group (symmetry class $\beta = 2$). On the ensemble average, the current is a linear function of the applied voltage $V$. The average conductance is just the classical series resistance of the two quantum point contacts: $\langle G \rangle = e^2 N_1 N_2/hN$ where $N = N_1 + N_2$. Weak localization corrections vanish when $\beta = 2$. Due to wave interference different members of the ensemble have a different linear conductance. For $N_1 = N_2 \gg 1$ the variance of the conductance fluctuations is

$$\text{var}G = \frac{e^4}{\hbar^2} \frac{1}{8\beta}.$$  

(9)

Similar to the fluctuations in the linear conductance an individual cavity exhibits a fluctuating nonlinear $I-V$ characteristics. Unlike the linear regime, which has been extensively studied, the quantum fluctuations of the nonlinear conductance have found much less attention (see, however, Refs. 6,31,32,33,34). The energy scale of quantum interference effects is determined by the energy $\hbar/\tau_d$ where $\tau_d$ is the dwell time for noninteracting electrons and $\delta$ the mean level spacing in the cavity. This energy is also the relevant energy scale for the nonlinear effects considered here.

We next discuss in detail how we determine the transport potential $U$. A Poisson equation needs to be solved to obtain the characteristic potentials. Here, we assume for simplicity that the potential can be described with a single variable $U$ neglecting its spatial fluctuations within the cavity. In the magnetic field range considered here, the cavity is effectively zero-dimensional due to its isotropic scattering properties. We treat interactions within a random phase approximation which determines the Hartree potential with the help of a self-consistent effective interaction. For the open cavity with ideal multi-channel quantum point contacts such an approach can be justified rigorously. In response to a voltage shift $V_\alpha$ in lead $\alpha$ a bare charge $Q^\text{bare}_\alpha = e^2 T_\alpha V_\alpha$ is injected into the cavity. This excess charge generates a potential response which in turn leads to a screening charge of opposite sign: $Q^\text{scr} = -e^2 DU$ determined by the total density of states $D = \overline{D}_1 + \overline{D}_2$. In addition, the dot is coupled capacitively to a gate at voltage $V_g$ and geometric capacitance $C$. The excess charge on the cavity can now be expressed in two ways: consideration of the total charge response gives

$$Q = \sum Q^\text{bare}_\alpha + Q^\text{scr} = e^2 T_1 V_1 + e^2 T_2 V_2 - e^2 DU ,$$  

(10)

and consideration of the Poisson equation gives

$$Q = C(U - V_g).$$  

(11)

Therefore, using these two expressions to eliminate the total charge, we find for the potential

$$U = \frac{e^2 T_1 V_1 + e^2 T_2 V_2 + CV_g}{e^2 D + C}.$$  

(12)

Note that $U$ is the deviation from the equilibrium value $U_{eq}$. Taking the derivatives of $U$ with respect to the voltage shifts $V_1, V_2, V_g$ gives the characteristic potentials $u_1, u_2, u_g$.

We remark that different nonlinear $I-V$ characteristics are measured depending on the way the cavity is biased. For the chaotic cavity considered here the characteristics differ just by a sample-to-sample fluctuation. Without loss of generality we consider $V_g = V_2 = 0$ and $V_1 = V$. Thus, the expression for the differential conductance $G$ effectively corresponds to the two terminal case, Eq. 9.

The approximation of a single potential used here neglects charge oscillations on the scale of the Fermi wave length. Therefore, we can use the WKB approximation to replace potential derivatives in Eq. 5 with derivatives with respect to energy. Then,

$$G_{111} = -\frac{e^3}{\hbar} \frac{dT}{dE} \bigg|_{\text{eq}} (1 - 2u_1) ,$$  

(13)

where $T \equiv \text{Tr}(s_1 s_2^\dagger)$ is the transmission probability. For the chaotic cavity the energy derivative of the transmission probability is random from one ensemble member to the other and for a sufficiently open contacts (large $N$-limit) is not correlated with the potential fluctuations. As a consequence, the ensemble average $\langle G_{111} \rangle = 0$ vanishes. A magnetic-field asymmetry can develop only due to quantum fluctuations, of the characteristic potential $u_1$,

$$u_1 = \overline{D}_1 \delta \frac{C_\mu}{C}.$$  

(14)

Here we have introduced the electrochemical capacitance $1/C_\mu = 1/C + 1/e^2 D$. For $e^2/C \gg \delta$ the fluctuations in the capacitance are very small and the actual density of states in the capacitance can be replaced by its ensemble average $\langle D \rangle = 1/\delta$. Here $\delta$ is the level spacing in the cavity if the contacts to the reservoirs were closed.

From the characteristic potential we obtain the fluctuations of the magneto-asymmetry of the screening potential, Eq. 5,

$$\text{var} \Phi_U = \frac{V^2 g^2}{4} \left( \frac{C_\mu}{C} \right)^2 \text{var}(\overline{D}_1 - \overline{D}_1) ,$$  

(15)

in terms of the fluctuations of the difference of the injectivity and the emissivity. Using the results of Ref. 37 for the unscreened emittance of a chaotic cavity in the large $N$ limit we find

$$\text{var} \Phi_U = \frac{N_1 N_2 V^2}{N^4} \frac{C_\mu}{2} \left( \frac{C_\mu}{C} \right)^2 .$$  

(16)
The size of the fluctuating magneto-asymmetry vanishes quickly with increasing mode number (proportional to $1/N^2$ for a symmetrically coupled cavity).

We are now in a position to determine the magneto-asymmetry of the fluctuating nonlinear conductance. First, in the limit of a large number of modes the traces arising in Eq. (13) can be decoupled, i.e. we can disregard correlations between $dT/dE$ and $u_i$.

The unscreened nonlinear conductance $-(e^2/h)dT/dE|_{\text{eq}}$ changes sign randomly on the ensemble so that its average is zero. We find for the fluctuations \( \text{var}(dT/dE) = 8\pi^2 N_1^2 N_2^2 / N^6 \). Using these results and Eqs. (13) and (16) we find

\[
\text{var} \Phi_G = \frac{16e^6}{h^2} \frac{N_1^3 N_2^3}{N^{10}} \left( \frac{V}{\delta} \right)^2 \left( \frac{C_{12}}{C} \right)^2.
\]  

(17)

A characteristic feature is that \( \text{var} \Phi_G \) is maximal for perfect screening \( C_{12} = C \), i.e., when the charging energy of the dot is much larger than the mean level spacing, \( e^2/C \gg \delta \). In the opposite limit of weak screening, the ratio \( C_{12}/C \) tends to zero as \( C \) tends to infinity. Hence, the fluctuations exhibit a magnetic field asymmetry only to the extent that the potential fluctuations are an uneven function of \( B \). Importantly, the fluctuations of \( \Phi_G \) have an energy scale given by the applied voltage and increase until \( V \) is of the order of \( \hbar / \tau_d \). We remark that the fluctuations become smaller with increasing number of channels, suggesting that the effect is observable in the quantum regime only (small number of modes). This result has been confirmed in Ref. 3 with a numerical simulation of the probability distribution of the rectification fluctuations (see Fig. 1). We have, thus, demonstrated that the fluctuations of the differential conductance are not symmetric under reversal of the applied magnetic field and that this is purely an interaction effect.

Compared to the fluctuations of the linear conductance, Eq. (9), the fluctuations of the differential conductance lack, notably, a universal feature. Equation (17) demonstrates that the fluctuations of \( \Phi_G \) depend on the microscopic details of the sample through the parameters \( \delta \) and \( C \). How could one distinguish between both types of fluctuations in a realistic experiment? We notice that Eq. (17) shows a \( \beta \) dependence and no magnetic-field asymmetry for the linear conductance while the nonlinear conductance magneto-asymmetry is zero for \( \beta = 1 \) (\( B = 0 \)) and maximal for \( \beta = 2 \) (\( B = \hbar / eS \), with \( S \) the dot’s area). In between, we expect a smooth crossover from low to high magnetic fields. Moreover, the fluctuations in the linear conductance saturate in the asymptotic limit of the mode number whereas the fluctuations of \( G \), as already emphasized, are vanishingly small for \( N_1, N_2 \gg 1 \).

IV. EFFECT OF DEPHASING

For the chaotic cavity considered here nonlinearity arises only due to quantum interference. Therefore, it is interesting to investigate the role of phase breaking. Suppose that electrons retain their phase memory for a time \( \tau_\phi \). Carriers dwell a time \( \tau_d = \hbar / N \) inside the cavity. Note that the dwell time depends crucially on the size of the contacts \( N = N_1 + N_2 \). For \( \tau_d \ll \tau_\phi \) we deal with a quantum coherent cavity described above, whereas for \( \tau_d \gg \tau_\phi \) carriers are in the cavity long enough to lose phase memory.

Dephasing is induced by means of interaction with the environment (phonons, radiation, impurities with internal dynamics, fluctuations of gate voltages, etc.) or even with other carriers. Here we simulate such processes in a phenomenological way by introducing a fictitious voltage probe attached to the cavity. The current flowing through the probe is zero, so charge conservation is maintained. Nevertheless, when an electron enters the probe, it loses its phase memory and the emerging electron is injected in the dot with an uncorrelated phase. The voltage probe is dissipative, carriers relax on average to the equilibrium distribution function at the voltage probe. Inelastic processes are not necessary for phase-breaking: phase breaking can occur through quasi-elastic processes. Here we will treat only the case of a dissipative voltage probe.

For simplicity, we assume full screening \( (C \rightarrow 0) \). With the voltage probe, we have now a three-lead cavity. Applying charge conservation [see Eq. (10)], we find the screening potential

\[
U = \frac{\mathbf{D}_1 V_1 + \mathbf{D}_2 V_2 + \mathbf{D}_\phi V_\phi}{D},
\]  

(18)

with \( D = \mathbf{D}_1 + \mathbf{D}_2 + \mathbf{D}_\phi \) the total DOS. Without loss of generality, we take \( V_1 = V \) and \( V_2 = 0 \). The probe is connected via a contact with \( N_\phi \) modes and \( V_\phi \) must be found by setting \( I_\phi = 0 \):

\[
V_\phi = \frac{G_{11} + G_{21}}{G_{11} + G_{12} + G_{21} + G_{22}} V,
\]  

(19)

where \( G_{\alpha\beta} \) are linear conductances of the three probe conductors. Upon inserting this result in Eq. (15) one can find the fluctuations of \( \Phi_U \) with

\[
\Phi_U = \delta \left[ \left( \mathbf{D}_1 - \mathbf{D}_\phi \right) V + \mathbf{D}_\phi V_\phi(B) - \mathbf{D}_\phi V_\phi(-B) \right] .
\]  

(20)

In addition to the injectivity fluctuations as in Eq. (15) there are in var \( \Phi_U \) contributions due to the probe’s voltage fluctuations and the correlations between the injectivity \( D_\alpha \) and the probe’s voltage \( V_\phi \). The calculation is lengthy and we just quote the final result for symmetric couplings \( (N_1 = N_2 = N/2) \):

\[
\text{var} \Phi_U = \left( \frac{V}{2} \right)^2 \frac{8N^3 + 8N_\phi^3 + 8N_\phi^2 N + N_\phi N^2}{16(N + N_\phi)^2}.
\]  

(21)

For \( N_\phi = 0 \) we recover our earlier expression, Eq. (16). When \( N \gg N_\phi \), the leading-order correction to Eq. (21)
is
\[
\varphi_U = \left( \frac{V}{2} \right)^2 \left[ \frac{1}{2N^2} - \frac{39 N_\phi}{16 N^3} \right] + O(1/N^4). \tag{22}
\]

As expected, weak dephasing leads to a reduction of the observed magneto-asymmetry. In the opposite limit for \(N_\phi \gg N\) the magnetic field symmetry vanishes as \(1/N^2\).

The full calculation of the magneto-asymmetry of the differential conductance \(G\) is rather involved and we have presently only numerical results.\(^{42}\) The results are in good agreement with the reduction of \(\varphi_U\) with increasing coupling to the fictitious probe. Therefore, experiments carried out in cavities attached to few-channel contacts and large dephasing times are most promising for the observation of the symmetry breaking discussed in this work.

V. CONCLUSIONS

In this work we have analyzed the magnetic-field asymmetries arising in nonlinear mesoscopic transport which signal a departure from Onsager’s reciprocity relations. We have presented a theory based on the scattering approach which predicts that the magneto-asymmetry of the fluctuating rectification current is caused exclusively by the fact that, in general, the response of the screening potential is not an even function of the magnetic field. This is an interaction effect. We have investigated the size of the effect in a generic mesoscopic conductor, an open quantum (chaotic) dot. We predict fluctuations of the differential conductance which are asymmetric with regard to magnetic field reversal. We have discussed the nonuniversal form of the fluctuations and its dependence on screening, applied voltage and energy. Importantly, the magneto-asymmetry decreases rapidly with increasing coupling to the reservoirs since for chaotic cavities the effect has a quantum origin. Consequently, as we have shown here, the fluctuations of the magneto-asymmetry are also suppressed with increasing dephasing.

We have illustrated our theory with the help of a chaotic cavity. However, the main conclusions are completely general. Similar effects might be found in molecular conduction junctions,\(^{10,11,12}\) quantum wires,\(^{13}\) and carbon nanotubes.\(^{14}\) Our discussion shows that the magneto-asymmetry represents an important test of our understanding of nonlinear transport and its measurements can reveal information on the interaction strength in these structures.

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Note added in proof

Two recent works present experimental data on magnetic field asymmetry in field effect transistors and in carbon nanotubes. The data point to a classical effect different from the mesoscopic effects discussed here.

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