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Shot noise of photon-excited electron-hole pairs in open quantum dots

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We investigate shot noise of photon-excited electron-hole pairs in open multi-terminal, multi-channel chaotic dots. Coulomb interactions in the dot are treated self-consistently giving a gauge-invariant expression for the finite frequency correlations. The Coulomb interactions decrease the noise, the strong interaction limit coincides with the non-interacting adiabatic limit. Inelastic scattering and dephasing in the dot are described by voltage and dephasing probe models respectively. We find that dephasing leaves the noise invariant, but inelastic scattering decreases correlations eventually down to zero.

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Investigation of noise$^{1-3}$ generally provides information not available from current measurements, such as effective charge and quantum statistical properties of the carriers. Most of the work has focused on noise in the presence of stationary (dc) applied voltages. Of interest here are shot noise measurements in photon-assisted transport.$^4$ Initial experiments investigated noise in the presence of both ac- and dc-potentials.$^2$ However, recently Reydellet et al.$^3$ reported shot noise experiments in the presence of ac-potentials only. There is no dc-current linear in voltage. Reydellet et al.$^3$ subject one of the contacts of a two-terminal conductor, a quantum point contact, to rf-radiation. Good agreement was found between experiment and theory.$^5,6$

The purpose of our work is to investigate a generic conductor with one or several contacts subject to ac-potentials and to investigate the effect of Coulomb interactions, dephasing and inelastic scattering on the photon-assisted noise. We consider a multi-terminal chaotic quantum dot$^7,8$ connected to electronic reservoirs via quantum point contacts $\alpha = 1, 2, \ldots, M$ with a large number of channels (see Fig. 1). The reservoirs are subject to oscillating potentials $V_\alpha(t) = V_\alpha \cos(\omega t + \phi_\alpha)$ at the same frequency $\omega$, with arbitrary phase $\phi_\alpha$ (this is in contrast to the widely investigated case of shape modulating potentials applied to gates$^9$). In the absence of a dc-bias, the elementary excitations generated by ac-potentials can be understood as electron-hole ($e-h$) pairs rather than single electrons.$^{10}$ The interpretation of the noise in terms of $e-h$ pairs generated in the contacts of the sample provides an intuitively appealing picture$^{11}$ of the resulting shot-noise. In the many-channel regime the noise correlator $S_{\lambda \mu}$ between the contacts $\lambda$ and $\mu$ fluctuates weakly from sample to sample, and we consider only the mesoscopically averaged shot-noise $\overline{S}_{\lambda \mu}$.

The effect of inelastic scattering and/or dephasing on photon-assisted noise has, to the best of our knowledge, not been addressed in the literature. For dc-biased chaotic cavities, it was shown that the ensemble averaged noise is insensitive to dephasing.$^{12,13}$ It was expected$^{14}$ and demonstrated$^{15,16}$ that this is true not only for the conductance and shot noise but also for higher order cumulants. To investigate inelastic scattering and dephasing we employ here two models. In the first, inelastic scattering is introduced by connecting the cavity to a voltage probe.$^{17}$ The low frequency charge current into the probe is zero. The voltage probe is characterized by a (dimensionless) rate $\gamma_\varphi = \hbar/(\tau_\varphi \Delta)$, where $\tau_\varphi$ is the inelastic scattering time and $\Delta$ the mean level spacing in the dot. In the second, dephasing probe model,$^{18}$ in addition to the charge current, the energy exchange with the probe is prohibited. The rate $\gamma_\varphi$ is then due to pure dephasing. In both models the scattering is spatially uniform, which is ensured by a large number $N_\varphi \to \infty$ of poorly transmitting channels, $\Gamma \ll 1$, in the lead to the probe, such that $\gamma_\varphi = N_\varphi \Gamma$ is a finite constant.$^{19}$ We compare the results from random matrix theory with a semi-classical approach and find that they lead to the same result.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{A four-terminal ($M = 4$) chaotic dot is subject to oscillating potentials $V_\alpha(t)$ at contacts $\alpha = 1, 2$ and coupled to a gate with a time-dependent potential $V_0(t)$, via a capacitance $C$. The internal potential of the dot is $U(t)$. Inelastic scattering/dephasing/ with a rate $\gamma_\varphi$ is introduced by connecting a probe $\varphi$ to the dot. An electron-hole pair excited in lead 1 and split into leads 3 and 4 is indicated.}
\end{figure}
It is convenient to express the noise in terms of an effective noise temperature $T^*$. For the correlator $S_{\lambda\mu}$ between current fluctuations in the $\lambda$th and $\mu$th leads, with $N_\lambda$ and $N_\mu$ channels respectively, we find for low frequencies $\hbar\omega \ll N\Delta$, with $N = \sum_\alpha N_\alpha$, and low temperatures, to leading order in $eV_\alpha/h\omega$

$$\overline{S}_{\lambda\mu}^{\text{deph}} = 2G_{\lambda\mu}k_BT_0^*, \quad \overline{S}_{\lambda\mu}^{\text{vol}} = 2G_{\lambda\mu}k_BT_0^{\ast}N + \gamma_e, \quad (1)$$

$$T_0^* = \frac{e^2}{k_Bh\omega} \left( \frac{\text{tr} V^2}{N} - \frac{\text{tr} Ve^{i\phi}e^{i\phi}^*}{N^2} \right),$$

where $T_0^*$ is the noise temperature corresponding to the coherent limit and $G_{\lambda\mu} = (e^2/h)(N_\lambda \delta_{\lambda\mu} - N_\lambda N_\mu/N)$, the ensemble averaged differential conductance. Here we introduce diagonal matrices of the amplitudes $V = \text{diag}(V_1, \ldots, V_M)$ and phase shifts $\phi = \text{diag}(\phi_1, \ldots, \phi_M)$, with $\Pi_\lambda$ a projector matrix on the $\lambda$th lead. Energy conserving dephasing processes do not affect the noise to leading order in $N$, in particular the dependence on the phases of the applied voltages is preserved. The results from the dephasing probe model and the semi-classical approach coincide, which demonstrates the consistency of the different approaches and in particular supports the dephasing probe model.

In contrast, inelastic scattering suppresses noise similarly to the suppression in conductors subject to a dc voltage only.\(^1\) The decrease of the noise temperature with increasing inelastic rate can be understood as follows: a sufficiently large voltage probe acts as an absorber of rare $e - h$ pairs created in the real leads. An absorption of these pairs creates no potential response. Only events in which a single electron or hole reaches the voltage probe gives rise to potential fluctuations and to the generation of $e - h$ pairs by the probe. With increasing coupling to the probe it is ever more likely that both the electron and the hole generated in a real lead end up in the voltage probe, rather then split into current-measuring leads. For the dephasing probe the balance of the energy currents is reached through a non-equilibrium distribution in the probe. The probe serves as a source of $e - h$ pairs which can be split into different outgoing leads, giving an additional contribution to the correlations which exactly compensates for the absorbed pairs.

AC-voltages can generate excess charge densities and, therefore, it is important to treat the effect of Coulomb interactions on the noise. Experiments can be performed at sufficiently high frequencies for which the conductance is energy-dependent. Then a non-interacting treatment gives an unphysical (not gauge-invariant) result. To have a meaningful result, also including possible external macroscopic gates, we consider Coulomb interactions on the level of the random phase approximation\(^2\) and find the dynamic self-consistent potential $U(t)$ in the sample. For open, many channel conductors, the neglected part of the Coulomb interactions (Fock-terms) gives only subleading corrections\(^2\) and is disregarded here.

We find a signature of Coulomb interactions when the period of the ac-excitation is comparable to the charge relaxation time $\tau$, rather than the dwell time $\tau_d$ of the quantum dot. A qualitatively similar result was found for the ac-conductance in a quantum dot.\(^23\) The interactions decrease the correlations: In the absence of interactions the current fluctuations in different leads are uncorrelated, since there is no requirement of charge conservation in the dot. With interactions, these fluctuations are suppressed by negative feedback from displacement currents due to the internal potential. We remark that for strong Coulomb interaction, $\tau \rightarrow 0$, the noise corresponds effectively to the low frequency limit $\hbar\omega \ll N\Delta$ for the non-interacting system. This explains why the expression for the noise in the ac-driven system of Refs. \([5,6]\) fits the experimental data \([2,3]\) so well: the dispersion of the noise is significant on a much larger frequency scale than applied in these experiments.

**Formalism** We first consider the current correlations in the leads $\lambda, \mu$ using scattering theory.\(^1,7\) The coherent open dot is fully characterized by its unitary $N \times N$ scattering matrix $\mathcal{S}$. The dot is in the chaotic regime, i.e., the dwell time of the dot $\tau_d = h/(N\Delta)$ is sufficiently large. Scattering is spin-independent and the results given below are presented for a single spin direction. We take $e = h = k_B = 1$ and express the sample-specific noise in terms of energy-dependent scattering matrices $\mathcal{S}(\varepsilon)$, amplitudes of applied voltages $V_{\alpha}$, their relative phase shifts $\phi_\alpha$ and electronic distributions in the leads $f_\alpha(\varepsilon)$:\(^11\)

$$S_{\lambda\mu} = \sum_{kl m, \alpha\beta} \int d\varepsilon \text{tr} \left( A_{\alpha\beta}(\lambda, \varepsilon - m\omega)A_{\beta\alpha}(\mu, \varepsilon) \right) \times J_k \left( \frac{V_\alpha}{\omega} \right) J_{k+m} \left( \frac{V_\alpha}{\omega} \right) J_{l+m} \left( \frac{V_\beta}{\omega} \right) J_l \left( \frac{V_\beta}{\omega} \right) \times e^{im(\phi_\beta - \phi_\alpha)} f_\alpha(\varepsilon - k\omega)(1 - f_\beta(\varepsilon - l\omega)), \quad (2)$$

where $A(\lambda, \varepsilon) = \Pi_\lambda - S^\dagger(\varepsilon) \Pi_\lambda S(\varepsilon)$ and $J_n$ is the nth order Bessel function. The noise power Eq. (2) differs from sample to sample according to its mesoscopic distribution $P(S_{\lambda\mu})$. However, for $N \gg 1$ the value of the ensemble averaged correlations $\overline{S}_{\lambda\mu}$ is representative\(^2\) (the average is carried following Refs. \([25,26]\)). We note that to leading order in $N \gg 1$ the averaged noise is unaffected by a time-reversal symmetry breaking magnetic field. Below we first consider the effect of Coulomb interaction and later find the role of decoherence (dephasing) on the averaged noise $\overline{S}_{\lambda\mu}$. We also show that the noise can be found within a semi-classical approach.

**Coulomb interactions** First we find the correlations in the non-interacting limit, when the internal potential of the dot and its coupling to external gates are not accounted for. We obtain the effective noise temperature $T_{\text{NI}}$, with $\overline{S}_{\lambda\mu} = 2G_{\lambda\mu}T_{\text{ NI}}$, as

$$T_{\text{ NI}} = \frac{1}{\omega} \left( \frac{\text{tr} V^2}{N} - \frac{|\text{tr} Ve^{i\phi}e^{i\phi}^*|^2}{N^2 + \omega^2/\Delta^2} \right). \quad (3)$$

In the limit $\omega \ll N\Delta$ the noise temperature $T_{\text{ NI}} \rightarrow T_0^*$, with $T_0^*$ defined in Eq. (1). Only in this limit is the noise gauge-invariant, i.e., unaffected by a uniform shift
of applied potentials. At finite frequencies it is necessary to consider the internal potential $U(t)$ to obtain a physically meaningful result. The interacting problem with $U(t) \neq 0$ is reduced to the noninteracting one by the global uniform shift $V(t) - V(t) - U(t)$ in Eq. (2). The potential $U(t)$ is found using gauge invariance of currents. Charge inflow into the dot shifts the potential $U(t)$ due to capacitive coupling to the gate, kept at a potential $V_0(t) = V_0 \cos(\omega t + \phi_0)$. The current $I_\alpha(\Omega)$ at finite frequency $\Omega$ is the sum of particle currents $\sum_\beta G_{\alpha\beta}(\Omega)V_\beta(\Omega)$ and the displacement current $\chi_\alpha(\Omega)U(\Omega)$ due to variations of the uniform potential in the dot, with $^\text{21}$

$$G_{\alpha\beta}(\Omega) = N_\alpha \delta_{\alpha\beta} - \int_0^\Omega \frac{d\varepsilon}{\Omega} \text{tr} \mathbb{1}_\alpha S^\dagger(\varepsilon - \Omega) \mathbb{1}_\beta S(\varepsilon). \quad (4)$$

Gauge invariance implies a susceptibility given by $^\text{21}$

$$\chi_\alpha(\Omega) = - \sum_\beta G_{\alpha\beta}(\Omega),$$

and the current $I_\alpha$ reads

$$I_\alpha(\Omega) = \sum_\beta G_{\alpha\beta}(\Omega)(V_\beta(\Omega) - U(\Omega)). \quad (5)$$

We note that to leading order in $V/\omega$ only $\Omega = \pm \omega$ contribute. From charge conservation $\sum_\alpha I_\alpha(t) = C(d/dt)[V_0(t) - U(t)],$ we find the sample-specific potential $U_\omega$. The potential $U_\omega$ is self-averaging, i.e., its sample-to-sample fluctuations can be neglected, if $N^2\text{tr}(V e^{i\phi})^2 \gg \text{tr}(V^2 e^{2i\phi})$ and $N \gg 1$. Here we assume that this is the case and hence the potential $U_\omega$ averages to

$$\bar{U}_\omega = V_0 e^{i\phi_0} + \frac{C_\mu/C}{1 - i\omega R_q C_\mu} \left( \frac{\text{tr} V e^{i\phi}}{N} - V_0 e^{i\phi_0} \right). \quad (6)$$

We have introduced the electrochemical capacitance $^\text{21}$

$$1/C_\mu = \Delta + 1/C, \text{ the series addition of the geometrical capacitance } C \text{ and the "quantum capacitance" determined by density of states of the cavity } 1/\Delta, \text{ as well as the charge relaxation resistance }^\text{21,23} R_q = 1/N. \text{ The Coulomb interaction thus leads to charging processes on the scale of the charge relaxation time } \tau = R_q C_\mu. \text{ Using this Hartree potential Eq. (6) we find that the Coulomb interaction modifies the non-interacting result (3), in that the density of states } 1/\Delta \text{ is substituted by the electrochemical capacitance } C_\mu. \text{ Taking into account the external gate potentials then yields a gauge-invariant noise temperature}

$$T_0^* = T_0^* + \frac{1}{\omega} \frac{|\text{tr}(V e^{i\phi})/N - V_0 e^{i\phi_0}|^2 (2\omega R_q C_\mu)^2}{1 + (\omega R_q C_\mu)^2}. \quad (7)$$

An important test of validity of Eq. (7) is obtained by considering the limit of synchronous voltages at all contacts, $V_\alpha = V, \phi_\alpha = 0$. This corresponds to a global shift of the potential and has no physical consequences.

Since typically $C_\mu \Delta \ll 1$, this implies that much higher frequencies are needed to observe the dispersion of the shot noise than one might naively expect from the non-interacting result (3). As a consequence, for the experimentally relevant limit $\omega \tau \ll 1$, the noise temperature $T_0^*$ coincides with the result $T_0^*$ for the adiabatic cycling of the potentials, $\omega \ll N\Delta$, in the non-interacting limit. Below we focus on this low frequency limit $\omega \tau \ll 1$ when deriving the result in Eq. (1) for dephasing and inelastic scattering. As described above, the dephasing and inelastic scattering is modeled by connecting the dot to voltage and dephasing probes. Both models require that the low frequency charge current into the additional lead $\varphi$ vanishes. For a real voltage probe this corresponds to a voltmeter with infinite impedance at zero frequency, dropping off at higher frequencies. If there is no parallel capacitance to the probe, then the potentials $V_\varphi$ of the probe and the dot must be equal and are obtained from Eq. (6) in the limit $V_0 = 0$, $C\Delta \to 0$, $\omega \to 0$.

**Voltage probe model** Scattering in the quantum dot gives rise to bare particle current fluctuations in the leads, $\delta_\alpha$. At low frequencies, the potential $\delta V_\varphi$ in the voltage probe fluctuations to maintain zero current. The real leads are however voltage controlled and $\delta V_\varphi = 0$. Thus the total fluctuation $\Delta I_\varphi(t)$ of the current in the real leads consists of particle current fluctuations $\delta_\alpha$, and additional displacement current fluctuations $G_{\alpha\varphi}\delta V_\varphi$ [Note that $G_{\alpha\varphi}$ is the conductance for the dot connected via a non ideal contact to the probe]. The conservation of current fluctuations into the probe, $\Delta I_\varphi = 0$, gives $\sum_\alpha G_{\alpha\varphi}\delta V_\varphi(-t) = -\Delta I_\varphi(t)$ from which $\delta V_\varphi$ is determined. We can then write the total fluctuations $\Delta I(t) = \mathbb{1}\delta I(t)$ with $\mathbb{1} = \mathbb{1}_\lambda + \mathbb{1}_\varphi G_{\lambda\varphi}/G_{\varphi\varphi}$, where we introduced a vector notation $\Delta I(t) = [\Delta I_1(t),...,\Delta I_M(t)]$. Using Eq. (2) in the low frequency limit, the total noise $S^{\varphi\lambda}_\mu$ is given by

$$S^{\varphi\lambda}_\mu = \sum_{\alpha\beta} \frac{|V_\alpha e^{i\phi_\alpha} - V_\beta e^{i\phi_\beta}|^2}{4\omega} \text{tr}(\mathbb{1}_\lambda)_{\alpha\beta} \mathbb{1}_\mu_{\beta\alpha}. \quad (8)$$

where $\mathbb{1}_\lambda = \mathbb{1}_\lambda - \mathbb{1}^\dagger S\mathbb{1}_\lambda S$. For a coherent dot, the effective noise temperature in the limit $\omega \ll N\Delta$ is given by $T_0^*$, while for a partially coherent dot the effective temperature is suppressed, given by $NT_0^*/(N + \gamma_\varphi)$, as stated in the r.h.s. of Eq. (1). This result is obtained from separate averaging of the matrices $\mathbb{1}$ and the 4-matrix correlators for a quantum dot with non-ideal leads $^\text{28}$.

**Dephasing probe model** We now compare the results for the inelastic voltage probe with the dephasing probe model. Current conservation at each energy now determines the non-equilibrium distribution function $f_\varphi(\varepsilon)$ in the dephasing probe. Mesoscopic fluctuations of $f_\varphi$ are small, so that we can characterize a dot by the mesoscopically averaged distribution $\bar{f}_\varphi(\varepsilon)$. The detailed balance of the currents at energy $\varepsilon$ leads to

$$\sum_m j_m^2 \frac{V_\varphi}{\omega} = \sum_{\beta, m} N_\beta \frac{\gamma_\beta}{N} j_m^2 \frac{V_\varphi}{\omega}, \quad (9)$$

where the summation is taken over real leads $\beta$ only. To
leading order in \(V/\omega\) the distribution is

\[
\overline{f}_\varphi(\varepsilon) = \Theta(-\varepsilon) + sgn \varepsilon \, \Theta(\omega - |\varepsilon|) \frac{T_0^*}{\omega}. \tag{10}
\]

The fluctuations of the distribution function are treated in a similar way as for the voltage probe. However, the correlations in the dephasing probe model have both the contribution (8) due to the potential \(V_\varphi\) and a term due to the non-equilibrium distribution \(\overline{f}_\varphi\). As a consequence, the noise is characterized by \(T_0^*\), i.e. elastic dephasing does not affect the noise.

We note that in an experiment, a finite temperature \(T\) affects the noise measurements. From Eq. (2) taken at finite temperature \(T\) the equilibrium noise temperature is readily obtained, \(T_{eq}^* = T\). Temperature effects can thus be neglected for inelastic scattering if \(T \ll NT_0^*/(N + \gamma_\varphi)\) and for \(T \ll T_0^*\) in the absence of inelastic scattering.

Semi-classical approach The fact that \(N \gg 1\) and the observation that the noise is insensitive to elastic dephasing suggests that, just as in the dc-biased case, a semi-classical description of the noise is possible. This is indeed the case, when \(V^2/\omega \gg \Delta\). The point contacts act as independent emitters of fast fluctuations into the dot. The distribution function \(f(\varepsilon,t) = f(\varepsilon) + \delta f(\varepsilon,t)\) of the dot responds with fluctuations \(\delta f(\varepsilon,t)\) to preserve current at each energy. The total charge current into the probe is zero, which gives the potential of the dot \(U(t) = \text{tr} V(t)/N\). The static part of the distribution function \(f(\varepsilon)\) coincides with the distribution function \(\overline{f}_\varphi(\varepsilon)\) in Eq. (10), since \(\overline{f}_\varphi(\varepsilon)\) was derived under the assumption that currents are conserved at each energy.

The total fluctuations of the current \(\Delta I_\alpha(\varepsilon)\) in lead \(\alpha\) at energy \(\varepsilon\) are the sum of bare fluctuations \(\delta I_\alpha(\varepsilon)\) and the displacement current fluctuations \(N_\alpha \delta f(\varepsilon,t)\), \(\Delta I_\alpha(\varepsilon) = \delta I_\alpha(\varepsilon) + N_\alpha \delta f(\varepsilon,t)\). Conservation of current at each energy in the dot gives \(\delta f(\varepsilon,t) = -(1/N) \sum_\alpha \delta I_\alpha(\varepsilon)\), yielding \(\Delta I_\alpha(\varepsilon)\) in terms of the bare correlators \(\delta I_\alpha(\varepsilon)\). The total noise \(\overline{S}_\mu = \langle \Delta I_\lambda \Delta I_\mu \rangle\) is thus given by a weighted sum of the correlators of the individual point contacts \(\langle \delta I_\alpha(\varepsilon, t) \delta I_\beta(\varepsilon, t) \rangle = \delta_{\alpha \beta} S_\alpha\), with \(S_\alpha\) given by Eq. (2) for the corresponding two terminal point contact \(\alpha\). Due to the non-equilibrium distribution function \(f(\varepsilon)\) in the dot, the noise \(S_\alpha\) is nonzero. Summing up all the terms in \(\overline{S}_\mu\) we arrive at the effective noise temperature \(T_0^*,\) the same as obtained within a random matrix approach. Similarly we can show that a voltage probe suppresses the correlations, giving a noise temperature \(NT_0^*/(N + \gamma_\varphi)\) in accordance with Eq. (1).

In conclusion, we have investigated the shot noise of \(e - h\) pairs in chaotic quantum dots subject to an ac bias at the contacts. The noise has been derived within a scattering random matrix approach as well as with a semi-classical approach. By including Coulomb interactions, a gauge invariant theory for finite frequencies was constructed. It was found that elastic dephasing does not affect the noise, however inelastic scattering leads to a suppression of the noise.

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20. For few-channel systems predictions of the dephasing probe model might differ from phase averaged results, see F. Marquardt and C. Bruder, Phys. Rev. B 70, 125305 (2004). See also C. W. J. Beenakker and B. Michaelis, cond-mat/0503347.
24. Although formally considering \(N_\alpha \gg 1\), we expect the result to be useful for few-channel dots (see discussion of conductance for \(N = 2\) in Refs. [9,22]).

M. Büttiker and M. L. Polianski, cond-mat/0508220.