Massive rotating stars : the road to supernova explosion

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Abstract

Durant les dix dernières années, les développements du logiciel d'évolution stellaire de Genève ont permis l'étude de l'évolution des étoiles massives en rotation depuis la séquence principale (SP) jusqu'à la fusion du carbone. Le logiciel permet de très bien reproduire les observations à diverses métallicités. Ce travail consiste à modéliser l'évolution des étoiles massives en rotation durant les phases avancées pré-supernova (fusions du néon, de l'oxygène et du silicium). Au premier chapitre, les étoiles massives sont introduites. Aux chapitres deux et trois, le logiciel d'évolution stellaire de Genève ainsi que les modifications effectuées sont décrits. Aux chapitres quatre et cinq, l'évolution des modèles depuis la SP jusqu'à la fin de la fusion du silicium est présentée pour des masses entre 12 et 60 masses solaires à métallicité solaire ainsi que leurs rendements stellaires. Les conclusions et perspectives sont données au chapitre six.

Reference


URN : urn:nbn:ch:unige-2877
DOI : 10.13097/archive-ouverte/unige:287

Available at:
http://archive-ouverte.unige.ch/unige:287

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Massive Rotating Stars: 
the Road to 
Supernova Explosion

THÈSE

présentée à la Faculté des sciences de l’Université de Genève
pour obtenir le grade de Docteur ès sciences,
mention astronomie et astrophysique

par

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de

Schangnau (BE)

Thèse N° 3550

GENÈVE
Atelier de reproduction de la Section de physique
2004
FACULTÉ DES SCIENCES

UNIVERSITÉ DE GENÈVE

Doctorat ès sciences
mention astronomie et astrophysique

Thèse de Monsieur Raphaël HIRSCHI

intitulée :

"Massive rotating stars :
the road to SuperNova explosion"

La Faculté des sciences, sur le préavis de Messieurs A. MAEDER, professeur ordinaire et directeur de thèse (Département d'Astronomie), G. BURKI, professeur ordinaire (Département d'Astronomie), G. MEYNET, docteur (Département d'Astronomie), et F. K. THIELEMANN, professeur (Universität of Basel - Institut für Physik - Basel, Schweiz), autorise l'impression de la présente thèse, sans exprimer d'opinion sur les propositions qui y sont énoncées.

Genève, le 27 septembre 2004

Le Doyen, Pierre SPIERER

N.B.- La thèse doit porter la déclaration précédente et remplir les conditions énumérées dans les "Informations relatives aux thèses de doctorat à l'Université de Genève".

Nombre d'exemplaires à livrer par colis séparé à la Faculté : - 7 -

Ce travail de thèse a donné lieu à des publications dont la liste se trouve à la page 135.
Thesis Outline

Over the last years, the development of the Geneva evolutionary code has allowed the study of rotating star evolution from the ZAMS until the end of the core Carbon burning phase. Various checks of the validity of the rotating stellar models have been made. In particular, it has been shown that rotating models well reproduce the observed surface enrichments (Heger & Langer 2000; Meynet & Maeder 2000), the ratios of blue to red supergiants in the Small Magellanic Cloud (Maeder & Meynet 2001), and the variations of the Wolf-Rayet (WR hereinafter) star populations as a function of the metallicity (Meynet & Maeder 2003). For all these features non-rotating models can not reproduce observations. The goal of this work is to follow the evolution of these models, which well reproduce the above observed features, during the pre-supernova evolution. So far, only one other research group (Heger et al. 2000) follows the pre-SN evolution including the effects of rotation. The comparison of the pre-SN models between the two groups is therefore useful for the community to understand the dependence of the results on the different physics. Even for models without rotation, there are discrepancies between the results of various groups (Woosley & Weaver 1995; Thielemann et al. 1996; Rauscher et al. 2002; Limongi & Chieffi 2003). We will therefore also compare our non-rotating models with the literature.

The organisation of this thesis is the following. In Chapter 2, the Geneva code and its physical ingredients are described. The modifications brought to the structure equations as well as the inclusion of dynamical shear are presented. In Chapter 3, the nuclear reaction network used in the advanced stages is described in details. In Chapter 4, the evolution of the models from the ZAMS until the end of Si-burning is described for stars with masses between 12 and 60 $M_\odot$ at solar metallicity. In Chapter 5, the yields from rotating massive stars at solar metallicity are presented. Conclusions and perspectives are given in Chapter 6.
Résumé

Il est actuellement bien établi que l’anglais est la langue la plus utilisée dans le monde scientifique. Néanmoins, il est important de conserver nos différentes cultures et langues. C’est pour cela que malgré le fait que la thèse soit écrite en anglais, ce résumé est écrit en français.

“Étoiles massives en rotation: en route vers l’explosion Supernova!”

Ce travail consiste à reproduire, sur ordinateur, l’évolution des étoiles massives depuis leur naissance jusqu’à leur mort (juste avant leur mort). Lors de leur mort, ces étoiles explosent en supernovae (SNe) et provoquent la naissance d’autres étoiles, créant ainsi un cycle de vie. Avant de présenter le travail effectué et les résultats obtenus, regardons comment se déroule la vie d’une étoile massive.

Introduction

Dans ce travail, une étoile est considérée comme massive si elle produit, à la fin des phases de fusions hydrostatiques, un noyau de fer. Ces étoiles ont au moins dix fois la masse du soleil ($M_\odot \simeq 2 \times 10^{33}$g). Le cœur de la plupart des étoiles moins massives deviendra, à la fin de leur vie, une naine blanche (NB, WD en anglais) composée de carbone et d’oxygène. Leur enveloppe produira une nébuleuse planétaire.

Les étoiles massives sont beaucoup moins nombreuses que les étoiles de plus faibles masses. Les observations montrent que le nombre d’étoiles d’une masse donnée est
Résumé

proportionnel à la masse de l’étoile à la puissance -2.35 (fonction initiale des masses, FIM, de Salpeter; IMF en anglais). Donc, il y a environ mille soleils pour une étoile de 20 $M_\odot$. Néanmoins, les étoiles massives sont très importantes. En effet, elles produisent la plupart des éléments chimiques et leur luminosité est si grande qu’elles contribuent de manière dominante à la luminosité totale des galaxies. Les étoiles massives donnent naissance aux SNe, étoiles de neutrons, trous noirs et peut-être aussi aux sursauts de rayonnement gamma longs et mous.

La vie d’une étoile massive

La naissance des étoiles massives n’est pas encore complètement comprise à l’heure actuelle. Il y a deux scénarios favoris pour leur formation. Le premier est que deux étoiles de plus faibles masses fusionnent (Bonnell et al. 1998). Le deuxième est que le taux d’accrétion de masse sur l’étoile en formation augmente au fur et à mesure que sa masse augmente (Behrend & Maeder 2001). L’étude de la formation est généralement séparée de l’étude de l’évolution de l’étoile. Dans ce travail (comme dans la plupart des autres études) l’évolution est étudiée à partir de la séquence principale d’âge zéro (SPAZ, ZAMS en anglais). Pendant la séquence principale (SP, MS en anglais), l’étoile consume son hydrogène en son cœur et produit de l’hélium. La SP représente la phase la plus longue de la vie de l’étoile (environ 90% du temps de vie total) et dure trois à quinze millions d’années pour des étoiles de 60 à 15 $M_\odot$. Leur vie est beaucoup plus courte que celle du soleil, qui dure environ neuf milliards d’années.

A la fin de la fusion de l’hydrogène (H), le coeur se contracte et l’enveloppe s’étend (effet appelé “effet miroir”). L’évolution de l’enveloppe dépend de la masse initiale de l’étoile et de la perte de masse qu’elle subit par vents stellaires. – Vu que l’étoile perd de la masse, sa masse totale évolue au cours de son évolution. Dans ce travail, si la masse d’une étoile est donnée avec un nombre entier (par exemple 25 $M_\odot$), il s’agit de la masse initiale de l’étoile. Si la masse est donnée par un nombre avec décimales, il s’agit de la masse de l’étoile durant son évolution ou à la fin de sa vie. En général, la masse initiale est utilisée. – Si la perte de masse est faible ($M \lesssim 20 M_\odot$ pour les étoiles en rotation), l’enveloppe s’étend jusqu’à que l’étoile devienne une supergéante rouge (SGR, RSG en anglais) et l’étoile finira sa vie en SGR. Si la perte de masse est faible durant la SP mais importante pendant le stade de SGR ($20 M_\odot \lesssim M \lesssim 50 M_\odot$ pour les étoiles en rotation), l’étoile finit par perdre son enveloppe riche en hydrogène. L’enveloppe se contracte alors et l’étoile devient une supergéante bleue (SGB, BSG en anglais). Une fois que l’étoile a perdu toute son enveloppe riche en hydrogène, elle devient une étoile Wolf-Rayet (WR). Si la perte de masse est déjà importante durant la SP ($M \gtrsim 50 M_\odot$ pour les étoiles en rotation), l’étoile devient déjà une WR pendant ou à la fin de la SP et ne deviendra jamais une SGR. Notons que l’évolution dépend de la métallicité de l’étoile et du fait qu’elle peut être dans un système binaire serré (où une étoile peut influencer la perte de masse de l’autre étoile). Ici l’évolution n’est décrite que pour la métallicité correspondante à celle du soleil et pour les étoiles simples.
Dans le cœur, la contraction continue jusqu’à que la température centrale soit assez élevée pour démarrer la fusion de l’hélium (He, $T_c \sim 1.5 \times 10^8$ K). La fusion He dure environ 10% de celle de l’hydrogène. Pendant cette phase, l’hélium est transformé en carbone et oxygène.

Les fusions H et He sont les phases lentes de l’évolution. En effet, à elles seules, elles contribuent à environ 99% de la durée de vie de l’étoile. Il est donc facile d’observer les étoiles durant ces stades. Durant ces phases, l’énergie produite par les réactions nucléaires au centre est évacuée par des photons (particules de lumière) qui vont interagir avec l’enveloppe. Après la fusion He, le cœur se contracte à nouveau et la température atteint à peu près 800 millions degrés. A partir de cet instant, la majeure partie de l’énergie produite dans le cœur est enlevée par des neutrinos (particules élémentaires de très faible masse). Étant donné que les neutrinos n’interagissent pratiquement pas avec la matière, l’évolution du cœur devient découplée de celle de l’enveloppe. Les propriétés de la surface de l’étoile ne changent donc pas durant les phases de fusions suivantes qui constituent les phases avancées de l’évolution. Cela veut dire que nous n’avons pas la possibilité d’observer directement ces phases avancées. Il faudra attendre l’explosion finale pour voir le résultat de ces phases. Pendant l’évolution avancée, les stades de fusion suivants ont lieu: carbone (produisant principalement du néon, sodium et magnésium), néon ($\rightarrow$ oxygène et magnésium), oxygène ($\rightarrow$ silicium, phosphore et soufre) et silicium ($\rightarrow$ nickel et fer). A la fin des phases avancées (stade pré-supernova), un cœur de fer (constitué d’élément du groupe du fer) est formé. Comme les éléments du pic du fer sont les plus stables, il n’y a plus de réactions nucléaires capables de produire de l’énergie. Le cœur s’effondre alors sous sa propre gravité. Lorsque le cœur atteint des densités nucléaires ($\rho \sim 10^{14}$ g cm$^{-3}$), la matière tombant sur le cœur rebondit. Cela crée un choc (choc instantané) qui s’arrête assez vite (après avoir voyagé sur seulement environ 150 km). Les neutrinos, émis par le cœur, redonne de l’énergie au choc grâce aux interactions avec les particules comme les protons et les neutrons (choc retardé) et ils déclenchent alors l’explosion en supernova (SN). Les neutrinos sont probablement aidé par la convection, la rotation et les champs magnétiques. Malgré plusieurs décennies de recherche, les modèles ont encore de la peine à reproduire des explosions en utilisant la physique la plus récente (Janka et al. 2003). L’explosion crée une onde de choc qui se propage à travers l’étoile. Le passage de cette onde de choc réchauffe les différentes couches de l’étoile à de très hautes températures (pour les régions centrales) et les différents stades de fusions vus précédemment ont à nouveau lieu dans des conditions similaires mais avec des durées beaucoup plus courtes. La liste de tous les éléments produits est très longue. On peut trouver notamment dans la littérature la quantité produite de chaque élément chimique par différentes étoiles ainsi que le processus qui a permis cette production (Woosley et al. 2002; Limongi & Chieffi 2003; Thielemann et al. 1996). En général:

- Les éléments plus légers que le silicium sont créés avant l’explosion (phases appelées hydrostatiques)
- Les éléments entre le silicium et le calcium sont produits avant et pendant l’explosion.
Les éléments du groupe du fer sont produits lors de l’explosion.

Les étoiles massives finissent donc leur vie en une explosion très énergétique (énergie cinétique d’environ $10^{51}$ erg) appelée supernova. Les SNe ont été classées par des observateurs. La classification (voir Fig. 1.1) est basée sur les propriétés spectrales et celles des courbes de lumière des SNe (voir Turatto 2003, pour les détails et des exemples). Il y a à l’heure actuelle quatre grands types de SNe:

- Ia: Ce sont des SNe produites par l’explosion thermonucléaire de naines blanches qui ont accrété assez de matière d’un compagnon binaire afin d’atteindre la masse de Chandrasekhar (voir Yoon & Langer 2004; Röpke & Hillebrandt 2004, pour de récents développements sur le sujet).
- II: SNe produites par des étoiles massives qui ont encore une partie de leur enveloppe riche en hydrogène au moment de l’explosion.
- Ib: SNe produites par des étoiles massives qui ont perdu toute leur enveloppe riche en hydrogène avant l’explosion (le type IIb est le cas intermédiaire entre les types II et Ib).
- Ic: SNe produites par des étoiles massives qui ont perdu non seulement toute leur enveloppe riche en hydrogène mais aussi toutes leurs couches riches en hélium avant l’explosion. Ce sont donc les étoiles WR qui produisent les SNe de types Ib et Ic.

Le cœur des étoiles massives, une fois effondré, donne naissance soit à une étoile de neutrons (EN, NS en anglais), soit à un trou noir (TN, BH en anglais). Notons encore que l’effondrement d’étoiles massives en rotation peut mener à la production de sursauts de rayonnement gamma (SRGs, GRBs en anglais) longs et mous.

Le logiciel d’évolution stellaire de Genève

Le logiciel d’évolution stellaire de Genève est un des descendants du logiciel à une dimension (1D) développé à Göttingen dans les années 60. Bien que la structure principale (décrite dans Kippenhahn et al. 1967) et certains noms de variable (encore en allemand) ont été conservés, le logiciel a aussi beaucoup évolué. Il comprend maintenant plusieurs dizaines de milliers de lignes écrites dans le langage Fortran 77.

La rotation dans le modèle de Genève

Les changements du logiciel pour tenir compte des effets de la rotation ont été effectuées durant les dix dernières années. Une revue de ces développements se trouve dans Maeder & Meynet (2000b). Les effets dus à la rotation peuvent être séparés en trois catégories:
• Effets hydrostatiques: La force centrifuge modifie l'équilibre hydrostatique de l'étoile. L'étoile devient oblate (un peu comme un disque en athlétisme). Les équations décivant la structure de l'étoile en 1D doivent être modifiées (voir Sect. 2.3).

• Anisotropie et augmentation de la perte de masse: La perte de masse dépend de l'opacité de la matière et de la gravité effective (addition des effets de la gravité et de la force centrifuge) à la surface de l'étoile. Plus l'opacité est grande, plus la perte de masse est grande. Plus la gravité effective est élevée, plus le flux radiatif (von Zeipel 1924) et la température effective (température mesurée par les observations) sont grands. La rotation, au travers de la force centrifuge, réduit la gravité effective à l'équateur par rapport au pôle de rotation. De ce fait, pour les étoiles massives, dont l'opacité est dominée par la diffusion par électrons libres (indépendante de la température), la rotation augmente la perte de masse au pôle de rotation. De plus, la rotation augmente la luminosité moyenne de l'étoile et donc augmente sa perte de masse moyenne.

• Les instabilités dues à la rotation: Les instabilités causées par la rotation considérées dans ce travail sont la turbulence horizontale, la circulation méridienne et les cisaillements dynamiques et séculaires. Le cisaillement dynamique a été introduit dans le logiciel dans le courant de ce travail.

La turbulence horizontale correspond à la turbulence le long des surface isobares (à pression constante). Si cette turbulence est forte, la rotation angulaire est constante sur les isobares et la situation est appelée “rotation en coquille”. Dans le modèle, on fait l'hypothèse que la turbulence horizontale est très forte et largement plus grande que la turbulence verticale. Cette hypothèse est justifiée car il n'y a pas de force de rappel due à la gravité le long des isobares (Maeder 2003).

La circulation méridienne, aussi appelée circulation d'Eddington–Sweet, découle du déséquilibre radiatif dans les étoiles en rotation. Ce déséquilibre est créé par le fait que les surfaces de températures constantes et de pressions constantes ne coïncident pas. En effet, la rotation étire les isobares le long de l'équateur et la température sur une même isobare est plus faible à l'équateur qu'au pôle de rotation. Cela induit un mouvement macroscopique de matière, dans lequel la matière monte en direction de la surface au pôle et descend à l'équateur (ou l'inverse). Cette circulation est un processus d'advection qui diffère du processus de diffusion par le fait que l'advection peut aussi renforcer des gradients de rotation ou de composition chimique (Maeder & Zahn 1998).

Le cisaillement dynamique a lieu lorsque l'excès d'énergie contenu dans la rotation différentielle est supérieure au travail à effectuer contre la force gravifique pour déplacer des éléments de matière. Ce cisaillement a la même échelle de temps que les autres processus dynamiques et donc c'est un processus très efficace. Comme mentionné plus tard, les zones de l'étoile qui subissent ce cisaillement sont très petites et donc l'effet du cisaillement dynamique reste très modeste.
Résumé

cisaillement séculaire a lieu lorsque le gradient de rotation est plus faible et s’appuie sur la turbulence thermique pour surmonter la force gravifique. Ce cisaillement a donc lieu sur une échelle de temps thermique, qui est beaucoup plus longue que l’échelle de temps dynamique.

Ces instabilités transportent du moment angulaire (Eq. 2.1) et les abondances chimiques (Eq. 2.2). Ces instabilités augmentent le mélange à l’intérieur des étoiles. Notons que la principale source de mélange reste la convection. Néanmoins, la rotation permet de mélanger lentement les zones radiatives de l’étoile, qui sans rotation ne seraient pas du tout mélangées.

Le logiciel de Genève incluant les effets dus à la rotation permettent de calculer des modèles d’évolution d’étoile qui reproduisent mieux les observations sur plusieurs points:

- La rotation permet de reproduire des enrichissements de surface notamment en azote pour des étoiles simples sur la SP (Meynet & Maeder 2000; Heger & Langer 2000).

- Le rapport entre SG rouges et bleues à basse métallicité (Maeder & Meynet 2001) ainsi que les populations de WR à diverses métallicités sont mieux reproduites (Meynet & Maeder 2003, 2004; Prantzos & Boissier 2003).

- Finalement, notons que, depuis Newton, la rotation des étoiles est observée et donc qu’il est important d’introduire la rotation dans les modèles d’étoile.

Les modifications apportées au modèle pendant cette thèse

Au début de la thèse, le modèle de Genève permettait de suivre l’évolution des étoiles massives jusqu’à la fusion du carbone. Les modifications apportées au modèle ont donc pour but de suivre l’évolution des étoiles pendant toutes les phases avancées. Les deux modifications principales sont la résolution des équations de structure interne de l’étoile et l’extension du réseau de réactions nucléaires afin d’inclure les réactions importantes durant les phases avancées.

Les équations de structure interne

Il y a quatre équations régissant l’évolution de la structure des étoiles: la conservation de la masse et de l’énergie ainsi que les équations du mouvement et de transfert de l’énergie (voir Sect. 2.3). Il y a aussi l’équation du changement des abondances chimiques dont nous allons parler plus tard et qui est traitée séparément des équations de structure.

Pendant les phases avancées, l’évolution s’accélère car les neutrinos emportent l’énergie sans interagir avec le reste de l’étoile. Les calculs effectués avec une méthode numérique
adaptée aux phases lentes (H et He) deviennent instables et il faut modifier la discrétisation des quatre équations mentionnées ci-dessus afin d'éviter ces instabilités. La méthode utilisée ici est celle décrite par Sugimoto (1970). L'expression détaillée des équations se trouve dans la Sect. 2.3.

**Le réseau de réactions nucléaires**

Le réseau de réactions nucléaires est un point crucial de la modélisation des phases avancées. En effet, les phases avancées comprennent 4 phases de fusion: carbone (C), néon (Ne), oxygène (O) et silicium (Si). Le réseau n'est pas seulement utilisé pour calculer les changements d'abondances chimiques mais aussi pour déterminer le taux de production d'énergie nucléaire. Le nombre d'éléments chimiques inclus dans les réseaux des réactions nucléaires varie d'une dizaine (Hix et al. 1998; Timmes et al. 2000) à quelques centaines (Rauscher et al. 2002; Limongi & Chieffi 2003). Le choix de la taille du réseau est déterminant pour le temps de calcul. Pour une séquence d'évolution pour une étoile, il faut environ 100'000 pas de temps à multiplier par le nombre de coquilles qui subdivisent l'étoile (environ 500) et à multiplier par le nombre d'itérations pour que le résultat converge (environ 5). Donc si un calcul dure un mois, le temps de calcul dans une coquille ne doit prendre plus d'un centième de seconde. Au début de la thèse, un réseau comprenant un grand nombre d'éléments et avec une méthode numérique sophistiquée, généreusement donné par Stéphane Goriely de l'ULB à Bruxelles, a été utilisé. Malheureusement, le temps de calcul était trop long. Après cet échec, un réseau minimal a été développé afin de minimiser le temps de calcul sans perdre de précision sur la production d'énergie et sur l'évolution des abondances chimiques principales. Les éléments inclus dans ce réseau sont présentés dans la table 3.1. Il comprend les éléments multiples de $\alpha$ (un ion d'hélium) entre le carbone et le nickel ainsi que les particules $\alpha$. Une fois les éléments chimiques à suivre choisis, il faut incorporer les réactions nucléaires les reliant. La liste de ces réactions est donnée dans la table 3.2. Les détails de la méthode numérique utilisée pour résoudre le changement des abondances sont décrits au chapitre 3.

Les abondances chimiques sont aussi modifiées par le mélange. Les deux processus, bien qu'ayant lieu simultanément, sont traités en série dans la plupart de modèles d'évolution stellaire. Il est néanmoins nécessaire de bien négocier l'interaction entre le mélange et les fusions nucléaires pour ne pas introduire d'erreur dans les calculs (voir Sect. 3.5.1).

** Modifications mineures**

Comme mentionné précédemment, le cisaillement dynamique a été introduit dans le modèle de Genève. La convection qui peut être considérée comme instantanée durant les phases lentes est traitées comme un processus de diffusion durant les phases avancées. Finalement, le contrôle du pas de temps a été modifié afin d'éviter des pas de temps trop grands durant les phases avancées.
Modèles numériques à métallicité solaire

Après avoir effectué les modifications brièvement décrites ci-dessus, des modèles d’évolution d’étoile ont été calculés à métallicité solaire pour des masses initiales de 12, 15, 20, 25, 40 et 60 $M_\odot$ avec et sans rotation. Pour les modèles en rotation, la vitesse de rotation initiale, $v_{\text{ini}}$, à la surface est de 300 km s$^{-1}$. Cette vitesse diminue légèrement durant la SP et correspond sur la SP à la moyenne des vitesses observées (Fukuda 1982), qui est aux alentours de 220 km s$^{-1}$. Les propriétés principales de ces modèles sont présentées dans les tables 4.1 et 4.2.

Evolution dans le diagramme de Herzsprung–Russell (HR)

L’évolution des étoiles, durant les phases lentes, est souvent représentée dans le diagramme HR qui montre l’évolution de la luminosité en fonction de la température effective de l’étoile (paramètres observables). L’évolution des modèles dans le diagramme HR ainsi que les caractéristiques des phases lentes de l’évolution sont décrites dans Meynet & Maeder (2003). Ici, seuls quelques points sont discutés. La figure 1 montre que la rotation a un effet important sur la position finale de l’étoile dans le diagramme HR pour les modèles de 20 $M_\odot$. En effet, le modèle sans rotation finit son évolution en SGR alors que les modèles en rotation avec $v_{\text{ini}}$= 200 et 300 km s$^{-1}$ finissent leur évolu-
Figure 2: Évolution des profils de la vitesse angulaire (gauche), $\Omega_r$, et du moment cinétique spécifique (droite), $j_r$, en fonction de la position en masse dans une étoile de $25\ M_\odot$ à différents stades de l'évolution. Le trait plein représente le profil au stade pré-supernova.

tion en SG jaune et bleue respectivement. Ces différences sont dues à l’augmentation de la perte de masse dues à la rotation qui réduit fortement (200 km s$^{-1}$) voire supprime complément (300 km s$^{-1}$) l’enveloppe riche en hydrogène. Le modèle avec $v_{ini} = 200$ km s$^{-1}$ a des ressemblances avec la SN 1993J bien que cette SN est probablement issue d’un système binaire serré (Podsiadlowski et al. 1993). En effet, ils ont une position dans le diagramme HR et une métallicité similaires et ils sont tous les deux des SNe de type IIb.

**Evolution de la rotation**

Comme mentionné précédemment, le cisaillement dynamique a été introduit dans le modèle. Malgré sa grande efficacité, l’étendue des zones où ce cisaillement a lieu est très restreinte. Le cisaillement n’a donc pour effet que d’adoucir les très forts gradients de rotation. Il n’induit pas un mélange à grande échelle.

La figure 2 montre l’évolution des profils de la vitesse angulaire (gauche), $\Omega_r$, et du moment cinétique spécifique (droite), $j_r = 2/3 \Omega_r \, r^2$, en fonction de la position en masse dans une étoile de $25\ M_\odot$ à différents stades de l’évolution. Pour la vitesse angulaire, on voit que, dans le cœur, la vitesse augmente lors des contractions qui ont lieu entre les différents stades de fusion. En surface, la perte de masse et le mélange réduisent la vitesse angulaire.
Pour le moment cinétique spécifique, \( j_r = \frac{2}{3} \Omega_r r^2 \), on voit que celui-ci décroît au cours de l'évolution. Notons que le moment cinétique est conservé lors de contractions et d'expansions. La plus grande décroissance a lieu pendant la séquence principale. Ceci est dû aux différents mécanismes de transport (convection, mélanges dus à la rotation) et à la perte de masse. Une perte plus faible a lieu durant la fusion de l'hélium pour les mêmes raisons. Durant les phases avancées, le temps d'évolution est trop court pour permettre un changement important du moment cinétique. Seules les zones de convection créent le profil denté car elle accumule le moment cinétique au bord extérieur de chaque zone convective (dû à la rotation solide dans les zones convectives). De ces résultats, on peut voir que le traitement du transport du moment cinétique durant les fusions H et He est très important et que ces phases déterminent la quantité de moment cinétique présent au stade pré-supernova. Ce moment cinétique est important dans le contexte des collapsars (Woosley 1993). En effet, si une étoile assez massive pour former une trou noir et qui a perdu son enveloppe riche en hydrogène contient assez de moment cinétique au moment de l’effondrement final, elle peut produire un disque d’accrétion en même temps que le trou noir. La matière tombant de ce disque sur le trou noir central permet de produire un jet bipolaire qui émet du rayonnement gamma. Ce modèle est l’un des modèles permettant d’expliquer les sursauts de rayonnement gamma longs et mous. D’autre part, pour quelques cas, les spectres de ces sursauts ressemblent après un certain temps à des spectres de SN de type Ic ou Ib. Il est donc très probable que les étoiles massives sont les précurseurs de ces sursauts. Le moment cinétique spécifique minimum que le coeur doit avoir pour former un disque d’accrétion est de \( 10^{16} \text{ cm}^2 \text{s}^{-1} \). On peut voir sur la figure 2 que le modèle de 25 \( M_\odot \) contient assez de moment cinétique pour produire un disque d’accrétion et donc probablement un sursaut. Notons que dans le modèle de Genève utilisé pour faire les calculs présentés ici, le champ magnétique n’est pas pris en compte. Les premiers modèles calculés avec un champ magnétique semblent montrer que le champ magnétique a un très grand effet et que, dans ce cas, le moment cinétique final serait trop faible (Heger et al. 2003; Maeder & Meynet 2004). Les prochains développements seront très intéressants pour les sursauts ainsi que pour les étoiles de neutrons pour lesquels les modèles prédisent une trop grande quantité de moment cinétique (Heger et al. 2000).

**Evolution de la structure et structure au stade pré-supernova**

L’évolution de la structure de l’étoile est bien visualisée dans un diagramme de Kippenhahn (voir figure 3). Ce diagramme représente les diverses zones convectives (zones noires) le long de la position en masse dans l’étoile (ordonnée) en fonction du temps d’évolution restant jusqu’à l’explosion en SN (en année). En comparant ce diagramme pour les modèles de 20 \( M_\odot \), on voit clairement les effets dus à la rotation. La rotation augmente la perte de masse et donc la masse finale de l’étoile qui tourne est nettement plus petite et l’enveloppe convective disparaît pendant la fusion He. Les coeurs convectifs lors des stades de fusion H et He sont plus massifs dans le modèle avec rotation. Ceci est dû au mélange induit par la rotation. Finalement, pour cette masse, la fusion C a
lieu dans un cœur convectif pour le modèle sans rotation alors que le cœur est radiatif dans le modèle avec rotation. Cela est dû au fait que dans le modèle avec rotation, durant la fusion He, plus de carbone est brûlé en oxygène et donc moins de carburant est disponible pour la fusion du carbone. Les effets de la rotation décrits ci-dessus sont les plus importants entre 15 et 30 $M_\odot$. Pour les étoiles plus massives ($\sim 60$ $M_\odot$), la perte de masse est l’effet dominant et la rotation a comme effet d’avancer le début de la perte de masse. Les masses finales, elles, ne diffèrent pas entre les modèles avec et sans rotation. Notons que le mélange reste important pour ces étoiles.

Au stade pré-supernova, l’étoile a une structure en “pelure d’oignon” avec un cœur composé d’éléments du groupe du fer et d’éléments de plus en plus légers au fur et à mesure qu’on se rapproche de la surface. La figure 4 nous permet d’apprécier les différences décrites ci-dessus entre un modèle avec et un sans rotation. Comme indiqué sur la figure, ces structures sont celles à la fin de la fusion Si. Les calculs n’atteignent pas l’effondrement final qui précède l’explosion. Plusieurs développements (agrandissement du réseau et modification de l’équation d’état) sont prévus pour que le modèle puisse suivre l’effondrement aussi loin que possible. Les résultats obtenus ont été comparés à la littérature (Rauscher et al. 2002; Heger et al. 2000; Limongi et al. 2000). Les différences entre les résultats présentés ici et les résultats d’autres groupes sont explicables par les différentes prescriptions adoptées pour le traitement de la convection ainsi que les taux de réactions nucléaires choisis. Les modèles avec rotation sont comparés avec les résultats de Heger et al. (2000). Dans ce cas, en plus des différences dans le traitement de la convection, le traitement de la rotation est légèrement simplifié dans les modèles de
Résumé

Figure 4: Profiles des abondances pour les modèles de 20 $M_\odot$ sans (gauche) et avec (droite) rotation à la fin de la fusion Si. L’abondance du $^{44}$Ti est multipliée par mille pour que cette courbe apparaîsse dans le graphique.

Heger et al. (2000). Même si les deux modèles conduisent à des différences quantitatives dans les résultats, ces derniers restent similaires qualitativement.

Production d’éléments chimiques dans les étoiles massives

Pour l’évolution chimique des galaxies, il est important de connaître la quantité d’éléments chimiques produite et éjectée par chaque étoile. C’est pour cette raison qu’à partir des modèles présentés au chapitre 4 et brièvement décrits ci-dessus, les rendements stellaires (Eq. 5.1; stellar yields en anglais) ont été calculés. Afin d’élargir la gamme de masse des résultats, les rendements des modèles de 9, 85 et 120 $M_\odot$, présentés dans Meynet & Maeder (2003) et dont l’évolution a été suivie seulement jusqu’à la fin de la fusion He, ont aussi été calculés.

Rendements stellaires dus aux vents et à la SN

Les éléments chimiques produits dans les étoiles sont éjectés par les vents stellaires et lors de leur explosion en SN. Les contributions de ces deux sources d’éjection ont été calculées séparément avant d’être additionnées pour obtenir les rendements stellaires totaux.

Les rendements provenant des vents stellaires sont présentés dans les tables 5.2 et 5.3. Les rendements provenant de l’explosion SN sont eux présentés dans les tables 5.4,
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Figure 5: Rendements stellaires divisés par la masse initiale en fonction de cette même masse initiale pour les modèles sans (gauche) et avec (droite) rotation.

5.5 et 5.6. Normalement, il faut aussi suivre l’explosion pour obtenir les rendements dus à la SN. Néanmoins, les éléments plus légers que le silicium ne sont pas (ou peu) affectés par l’explosion. Il est donc possible de calculer la contribution de la SN au stade pré-SN pour ces éléments. De plus, les incertitudes concernant la contribution due à la SN sont discutées en détails à la Sect. 5.7.

Quelle est l’importance relative des contributions des vents et de la SN? La figure 5 montre les rendements stellaires totaux divisés par la masse initiale de l’étoile en fonction de cette même masse initiale, pour les modèles sans (gauche) et avec (droite) rotation. Les rendements totaux des différents éléments chimiques sont empilés. Les rendements pour l $^4$He sont délimités au-dessus par la courbe continue et au-dessous par la courbe traitillée (surface colorée supérieure). Les rendements pour le $^{12}$C sont délimités au-dessus par la courbe traitillée et au-dessous par la courbe avec des traitillés longs et courts. Les rendements pour l $^{16}$O sont délimités au-dessus par la courbe avec des traitillés longs et courts et au-dessous par la courbe avec des traits et points. Les rendements pour le reste des éléments lourds sont délimités au-dessus par la courbe avec des traits et points et au-dessous par la courbe continue du bas. Cette courbe continue décrit aussi la fraction de l’étoile qui va rester bloquée dans le reste de SN (l’étoile de neutrons ou le trou noir). Le type de la SN qui se produit le plus de la mort de l’étoile est aussi indiqué. Les contributions dues aux vents stellaires uniquement sont superposées aux rendements totaux entre leur limite inférieure et la courbe pointillée au-dessus. Une surface pointillée aide à quantifier la fraction des rendements dus aux vents. Pour l $^4$He, les rendements totaux sont plus petits que les rendements dus aux vents car la contribution de la SN est négative pour les étoiles très massives (ceci s’explique par le
Résumé

fait que pour le calcul du rendement, l’abondance initiale est soustraite à l’abondance finale).

Pour l$^{14}$He (comme pour les autres produits de la fusion H), la contribution due aux vents augmente avec la masse initiale de l’étoile et domine pour $M \gtrsim 22M_\odot$ pour les modèles avec rotation et pour $M \gtrsim 35M_\odot$ pour les modèles sans rotation. Ces limites en masse correspondent aux masses limites inférieures pour former des étoiles WR. Pour les étoiles très massives, comme mentionné ci-dessus, la contribution des vents est plus grande que le total à cause de la contribution négative des SN. Pour le $^{12}$C, la contribution des vents n’est importante que pour les étoiles WR. Ceci est normal car une étoile doit d’abord éjecter ses couches riches en hélium avant de pouvoir éjecter des éléments plus lourds (voir figure 4). Pour l’oxygène et les éléments plus lourds, la contribution due aux vents reste négligeable.

Rendementsstellaires totaux

Les rendements stellaires totaux sont présentés dans les tables 5.7, 5.8 et 5.9. Ces rendements ont été comparés aux résultats d’autres groupes (Rauscher et al. 2002; Limongi & Chieffi 2003; Thielemann et al. 1996). Les résultats sont comparables. Les différences sont explicables par le traitement différent de la convection et des taux de réactions nucléaires (surtout pour la réaction $^{12}$C($\alpha, \gamma$)$^{16}$O). Cela vérifie la précision de nos calculs et donne une base solide pour les rendements des modèles incluant les effets dus à la rotation.

Les rendements totaux pour l$^{14}$He sont en général plus grands pour les modèles avec que sans rotation. Ceci est dû aux coeurs convectifs et aux pertes de masse plus importants. Cependant, entre environ 15 et 25 $M_\odot$, les rendements des modèles avec rotation sont plus bas. Dans cette gamme de masse, les vents n’éjectent pas encore beaucoup d$^{14}$He alors qu’il est en grande partie brûlé durant les phases avancées (donnant des rendements SN négatifs). Pour les étoiles très massives ($M \gtrsim 60M_\odot$), les modèles d’étoile en rotation entre dans le régime WR durant la SP. La longue durée du stade WR résulte en l’éjection de larges quantités d$^{14}$He. La rotation augmente donc les rendements pour l$^{14}$He (comme pour les autres produits de la fusion H).

Les rendements pour le $^{12}$C proviennent, au-dessous de 40 $M_\odot$, de la contribution de la SN. Dans ce cas, les modèles avec rotation ayant des cœurs plus gros ont des rendements plus grands. Pour les étoiles très massives ($M \gtrsim 60M_\odot$), la situation est inversée à cause de l’historique de la perte de masse. Comme mentionné plus haut, les modèles d’étoile en rotation entre plus tôt dans le régime WR. Ils perdent donc plus de masse tôt dans leur évolution et ont des cœurs plus petits. La situation pour l$^{16}$O et les autres métaux lourds est similaire au carbone. Les rendements pour ces éléments sont donc en général plus grands (1,5 à 2,5 fois, au-dessous de 40 $M_\odot$) pour les modèles avec rotation.
Conclusions et perspectives

Durant cette thèse, le logiciel d’évolution stellaire de Genève a été amélioré afin de pouvoir suivre l’évolution des étoiles en rotation pendant les phases avancées. Un nouveau réseau de réactions nucléaires a été mis en place et permet de modéliser les quatre phases de fusion des phases avancées (C, Ne, O et Si). Ce réseau est à l’heure actuelle de taille réduite mais assure en plus de la rapidité de calcul, la précision suffisante pour la production d’énergie et les abondances des éléments chimiques principaux. La discrétisation des équations de structure interne a été modifiée afin d’éviter les instabilités numériques qui se développent durant les phases avancées. D’autres modifications ont été apportées: le cisaillement dynamique, le traitement de la convection comme un processus de diffusion et le contrôle du pas de temps.

A l’aide du logiciel modifié, une série de modèles avec et sans rotation à métallicité solaire ont été calculés pour des masses de 12, 15, 20, 25, 40 et 60 $M_\odot$. L’évolution de la rotation durant toute l’évolution pré-supernova a été suivie en détails. Un des résultats très intéressants est que les modèles contiennent assez de moment cinétique au stade pré-supernova afin de créer des collapsars. Ceci veut dire que les étoiles massives en rotation remplissent les conditions nécessaires pour former des sursauts de rayonnement gamma longs et mous. Néanmoins, dans ce travail, les effets dus aux champs magnétiques ne sont pas inclus et des études préliminaires montrent que ces effets sont suffisants pour empêcher les modèles de garder assez de moment cinétique pour créer des sursauts gamma (Heger et al. 2003; Maeder & Meynet 2004). De futurs développements devraient apporter des précisions à ce sujet.

La rotation change de manière importante les modèles pré-SN par l’impact qu’elle a sur les fusions H et He. On distingue deux gammes de masses où, soit le mélange induit par la rotation domine ($M < 30 M_\odot$), soit la perte de masse augmentée par la rotation domine ($M > 30 M_\odot$). Les effets de la rotation sont les plus importants entre 15 et 25 $M_\odot$. En effet, la rotation change le type de SN (IIB ou Ib à la place de II), la taille totale de l’étoile (SGB à la place de SGR) et la taille des coeurs des fusions H et He par un facteur 1,5 (plus gros avec rotation).

Les rendements stellaires de ces modèles ont été calculés. Les contributions séparées des vents stellaires et de l’explosion ont été déterminées. En général, la contribution des vents est importante pour les produits des fusions H et He ($^4$He et $^{12}$C). Pour les éléments plus lourds ($^{16}$O, ...), seule la contribution de la SN est importante. Pour les rendements totaux, la rotation augmente sensiblement la production d’éléments lourds ($^{12}$C, $^{16}$O, ...) au-dessous de 30 $M_\odot$. Au-dessus de 30 $M_\odot$, c’est surtout la production d’$^{4}$He qui est augmentée par la rotation.

Acknowledgments

This is my opportunity to thank all the people who have helped me during the course of this work. First of all, I wish to warmly thank my two supervisors, Prof. André Maeder and Dr. Georges Meynet. André Maeder for his scientific advice, availability, trust, kindness and the freedom he gave me to do my work. Georges Meynet for his scientific advice, availability, generosity and friendship. Their work and help contribute greatly to the quality of the results of this thesis. I enjoyed working with them and appreciated the atmosphere in the group.

I thank the experts, Prof. Friedel Karl Thielemann and Prof. Gilbert Bürki, for taking the time to assess my work and for attending the oral presentation. I also wish to thank other scientists who helped me in this work: Dr. Stéphane Goriely for his help in the development of the numerical method for the nuclear reaction network, Prof. F.–K. Thielemann and Dr. W. R. Hix for their help concerning silicon burning, N. Langer for the discussions about angular momentum, Dr. A. Heger for the information concerning his models and the Universiti Malaya in Kuala Lumpur, in particular Dr. Hasan Abu Kassim for their hospitality during my visits to their physics department.

I thank all the other people working at the Observatoire de Genève. It would be to long to thank them all personally. Thanks to Dr. Raoul Behrend, with whom I shared an office, and who helped me on various work related and private topics. One nice example is the Pascal code to write BMP images. I wish to mention Dr. Paul Bartholdi, who helped me with computer related issues. Most plots presented in this work were built with his help using SuperMongo. I will also remember his help for setting up my private workstation and the cross country trip we made together in the Jura. Thanks to the other computers guys who helped me and with whom I enjoyed playing volleyball during the summer. Thanks to Mirka and Yves for their longterm friendship.

Thanks to my family and especially my parents who supported me and gave me their advice all these years. Thanks also to my friends who make life fun. A huge thank you to my wife, Ramona, for the professional and personal sacrifices she makes for me. Finally, I wish to praise the Lord for all his blessings and for making such beautiful objects as massive stars!
Acknowledgments

This work is dedicated to my son, Ezra Shawn, whose birth has coincided with the completion of this thesis.
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Chapter 1

Introduction

This introductory chapter sets out the framework of this thesis. The relevance of massive stars is highlighted. The life of massive stars as well as the nucleosynthesis occurring in their interior are described. Then, observational constraints that models ought to reproduce are listed. A short history of the development of stellar evolution models is given with an emphasis on models including rotation. Finally, the motivation for and the organisation of this work are presented.

1.1 Few stars, big impact

Stars are considered as massive in this work when they can produce an iron core after undergoing all the hydrostatic burning stages, which will be described later. The lower mass limit for this definition of massive stars is around $12 M_\odot$. This limit depends on the treatment of convection and the inclusion or not of overshooting. Woosley et al. (2002) describe the evolution of stars near and below this limit. Most stars with an initial mass below $12 M_\odot$ will form a degenerate core of carbon and oxygen (or oxygen-neon-magnesium around $10 M_\odot$). A small fraction of them (around $11 M_\odot$) may end up as electron capture supernovae (see Woosley et al. 2002; Nomoto & Hashimoto 1988, for more details).

Massive stars are much less numerous than lower mass stars. Indeed, observations show that the distribution of the number of stars as a function of their initial mass, $dN/dM$, also called the initial mass function (IMF), scales as $M^{-2.35}$ (for Salpeter’s IMF, and $M^{-2.7}$ for Scalo 1986). This means that for every $50 M_\odot$ star, there are about 10 000 suns, 44 stars with $10 M_\odot$ and 8.6 stars with $20 M_\odot$. However, as we shall see in this work, massive stars are of first importance. They are the major contributors
1.2. The life of massive stars

To the nucleosynthesis of helium and of most heavy elements (see Sect. 1.3). Massive stars are very bright and contribute largely to the luminosity of a galaxy. Individual massive stars are even observable in distant galaxies. Massive stars are the progenitors of most types of supernova (see Sect. 1.2) and the cores left after the explosions are the progenitors of neutron stars or black holes. Lately, a connection has been established between long soft gamma-ray bursts (GRBs) and very energetic supernovae, hinting that massive stars could be progenitors of these long soft GRBs (see Sect. 1.4).

1.2 The life of massive stars

The life of the star is divided into several stages. The first stage is the formation of the star, not yet completely understood for massive stars. Current thinking is that they are formed either by the merger of smaller mass stars (Bonnell et al. 1998) or that they are formed by a mass accretion that increases with the instant mass of the star (Beltrán & Maeder 2001). The study of the formation is usually separated from the study of the evolution and is beyond the scope of this work. Stellar evolution studies usually start on the zero age main sequence (ZAMS). During the main sequence (MS), the star burns hydrogen into helium in its core. This is the longest stage of the life of the star (∼90% of the total lifetime) and it lasts for three to 15 million years for masses between 60 and 15 $M_\odot$ respectively. Note that this is much shorter than the lifetime of the sun which is around nine billion years.

At the end of hydrogen (H) burning, the core contracts and the envelope expands (effect called the “mirror effect”). The evolution of the envelope varies with the initial mass of the star and depends on mass loss. Since mass loss occurs, the total mass of the star varies during the evolution. The following convention will be used in this work. If the mass of the star is given as an integer, it corresponds to the initial mass of the star. If it is given with decimals, it corresponds to the mass of the star during the evolution or to its final mass. If mass loss is small ($M \lesssim 20 M_\odot$ at solar metallicity), the envelope expands until the star becomes a red supergiant (RSG) and the star ends up as a RSG. If mass loss is small during the MS but important in the RSG stage ($20 M_\odot \lesssim M \lesssim 50 M_\odot$ for rotating stars at solar metallicity), the star eventually loses most of its hydrogen rich envelope. The envelope then contracts and the star becomes a blue supergiant (BSG). When the entire hydrogen rich envelope is lost ($M \gtrsim 21 M_\odot$ for rotating stars at solar metallicity), the star becomes a Wolf-Rayet (WR) star. If mass loss is already important during the MS ($M \gtrsim 50 M_\odot$ for rotating stars at solar metallicity), the star already becomes a WR star near or at the end of the MS and never becomes a RSG. Note that if the star is in a binary system, the mass loss history can be significantly changed.

In the core, contraction continues until the central temperature is high enough to start helium (He) burning ($T_c \sim 1.5 \times 10^8$ K). He-burning lasts for about 10% of H-burning lifetime. During this stage, helium is transformed into carbon and oxygen. Note that He-burning is sensitive to the treatment of convection because the He-burning core grows in mass and faces a mean molecular weight ($\mu$) gradient. He-burning is
also sensitive to the rate of the nuclear reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ which competes with the triple-$\alpha$ reaction (transforming helium into carbon). See Schlattl et al. (2004) for a recent discussion on this topic.

H and He–burnings are called the early stages. They last for about 99% of the life of the massive star. It is therefore easy to observe stars in these early stages and observational tests for models are described in Sect. 1.4. During these stages, the energy produced by nuclear reactions in the core is evacuated by photons which interact with the envelope. After He–burning, the core contracts again and the central temperature reaches about half a billion degrees Kelvin. From now onwards, most of the energy created in the core is carried away by neutrinos (photo–neutrinos, pair annihilation, plasma and bremsstrahlung neutrinos). Since neutrinos hardly interact with matter, the evolution of the core becomes decoupled from the envelope. The surface properties of stars therefore do not change during the following burning stages which constitute the advanced stages. This means that we have no possibility of directly observing these advanced stages. During the advanced stages, the following burning phases occur: carbon, neon, oxygen and silicon. They will be described in Sect. 1.3. At the end of the advanced stages (called the pre–supernova stage), an iron core (composed of iron group elements) is formed. Since iron group elements are the most stable elements, no further nuclear reaction can produce energy. The core collapses due to its own gravity. When the core reaches nuclear densities, additional infalling layers bounce on the core. This creates a shock (prompt shock) that stalls very quickly (after traveling only about 150 km). Neutrinos, emitted in the core, are believed to revive this shock by transferring their energy to particles via interactions (delayed explosion) and trigger the supernova (SN) explosion. Note that they are probably helped by convection, rotation and magnetic fields. However, even after many years of research, models struggle to reproduce the explosion using the most up–to–date physics (Janka et al. 2003). SN explosions create a shock wave that will travel through the star. The passage of the shock wave heats up layers to high temperature and nuclear burnings (called explosive) similar to the pre–explosive burning stages (called hydrostatic) occur.

SN explosions have been classified by observers and their classification (shown in Fig. 1.1) depends on spectral and light curve properties (see Turatto 2003, for details and examples. Here only an overview is presented). The observers first separate SNe with hydrogen lines (Type II) in their spectra and those without (Type I). For type I SNe, they then look at the presence of helium and silicon. If the spectra contain silicon (SiII), then the SN is a type Ia. Type Ia SNe are the thermonuclear explosion of a white dwarf (WD) which accretes enough mass from its binary companion in order to reach the Chandrasekhar mass limit (see Yoon & Langer 2004; Röpke & Hillebrandt 2004, for recent developments on the subject). If the spectra of a type I SN do not contain silicon lines, it is either a type Ib if the spectra contain helium lines (HeI) or a type Ic if the spectra does not contain helium lines. Type Ib/c are associated with the core collapse of massive stars which have lost their hydrogen rich envelope (type Ib) or both their hydrogen and helium rich envelope (type Ic) through mass loss. In other words, WR stars are their progenitors. A few SNe appear like type II SNe in their early phase but
1.2. The life of massive stars

![Diagram of SN classification]

Figure 1.1: SNe classification from Turatto (2003).

their spectra evolved into type Ib/c like spectra. This class of object is therefore called type IIb SNe and is thought to be the intermediate case between stars which retain their envelope and those which lose it. The progenitors of these intermediate cases are massive stars which retain only a small fraction of their hydrogen rich envelope.

For type II SNe, the shape of the light curve (luminosity of the SN as a function of time) is used to classify them. After a maximum, the luminosity of SNII decreases either exponentially (III, L for linear in logarithmic scale) or has a plateau phase (IIP). The luminosity with exponential decrease is powered by the radioactive decay of nickel (half life of 6.1 days) into cobalt and of cobalt (half life of 77.1 days) into iron. The plateau is produced by a recombination wave moving inwards in mass through the massive hydrogen envelope, releasing its internal energy. The progenitors of SNII are massive stars ($12 M_\odot \lesssim M \lesssim 20 M_\odot$ for rotating stars at solar metallicity) which have lost only a small fraction of their envelope for the IIP and a large fraction of it for the III. Considering single massive stars, the explosion type evolves in the following order with an increasing initial mass: IIP − III − IIb − Ib − Ic. SNe are also classified with the explosion energy. Recently, very energetic explosions (called hypernovae, HNe) as well as faint SNe have been discovered (see for example Nomoto et al. 2003).

The collapsed cores will either form a neutron star (NS, see Lattimer & Prakash 2004, for a review) or a black hole (BH). It is usually believed that the maximum mass for a neutron star (classically known as the Oppenheimer–Volkov mass) is between two and three solar masses. However, this result depends on the equation of state. Observed masses of NSs measured in binary systems all lie very close to 1.4 $M_\odot$ (Thorsett and Chakrabarty 1999). It is also difficult to estimate the final remnant mass of a given star since the explosion mechanism is not yet fully understood and not all evolutionary models give the same final iron core at the pre-supernova stage. Finally, a fraction of the star might be initially pushed away but, failing to escape, eventually falls back on the core after some time (see Woosley et al. 2002, for a discussion and references). The limit between NS and BH will be further discussed in Chapt. 5 (stellar yield calculation).
The possibility of producing GRBs from the collapse of rotating massive stars will be discussed in Sect. 1.4.

1.3 Nucleosynthesis

Nucleosynthesis occurs in massive stars through hydrostatic (Pre–SN) and explosive burnings as well as through the so-called s and r (and p, rp and op) processes.

Hydrostatic burnings consists of hydrogen (H), helium (He), carbon (C), neon (Ne), oxygen (O) and silicon (Si) burnings. These burnings take place successively in the core as well as in the shells above the core (see the Kippenhahn diagrams in Figs. 4.11–4.12).

H–burning contains two main groups of reactions, which are the p–p chains and the CNO cycle. The CNO cycle transforms most of the CNO isotopes into $^{14}$N (due to the fact that the reaction $^{14}$N($p$, $\gamma$)$^{15}$O is the slowest reaction in the cycle). Note that lower mass stars also contribute significantly to the production of $^{13}$C, $^{14}$N (as well as $^{23}$Na).

In He–burning, the main reactions are the triple–$\alpha$ reaction, $^4$He(2$\alpha$, $\gamma$)$^{12}$C, and $^{12}$C($\alpha$, $\gamma$)$^{16}$O. The reaction rate of $^{12}$C($\alpha$, $\gamma$)$^{16}$O is still uncertain and is therefore a remaining problem. There is also a production of neutrons during He–burning by the chain reaction $^{14}$N($\alpha$, $\gamma$)$^{18}$F($\beta^+$)$^{18}$O($\alpha$, $\gamma$)$^{22}$Ne($\alpha$, $n$)$^{25}$Mg and by the reaction $^{13}$C($\alpha$, $n$)$^{16}$O. This neutron production induces the s–process. The s–process (s for slow) produces nuclei up to Pb and Bi starting with pre–existing seed nuclei up to Fe through a series of neutron captures and $\beta$–decays alternatively. Neutron captures are followed by $\beta$–decays because the neutron capture rate is slow compared to the $\beta$–decay lifetime (explaining the name of slow process). Another site for s–process is helium–shell flashes during the asymptotic giant branch (AGB) phase in intermediate mass stars.

Concerning core C–burning (ignition temperature: 5 to $8 \times 10^8$K and lifetime of about 1000 years), the main reactions are $^{12}$C($^{12}$C, $\alpha$)$^{20}$Ne and $^{12}$C($^{12}$C, $p$)$^{23}$Na. Then most of the $^{23}$Na is transformed via $^{23}$Na($p$, $\alpha$)$^{20}$Ne and $^{23}$Na($p$, $\gamma$)$^{24}$Mg. Hence the main products are $^{20}$Ne and $^{24}$Mg followed by $^{23}$Na. Core Ne–burning (ignition temperature: $1 \times 10^9$ K and lifetime of about one year) consists of photodisintegrations and $\alpha$–captures. Main reactions are $^{20}$Ne($\gamma$, $\alpha$)$^{16}$O, $^{20}$Ne($\alpha$, $\gamma$)$^{24}$Mg and $^{24}$Mg($\alpha$, $\gamma$)$^{28}$Si. During core O–burning (ignition temperature: $2 \times 10^9$ K and lifetime of about one year), main reactions are, by order of importance, $^{16}$O($^{16}$O, $p$)$^{31}$P, $^{16}$O($^{16}$O, $\alpha$)$^{28}$Si, $^{16}$O($^{16}$O,n)$^{31}$S and $^{16}$O($^{16}$O,d)$^{30}$P. Further reactions imply that the main products are $^{28}$Si and $^{32}$S.

In early core Si–burning (ignition temperature: $2-4 \times 10^9$ K and lifetime of about one day) small quasi-equilibrium (QSE) groups form around $^{28}$Si. Then two QSE clusters are built up around silicon and iron respectively. An important dividing line is caused by the proton magic number Z=20 (this region is sometimes called the "bottleneck"). These two clusters are linked together by several reactions and not only by $^{45}$Sc($p$, $\gamma$)$^{46}$Ti as first thought. It can be noted that isotopes having Z< 14 (Si) are still linked by reactions far from equilibrium. At the end of Si–burning, the nuclear statistical equilibrium (NSE) is reached. Main products are $\alpha$–particles and species from the iron group (Thielemann & Arnett 1985).
1.3. Nucleosynthesis

On top of the abundance of the main elements, it is useful to follow the evolution of the neutron excess. The neutron excess, $\eta = \sum_i (N_i - Z_i)Y_i$, is linked to the electron mole number, $Y_e = \sum_i Z_i Y_i$, by the following relation: $Y_e = (1 - \eta)/2$ ($N_i$, $Z_i$ and $Y_i$ are respectively the number of neutrons, protons and the number abundance of element $i$; $Y_i = X_i/A_i$, where $X_i$ and $A_i$ are the mass fraction and the mass number of element $i$). Although neutron excess does not affect the evolution of the structure of the star until Si–burning and does not change the abundance of main elements (multiple–α elements), it determines the amount of s-process elements produced during He–burning and the neutron richness of the products of explosive burnings. The neutron excess increases every time neutrons are produced via a weak interaction starting with a proton ($\beta^+\text{-decay, electron capture,}$ ...). As neutrons are produced by elements which are not the most abundant ($10^{-2}$ of the most abundant), $\eta$ reflects subtle details and small abundances. Note that the maximum white dwarf mass also depends on $Y_e$ since it corresponds to the maximum mass supported against gravity by a gas of degenerate electrons. An enhancement in $\eta$ takes place during He–burning (due to the chain reaction $^{14}\text{N}(\alpha,\gamma)^{18}\text{F}(\beta^+)^{18}\text{O}(\alpha,\gamma)^{22}\text{Ne}(\alpha,\gamma)^{25}\text{Mg}$). $\eta$ increases slightly during C–burning and is not really affected during Ne–burning. A second increase is seen in O–burning (due to electron captures). All these increases are however small in comparison to the largest enhancement occurring during Si–burning (due to electron capture again). See Thielemann & Arnett (1985).

Explosive burnings occur when the shock wave from the SN explosion heats up the different layers of the star. Important parameters are the peak temperature reached and the duration of that peak as well as the pre–explosive neutron excess. Because the helium and hydrogen–rich layers are far from the centre of the star, explosive He and H–burnings do not occur. Carbon and neon rich layers are sometimes reprocessed by explosive burning, sometimes not. It depends on the explosion energy and the compactness of the star. If the energy is higher or the star more compact, C and Ne–burnings have a higher probability to occur (Arnett, 1969). Oxygen and silicon explosive burnings always occur. The products of explosive burnings are similar to the products of hydrostatic burnings. However, the duration of explosive burnings is much shorter than hydrostatic ones. Therefore, no neutron excess increase can occur by $\beta$–decays in explosive stages. The list of all the elements produced is long and the reader can look at the following references to know the exact quantities of each chemical element produced in a star (Thielemann et al. 1996; Limongi & Chieffi 2003; Rauscher et al. 2002). Woosley et al. (2002) also give in table III the production site of elements up to Zn. In general, one can say that: a) Elements lighter than silicon are produced by hydrostatic burnings, b) Elements between silicon and calcium are produced by both hydrostatic and explosive burnings and c) Elements of the iron–group ejected by the star are only made explosively. Note that the production of the different chemical elements depends on the initial mass of the star.

Another important nucleosynthetic production is done via the r–process (r for rapid). The r–process occurs when the neutron density is so high that many neutron captures can take place before a $\beta$–decay. An equilibrium is even reached between $(n, \gamma)$ and
(γ, n) before a β-decay occurs. The site of r-process is not yet known with certainty, though a recent work by (Argast et al. 2004) shows that SNe (neutrino–powered wind or asymmetric explosions) are favoured above neutron star mergers (see Woosley et al. 2002, for more details and references). The other processes producing heavy elements are the p-process, rp-process and the αp-process, which concern proton and α-particle captures (see Arnould & Takahashi 1999, for more details).

1.3.1 26Al, 60Fe and 44Ti

26Al, 60Fe and 44Ti have a low abundance and would go unnoticed if they were not radioactive isotopes. The half lives of 26Al, 60Fe and 44Ti are 1.1 10^6, 2.2 10^6 and 89 years respectively. Their common point is that they all emit gamma-rays with energies around 1 MeV during their decay. Therefore, their decay can be observed with gamma–ray satellites (the latest being Integral, see Sect. 1.4). Two situations lead to the production and especially ejection of 26Al in massive stars. The first situation is the production during hydrostatic H–burning and the ejection by winds of WR stars (Vuissoz et al. 2004). The second one is the production during static and explosive Ne–burnings as well as during neutrino processes and the ejection by the SN explosion (Limongi & Chieffi 2003; Rauscher et al. 2002; Thielemann et al. 1996; Woosley & Weaver 1995). 60Fe is only produced deep inside the massive stars by neutron captures and is ejected by the SN explosion. 44Ti is produced during the α–rich freeze–out and is ejected by the SN explosion.

1.4 Observations

The first goal of a model is to reproduce observations. Before delving into stellar evolution models, it is important to know what observational constraints are present.

1.4.1 Rotation

Rotation of celestial bodies has been observed by Galilee at the beginning of the seventeenth century. Furthermore, anyone, with a suitable equipment (telescope with a solar filter), can see that the surface of the Sun rotates by looking at the slow displacement of sunspots. Nowadays, we have measurements of the surface rotational velocity of many stars (Fukuda 1982; Howarth et al. 1997). The surface rotational velocity, averaged over the different observations of MS stars, is about 220 km s^{-1} for OB stars. Observations of Be stars (stars presenting emission lines in their spectra explained by the presence of a disk) have an average surface velocity as high as 400 km s^{-1}. The VLTI has allowed to reproduce the shape of a star where we can see that rotation makes the star shape oblate (see Fig. 1.2). Finally, axial symmetries in planetary nebulae, in winds from massive stars (for example the shape of eta carinae) and in SN are indirect proof that the progenitor star rotated.
1.4. Observations

Figure 1.2: Shape reconstruction of the rapidly rotating star Achenar, as deduced from observations with the VLT Interferometer (VLTI, http://www.eso.org/outreach/press-rel/pr-2003/pr-14-03.html).

1.4.2 Early stages

The early stages (H and He–burnings) of the life of stars are the longest and are therefore the easiest to observe. Except for the sun and a few other stars (for example α Centauri, Eggenberger et al. 2004) where asteroseismology is able to study the interior of stars, our observations are limited to the surface properties of the stars (total luminosity, effective temperature and surface chemical abundances). These surface properties are usually determined by photometry or by fitting an observed spectrum with complex atmospheric models. Once the observed properties are determined, they are compared to stellar evolution models. The comparison is in general satisfactory. However, observations in MS stars of nitrogen enrichment (in comparison to the surrounding interstellar medium) accompanied with low surface abundances of carbon and oxygen (Gies & Lambert 1992) can only be reproduced by models where mixing occurs between the surface of the star and its interior where abundances can be affected by nuclear burning. We will see later that differentially rotating stars can naturally induce such mixing (Maeder & Meynet 2000b; Heger & Langer 2000). These enrichments could come from binary interaction but evidence of boron depletion (Venn et al. 2002) in stars which are not yet enriched in nitrogen on their surface supports only rotation induced mixing (at least some form of internal mixing). Other observational constraints come from the ratios of
the number of stars in different evolutionary stages. For example, the ratio of blue to red supergiant (at various metallicities) is very sensitive to different parameters of the models like the treatment of convection, overshooting and mass loss. Standard models (without rotation) cannot reproduce the observations Langer & Maeder (1995). Once again, recent models of differentially rotating stars are able to reproduce this strong observational constraint (Maeder & Meynet 2001). The ratio between normal B stars and fast rotating Be star as a function of the metallicity can be an indicator of the dependence of rotation as a function of metallicity. WR star populations at different metallicities as well as the number ratio between different subgroups of WR stars also put constraints on models (Meynet & Maeder 2003, 2004). As stated earlier, WR stars explode as SNIIb/c such that we can also use the ratio of SNIIb/c to SNII as a function of metallicity and compare it to the fraction of WR stars (Prantzos & Boissier 2003).

1.4.3 SN and neutron stars

As stated earlier, the advanced stages of the evolution have a very short lifetime and during this time, the surface properties do not evolve (except in some exotic cases where pulsation could eject a significant part of the envelope). We therefore observe the impact of the advanced stages indirectly in SN explosions, the related nucleosynthesis and the remnants.

Nowadays, SNe are observed in wavelength ranging from radio to gamma-rays. The two most common observations are the spectra and the light curve. By fitting models to these observations, one can deduce the energy of the explosion, the quantity of radioactive nickel ejected, the chemical composition of the ejecta and the mass of the star at the pre–SN stage as well as the mass of the circumstellar material ejected by winds (with varying degrees of precision). Of course, most of these quantities depend solely on the explosion but we can still get information about the pre–SN evolution. Indeed, the mass of the star and composition of the ejecta which are not affected by explosive nucleosynthesis (H and He–burning products) can help constrain the pre–SN models. This is especially true if we have observations of the progenitor like for SN1987A (Arnett et al. 1989, for a review). However, SN1987A is a peculiar case. It is hard to explain it within a single star scenario (Langer 1991) and SN1987A is probably part of a binary system (Podsiadlowski 1992).

The observed rotation rate of pulsars, which tell us the rotation rate of neutron stars can be compared to the angular momentum present in the core at the pre–SN stage (Heger et al. 2000). At the present time, the rotation rate predicted by models is much faster than the observed rate for young pulsars. The question is whether the discrepancy is due to the effects of magnetic field during the pre–SN evolution (Heger et al. 2003) or to mechanisms which slow down the core during the core collapse (Fryer & Warren 2004) or both. The neutron stars could also lose angular momentum during its young age.

At the same time, long soft gamma–ray bursts (GRBs) have recently been connected with SNe (see for example Matheson 2003). A model called collapsar (Woosley 1993)
1.4. Observations

explains the production of GRBs in the following way: a star which has lost its hydrogen rich mantle collapses into a black hole and an accretion disk due to the high angular momentum of the core. Accretion from the disk onto the central black hole then produces bi-polar jets. These jets can only reach the surface of the star (and be detected) if the star has previously lost its hydrogen rich envelope. WR stars are therefore good candidates for collapsar progenitors since they lose their hydrogen rich envelope. The question to answer is whether or not their core contains enough angular momentum \( j \gtrsim 10^{16} \text{ cm}^2 \text{ s}^{-1} \) for the material just outside the core in order to produce a black hole with an accretion disk. The answer to this question will be addressed in this work.

1.4.4 INTEGRAL

INTEGRAL is the latest and most powerful gamma–ray satellite. Before that, other satellite and balloon borne telescopes have been sent in order to observe the very energetic part of the spectrum of light. Gamma–rays are emitted by the decay of several radioactive nuclei produced in massive stars. The main radioactive nuclei are \(^{56}\text{Ni},^{56}\text{Co},^{57}\text{Co},^{44}\text{Ti},^{26}\text{Al}\) and \(^{60}\text{Fe}\) (see Prantzos 2004, for more details and references). All are produced by explosive burnings, while only \(^{26}\text{Al}\) is also produced by hydrostatic burnings (H–burning in the case of \(^{26}\text{Al}\)). As stated earlier, these elements need to be ejected before they decay in order to be detected.

\(^{56}\text{Ni},^{56}\text{Co},^{57}\text{Co}\) and \(^{44}\text{Ti}\) are all short–lived nuclei (lifetimes between one year and 100 years) and are ejected by the SN explosions. They are therefore detectable in young SN remnants like SN1987A and CAS A (Vink et al. 2001a). In the case of SN1987A, both \(^{56}\text{Co}\) and \(^{57}\text{Co}\) gamma–ray lines were measured, while for \(^{44}\text{Ti}\), light curve fitting was used to derive its abundance. For CAS A, the \(^{44}\text{Ti}\) gamma–ray line has been measured. If all observations (made by COMPTEL and Beppo–Sax) are gathered together, a wide range of values for the \(^{56}\text{Ni}\) produced by core collapse SNe is found. For SN1987A, the measured value is \(0.07 \ M_\odot\). For \(^{44}\text{Ti}\), deduced values for both SN1987A and CAS A are around \(1 \times 10^{-4} \ M_\odot\). The reproduction of these values (especially for \(^{44}\text{Ti}\)) seems to require asymmetric effects like those studied by Maeda & Nomoto (2003). The measurements of the \(^{44}\text{Ti}\) gamma–ray line in CAS A as well as measurements of Ni and Co lines by INTEGRAL SPI (spectrograph) should bring even stronger constraints.

\(^{26}\text{Al}\) and \(^{60}\text{Fe}\) are long–lived radio nuclei (lifetimes of 1–2 million years). Thus, the observed quantities come from the cumulative production of many objects. \(^{26}\text{Al}\) production is also possible in AGB stars and novae. However, the galactic map of \(^{26}\text{Al}\) production shows “hot–spots” which are correlated with the spiral arms. The production of \(^{26}\text{Al}\) by massive stars is therefore favoured (Diehl et al. 2004). We expect INTEGRAL to constrain the relative importance of \(^{26}\text{Al}\) production by WR stars and SNe. Recently there were some interesting results with RHESSI by Smith (2004) who detected both \(^{26}\text{Al}\) and \(^{60}\text{Fe}\) in the centre of the Milky Way. They obtained a flux ratio \(^{60}\text{Fe}/^{26}\text{Al}\) of 10%. This is consistent with calculations by Timmes et al. (1995) who found a ratio of 0.16 for SNII. However, recent yield calculations by Rauscher et al. (2002); Limongi & Chieffi (2003) obtain a much higher ratio \(\gtrsim 1\) in which case a dominant contribution
from WR stars is needed in order to reconcile theory and observation. Furthermore, observations of the active star forming region Cygnus X show a large production of $^{26}$Al (Knödlseder et al. 2004) and favour a large contribution from WR stars because only few SNe could have exploded by now. However, yields of non-rotating WR stars are not sufficient on their own to produce the quantity of $^{26}$Al observed. Rotating models (Vuissoz et al. 2004; Palacios et al. 2004) produce more $^{26}$Al and help close the gap. INTEGRAL should bring further constraints on this topic over the next few years.

1.5 Stellar evolution models

The development of models is linked to the development of computers. Indeed, since the 1950’s, most stellar evolution models are the result of numerical simulations on computers. Computers being more and more powerful, the numerical simulations can be more and more complex and demanding in both calculation time and memory. In the present work, a typical simulation has 100,000 time steps with 500 zones in spatial resolution and the output files take a few GBytes of memory. This would not have been possible ten years ago.

In fact, we arrive now at a stage where computers are fast enough for detailed models in one dimension (1D). Unfortunately, computers will not allow detailed 3D simulations of hydrostatic stellar evolution for several decades. However 3D simulations of SN explosion and other phenomena occurring on the hydrodynamic time scale are already possible (Janka et al. 2003; Reinecke et al. 2002; Eggleton et al. 2002). It should also soon be possible to have 3D simulations of multi-dimensional effects like convection in order to obtain prescriptions for 1D models.

H and He-burnings have been the first and the most widely studied phases since they are the longest-lasting phases. By contrast, the late stages of hydrostatic burning have only been studied by several groups recently (Limongi et al. 2000; Woosley et al. 2002; Heger et al. 2000; Umeda & Nomoto 2003; Hix et al. 1998; Thielemann & Arnett 1985; Bazan & Arnett 1998).

1.5.1 Rotation

The theoretical work for the study of rotation in numerical models was undertaken in the 1970’s by Kippenhahn et al. (1970); Endal & Sofia (1978). Then Zahn (1992) and Maeder & Zahn (1998) further developed the theory of turbulent mixing due to differential rotation (other references can be found in Heger 1998). Nowadays, two main research groups include the theory of differential rotation in stellar evolution models of massive stars (Meynet & Maeder 2003; Heger & Langer 2000). Maeder and Meynet have been developing the Geneva evolution code over the last ten years in order to take into account the effects of rotation. The developments are described in a series of papers on the subject (Meynet & Maeder 1997, 2004). The model follows the evolution of stars until C-burning and succeed in reproducing many observational constraints like nitrogen enrichment at the surface of MS stars (Meynet & Maeder 2000; Heger & Langer...
2000), the ratio of blue to red supergiants at low metallicity (Maeder & Meynet 2001) as well as WR star populations at different metallicities (Meynet & Maeder 2003, 2004). Heger has used the codes developed by Langer (STERN) and by Woosley and Weaver (KEPLER) in order to follow the evolution of differentially rotating massive star from the ZAMS until core collapse (Heger et al. 2000).

Some people, however, think that magnetic fields would enforce solid body rotation (Roxburgh 1991). Recently, Heger et al. (2003) have included the effects of magnetic fields according to the formula from Spruit (2002). Maeder & Meynet (2004) have included the effects of magnetic fields using general equations for the transport by Tayler–Spruit dynamo. Both groups find that the effects of magnetic fields are important but that no surface enrichment is produced during the MS. Further developments are underway and observational constraints like the ones cited in Sect. 1.4 will be the judge. Note that, in this work, the effects of magnetic fields and binarity (see Podsiadlowski et al. 2003; Dessart et al. 2003, for recent works on the subject) are not taken into account.

1.6 This work

The aim at the start of this work was to develop the Geneva code in order to follow the evolution of stars until the core collapse. The motivation is obvious. The Geneva model is able to reproduce observational constraints in the early stages of the evolution and it is therefore interesting to see what SN progenitors, yields and angular momentum of remnant would be produced with this model. So far, only one other research group (Heger et al. 2000) follows the pre–SN evolution including the effects of rotation. The comparison of the pre–SN models between the two groups is therefore useful for the community to understand the dependence of the results on the different physics. Even for models without rotation, there are discrepancies between the results of various groups (Woosley & Weaver 1995; Thielemann et al. 1996; Rauscher et al. 2002; Limongi & Chieffi 2003). We will therefore also compare our non–rotating models with the literature.

The organisation of this thesis is the following. In Chapter 2, the Geneva code and its physical ingredients are described. The modifications brought to the structure equations as well as the inclusion of dynamical shear are presented. In Chapter 3, the nuclear reaction network used in the advanced stages is described in details. In Chapter 4, the evolution of the models from the ZAMS until the end of Si–burning is described for stars with masses between 12 and 60 $M_{\odot}$ at solar metallicity. In Chapter 5, the yields from rotating massive stars at solar metallicity are presented. Conclusions and perspectives are given in Chapter 6.
Chapter 2

Geneva stellar evolution model

This chapter describes the Geneva stellar evolution model as well as several modifications brought to it during the course of this work. The various physical ingredients of the code are described. The implementation of dynamical shear instabilities is presented. The discretisation of the structure equations is described in detail. Finally, minor modifications are presented.

The Geneva stellar evolution code comes from the Göttingen code (Kippenhahn et al. 1967). It keeps to this date some souvenirs of its origins with variable names still expressed in the german language. The code has nowadays about 50'000 lines (including comments and different versions of the same subroutines). It is therefore not possible to describe all of it in detail. In this chapter, only a general overview and the relevant references are presented. Note that the main body of the code is very well described in Kippenhahn et al. (1967).

2.1 The Geneva stellar evolution model and its physical ingredients

2.1.1 Rotation

The physics of rotation included in the Geneva model has been developed over the last ten years. A review of this development can be found in Maeder & Meynet (2000b). The effects induced by rotation can be divided into three categories.

1) Hydrostatic effects: The centrifugal force changes the hydrostatic equilibrium of the star. The star becomes oblate and the equations describing the stellar structure have to be modified. This point is developed in Sect. 2.3.

2) Mass loss enhancement and anisotropy: Mass loss depends on the opacity
2.1. The Geneva stellar evolution model and its physical ingredients

and the effective gravity (sum of gravity and centrifugal force) at the surface. The larger the opacity, the larger the mass loss. The higher the effective gravity, the higher the radiative flux (von Zeipel 1924) and effective temperature. Rotation through the centrifugal force reduces the surface effective gravity at the equator compared to the pole. As a result, the radiative flux of the star is larger at the pole than at the equator. In massive stars, since the opacity is dominated by the temperature-independent electron scattering, rotation enhances mass loss at the pole. If the opacity increases when the temperature decreases, mass loss can be enhanced at the equator. The mass loss prescriptions used in the Geneva model are described in Sect. 2.1.2.

3) Rotation driven instabilities The rotation driven instabilities considered in this work are the horizontal turbulence, meridional circulation and dynamical and secular shear. Dynamical shear was included in the model during this work and is described in detail in Sect. 2.2.

Horizontal turbulence corresponds to turbulence along the isobars. If this turbulence is strong, rotation is constant on isobars and the situation is usually referred as “shellular rotation” (see Sect. 2.3). The horizontal turbulence is expected to be stronger than the vertical turbulence because there is no restoring buoyancy force along isobars (see Maeder 2003, for the latest development on this topic).

Meridional circulation, also referred to as Eddington–Sweet circulation, arises from the local breakdown of radiative equilibrium in rotating stars. This is due to the fact that surfaces of constant temperature do not coincide with surfaces of constant pressure. Indeed, since rotation elongates isobars at the equator, the temperature on the same isobar is lower at the equator than at the pole. This induces large scale circulation of matter, in which matter usually rises at the pole and descends at the equator. In this situation, angular momentum is transported inwards. It is however also possible for the circulation to go in the reverse direction and, in this second case, angular momentum is transported outwards. Circulation corresponds to an advective process, which is different from diffusion because the latter can only erode gradients. Advection can either build or erode angular velocity gradients (see Maeder & Zahn 1998, for more details).

Dynamical shear occurs when the excess energy contained in differentially rotating layers is larger then the work that needs to be done to overcome the buoyancy force. This instability is describes in Sect. 2.2 and it occurs on a dynamical time scale. If the differential rotation is not strong enough to induce dynamical shear, it can still induce the secular shear instability when thermal turbulence reduce the effect of the buoyancy force. The secular shear instability occurs therefore on the thermal time scale, which is much longer than the dynamical one. Note that the way the inhibiting effect of the molecular weight ($\mu$) gradients on secular shear is taken into account impacts strongly the efficiency of the shear. In some work, the inhibiting effect of $\mu$–gradients is so strong that secular shear is suppressed below a certain threshold value of differential rotation (Heger et al. 2000). In this model, thermal instabilities and horizontal turbulence reduce the inhibiting effect of the $\mu$–gradients. As a result, shear is not suppressed below a threshold value of differential rotation but only decreased when $\mu$–gradients are present.
(Maeder 1997).

There are other instabilities which are not included in this work. However, they are not believed to be significant. From these, we can mention the GSF instability (Goldreich & Schubert 1967; Fricke 1968), the ABCD instability (Knobloch & Spruit 1983) and the Solberg–Høiland instability (Kippenhahn & Weigert 1990). The GSF instability is induced by axisymmetric perturbations. The ABCD instability is a kind of horizontal convection. Finally, Solberg–Høiland stability criterion is the criterion that should be used instead of the Ledoux or Schwarzschild criterion in rotating stars. See Sect. 2.2 for an interesting result concerning Solberg–Høiland instability.

**Transport of angular momentum**

For shellular rotation, the equation of transport of angular momentum (Zahn 1992) in the vertical direction is (in lagrangian coordinates):

\[
\rho \frac{d}{dt} (r^2 \Omega)_{M_r} = \frac{1}{5r^2} \frac{\partial}{\partial r} \left( \rho r^4 \Omega (r) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho D \frac{\partial \Omega}{\partial \Omega} \right),
\]

(2.1)

where \( \Omega (r) \) is the mean angular velocity at level \( r \), \( U(r) \) the vertical component of the meridional circulation velocity and \( D \) the diffusion coefficient due to the sum of the various turbulent diffusion processes (convection and shears). The factor \( \frac{1}{5} \) comes from the integration in latitude. Note that angular momentum is conserved in the case of contraction or expansion. The first term on the right hand side, corresponding to meridional circulation, is an advective term. The second term on the right hand side, which corresponds to the diffusion processes, is a diffusive term. One can see again that advection and diffusion are different. The correct treatment of advection is very costly numerically because Eq. 2.1 is a fourth order equation (the expression of \( U(r) \) contains third order derivatives of \( \Omega \), see Zahn 1992). This is why some research groups treat meridional circulation in a diffusive way (see for example Heger et al. 2000) with the risk of transporting angular momentum in the wrong direction (in the case meridional circulation builds gradients).

**Transport of chemical species**

The transport of chemical elements is also governed by a diffusion–advection equation like Eq. 2.1. However, if the horizontal component of the turbulent diffusion is large, the vertical advection of the elements (and not that of the angular momentum) can be treated as a simple diffusion (Chaboyer & Zahn 1992) with a diffusion coefficient \( D_{\text{eff}} \),

\[
D_{\text{eff}} = \frac{|rU(r)|^2}{30D_h},
\]

(2.2)

where \( D_h \) is the coefficient of horizontal turbulence, for which the estimate is \( D_h = |rU(r)| \) (Zahn 1992). Equation 2.2 expresses that the vertical advection of chemical
2.1. The Geneva stellar evolution model and its physical ingredients

elements is severely inhibited by the strong horizontal turbulence characterized by $D_h$. Thus, the change of the mass fraction $X_i$ of the chemical species $i$ is simply

$$ \left( \frac{dX_i}{dt} \right)_{M_r} = \left( \frac{\partial}{\partial M_r} \right)_t \left( 4\pi r^2 \rho \right)^2 D_{\text{mix}} \left( \frac{\partial X_i}{\partial M_r} \right)_t + \left( \frac{dX_i}{dt} \right)_{\text{nucl}}, \tag{2.3} $$

where the second term on the right accounts for composition changes due to nuclear reactions. The coefficient $D_{\text{mix}}$ is the sum $D_{\text{mix}} = D + D_{\text{eff}}$, where D is the term appearing in Eq. 2.1 and $D_{\text{eff}}$ accounts for the combined effect of advection and horizontal turbulence.

2.1.2 Mass loss rates

The changes of the mass loss rates $\dot{M}$ with rotation are taken into account as explained in Maeder & Meynet (2000a). As reference mass loss rates, we adopt the mass loss rates of Vink et al. (2000, 2001b) which account for the occurrence of bi-stability limits which change the wind properties and mass loss rates. For the domain not covered by these authors, we use the empirical law devised by de Jager et al. (1988). Note that this empirical law, which presents a discontinuity in the mass flux near the Humphreys–Davidson limit, implicitly accounts for the mass loss rates of LBV stars. For the non-rotating models, since the empirical values for the mass loss rates are based on stars covering the whole range of rotational velocities, we must apply a reduction factor to the empirical rates to make them correspond to the non rotating case. The same reduction factor was used as in Maeder & Meynet (2001). During the Wolf–Rayet phase we use the mass loss rates by Nugis & Lamers (2000). These mass loss rates, which account for the clumping effects in the winds, are smaller by a factor 2–3 than the mass loss rates used in the previous non-rotating “enhanced mass loss rate” stellar grids (Meynet et al. 1994). Wind anisotropy (described in Meynet & Maeder 2003) was only taken into account for $M \geq 40 M_{\odot}$. The wind anisotropy is important: a) For very massive stars but not too massive ($40 M_{\odot} \leq M \leq 80 M_{\odot}$) so that the couple has the time to act before the mass is ejected away from the star by the winds. b) For high initial rotation velocities ($\gtrsim 400 \text{ km s}^{-1}$ at solar metallicity).

2.1.3 Treatment of convection

Which criterion should be used to determine the limit of convective zones: Schwarzschild or Ledoux? In other words, how efficient is semi-convection? Does overshooting occur? All these questions do not have a final answer yet.

The strict criterion to determine the edge of convective zones (see for example Kippenhahn & Weigert 1990) is the Ledoux criterion including the stabilising effect of the $\mu$-gradients. The Schwarzschild criterion, on the other hand, does not take into account the $\mu$-gradients. The zone in between these two limits is called the semi-convective zone. In the semi-convective zone, thermal turbulence, eddy after eddy, mixes material and reduces the effects of the $\mu$-gradients. The crucial question is: how fast is this process?
CHAPTER 2. GENEVA STELLAR EVOLUTION MODEL

Note that if semiconvection is infinitely efficient, Schwarzschild criterion is recovered. There is no precise theory of semiconvection at the present time and different authors use different prescriptions for the treatment of convection. In this model, Schwarzschild criterion is used. This means that semiconvection is considered as extremely efficient. Limongi & Chieffi (2003) and Thielemann et al. (1996) also use Schwarzschild criterion. On the other hand, Woosley & Weaver (1995) and Rauscher et al. (2002) use Ledoux criterion for convection with semiconvection. They use a relatively large diffusion coefficient for modeling semiconvection. Heger et al. (2000) use Ledoux criterion for convection with semiconvection using a small diffusion coefficient (about one percent of the coefficient used by Woosley & Weaver 1995).

Another part of the treatment of convection is overshooting. Overshooting as its name indicate corresponds to gas traveling beyond the limit of the convective zones (determined by either criterion). This means that the layers just above the limit zone are also mixed. Again, different authors use different prescriptions for overshooting. In this model, we consider overshooting for core H and He–burnings. The overshooting length is 0.1 Hₚ, where Hₚ is the pressure height scale determined at the border of the convective core given by the Schwarzschild criterion. In the models of Woosley & Weaver (1995) and Rauscher et al. (2002), non–convective zones immediately adjacent to convective regions are slowly mixed on the order of a radiation diffusion time scale to approximately allow for the effects of convective overshoot. Limongi & Chieffi (2003), Heger et al. (2000) and Thielemann et al. (1996) do not include overshooting.

The choices concerning the treatment of convection are made by comparing the models with observations. Strict constraints on models come from the observed surface chemical composition and the ratio between the numbers of stars in different evolutionary stages (RSG, BSG, WR, ...). In this context, it has been shown that the rotating models calculated with the Geneva code well reproduce the observed surface enrichments (Meynet & Maeder 2000), the ratios of blue to red supergiants in the Small Magellanic Cloud (Maeder & Meynet 2001), and the variations of the Wolf–Rayet (WR hereinafter) star populations as a function of the metallicity (Meynet & Maeder 2003). We can also obtain constraints from observations of SNe, especially close ones (see introduction). In this model, the extent of overshooting is determined by reproducing the correct width of the MS (in the mass range 1.3–3 Mₒ) in HR–diagrams of clusters. This comparison gives an overshooting length of 0.1–0.2 Hₚ (for many years Maeder & Mermilliod 1981).

2.1.4 Opacities

In this model, we use the opacity tables from the OPAL group (Iglesias & Rogers 1996), complemented at low temperatures with the molecular opacities of Alexander (http://web.physics.twsu.edu/alex/wwwdra.htm). One of the great successes of these new opacities by OPAL is that they permitted to understand what has been a major challenge of stellar pulsation theory, namely the cause of β-Cephei variability (see for example Dziembowski & Pamiatnykh 1993). Note that another team, known as the Opacity Project group (OP), obtained results generally in close agreement with OPAL.
2.1. The Geneva stellar evolution model and its physical ingredients

(Seaton 1996).

Recent results show that the present opacities have to be increased (Seaton & Badnell 2004). This can compensate for the decrease in solar abundances in C, N, O and Ne found by Asplund et al. (2004).

2.1.5 Equation of state

The equation of state for a perfect gas, which is totally ionised, is simple. However the equation of state is more complex when the gas becomes partially degenerate or partially ionised. The gas can be partially ionised near the surface of the star. In this case, we use the method developed by Schaerer in 1990. The gas becomes partially degenerate in the core of the star during the advanced stages. The equation of state used in this model for partially degenerate gas is described in Kippenhahn et al. (1967). The approximations made by Kippenhahn et al. (1967) are precise for any degree of relativity when the degeneracy parameter $\psi$ is smaller than 7. If the gas is more degenerate, the complete degeneracy approximations from Chandrasekhar (1939) are used. The precision of this equation of state might be less good when the degeneracy becomes larger than 7. During most of the pre–SN evolution, the degeneracy parameter is less than 7 and the equation of state is precise enough. However, after core or shell Si–burning, the degeneracy can become similar or larger than 7. It is the case in the 15 $M_\odot$ models. The limitation due to the use of this equation of state is therefore a possible cause of the failure of our models to follow the final collapse. It is therefore planned in the near future to update the equation of state and to use the one derived by Blinnikov et al. (1996) which is precise for any degree of relativity and degeneracy.

2.1.6 Neutrino energy losses

During the advanced stages, most of the energy produced by nuclear reactions is evacuated by neutrinos. Neutrinos are produced in weak interactions during Si–burning but are mainly produced by the four following leptonic processes throughout the advanced stages:

a) pair annihilation $e^+ + e^- \rightarrow \nu + \bar{\nu}$

b) $\nu$-photoproduction $\gamma + e^\pm \rightarrow e^\pm + \nu + \bar{\nu}$

c) plasmon decay $\gamma^* \rightarrow \nu + \bar{\nu}$

d) bremsstrahlung on nuclei $e^\pm + Z \rightarrow e^\pm + Z + \nu + \bar{\nu}$

Each process is the dominant contribution to the neutrino energy losses in different regions of the core density–temperature plane (Esposito et al. 2003). For very large core temperatures, $T \gtrsim 10^9$K, and relatively low density, $\rho \lesssim 10^5$ g cm$^{-3}$, the pair annihilation is the most efficient cooling process. For the same values of densities but lower temperatures, $10^8$K$\lesssim T \lesssim 10^9$K, the $\nu$–photoproduction gives the leading contribution. These density-temperature ranges are the typical ones for very massive stars.
in their late evolution. Finally, plasmon decay and bremsstrahlung on nuclei are mostly important for large ($\rho \gtrsim 10^6$ g cm$^{-3}$) and extremely large ($\rho \gtrsim 10^9$ g cm$^{-3}$) core densities, respectively, and temperatures of the order of $10^8$ K $\lesssim T \lesssim 10^{10}$ K. Such conditions are typically realized in white dwarfs.

In this model, we use the neutrino emission rates from Itoh et al. (1989) for pair, photo and plasmon–neutrinos and from Dicus et al. (1976) for bremsstrahlung. Note that Itoh et al. updated their neutrino energy loss rates (Itoh et al. 1996). More recently, Esposito et al. (2003) recalculate the various neutrino energy loss rates and found similar results to those of Itoh et al. (1996).

### 2.2 Dynamical shear

The criterion for stability against dynamical shear instability is the Richardson criterion:

$$Ri = \frac{N^2}{(\partial U/\partial z)^2} > \frac{1}{4} = Ri_c,$$  \hspace{1cm} (2.4)

where $U$ is the horizontal velocity, $z$ the vertical coordinate and $N^2$ the Brunt–Väisälä frequency:

$$N^2 = \frac{g\delta}{H_P} [\nabla_{ad} - \nabla + \frac{\varphi}{\delta} \nabla_{\mu}],$$  \hspace{1cm} (2.5)

where $g$ is the gravity, $\delta = -d\ln \rho / d\ln T)_{\mu,P}$, $H_P$ is the pressure scale height, $\nabla_{ad} = d\ln T / d\ln P)_{ad}$, $\nabla = d\ln T / d\ln P$, $\nabla_{\mu} = d\ln \mu / d\ln P$ and $\varphi = d\ln \rho / d\ln \mu)_{T,P}$.

The critical value, $Ri_c = 1/4$, corresponds to the situation where the excess kinetic energy contained in the differentially rotating layers is equal to the work done against the restoring force of the density gradient (also called buoyancy force). It is therefore used by most authors as the limit for the occurrence of the dynamical shear. However, recent studies by Canuto (2002) show that turbulence may occur as long as $Ri \lesssim Ri_c \sim 1$. This critical value is consistent with numerical simulations done by Brüggen & Hillebrandt (2001) where they find shear mixing for values of $Ri$ greater than 1/4 (up to about 1.5).

Different dynamical shear diffusion coefficients, $D$, can be found in the literature. Heger et al. (2000) use:

$$D = [\min\{d_{inst}, H_P\}(1 - \max\{\frac{Ri}{Ri_c}, 0\})]^2 / \tau_{dyn}$$  \hspace{1cm} (2.6)

where $\tau_{dyn} = \sqrt{r^3/(Gm_r)}$ is the dynamical time scale and $d_{inst}$ the spatial extent of the unstable region, which is limited to one $H_P$.

Brüggen & Hillebrandt (2001) use another formula and they do numerical simulations to study the dependence of $D$ on $Ri$. They find the following result:
2.2. Dynamical shear

Figure 2.1: Ω variations as a function of the radius inside 15 $M_\odot$ models: the dashed line is a profile from a model without dynamical shear and the solid line from a model with dynamical shear and $Ri_c = 1$ during core O–burning. The long and short arrows indicate the zones where $Ri < 1$ in the model without and with dynamical shear respectively. Note that the profiles do not differ significantly.

\[ D = \frac{0.6 \times 10^{10}}{Ri} \]  \hspace{1cm} (2.7)

2.2.1 The recipe

The following dynamical shear coefficient is used, as suggested by J.-P. Zahn (priv. comm.):

\[ D = \frac{1}{3} vl = \frac{1}{3} \frac{v}{l} l^2 = \frac{1}{3} r \frac{d\Omega}{dr} \Delta r^2 = \frac{1}{3} r \Delta\Omega \Delta r \]  \hspace{1cm} (2.8)

where $r$ is the mean radius of the zone where the instability occurs, $\Delta\Omega$ is the variation of $\Omega$ over this zone and $\Delta r$ is the extent of the zone. The zone is the reunion of consecutive shells where $Ri < Ri_c$. This is valid if $P_e > 1$, where $P_e$, the Peclet Number is the ratio of cooling to dynamical time scale of a turbulent eddy. We calculated three $v_{ini} = 300 \text{km s}^{-1}$ 15 $M_\odot$ models to see the impact of dynamical shear and the importance of the value of $Ri_c$ (Hirschi et al. 2003a): one without dynamical shear, one with $Ri_c = 1/4$ and the last one with $Ri_c = 1$. In Fig. 2.1, the variation of the angular
velocity, $\Omega$, as a function of the radius is shown inside 15 $M_\odot$ stellar models in the core O-burning phase. Arrows indicate the zones which are unstable against dynamical shear instability. These zones remain unstable during the whole post core He-burning phase. Our simulations show that the characteristic time scale of the dynamical shear ($\propto R^2/D$) is always very short when using Eq. (2.8) for the dynamical shear diffusion coefficient. Indeed, we obtain diffusion coefficients between $10^{12}$ and $10^{14}$ cm$^2$ sec$^{-1}$. This is in general one or two orders of magnitude larger than using the expressions given by Brüggen & Hillebrandt (2001) or Heger et al. (2000). However, the extent of the unstable zones is very small, a few thousandths of a solar mass. Therefore the shear mainly smoothens the sharp $\Omega$-gradients as can be seen in Fig. 2.1 but does not transport angular momentum or chemical species over long distances. The general structure and the convective zones are similar between the model without dynamical shear and the one with dynamical shear.

Concerning the Richardson criterion, there is no significant difference between the models using $\dot{R}_i c = 1/4$ and $\dot{R}_i c = 1$. Except for the 15 $M_\odot$ model discussed in this subsection, all the other models were computed with $\dot{R}_i c = 1/4$.

### 2.2.2 Solberg–Høiland instability

Solberg–Høiland stability criterion corresponds to the inclusion of the effect of rotation (variation of centrifugal force) in the convective stability criterion. It is more general and contains the Ledoux (or possibly Schwarzschild) and the Rayleigh criteria (Maeder & Meynet 2000b; Heger et al. 2000). Both the dynamical shear and Solberg–Høiland instabilities occur in the case of a very large angular velocity decrease outwards (usual situation in stars, see Fig. 2.1). Note that if there is a large increase outwards, dynamical shear instability occurs but not the Solberg–Høiland instability.

Both instabilities, shear instability and Solberg–Høiland stability, occur on the dynamical time scale. We therefore expect them to have similar effects. The question is which instability sets in first? By comparing the stability criteria of the dynamical shear and of the Solberg–Høiland instability:

$$1/4 (d\Omega/dr)^2 > N^2 \text{ dynamical shear}$$

$$-2\Omega[2\Omega + (d\Omega/dr) r] < N^2 \text{ Solberg–Høiland},$$

where $\Omega$ is the angular velocity, $r$ the radius and $N^2$ the Brunt–Väisälä frequency, it can be demonstrated that whenever a zone is unstable towards the Solberg–Høiland instability, it is also unstable towards the dynamical shear instability. Indeed:

$$1/4 (d\Omega/dr)^2 r^2 > -2\Omega[2\Omega + (d\Omega/dr) r]$$

because

$$1/4 (d\Omega/dr)^2 r^2 + 2\Omega[2\Omega + (d\Omega/dr) r] =$$

$$1/4[(d\Omega/dr) r + 4\Omega]^2 > 0$$

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2.2. Dynamic shear

This means that the treatment of the dynamical shear instability alone is sufficient (since the time scales are similar). We therefore did not include explicitly the Solberg–Høiland instability in our model.

Note that these criteria were compared at the equator. If a constant angular velocity is assumed on isobar, the stability criteria will first be violated at the equator and then at other latitude.
2.3 Stellar structure equations

There are four equations describing the evolution of the structure of the star: the mass and the energy conservations and the momentum and the energy transport equations. On top of that, the equations of the evolution of chemical elements abundances are to be followed. In the Geneva model, as in the original version of Kippenhahn and Weigert (Kippenhahn et al. 1967; Kippenhahn & Weigert 1990), the problem is treated in one dimension (1D) and the equations of the evolution of chemical elements abundances are calculated separately from the structure equations. These last equations are discussed in the next chapter. Since rotation is included in the Geneva model, spherical symmetry is no longer valid. In fact, the effective gravity (sum of the centrifugal force and gravity) can no longer be derived from a potential and the case is said to be non-conservative. The problem can still be treated in 1D by assuming that the angular velocity is constant on isobars. This assumes that there is a strong horizontal (along isobars) turbulence which enforces constant angular velocity on isobars (Zahn 1992). The case is referred to as ‘shellular’ rotation and is described in Meynet & Maeder (1997). In this scheme, the four structure equations are the following:

- Energy conservation:
  \[
  \frac{\partial L_P}{\partial M_P} = \epsilon_{\text{nucl}} - \epsilon_\nu + \epsilon_{\text{grav}} = \epsilon_{\text{nucl}} - \epsilon_\nu - \epsilon_P \frac{\partial T}{\partial t} + \frac{\delta \partial P}{P \partial t} \quad (2.9)
  \]

- Momentum equation:
  \[
  \frac{\partial P}{\partial M_P} = - \frac{GM_P}{4\pi r^4} f_P \quad (2.10)
  \]

- Mass conservation (or continuity equation):
  \[
  \frac{\partial r_P}{\partial M_P} = \frac{1}{4\pi r^2 \rho} \quad (2.11)
  \]

- Energy transport equation:
  \[
  \frac{\partial \ln T}{\partial M_P} = \frac{GM_P}{4\pi r^4} f_P \min [\nabla_{\text{ad}}, \nabla_{\text{rad}} \frac{f_T}{f_P}] \quad (2.12)
  \]

where

\[
\nabla_{\text{ad}} = \frac{P\delta}{T\rho c_P} \quad \text{(convective zones)},
\]

\[
\nabla_{\text{rad}} = \frac{3}{16\pi acG} \frac{\kappa P}{mT_4} \quad \text{(radiative zones)}.
\]
2.3. Stellar structure equations

\[ f_P = \frac{4\pi r_P^4}{GM_P S_P} \langle <g^{-1}> \rangle, \]

\[ f_T = \left( \frac{4\pi r_T^2}{S_P} \right)^2 \frac{1}{\langle g \rangle < g^{-1}> } . \]

\(< x >\) is \( x \) average on an isobaric surface, \( \bar{x} \) is \( x \) average in the volume separating two successive isobars and the index \( P \) refers to the isobar with a pressure equal to \( P \) and the other variables have their usual meaning (see Meynet & Maeder 1997, for more details).

2.3.1 Hydrodynamic equations with shellular rotation law:

In the above formulation, hydrostatic equilibrium is assumed. If this assumption is not made, the momentum equation, Eq. 2.10, including the acceleration term (Heger et al. 2000), is:

\[
\frac{\partial P}{\partial M_P} = -\frac{G M_P}{4\pi r_P^4} f_P - \frac{1}{4\pi r_P^2} \frac{\partial^2 r_P}{\partial t^2} .
\]  

(2.13)

The equations for mass and energy conservations are unchanged and the energy transfer equation, Eq. 2.12, in the case of convection only, becomes:

\[
\frac{\partial \ln T}{\partial M_P} = -\frac{\nabla_{ad}}{\bar{P}} \left[ -\frac{G M_P}{4\pi r_P^4} f_P - \frac{1}{4\pi r_P^2} \frac{\partial^2 r_P}{\partial t^2} \right] .
\]  

(2.14)

Equation 2.14 is different from (Heger et al. 2000) because they consider \( \mathrm{dlnT/dlnP} \) in their set of equations. We note that other quantities like \( H_P \) should be affected by the acceleration term as well.

It is instructive to look at the time scale for the dynamical adjustment of the structure, called \( \tau_{\text{hydr}} \) (see Kippenhahn & Weigert 1990, page 11), in order to see whether hydrodynamic equations are necessary.

\[ \tau_{\text{hydr}} \approx \left( \frac{R^3}{GM} \right)^{1/2} . \]

At the end of core silicon burning, the radius of the iron core is about \( 4 \times 10^8 \) cm and the mass of the core is about \( 3 \times 10^{32} \) g. Using \( G=6.67 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2} \), we obtain \( \tau_{\text{hydr}} \sim 0.5 \) s. It is still very small compared to the silicon burning lifetime which is of the order of \( 10^5 \) s (20 hours). Note that during O–burning, \( \tau_{\text{hydr}} \sim 2.2 \) s. Therefore, hydrostatic equations still provide a good description until silicon burning. Note however that the hydrodynamical term can influence crossing times in the HR–diagram. The present version of the Geneva code does not include the acceleration terms. Limongi & Chieffi (2003) also use hydrostatic equations to model the pre–supernova evolution.
2.3.2 Numerical method

The numerical methods available to solve the system of equations described above are numerous and can differ significantly from one to the other. In this paragraph, the aim is to present the numerical method used in the Geneva model. The general method is close to the original one and is well described in Kippenhahn et al. (1967). Here we only present the main points. The method used to solve the structure equations is a finite difference method. This means that the equations are verified exactly in every point of a mesh. The general solution is then obtained by interpolating the solution between the mesh points. Other methods, like the finite elements methods, require the best approximation over the calculation domains.

The method is implicit in time. In such a method, the solution at time $t + \Delta t$ is calculated using time derivatives calculated from the solution at time $t + \Delta t$, solution which is not known. A good example of an implicit method is the method used to solve the equations of the chemical elements abundances (see next chapter). The other possibility is to use an explicit method. In an explicit method, the solution at time $t + \Delta t$ is calculated using time derivatives calculated from the solution at time $t$, thus an already known solution. Implicit methods are more complex to calculate than explicit methods. However, they have the huge advantage to be stable whatever time step is used. In the explicit method, the time step is limited by the so called Courant condition. Stability does not guaranty precision. In order to ensure the precision of the solution, a Newton–Raphson method adapted to stellar evolution is used. This method is usually called the Henyey method (see Kippenhahn & Weigert 1990, page 78). In this method, an approximate initial solution of the structure is refined at each time step of the method by applying corrections to every variable at every mesh point (see Kippenhahn et al. 1967, page 145). We note that in between each time step of the Henyey method, the changes of the abundance of the chemical elements are recalculated using the updated structure (temperature and density).

The set of equation has mixed boundary conditions. At the centre of the star, $M_p = 0$, the conditions are that both the radius and the luminosity must vanish. At the surface of the star, simplified boundary conditions are that both the pressure and the temperature vanish. However, the surface boundary conditions used are more complex. In fact, the star is divided into three parts, the interior described by the equations given above, the envelope and the atmosphere (see Kippenhahn et al. 1967, page 133). The total luminosity of the star and its effective temperature, $T_{\text{eff}}$, are first approximated. From the integration of the atmosphere equations (from the top, where the pressure is zero, down to the bottom of it), one can obtain the value of the pressure at the photosphere (point where $T = T_{\text{eff}}$ and limit between the atmosphere and the envelope in the model). The radius and luminosity are assumed to be respectively equal to the total radius and total luminosity at this point. The radius can be obtained from the relation between the luminosity and the effective temperature ($L \sim R^2 T_{\text{eff}}^4$). Using the value of the four dependent variables (pressure, temperature, radius and luminosity, the mass being the independent variable) the boundary conditions for the interior is obtained.
by integrating the equations of the structure through the envelope (downwards). The boundary conditions and the approximate solution for the interior are used to calculate the corrections of the Henyey method. After applying the corrections to the interior, the integration of the equations of the structure through the envelope (upwards) yields new values for the effective temperature and luminosity. If these two new values are too different from the previous ones, these values are changed and the process is repeated until the convergence of the solution of the interior and the envelope. A flow chart of the model can be found in Kippenhahn et al. (1967) at page 154.

In parallel to the resolution of the structure equations, the fourth order equation for the transport of the angular momentum (Eq. 2.1) has to be solved. A Henyey-type method is used to solve this complex equation. The development of the method has been made by Meynet and Maeder.

### 2.3.3 Discretisation of the equations and Sugimoto’s prescription

#### Discretisation of the equations

Before the equations are discretised, the following changes of variables need to be done:

- \( Q = \ln(1 - M_P/M_f) \): the new mass coordinate,
- \( M_f \) is the mass limit between the interior and the envelope of the star,
- \( P = \ln P \): the new pressure variable,
- \( T = \ln \bar{T} \): the new temperature variable,
- \( R = \ln r_P \): the new radius coordinate,
- \( RH = \ln \bar{r} \) the new density coordinate,
- \( CAP = \ln \kappa \), \( \kappa \) is the opacity,
- \( S = \ln(1 + L_P/(f L_{tot})) \): the new luminosity coordinate,
- \( L_{tot} \) is the total luminosity of the star and
- \( f \) is a factor to ensure that \( L_P/(f L_{tot}) > -1 \) so that \( S \) is define. This factor is important when the neutrino energy losses are important.

Applying this to the equations above, one finds:

- **Energy conservation:**

\[
\frac{\partial S}{\partial Q} = - \frac{M_f}{f L_{tot}} e^{(Q - S)} (\epsilon_{\text{nucl}} - \epsilon_{\nu} + \epsilon_{\text{grav}}),
\]  
\hspace{1cm} (2.15)

- **Momentum equation:**

\[
\frac{\partial P}{\partial Q} = \frac{GM_f^2}{4\pi} f_P e^{(Q - P - 4R)} (1 - e^Q),
\]  
\hspace{1cm} (2.16)

- **Mass conservation:**

\[
\frac{\partial R}{\partial Q} = \frac{M_f}{4\pi} e^{(Q - 3R - RH)},
\]  
\hspace{1cm} (2.17)
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- Energy transport equation:

**Radiation:**

\[
\frac{\partial T}{\partial Q} = \frac{3M_f f L_{tot}}{256\pi^2 \sigma} f_T (e^S - 1) e^{[Q+CAP-4(R+T)]},
\]

**Convection:**

\[
\frac{\partial T}{\partial Q} = \frac{GM_f^2}{4\pi} f_P e^{(Q-P-4R)} (1 - e^Q) \frac{\delta}{c_P}
\]  
(2.18)

Now if we discretise the equations, we obtain by approximating the spatial derivatives in the middle of the mass zone (and replacing \( J + 1 \) by \( J_1 \)):

- Energy conservation:

\[
S(J) - S(J_1) \over Q(J) - Q(J_1) + M_f \over f L_{tot} e^{0.5 (Q(J) + Q(J_1) - S(J) - S(J_1))}
\]

\[
x \left( \sqrt{\epsilon_{\text{nucl}}(J_1)} \epsilon_{\text{nucl}}(J) - \sqrt{\epsilon_{\nu}(J_1)} \epsilon_{\nu}(J) + \epsilon_{\text{grav}} \right) = 0,
\]

\[
\epsilon_{\text{grav}} = \frac{1}{4\Delta t} e^{0.5 (P(J_1) + P(J) - RH(J_1) - RH(J))}
\]

\[
x \left[ -(RHT(J_1) + RHT(J)) (VP(J_1) + VP(J)) + \left( \frac{RHT(J_1)}{ADI(J_1)} + \frac{RHT(J)}{ADI(J)} \right) (VT(J_1) + VT(J)) \right]
\]

- Momentum equation:

\[
\frac{P(J) - P(J_1)}{Q(J) - Q(J_1)} - \frac{GM_f^2}{4\pi} f_P e^{0.5 (Q(J_1) + Q(J) - P(J_1) - P(J) - 4R(J_1) - 4R(J))} (1 - e^{0.5 (Q(J_1) + Q(J))}) = 0
\]

(2.20)

- Mass conservation:

\[
\frac{R(J) - R(J_1)}{Q(J) - Q(J_1)} + M_f \over 4\pi e^{0.5 (Q(J_1) + Q(J) - 3R(J_1) - 3R(J) - RH(J_1) - RH(J))} = 0
\]

(2.21)

- Energy transport equation:

**Radiation:**

\[
\frac{T(J) - T(J_1)}{Q(J) - Q(J_1)} - \frac{GM_f^2}{4\pi} f_T e^{0.5 (Q(J_1) + Q(J) - P(J_1) - P(J) - 4R(J_1) - 4R(J))}
\]

\[
x (1 - e^{0.5 (Q(J_1) + Q(J))}) 0.5 (RAD(J_1) + RAD(J)) = 0
\]

(2.22)

**Convection:**

\[
\frac{T(J) - T(J_1)}{Q(J) - Q(J_1)} - \frac{GM_f^2}{4\pi} f_T e^{0.5 (Q(J_1) + Q(J) - P(J_1) - P(J) - 4R(J_1) - 4R(J))}
\]

\[
x (1 - e^{0.5 (Q(J_1) + Q(J))}) 0.5 (ADI(J_1) + ADI(J)) = 0
\]

(2.23)
2.3. Stellar structure equations

where $RHT = \partial RH/\partial T = -\delta$, $ADI = \nabla_{ad}$ is the adiabatic gradient $\partial \ln T / \partial \ln P|_{ad}$, $RAD = \nabla_{rad}$ is the radiative gradient $\partial \ln T / \partial \ln P|_{rad}$,

Sugimoto's prescription

As said above, the equations are implicit in time. This means that large time steps are allowed without compromising the stability of the equations. This is very useful to model the evolution on the main sequence and during He-burning. In the advanced stages, the evolution of the core accelerates due to neutrino losses. In this case, an explicit method is more suitable. However, during the advanced stages, the evolution of the envelope is decoupled from that of the core and the envelope still evolves on a slow time scale. This means that an explicit method needs to be used for the core and an implicit method needs to be used for the envelope. The problem becomes stiff and the correct solutions are hard to find. The problem with the implicit method arises when the time step becomes smaller than the time scale of heat conduction through the layer:

$$\Delta t < \tau_h(r) = \frac{3C_P \rho r^2}{4acT^3} = \frac{r^2}{K}$$

(2.24)

where $K$ is the thermal diffusivity.

Sugimoto (1970) finds a recipe to avoid this problem:

"It is shown that, with a properly formulated Henyey method, we can obtain physically significant branches of solutions by avoiding the unstable branches of solution which the original system of four differential equations has in the above limit (limit of the short time steps during the advanced stages). In this formulation the difference equation for energy flow and the difference equation for energy conservation should be off center in opposite directions, in mass shells where the stability condition for the implicit method (here the instability criterion is given in Eq. 2.24) is not satisfied."

Therefore, during the advanced stages, the two following equations replace the ones given above:

- Energy conservation:

$$\frac{S(J1) - S(J)}{Q(J1) - Q(J)} + \frac{M_f}{\int L_{tot}} \left[ \beta_{Sugimoto} e^{(Q(J)-S(J))}(\epsilon_{\nu cl}(J) - \epsilon_{\nu}(J) + \epsilon_{grav}(J)) 
+ (1 - \beta_{Sugimoto}) e^{(Q(J1)-S(J1))}(\epsilon_{\nu cl}(J1) - \epsilon_{\nu}(J1) + \epsilon_{grav}(J1)) \right] = 0,$$

(2.25)

$$\epsilon_{grav}(J) = -\frac{1}{\Delta t} e^{(P(J)-RH(J))} RHT(J) VP(J) + \frac{1}{\Delta t} e^{(P(J)-RH(J))} \frac{RHT(J)}{ADI(J)} VT(J)$$
• Energy transport equation:

\[
\begin{align*}
\frac{T(J1) - T(J)}{Q(J1) - Q(J)} & - \frac{GM_J^2}{4\pi} f_T \left[ (1 - \beta_{Sugimoto}) e^{(Q(J) - P(J) - 4R(J))} (1 - e^{Q(J)}) \right] RAD(J) \\
+ \beta_{Sugimoto} e^{(Q(J1) - P(J1) - 4R(J1))} (1 - e^{Q(J1)}) \right] RAD(J1) & = 0 \\
\text{Convection:} & \\
\frac{T(J1) - T(J)}{\bar{Q}(J1) - \bar{Q}(J)} & - \frac{GM_J^2}{4\pi} f_P \left[ (1 - \beta_{Sugimoto}) e^{(Q(J) - P(J) - 4R(J))} (1 - e^{Q(J)}) \right] ADI(J) \\
+ \beta_{Sugimoto} e^{(Q(J1) - P(J1) - 4R(J1))} (1 - e^{Q(J1)}) \right] ADI(J1) & = 0
\end{align*}
\]

(2.26)

where \( \beta_{Sugimoto} \) is the parameter used to shift the discretisation of the two equations above in opposite direction. The possible values for \( \beta_{Sugimoto} \) are zero and one.

The implementation of this method was not trivial. The first attempt was to use the normal discretisation (Eqs. 2.19–2.22) until the instabilities occur (after core C–burning) and then to use Sugimoto’s prescription in the unstable zones (zones where Eq. 2.24 is verified). In practice, it was found that, once C–burning starts, Eq. 2.24 is verified in most of the star. This means that the structure equations had to be changed in a large fraction of the star at once. The Henyey method did not agree with such an abrupt change. Sugimoto’s prescription was also tested during H–burning, using both possible values \( \beta_{Sugimoto} = 1, 0 \). A slightly (about one percent) shorter and longer lifetime of the main sequence was found using \( \beta_{Sugimoto} = 1 \) and 0 respectively. This shows the weakness of this method where the energy production rate is not evaluated in the centre of the mass zone but at its edge. In order to solve this problem, one would need to discretise the equations above using shifted intensive and extensive variables as developed in Meynet’s thesis (Meynet 1990). The difference being small, this important change was not undertaken here. Finally, the following method was adopted. At the start of C–burning, Sugimoto’s prescription is applied to the central shell. Then, at each iteration of the Henyey method, Sugimoto’s prescription is applied to one additional shell until eventually Sugimoto’s prescription is applied to the entire part of the star where Eq. 2.24 is verified. This smoother transition allows convergence of the structure within a few time steps. In the present calculations, \( \beta_{Sugimoto} = 1 \) was used. Note that, during the advanced stages, inequation 2.24 is verified throughout the star, such that Sugimoto’s prescription is used in the entire interior. Note as well that Sugimoto’s prescription was not applied at the centre and at the limit between the interior and the envelope because instabilities do not develop in these two zones. Finally, note that on top of the equations derived above, the Jacobian (partial derivatives of the equations with respect to the variable) were updated as well. One new subroutine was included in the code for the Sugimoto’s prescription (ginsu.f).
2.4 Minor modifications

2.4.1 Convection

Convective diffusion replaces convection during O-burning because the mixing time scale \( \tau_{\text{mix}} = R_{\text{core}}^2 / D_{\text{conv}} \sim 10^4 - 10^5 \, \text{s} \) becomes longer than the evolution time steps \( \Delta t \sim 10^2 - 10^3 \, \text{s} \) at that point. The theory of the mixing length is used to derive the corresponding diffusion coefficient, \( D_{\text{conv}} \). The interplay between mixing and the nuclear burning will be discussed in the next chapter.

2.4.2 Time step control

The time step control was updated. Previously, the time step was controlled by verifying that the central abundance of the main burning fuel was not varying by more than 0.2\% (in mass fraction) between two time steps. This method is suitable for H and He-burnings because during these two stages, the largest changes occur in the center. During the advanced stages, however, several episodes of shell burnings occur (releasing a large amount of nuclear energy) while, in the core, abundances do not evolve. It is therefore crucial to verify that the time step is suitable throughout the star. By monitoring the various energy production rates (nuclear, neutrinos and internal energy) throughout the evolution and throughout the star, it was found that the maximum energy production at a given time always corresponds to the contribution from nuclear reactions. We therefore use now the maximum nuclear energy production rate in order to fix the time step. We scale the time step so that the maximum change of abundances anywhere in the star (using a relation between the energy production and the change in abundance) varies by about 0.2\% (in mass fraction) between two time steps.
Chapter 3

Nuclear reaction network

This chapter describes the nuclear reaction network. This is a crucial part of the modeling of the advanced stages. First, the four advanced burning stages are presented. After a brief summary of reaction rates theory, the construction of the nuclear reaction network is described. The network (species and reactions) used in this work is then presented. Finally, the interplay between mixing and burning is discussed.

3.1 Generalities

The nuclear reaction network is a crucial part of the modeling of the advanced stages. Indeed, the advanced stages include 4 burning phases: carbon (C), neon (Ne), oxygen (O) and silicon (Si). The network is not only used to calculate the nucleosynthesis occurring during these burning stages but also to calculate the nuclear energy production rate. Before describing the network used in this work, it is useful to present each of the four burning phases that need to be modeled. Summaries can be found in Arnett & Thielemann (1985); Thielemann & Arnett (1985); Chieffi et al. (1998); Audouze & Woosley (1986).

3.1.1 Carbon burning

At the end of He-burning, the most abundant elements are $^{12}$C and $^{16}$O and the ratio C/O decreases when the mass of the star increases (Audouze & Woosley 1986) and for very massive stars ($60-70 \, M_\odot$) the most abundant elements can even be $^{16}$O and $^{20}$Ne (Meynet 1990). The next main fusion process is then the carbon one since it has the lowest Coulomb barrier: $^{12}$C + $^{12}$C $\rightarrow$ $^{24}$Mg*. The compound nuclear state of
24Mg can then decay via the canals:

\[ 24\text{Mg} + \gamma \quad (Q = 13.930 \text{ MeV}) \]
\[ 23\text{Mg} + n \quad (-2.605 \text{ MeV}) \]
\[ 23\text{Na} + p \quad (2.238 \text{ MeV}) \]
\[ 20\text{Ne} + \alpha \quad (4.616 \text{ MeV}) \]
\[ 16\text{O} + 2\alpha \quad (-0.114 \text{ MeV}) \]

At the temperature range of C-burning (0.7-1.3 \(10^9\) K) and for a mass range from 15 to 40 \(M_\odot\), the most important decay canals are the \(p\) and \(\alpha\) canals. The \(n\) canal may also be important at the end of C-burning (it cannot occur at low temperature because of the negative energy balance, \(Q\), of the reaction) because, even if it is a much less probable decay canal, \(23\text{Mg}\) can increase the neutron excess by \(\beta^+\)-decaying into \(23\text{Na}\). \(23\text{Na}\) then decays via \((p, \alpha)\)20Ne and \((p, \gamma)\)24Mg and therefore the main products of C-burning are 20Ne, 24Mg and 23Na.

The other reactions producing energy are: \(16\text{O}(\alpha, \gamma)\)20Ne and \(20\text{Ne}(\alpha, \gamma)\)24Mg and to a lesser extent: \(20\text{Ne}(n, \gamma)\)21Ne, \(26\text{Mg}(p, \gamma)\)27Al (only at low temperatures) and \(22\text{Ne}(p, \gamma)\)23Na.

Other reactions are important for the neutron production or captures and the nucleosynthesis of minor elements (Chieffi et al. 1998):

- At the start of C-burning:

\[ 12\text{C}(p, \gamma)\]13N(\(\beta^+\))13C(\(\alpha, n\))16O, \[ 12\text{C}(n, \gamma)\]13C(\(\alpha, n\))16O,
\[ 24\text{Mg}(p, \gamma)\]25Al(\(\beta^+\))25Mg and \[ 25\text{Mg}(n, \gamma)\]26Mg

- At the end of the C-burning:

\(23\text{Na}(\alpha, p)\)26Mg, \(22\text{Ne}(\alpha, n)\)25Mg(\(p, \gamma\))26Al(\(\beta^+\))26Mg,
\(21\text{Ne}(\alpha, n)\)24Mg and \[ 12\text{C}(n, \gamma)\]13C(\(\alpha, n\))16O.

The best neutron producers are \(13\text{C}(\alpha, n)\)16O at low temperature and \(22\text{Ne}(\alpha, n)\)25Mg at higher temperature. The best neutron poisons are 20Ne, 25Mg and 56Fe.

Therefore the important isotopes to include in a network modeling C-burning are \(n, p, \alpha, 12, 13\text{C}, 16\text{O}, 20, 21, 22, 23\text{Ne}, 22, 23, 24\text{Na}, 24, 25, 26\text{Mg}\) and \(25, 26, 27\text{Al}\).
3.1.2 Neon burning (or Neon photodisintegration)

At the end of C-burning, the most abundant elements are $^{20}$Ne, $^{24}$Mg and $^{23}$Na, produced in C-burning, and $^{16}$O, which is produced during He-burning and is only slightly burnt during C-burning via $^{16}$O($\alpha$, $\gamma$)$^{20}$Ne. Therefore the oxygen is the next element expected to burn since it has the lowest Coulomb barrier. However, it is stable because it has a double magic number, N = Z = 8. Thus, the next burning stage to occur is Ne-burning. At the start of the fusion ($T_c \approx 1.2 \times 10^9$ K), the most important reaction is $^{20}$Ne($\gamma$, $\alpha$)$^{16}$O (Neon photodisintegration). It is due to the fact that Neon has the lowest $\alpha$–separation energy (4.73 MeV compared to 7.16 and 9.32 MeV for $^{16}$O and $^{24}$Mg respectively). The reverse reaction rate increases during the stage and the equilibrium is almost reached at the end. The $\alpha$–particles produced by the photodisintegration are captured by the remaining $^{20}$Ne and it produces $^{24}$Mg via ($\alpha$, $\gamma$). At the end of the stage, Most of the $^{20}$Ne is transformed into $^{16}$O and $^{24}$Mg. Another reaction important for the energy generation is $^{24}$Mg($\alpha$, $\gamma$)$^{28}$Si. Even if the energy balance, Q, of $^{20}$Ne($\gamma$, $\alpha$)$^{16}$O is negative (-4.73 MeV), the different $\alpha$–captures allow the stage to generate energy. The most important products of Ne–burning are $^{16}$O, $^{24}$Mg and $^{28}$Si.

The other reactions, which are not relevant for energy production but only for the nucleosynthesis of minor elements and neutron excess are the following:

$$21\text{Ne}(\alpha, n)^{24}\text{Mg}(n, \gamma)^{25}\text{Mg}(\alpha, n)^{28}\text{Si}(\alpha, \gamma)^{32}\text{S},$$

$$23\text{Na} \left\{ \begin{array}{l} (p, \alpha)^{20}\text{Ne} \\ (\alpha, p)^{26}\text{Mg} \left\{ \begin{array}{l} (\alpha, n)^{20}\text{Si} \\ (p, \gamma)^{27}\text{Al} \left\{ \begin{array}{l} (\alpha, p)^{30}\text{Si} \\ (n, \gamma)^{28}\text{Al}(\beta^-)^{28}\text{Si} \end{array} \right. \end{array} \right. \\ 22\text{Ne}(\alpha, n)^{25}\text{Mg}, \quad 20\text{Ne}(n, \gamma)^{21}\text{Ne}, \quad 28\text{Si}(n, \gamma)^{29}\text{Si}(n, \gamma)^{30}\text{Si} \quad \text{and} \quad 25\text{Mg}(n, \gamma)^{26}\text{Mg}. \end{array} \right.$$  

It is noted that $^{20}$Ne does not react with $^{12}$C not because of the low cross section but because they are never both in a sufficient quantity for the flux of this reaction to be important.

The isotopes, which have to be added to the nuclear network listed at the end of the previous section, are $^{27}$Mg, $^{28}$Al, $^{28,29,30}$Si and $^{32}$S.

3.1.3 Oxygen burning

As temperature rises above $2 \times 10^9$ K, O–burning is favoured over the photodisintegrations of heavier species and the $\alpha$–captures (but these reactions still occur). $^{16}$O + $^{16}$O produces the compound nuclear state $^{32}$S* which can decay via the following canals:
3.1. **Generalities**

\[ ^{31}\text{S} + n \quad (Q = 1.45 \, \text{MeV}) \]

\[ ^{31}\text{P} + p \quad (7.68 \, \text{MeV}) \]

\[ ^{30}\text{P} + d \quad (-2.41 \, \text{MeV}) \]

\[ ^{28}\text{Si} + \alpha \quad (9.59 \, \text{MeV}) \]

The most important canals are the \( p \) and \( \alpha \) ones (the \( d \) canal can become important at high temperature when this endoenergetic canal is open). The other reactions, only important for nucleosynthesis, are:

\[
^{31}\text{P} \left\{ \begin{array}{c}
(\gamma, p)^{30}\text{Si} \\
(p, \gamma)^{32}\text{S} \\
(p, \alpha)^{28}\text{Si}(\alpha, \gamma)^{32}\text{S}
\end{array} \right.
\]

\[ ^{28}\text{Si}(\gamma, \alpha)^{24}\text{Mg}(\alpha, p)^{27}\text{Al}(\alpha, p)^{30}\text{Si} \]

\[ ^{32}\text{S}(n, \gamma)^{33}\text{S}(n, \alpha)^{30}\text{Si}(\alpha, \gamma)^{34}\text{S} \]

\[ ^{28}\text{Si}(n, \gamma)^{29}\text{Si} \left\{ \begin{array}{c}
(\alpha, n)^{32}\text{S}(\alpha, p)^{35}\text{Cl} \\
(p, \gamma)^{30}\text{P}(\beta^+)^{30}\text{Si} \\
(n, \gamma)^{30}\text{Si}
\end{array} \right.
\]

\[ ^{31}\text{S}(\gamma, p)^{30}\text{P} \]

- electron captures:

\[ ^{33}\text{S}(e^-, \bar{\nu}_e)^{33}\text{P}(p, n)^{33}\text{S} \]

\[ ^{35}\text{Cl}(e^-, \bar{\nu}_e)^{35}\text{S}(p, n)^{35}\text{Cl} \]

- In massive stars \((M \simeq 40 \, M_\odot)\):

\[ ^{32}\text{S}(\alpha, \gamma)^{36}\text{Ar} \left\{ \begin{array}{c}
(\alpha, p)^{39}\text{K} \\
(\gamma, p)^{35}\text{Cl} \\
(n, \gamma)^{37}\text{Ar} \left\{ (n, \alpha)^{34}\text{S} (\beta^+)^{37}\text{Cl} \right. \right. \right. \]
\[
35\text{Cl} \left\{ \begin{array}{l}
(e^-, \bar{\nu}_e)^{35}\text{S}(\gamma, n)^{34}\text{S} \\
(\gamma, p)^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}
\end{array} \right\} \left\{ \begin{array}{l}
(p, \gamma)^{39}\text{K}(p, \gamma)^{40}\text{Ca} \\
(\alpha, \gamma)^{46}\text{Ti} \\
(\alpha, p)^{45}\text{Sc}(p, \gamma)^{46}\text{Ti}
\end{array} \right\}
\]

- In lower mass stars \((M \simeq 15 M_\odot)\):

\[31\text{S}(e^-, \bar{\nu}_e)^{31}\text{P}, \quad 31\text{P}(n, \gamma)^{32}\text{P}, \]

\[32\text{S}(e^-, \bar{\nu}_e)^{32}\text{P}(p, n)^{32}\text{S} \quad \text{and} \quad 33\text{P}(p, \alpha)^{30}\text{Si}.\]

It can be seen in the list of reactions above, that massive stars (relatively high temperature) tend to produce heavier nuclei. For lower mass stars (lower temperature and higher density), electron captures are more important (due to a higher density) and lead to greater neutron excess, \(\eta\), increase. Indeed, during oxygen burning, \(\eta\) increases due to the different electron captures caused by the high density. The main products of O–burning are \(^{28}\text{Si}, \quad ^{32}\text{S}\) and some further multiples of \(\alpha\)–particles in the case of very massive stars. Two remarks can be made on this burning stage:

- The elements produced by the \(s\)–process during previous burning phases, and especially He–burning, are photodisintegrated during O–burning into isotopes of the iron group.

- There is formation of small Quasi Statistical Equilibrium, QSE, clusters which, along the stage, become larger and larger. At the end of O–burning, it extends between silicon and scandium.

Finally, the elements to add to the nuclear network listed at the end of the previous sections are: \(^{30,31,32,33}\text{P}, \quad ^{31,33,34,35}\text{S}, \quad ^{35,37}\text{Cl}, \quad ^{36,37,38}\text{Ar}, \quad ^{39}\text{K}, \quad ^{40,42}\text{Ca}, \quad ^{45}\text{Sc} \quad \text{and} \quad ^{44,46}\text{Ti}.

### 3.1.4 Silicon burning and NSE

Silicon burning resembles Ne–burning in the sense that it includes a combination of photodisintegrations and \(\alpha\)–captures which eventually transform silicon group elements into iron group elements (instead of two \(^{28}\text{Si}\) atoms merging and producing one atom of \(^{56}\text{Fe}\)). The number of nuclei involved increases and includes all stable and mildly unstable isotopes between silicon and zinc. Both strong and weak reactions are important in this phase. The QSE group around silicon, formed at the end of O–burning, extends up to scandium or titanium and another QSE group forms around iron and extends down to chromium. The two QSE groups are linked by several reactions which are not at equilibrium with their reverse reaction until the Nuclear Statistical Equilibrium phase, NSE, just before the collapse. The group of elements linking the two QSE clusters is called the bottleneck and includes calcium, scandium and titanium isotopes.
3.1. Generalities

When NSE is established, all reaction rates are equal to their reverse reaction rates and the abundance of the elements is determined by the nuclear equilibrium values which depend on the temperature, the density and the neutron richness.

Even though nucleosynthesis involves many isotopes, Hix et al. (1998) and Timmes et al. (2000) show that the energy production comes mainly from $\alpha$–captures and that a network including only the multiple $\alpha$–elements between $^{28}\text{Si}$ and $^{56}\text{Ni}$ is sufficient to follow the energy production. Note that for nucleosynthesis itself, assuming QSE for many elements can also simplify greatly the calculation.

3.1.5 Network sizes and applications

From the description above, one can see that the minimum network must at least include $^{4}\text{He}$ and the multiple $\alpha$–elements between $^{12}\text{C}$ and $^{56}\text{Ni}$. This network allows to follow the energy generation as well as the evolution of the main isotopes. However, it does not allow to model the nucleosynthesis of minor elements or the neutron excess. As said in the introduction, neutron excess is not important for the structure evolution and the energy production before core Si–burning (Thielemann & Arnett 1985). Even during core Si–burning, its main effect on the structure is a mean molecular weight gradient which prevents the growth of the core if one uses Ledoux criterion for convection. Nevertheless, neutron excess is important for the nucleosynthesis of neutron rich elements (s and r–processes) and the star collapse. Electron captures during Si–burning increases neutron excess and also reduces the electron pressure and this (with photodisintegration) will allow the core to collapse (Woosley et al. 2002). In order to follow the nucleosynthesis of minor elements and the neutron excess, a network of about 50 elements (see the list of the elements at the end of each burning stage) is necessary assuming QSE during Si–burning. There is no maximum size since the larger the network, the more element abundances can be derived. Nowadays, networks include up to 200–300 isotopes and thousands of reactions (Limongi & Chieffi 2003; Rauscher et al. 2002).

The only upper limit for the size of the network is the time that one wants to spend on the calculations. The total calculation time spent in the present version of the Geneva code is the following: the time spent for one zone, multiplied by the number of zones (about 500), multiplied by the number of iterations of the Henyey method (at least five), multiplied by the number of time steps (about 100 000). Therefore if one model is calculated for one month (2.6 million seconds), the time spent for one zone is one hundredth of a second.

At the beginning of the thesis, it was planned to merge a large nuclear network program with the Geneva code. The large nuclear network code, Hydroburn was kindly provided by Stéphane Goriely (Université Libre de Bruxelles) whom I warmly thank. Unfortunately, after several months, I was still not able to reduce the calculation time to the desired one hundredth of a second. Possible reasons for this failure are that the network code was designed to be used for post-processing with a large network and included a sophisticated numerical method called backward differentiation formula.
(BDF) or that I was not able to use it efficiently enough. After this drawback, I took the opposite approach: the minimal network. The network will be described in Sect. 3.4.

3.2 Thermonuclear reactions and reaction network

A general summary about thermonuclear reactions can be found in the two following references Maeder (2000); Clayton (1968). Here we will only recall what is necessary for this work.

3.2.1 Thermonuclear reaction rates

The main inputs of nuclear reaction networks are the thermonuclear reaction rates. Thermonuclear reactions can be divided in three main categories:

1. Decays, photodisintegrations, electron and positron captures and neutrino induced reactions (one massive particle involved)

2. Two–particles reactions

3. Three–particles reactions

In the second category, a reaction can be represented by:

\[ a + X \rightarrow b + Y \]

in which a particle \( a \) strikes a nucleus \( X \) and they are transformed into a new nucleus \( Y \) and a particle \( b \). In this case and assuming that \( a \) and \( X \) are gases with a uniform density, the reaction rate, \( r \), is given by:

\[ r = (N_a v)(\sigma(v) N_X) \]  

(3.1)

where \( \sigma(v) \) is the nuclear cross section and is defined by:

\[ \sigma(v) = \frac{\text{number of reactions/target } X/\text{unit time}}{\text{flux of incoming particles } a} = \frac{r/N_X}{N_a v} \]

with \( N_a \) and \( N_X \) being the number densities of elements \( a \) and \( X \) respectively and \( v \) being the relative velocity of \( a \) and \( X \) assuming that the two gases have a uniform relative motion. Obviously, this assumption is not verified. In stellar interiors, except for neutron stars, the nuclei are in a non–degenerate configuration. Therefore, in the astrophysical plasma, which is in thermodynamic equilibrium, the relative velocities follow
3.2. Thermoneutral reactions and reaction network

Maxwell-Boltzmann distribution of speed. Thus Eq. (3.1) becomes:

\[ r_{a,X} = N_a N_X \langle \sigma v \rangle = N_a N_X \frac{(8/\pi)^{1/2}}{\mu^{1/2} (k_B T)^{3/2}} \int_0^\infty E \sigma(E) \exp(-E / k_B T) dE \]  

(3.2)

In this equation, \( \mu \) represents the reduced mass of the system \((a, X)\) and the velocity distribution is replaced by an energy distribution. An important factor, \( f \text{screening} \), can be added in Eq. 3.2 to take into account the electron screening which occurs at high densities and which reduces the Coulomb barrier between the two interacting nuclei (see Sect. 3.2.2). Another factor is added in case the two interacting particles are the same in order to avoid double counting of the particles and is equal to \((1 + \delta_{aX})^{-1}\). The general form of a reaction rate, for a two particle reaction, is then:

\[ r_{a,X} = \frac{N_a N_X}{1 + \delta_{aX}} f \text{screening} \langle \sigma v \rangle_{aX} \]  

(3.3)

Let's go back now to the first category and more particularly to the case of photodisintegration. In this situation, particle \( a \) is a photon and therefore the relative velocity is always \( c \). Thus it simplifies the equation for the reaction rate to \( r = \lambda_{\gamma,X} N_X \) (see Thielemann et al. 1998). The photons follow Planck distribution at temperature \( T \) and an expression can be obtained for \( \lambda_{\gamma,X} \). However, this calculation does not need to be done since this rate can be expressed using the cross section of the inverse reaction and other factors like the reaction Q-value, the temperature and partition functions. This technique is used in the program Hydroburn. A similar technique can be used for electron and positron captures as well as for neutrino captures (see Thielemann et al. 1998, for references concerning these techniques). For decays, either experimental measurements have to be gathered or models like the so-called "Quasi-Particle Random-Phase Approximation" (QRPA) have to be used. This model can be used in conjunction with the FRDM and ETFSI models in order to do systematic calculations of nuclear masses.

In the third category, the only important reaction in later burning stages is the triple-\( \alpha \) process. In this case, as the main production and destruction channel of \( ^8\text{Be} \) is the \( 2\alpha \) one, one can use an equation of the Saha equation type in order to determine the small equilibrium abundance of \( ^8\text{Be} \) which serves as a target for the capture of a third \( \alpha \)-particle. Therefore it is noted that most reaction rates, except for some decays, can be expressed with the aid of the reaction rates of the reactions of the second category, i.e. the two particle reactions and from now on, only such reactions will be treated.

In order to calculate the reaction rate using equation (3.2), \( \sigma(E) \) still needs to be determined. There are two main kinds of contributions to \( \sigma(E) \), the non-resonant and resonant contributions. As it is not the scope of this project to study the details of the reaction rate calculations, the reader can look at the two references for more details on reaction rates calculation Maeder (2000); Clayton (1968).
3.2.2 Electron shielding or screening

As said above, an important factor can be added in Eq. 3.2 to take into account the presence of the electron gas around the ions. In the Geneva code, the method of Grabske et al. (1973) is used. In this scheme, the screening factor, $f_{\text{screening}}$, is given by the following formula (Grabske et al. 1973):

$$f_{\text{screening}} = \exp(k_b \eta_b \zeta_b \Lambda_0), \quad \Lambda_0 = 1.88 \cdot 10^8 (\rho/\mu_1 T^3)^{1/2} \quad (3.4)$$

where $k_b$ and $b$ are numerical parameters of the order of unity and $\eta_b$ and $\zeta_b$ are functions of the electric charge, $Z$, of the reacting elements. $\rho$ is the density, $\mu_1$ is the mean molecular weight of ions and $T$ is the temperature. The strength of the screening can be weak (usual case in astrophysics), intermediate or strong. The values of the four parameters, $k_b$, $\eta_b$, $\zeta_b$ and $b$, are given for each strength of the screening in Table 4 in Grabske et al. (1973). For example, in the case of weak electron screening between two nuclei of charge numbers $Z_1$ and $Z_2$ respectively, the formula for the screening factor becomes:

$$f_{\text{screening}} = \exp(1/2 \tilde{Z} Z_1 Z_2 \Lambda_0), \quad (3.5)$$

where $\tilde{Z}$ is the RMS charge average (see Dewitt et al. 1973, for the detailed definition).

3.2.3 Energy production rates

Elements between hydrogen and iron have a decreasing binding energy, BE, with increasing mass number. Elements heavier than iron have a slightly increasing BE with increasing mass number. The most stable elements therefore belong to the iron group. The nuclear reactions described in Sect. 3.1 transform ions (starting with carbon) into heavier ones (ending with iron group elements). They therefore release BE. The fact that iron group elements are the most stable explains why, once the iron core is formed after Si-burning, no more BE can be released and the core collapses (with the help of neutrinos and photodisintegrations and electron captures). The total energy released corresponds to the $Q$–values (given in the brackets next to the main reactions in Sect. 3.1) and is carried away by photons, $Q_\gamma$, and neutrinos, $Q_\nu$. Neutrinos escape freely while photons interact with the environment and heats it up. The effective energy transmitted to the environment is therefore the energy carried away by photons, $Q_\gamma$. This is why, on the NACRE website (http://pntpm.ulb.ac.be/nacre.html), both $Q_\gamma$ and $Q_\nu$ are given separately. Note that some reactions can have negative $Q$–values like $^{20}\text{Ne}(\gamma, \alpha)^{16}\text{O}$.

There are two ways to calculate the total nuclear energy production rate in the models. The first way is to multiply each reaction rate by the effective energy produced in the reaction, $Q_\gamma$, and sum over all the reactions:

$$\epsilon = \sum_i r^i Q_{\gamma}^i \quad (3.6)$$
3.2. Thermonuclear reactions and reaction network

where \( r^i \) is the reaction rate of reaction \( i \) and \( Q_i^j \) is its effective energy production. The advantage of this method is that it is easy to calculate the jacobian terms of the Henyey method \((\partial \epsilon / \partial \rho, \partial \epsilon / \partial T)\). The disadvantage of this method is that small time steps must be used to ensure that the energy production rates are constant over a time step \( \Delta t \).

The other way is to multiply the abundance variation of each element by its atomic mass excess and sum over all the chemical elements (Thielemann et al. 1998):

\[
\epsilon = - \sum_i \dot{Y}_i N_A \Delta M_i
\]  

(3.7)

where \( \dot{Y}_i = dY_i/dt \) is the abundance variation of element \( i \), \( Y_i = n_i/(\rho N_A) \), \( n_i \) is the number of species \( i \) [cm\(^{-3}\)], \( \rho \) is the density [g cm\(^{-3}\)] and \( N_A \) is Avogadro’s number. \( \Delta M_i \) is the atomic mass excess of element \( i \) in units of energy:

\[
\Delta M_i = (M_i [\text{amu}] - A_i) M_u c^2 = 931.478 (M_i [\text{amu}] - A_i) \quad [\text{MeV}]
\]

where \( M_u \) is the atomic mass unit (amu), defined as one-twelfth of the mass of one neutral \(^{12}\)C atom, \( M_i \) is the mass of ion \( i \) in atomic mass unit and \( A_i \) is its mass number. This method has the advantage that the nuclear energy produced is exactly equal to the effective energy released by the transformation of the elements. The two disadvantages are that the neutrino losses are to be computed separately and that the jacobian terms of the Henyey method are much harder to calculate in the Geneva code.

The first method is used at the present time in the Geneva code, especially for the convenience of the calculation of the jacobian terms for the Henyey method. This is done using a standard routine from Numerical Recipes, called \( e02acf \), which allows to calculate the derivatives of the reaction rates as a function of the temperature. The numerical derivatives are used because the NACRE compilation (Angulo et al. 1999) has been fitted numerically and therefore the numerical tables are better than the analytical fit provided. The derivatives of the reaction rates as a function of the density are straightforward to obtain. Small time steps are used to ensure that the energy generation is constant over a time step. Note that the nuclear energy production rate in the Henyey method is in [erg g\(^{-1}\) s\(^{-1}\)] and the suitable conversion factors (like MeV to erg) must be added to the expressions given in Eqs. 3.6 and 3.7 when necessary.

3.2.4 Nuclear reaction network

As said earlier, thermonuclear reactions can be divided in three main categories:

1. Decays, photodisintegrations, electron and positron captures and neutrino induced reactions \((\rightarrow \lambda_j)\)
2. Two–particles reactions \((\rightarrow N_A \langle \sigma v \rangle_{j,k})\)
3. Three–particles reactions \((\rightarrow N_A^2 \langle \sigma v \rangle_{j,k,l})\)
CHAPTER 3. NUCLEAR REACTION NETWORK

In the brackets, the expressions (see also above) corresponding to the reaction rates given in the literature are given.

These reactions can create or burn elements and therefore the following equation is the usual form obtained for the equations of the abundance evolution of element \( i \) (Thielemann et al. 1998):

\[
\frac{dY_i}{dt} = \sum_j N^i_j \lambda_j Y_j + \sum_{j,k} N^i_{j,k} f_{\text{screening}} \rho N_A (\sigma v)_{j,k} Y_j Y_k + \sum_{j,k,l} N^i_{j,k,l} f_{\text{screening}} \rho^2 N_A^2 (\sigma v)_{j,k,l} Y_j Y_k Y_l
\]  

(3.8)

where \( Y_i = X_i / A_i \), \( X_i \) being the mass fraction and \( A_i \) the atomic mass number \([g \text{ mole}^{-1}]\) is the abundance in unit \([\text{mole g}^{-1}]\). Therefore the unit of these equations is \([\text{mole g}^{-1} \text{ s}^{-1}]\). The numerical factors are determined in the following way:

\( N^i_j = N_i \),

\( N^i_{j,k} = N_i / (|N_j||N_k|!|) \),

\( N^i_{j,k,l} = N_i / (|N_j||N_k|!|N_l|!) \),

where the numbers, \( N_m \), are positive or negative and specify the number of isotopes \( m \) created (positive) or destroyed in the considered reaction.

As an example, we can look at the variation of the abundance of \( \alpha \), \( ^{12}\text{C} \) and \( ^{20}\text{Ne} \) due to the reaction \( ^{12}\text{C}(^{12}\text{C}, \alpha) ^{20}\text{Ne} \):  

\[
\frac{dY_\alpha}{dt} = \frac{1}{| -2|!0!} [12, 12]_\alpha \ Y_{^{12}\text{C}}^2 = \frac{1}{2} [12, 12]_\alpha \ Y_{^{12}\text{C}}^2
\]  

(3.9)

\[
\frac{dY_{^{12}\text{C}}}{dt} = \frac{-2}{| -2|!0!} [12, 12]_\alpha \ Y_{^{12}\text{C}}^2 = -[12, 12]_\alpha \ Y_{^{12}\text{C}}^2
\]  

(3.10)

\[
\frac{dY_{^{20}\text{Ne}}}{dt} = \frac{1}{| -2|!0!} [12, 12]_\alpha \ Y_{^{12}\text{C}}^2 = \frac{1}{2} [12, 12]_\alpha \ Y_{^{12}\text{C}}^2
\]  

(3.11)

where \([12, 12]_\alpha = \rho N_A (f_{\text{screening}} (\sigma v))_{^{12}\text{C}(^{12}\text{C}, \alpha) ^{20}\text{Ne}}\). In order to verify that the equations are written correctly, the test of the sum of the mass fraction can be applied. Indeed, since \( \sum_i X_i = 1 \), it implies that \( \sum_i dX_i / dt = 0 \). Using \( X_i = A_i Y_i \), this test is easily verified for the example given above \((4/2-12+20/2-0)\).

3.3 Numerical method

As for the structure equations, there are several numerical methods to solve the set of differential equations of the abundances of the chemical species. The most popular one is a Newton–Raphson like method implicit in time (see for example Thielemann et al. 1998). These methods are stable (implicit in time), precise (thanks to the iterative process), supple and fast. In the code Hydroburn, a more sophisticated method, called BDF (see Sect. 3.1.5), is used for post-processing. It is also stable and precise but less supple and probably too heavy to be implemented in a stellar evolution code. The
3.3. Numerical method

The fastest method is the linearisation of the equations followed by a matrix inversion, method devised by Arnett & Truran (1969). This last method is fast, stable (if implicit in time) and precise if small enough time steps are used. The Arnett & Truran (1969) method is used in the Geneva code.

In order to linearise the equations and keep them implicit in time, the following approximations are used (Arnett & Truran 1969):

1. For the second order terms:

\[
Y_{i,j}^{n+1} \approx Y_{i,j}^{n+1} - Y_{i,j}^{n-1} + \Delta t (Y_{i,j}^{n+1} - Y_{i,j}^{n-1}) + Y_{i,j}^{n+1} - Y_{i,j}^{n} + O(\Delta Y^2)
\]

2. For the third order terms:

\[
Y_{i,j}^{n+1} Y_{k,i}^{n+1} \approx Y_{i,j}^{n+1} Y_{k,i}^{n+1} + Y_{i,j}^{n+1} Y_{i,k}^{n+1} + Y_{i,j}^{n+1} Y_{k,j}^{n+1} - 2 Y_{i,j}^{n+1} Y_{k,j}^{n} + O(\Delta Y^2)
\]

3. For the time derivative:

\[
\frac{dY_i}{dt} \approx \frac{Y_{i}^{n+1} - Y_{i}^{n}}{\Delta t}
\]

where superscripts \( n \) and \( n + 1 \) correspond to times \( t_n \) and \( t_{n+1} \) respectively and \( \Delta t = t_{n+1} - t_n \), is the time step. Once this is done, we write a matrix equation of the form \( A \vec{Y}^{n+1} = \vec{B} \) and we obtain the new set of abundances \( \vec{Y}^{n+1} \) by multiplying \( \vec{B} \) by the inverse of matrix \( A \). The second order terms, \( O(\Delta Y^2) \) are neglected and the method is precise if the variation of the abundance of the elements is small (\( \Delta Y_i^2 << 1, \Delta Y_i = Y_i^{n+1} - Y_i^n \)). Note that the approximation in time (number three) demands anyway that the variation of the abundance of the elements be small (see Arnett 1996, for more details). Small enough time steps are used (on average 10000 time steps per burning stage) to ensure that the method was precise. During the calculations, it was verified that the sum of all the mass fractions stays equal to one. At the end of Silicon burning the accuracy is of the order of \( 10^{-5} \) or less.

A test was done to compare the speed of our advanced stages network (one thousandth of a second per zone) with the \textit{a7net} (Hix et al. 1998) network kindly provided by Raphael Hix whom I warmly thank. It was found that our network was ten times faster. Note however that \textit{a7net} is a Newton–Raphson method and has other advantages.

Coming back to the example above, the Arnett & Truran (1969) method applies to the reaction \( ^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne} \) for \( ^{12}\text{C} \) in the following way:

\[
\frac{Y_{12c}^{n+1} - Y_{12c}^{n}}{\Delta t} = -[12, 12]_\alpha \left( 2Y_{12c}^{n+1} Y_{12c}^{n} - (Y_{12c}^{n})^2 \right)
\]

which becomes, in the form \( A \vec{Y}^{n+1} = \vec{B} \):

\[
(1/\Delta t + 2 [12, 12]_\alpha Y_{12c}^{n}) Y_{12c}^{n+1} = Y_{12c}^{n}/\Delta t + [12, 12]_\alpha (Y_{12c}^{n})^2
\]
CHAPTER 3. NUCLEAR REACTION NETWORK

For $^{20}$Ne, the equation, in the form $A Y_{n+1} = \bar{B}$, is:

$$
1/\Delta t Y_{20Ne}^{n+1} - [12, 12]_\alpha Y_{12C}^{n} Y_{12C}^{n+1} = Y_{20Ne}^{n} / \Delta t - 1/2 [12, 12]_\alpha (Y_{12C}^{n})^2
$$

(3.17)

For $\alpha$, the equation is:

$$
1/\Delta t Y_{\alpha}^{n+1} - [12, 12]_\alpha Y_{12C}^{n} Y_{12C}^{n+1} = Y_{\alpha}^{n} / \Delta t - 1/2 [12, 12]_\alpha (Y_{12C}^{n})^2
$$

(3.18)

In order to verify that the equations are written correctly, the following test can be used (since $\sum_i dX_i / dt = 0$): $\sum_{i,j} A_i A_j Y_i^n - B_i = 0$ where $Y_j^n$ is used in the equation instead of $Y_j^{n+1}$. This test is verified in the example.

3.4 Network used in the advanced stages

Using the numerical method described in the previous section, an adaptive nuclear network was built. It is adaptive in the sense that the number of species and reactions included in the network can easily be changed. The list of species (and their initial abundance) and the list of reactions (and their reaction rates in a tabular form taken from the NACRE website, http://pntpm.ulg.ac.be/nacre.htm) to be included in the network are read from input files at the start of a calculation. The program then automatically calculates the matrix equation $A Y_{n+1} = \bar{B}$ and calculates the new abundances $Y_{n+1}$.

At the time when the reaction rates are calculated, the energy production rates and their derivatives with respect to the temperature and the density are also calculated from each individual reaction for C, Ne and O–burning stages. During Si–burning, two quasi–equilibrium groups form around $^{28}$Si and $^{56}$Ni respectively. Hix et al. (1998) therefore only follow explicitly the reactions between $^{44}$Ti and $^{48}$Cr and assume nuclear statistical equilibrium between the other elements heavier than $^{28}$Si. They choose the reaction between $^{44}$Ti and $^{48}$Cr because it is the link (in the chain of the multiple–alpha elements chain) in the bottleneck between the two quasi–equilibrium groups. For the energy production, the method of Hix et al. (1998) is followed during Si–burning. Therefore, only the reaction rate between $^{44}$Ti and $^{48}$Cr is considered and multiplied by the energy produced by the transformation of $^{28}$Si into $^{56}$Ni.

Even though the network is adaptive, the programs have so far only been made with a restricted number of species during the advanced stages for practical reasons. The list of elements followed during C, Ne, O and Si–burnings is: $\alpha$, $^{12}$C, $^{16}$O, $^{20}$Ne, $^{24}$Mg, $^{28}$Si, $^{32}$S, $^{36}$Ar, $^{40}$Ca, $^{44}$Ti, $^{48}$Cr, $^{52}$Fe and $^{56}$Ni. It is basically all the multiple–$\alpha$ elements except $^{8}$Be. It is rather small but it is enough to reasonably follow the energy production and the evolution of the main element abundances (see Timmes et al. 2000; Hix et al. 1998). During H and He–burning, the list of elements followed is: $^1$H, $^3$He, $^4$He, $^{12}$C, $^{13}$C, $^{14}$N, $^{15}$N, $^{16}$O, $^{17}$O, $^{18}$O, $^{20}$Ne, $^{22}$Ne, $^{24}$Mg, $^{25}$Mg and $^{26}$Mg.

The initial composition of the models is given in Table 3.1. For a given metallicity Z (in mass fraction), the initial helium mass fraction, $Y_e$, is given by the relation $Y = Y_p + \Delta Y / \Delta Z \cdot Z$, where $Y_p$ is the primordial helium abundance and $\Delta Y / \Delta Z$ the slope of the helium–to–metal enrichment law. The same values as in Maeder & Meynet (2001)
Table 3.1: Initial abundance (in mass fraction) of the chemical elements

<table>
<thead>
<tr>
<th>Element</th>
<th>Mass fraction</th>
<th>Element</th>
<th>Mass fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1$H</td>
<td>0.705</td>
<td>$^{24}$Mg</td>
<td>5.861D-4</td>
</tr>
<tr>
<td>$^3$He</td>
<td>2.915D-5</td>
<td>$^{25}$Mg</td>
<td>7.70D-5</td>
</tr>
<tr>
<td>$^4$He</td>
<td>0.275</td>
<td>$^{26}$Mg</td>
<td>8.84D-5</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>3.4245D-3</td>
<td>$^{28}$Si</td>
<td>6.5301D-4</td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>4.12D-5</td>
<td>$^{32}$S</td>
<td>3.9581D-4</td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>1.0589D-3</td>
<td>$^{36}$Ar</td>
<td>7.7402D-5</td>
</tr>
<tr>
<td>$^{15}$N</td>
<td>4.1D-6</td>
<td>$^{40}$Ca</td>
<td>5.9898D-5</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>9.6195D-3</td>
<td>$^{44}$Ti</td>
<td>0</td>
</tr>
<tr>
<td>$^{17}$O</td>
<td>3.9D-6</td>
<td>$^{48}$Cr</td>
<td>0</td>
</tr>
<tr>
<td>$^{18}$O</td>
<td>2.21D-5</td>
<td>$^{52}$Fe</td>
<td>0</td>
</tr>
<tr>
<td>$^{20}$Ne</td>
<td>1.8222D-3</td>
<td>$^{56}$Ni</td>
<td>0</td>
</tr>
<tr>
<td>$^{22}$Ne</td>
<td>1.466D-4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

are used i.e. $Y_0 = 0.23$ and $\Delta Y/\Delta Z = 2.25$. For the solar metallicity, $Z = 0.02$, one thus has $X = 0.705$ and $Y = 0.275$. For the mixture of the heavy elements, the same mixture as the one used to compute the opacity tables for solar composition are adopted. For elements heavier than Mg, the values from Anders & Grevesse (1989) are used.

The nuclear reactions included in the model during the advanced stages are listed in Table 3.2. The reactions taken into account explicitly (column one of Table 3.2) are all the reactions linking directly the elements of the network used in the advanced stages. Concerning the reactions taken into account implicitly (column two of Table 3.2), they can be divided into two groups:

- The various reactions ($\alpha, p$) have been added to the ($\alpha, \gamma$) rates. This assumes that an instantaneous proton capture is assumed (for example $^{35}$Cl($p, \gamma)^{36}$Ar). It allows to include in our restricted network the flow of reactions which are not directly following the multiple-$\alpha$ elements chain.

- The other reactions concern the $p$-decay canal of the main reaction of the different burning stages described in Sect. 3.1. For the compound elements ($^{24}$Mg* for C-burning and $^{32}$S* for O-burning), the most probable decay canals are the $\alpha$ and $p$ ones. The $\alpha$-decays are explicitly taken into account in our network. The $p$-decays are not. However, the $p$-decays produce elements ($^{23}$Na for C-burning and $^{31}$P for O-burning) which quickly capture a proton. By comparing the ($p, \gamma$) and ($p, \alpha$) rates for these elements, one can see that the ($p, \alpha$) is usually at least ten times more important. It is therefore justified to add the $p$-decay rates to the $\alpha$-decay ones.
Table 3.2: Reactions included in the network for C, Ne, O and Si–burnings. The first column lists the reactions which are explicitly taken into account in the network. The second column lists the reactions that are implicitly taken into account by adding their reaction rate to the rate of the reaction on their left. For example, the reaction rate of $^{32}\text{S}(\alpha, p)^{35}\text{Cl}$ was added to the rate of $^{32}\text{S}(\alpha, \gamma)^{36}\text{Ar}$.

<table>
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<tr>
<th>&quot;Explicit&quot; reactions</th>
<th>&quot;Implicit&quot; reactions</th>
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<tr>
<td>$^{4}\text{He}(2\alpha, \gamma)^{12}\text{C}$</td>
<td>$^{4}\text{He}(2\alpha, p)^{11}\text{B}$</td>
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<tr>
<td>$^{12}\text{C}(\gamma, 2\alpha)^{4}\text{He}$</td>
<td>$^{12}\text{C}(^{12}\text{C}, p)^{23}\text{Na}$</td>
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<tr>
<td>$^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}$</td>
<td>$^{16}\text{O}(^{12}\text{C}, p)^{27}\text{Al}$</td>
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<tr>
<td>$^{16}\text{O}(^{12}\text{C}, \alpha)^{24}\text{Mg}$</td>
<td>$^{20}\text{Ne}(^{12}\text{C}, p)^{31}\text{P}$</td>
</tr>
<tr>
<td>$^{20}\text{Ne}(^{12}\text{C}, \alpha)^{28}\text{Si}$</td>
<td>$^{16}\text{O}(^{16}\text{O}, p)^{31}\text{P}$</td>
</tr>
<tr>
<td>$^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$</td>
<td>$^{12}\text{C}(\alpha, p)^{15}\text{N}$</td>
</tr>
<tr>
<td>$^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$</td>
<td>$^{16}\text{O}(\alpha, p)^{19}\text{F}$</td>
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<tr>
<td>$^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$</td>
<td>$^{20}\text{Ne}(\alpha, p)^{23}\text{Na}$</td>
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<tr>
<td>$^{24}\text{Mg}(\gamma, \alpha)^{20}\text{Ne}$</td>
<td>$^{24}\text{Mg}(\alpha, p)^{27}\text{Al}$</td>
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<td>$^{28}\text{Si}(\alpha, p)^{31}\text{P}$</td>
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<td>$^{28}\text{Si}(\gamma, \alpha)^{24}\text{Mg}$</td>
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</tr>
<tr>
<td>$^{32}\text{S}(\gamma, \alpha)^{28}\text{Si}$</td>
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<td>$^{32}\text{S}(\alpha, \gamma)^{36}\text{Ar}$</td>
<td>$^{32}\text{S}(\alpha, p)^{35}\text{Cl}$</td>
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<td>$^{36}\text{Ar}(\alpha, p)^{39}\text{K}$</td>
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<td>$^{40}\text{Ca}(\gamma, \alpha)^{36}\text{Ar}$</td>
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</tr>
<tr>
<td>$^{40}\text{Ca}(\alpha, \gamma)^{44}\text{Ti}$</td>
<td>$^{40}\text{Ca}(\alpha, p)^{43}\text{Sc}$</td>
</tr>
<tr>
<td>$^{44}\text{Ti}(\gamma, \alpha)^{40}\text{Ca}$</td>
<td></td>
</tr>
<tr>
<td>$^{44}\text{Ti}(\alpha, \gamma)^{48}\text{Cr}$</td>
<td>$^{44}\text{Ti}(\alpha, p)^{47}\text{V}$</td>
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<td>$^{48}\text{Cr}(\gamma, \alpha)^{44}\text{Ti}$</td>
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</tr>
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<td>$^{48}\text{Cr}(\alpha, \gamma)^{52}\text{Fe}$</td>
<td>$^{48}\text{Cr}(\alpha, p)^{51}\text{Mn}$</td>
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<td>$^{52}\text{Fe}(\gamma, \alpha)^{48}\text{Cr}$</td>
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<td>$^{52}\text{Fe}(\alpha, \gamma)^{56}\text{Ni}$</td>
<td>$^{52}\text{Fe}(\alpha, p)^{55}\text{Co}$</td>
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<tr>
<td>$^{56}\text{Ni}(\gamma, \alpha)^{52}\text{Fe}$</td>
<td></td>
</tr>
</tbody>
</table>

45
3.5. Interaction between nuclear reactions and the rest of the code

Note that the implicit inclusion of reaction rates used here is very similar to the method used in Hix et al. (1998) and Timmes et al. (2000).

The reaction rates are taken from the NACRE compilation (Angulo et al. 1999) for the early stages and from the NACRE website (http://pntpm.ulb.ac.be/nacre.htm) for the advanced stages.

Note that the network for the advanced stages is small at the present time. It is nevertheless very instructive to do the calculations with a small network because the calculations are very fast and it still provides the correct energy production and the evolution of the main elements. It is therefore very useful to use such a small network in the development phase of a code. It is planned to use a larger network in the near future to study the nucleosynthesis of minor elements and the evolution of the neutron excess.

3.5 Interaction between nuclear reactions and the rest of the code

In order to include the new network for the advanced stages, two new subroutines were added to the code (netrates.f and netburn.f). In addition, 23 other subroutines had to be modified, mainly to take into account the abundance of the new elements. For example, the new elements had to be included in the calculation of the mean molecular weight.

3.5.1 Mixing and burning

The physical process interacting or disturbing nuclear processes the most is mixing. Indeed, in a convective zone, the medium is turbulent and the whole zone is mixed on a short time scale. Ideally, these two processes should be treated simultaneously by solving Eq. 2.3 in the whole star at once (like for the structure equations). This is unfortunately very costly numerically. The computers are just starting to be fast enough to treat simultaneously mixing and burning for small networks. In practice, the two processes are decoupled.

In order to understand the best way to treat these two processes, the following time scales are to be looked at:

- Evolution time scales: \( \tau_{\text{burn}} \) is the burning stage duration and \( \Delta t \) is the time step.

- Mixing time scales: \( \tau_{\text{mix}}(r) = r^2/D_{\text{conv}} \) is the mixing time scale over a distance \( r \) and \( D_{\text{conv}} \) is the convective diffusion coefficient (usually of the order of \( 10^{14} \) cm s\(^{-1}\); other mixing processes have smaller diffusion coefficients). \( r \) can be the size of the convective core (or shell), the mixing length or the distance between two mesh points \( \Delta r \).

- Nuclear time scales (or lifetimes): \( \tau_{\text{nucl}} \) is the nuclear lifetime of an element determined by the fastest destructive reaction rate. The nuclear lifetime is different
for each chemical element.

For a 20 $M_\odot$ star, the burning lifetimes are about 10 million years for H–burning, 1 million years for He–burning, 300 years for C–burning, 0.1 to one year for both Ne and O–burnings and 10 to 30 hours for Si–burning.

The time steps, $\Delta t$, are chosen to be 1000 to 10000 times smaller than the burning lifetimes, $\tau_{\text{burn}}$, at every stages. This is to ensure that the linearised method is precise. The burning lifetimes, $\tau_{\text{burn}}$, correspond to the nuclear lifetimes of the main elements $\tau_{\text{nucl}}^{\text{main el}}$: taking part in the burning stages. Other elements, which are not affected by the burning, have a much longer nuclear lifetime. Unstable particles, like $p$, $n$ and $\alpha$ in the advanced stages, have a much shorter lifetime than the main products. They may also be minor elements with a nuclear lifetime that is shorter than the evolution time steps but not so short so that it can be comparable to the mixing time scale. One important example is $^{20}\text{Ne}$ during O–burning. Note that $^{20}\text{Ne}$ is a main element until Ne–burning but it becomes a minor one afterwards.

As said in Sect. 2.4.1, the mixing time scale of the convective cores is much shorter than the evolution time steps in the early stages but longer during the last stages (during O–burning, $\tau_{\text{mix}}(\text{core}) \sim 10^3 - 10^5$ and $\Delta t \sim 10^2 - 10^3$ s).

The comparison of the mixing time scales and the evolution time steps shows that, for main elements, instantaneous convection can be assumed during the early stages of the evolution. In the Geneva code, the convective zones are treated as one zone. First the average composition and reaction rates over the convective zones are calculated (assuming complete mixing). Then the nucleosynthesis is calculated over the time step once for the entire zone. After that, diffusion is applied and finally the abundance is again homogenised over the convective zone. However, during the last stages of the evolution, convection must be treated as a diffusive process. This is why, in the present version of the Geneva code, convection is treated in a diffusive way since O–burning (the elements are first burnt and then mixed). Note, however, that the mixing time scale over the mixing length stays shorter than the evolution time steps ($\tau_{\text{mix}}(l) \lesssim 10^{-3}$ s during O and Si–burnings). This means that the chemical profiles are still rather flat in convective zones during Si–burning.

Unstable particles, which have a nuclear lifetime much shorter than the other time scales, can be considered at local nuclear equilibrium. They are therefore not mixed. In the present network, this concerns $\alpha$ particles during every advanced stages.

The situation becomes trickier with the elements which have nuclear lifetimes similar to the mixing time scales. For these elements a simultaneous treatment of mixing and burning would be ideal. As said above, one important example included in our network is $^{20}\text{Ne}$ during O–burning. The fact that, in the code, the processes are treated one after the other (in a serial way) makes their abundances vary slightly around the abundance they would have with a simultaneous treatment. Over many time steps, the small variations cancel each other so that the imprecision is very small for nucleosynthesis (although, for AGB stars, the simultaneous treatment is probably crucial for the $s$–process). Furthermore, in the case of $^{20}\text{Ne}$ during O–burning, $^{20}\text{Ne}$ is slowly burnt and is not involved in an important reaction chain. The fact that the abundances
3.5. *Interaction between nuclear reactions and the rest of the code*

vary slightly around the abundance they would have with a simultaneous treatment can nevertheless cause instabilities for the energy generation calculation. Indeed, if the abundance of $^{20}\text{Ne}$ is just above what it should be (in a nuclear equilibrium), its photodisintegration rate becomes high. Since photodisintegrations are energy sinks, the energy generation during O-burning may be reduced compared to its real value and it may even become negative (causing the reduction or even suppression of the convective core). In order to improve the serial treatment and to damp the induced instability, the following method was adopted: The elements are first burnt on half a time step, then they are mixed on a full time step and they are finally burnt on another half time step. Another way to improve the treatment is to use the method of the mass excesses (See Sect. 3.2.3) in order to calculate the energy production rates. The best method would be to treat the mixing and burning simultaneously, of course. Note that a few groups have started to treat the two processes simultaneously for AGB star simulations (Langer et al. 1999; Goriely & Siess 2004).
Chapter 4

Models at solar metallicity

This chapter describes the evolution of rotating and non-rotating models at solar metallicity calculated with the Geneva stellar evolution code described in the previous chapters. It is extracted from the article Hirschi et al. (2004). First, the evolution in the HR-diagram and the lifetimes are presented. The evolution of rotation and of the internal structure is described. Finally, the pre-supernova models are presented and compared with the literature.

4.1 Abstract

We describe the latest developments of the Geneva stellar evolution code in order to model the pre-supernova evolution of rotating massive stars. Rotating and non-rotating stellar models at solar metallicity with masses equal to 12, 15, 20, 25, 40 and 60 $M_\odot$ were computed from the ZAMS until the end of the core silicon burning phase. We took into account meridional circulation, secular shear instabilities, horizontal turbulence and dynamical shear instabilities. We find that dynamical shear instabilities mainly smoothen the sharp angular velocity gradients but do not transport angular momentum or chemical species over long distances.

Most of the differences between the pre-supernova structures obtained from rotating and non-rotating stellar models have their origin in the effects of rotation during the core hydrogen and helium burning phases. The advanced stellar evolutionary stages appear too short in time to allow the rotational instabilities considered in this work to have a significant impact during the late stages. In particular the internal angular momentum does not change significantly during the advanced stages of the evolution.
4.2. Hertzsprung–Russell (HR) diagram and lifetimes

We can therefore have a good estimate of the final angular momentum at the end of the core helium burning phase.

The effects of rotation on pre–supernova models are significant between 15 and 30 $M_\odot$. Indeed, rotation increases the core sizes (and the yields) by a factor $\sim 1.5$. Above 20 $M_\odot$, rotation may change the radius or colour of the supernova progenitors (blue instead of red supergiant) and the supernova type (Ibc instead of II). Rotation affects the lower mass limits for radiative core carbon burning, for iron core collapse and for black hole formation. For very massive stars ($M \gtrsim 30 M_\odot$), the pre–supernova structures are mostly affected by the intensities of the stellar winds and less by rotational mixing.

4.2 Hertzsprung–Russell (HR) diagram and lifetimes

Stellar models of 12, 15, 20, 25, 40 and 60 $M_\odot$ at solar metallicity, with initial rotational velocities of 0 and 300 km s$^{-1}$ respectively have been computed. The value of the initial velocity corresponds to an average velocity of about 220 km s$^{-1}$ on the Main Sequence (MS) which is very close to the observed average value (see for instance Fukuda 1982). The calculations start at the ZAMS for the 12, 15, 20 and 25 $M_\odot$ models and at the end of central He–burning for the 40 and 60 $M_\odot$ models (for these models, we take over the calculations done by Meynet & Maeder 2003). The calculations reach the end of central Si–burning with models of rotating stars and the end of shell Si–burning with models of non–rotating stars. For the non–rotating 12 $M_\odot$ star, Ne–burning starts at a fraction of a solar mass from the centre but does not reach the centre and the calculations stop there. For the rotating 12 $M_\odot$ star, the model stops after O–burning.

The major characteristics of the models are summarised in Tables 4.1 and 4.2. In order to calculate lifetimes of the central burning stages, we take the start of a burning stage when 0.003 in mass fraction of the main burning fuel is burnt. We consider that a burning stage is finished when the main fuel mass fraction drops below $10^{-5}$. The results would be the same if we had chosen $10^{-4}$ or $10^{-6}$. Neon burning is an exception because neon abundance does not drop significantly before the end of oxygen burning. We therefore consider the end of Ne–burning when its abundance drops below $10^{-3}$. Therefore the lifetimes for Ne–burning are to be considered as estimates. Other authors use the duration of the convective core as the lifetime. We note that using the duration of convective cores as central burning lifetimes instead of threshold values of the central abundance of the main fuel would yield results very similar to those given in Table 4.1 for H, He, O and Si–burning stages. The core sizes are given at the end of central silicon–burning and at the last model calculated (which corresponds to a different evolutionary stage in the non–rotating and the rotating models as seen above). The inner limit of each core is the star centre. The outer limit is the point in mass where the sum of the mass fraction of the main burning products (helium for $M_\alpha$, carbon and oxygen for $M_{\text{CO}}$, $^{28}\text{Si}–^{44}\text{Ti}$ for $M_{\text{Si}}$ and $^{48}\text{Cr}–^{56}\text{Ni}$ for $M_{\text{Fe}}$) becomes less than 0.75 (superscript 75) or 0.50 (superscript 50). Another possibility to define the outer limit of a core is to consider the lagrangian mass where the mass fraction of the main fuel (helium for the CO cores) drops below $10^{-2}$. 

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Table 4.1: Initial properties and lifetimes of central burning stages of solar metallicity models. Also given are the total mass and the different core masses at the end of central silicon burning as well as at the last time step of our calculations. These last models correspond approximatively to the end of the first shell silicon burning for non-rotating models and slightly later then central silicon burning for rotating ones. All masses are in solar mass units. Lifetimes are in years with exponent in brackets \((2.14 \times 10^{-2})\). Velocities are in \(\text{km s}^{-1}\).

<table>
<thead>
<tr>
<th>Initial model properties</th>
<th>(M_{\text{ZAMS}})</th>
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<th>15</th>
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<th>20</th>
<th>25</th>
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<td>0</td>
<td>300</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>Lifetime of burning stages</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(\tau_{\text{H}})</td>
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<td>1.43 (7)</td>
<td>7.95 (6)</td>
<td>1.01 (7)</td>
<td>6.55 (6)</td>
<td>7.97 (6)</td>
<td>4.56 (6)</td>
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<td>3.62 (6)</td>
<td>4.30 (6)</td>
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<td>1.13 (6)</td>
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<td>4.83 (5)</td>
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<td>9.56 (2)</td>
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<td>3.17 (2)</td>
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4.2. Hertzsprung–Russell (HR) diagram and lifetimes

Table 4.2: Same as Table 4.1 for the 12 $M_\odot$ models. The non-rotating model starts Ne-burning off-centre and the burning never reaches the centre. The unburnt Ne–O core, is given by $M_{\text{Ne–O}}$.

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<tr>
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<td>$\tau_C$</td>
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<td>$\tau_O$</td>
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</tbody>
</table>

The CO cores thus obtained are given in Tables 1 and 2 (superscript 01). These limits are suitable for most masses (see Fig. 4.15). However, for very massive stars (see Fig. 4.13), shell He-burning transforms most helium into carbon and oxygen and one could also consider that $M_{\text{CO}}$ includes the whole star. In that case we suggest another definition of $M_{\text{CO}}$, which we name $M_{\text{int}}^\text{CO}$, defined by $M_{\text{CO}}^\text{int} = M_{\text{CO}}^{01} + \int M_{\text{CO}}^{01} X_{\text{CO}} \, dm$, where $X_{\text{CO}}$ is the sum of $^{12}$C and $^{16}$O mass fractions. This definition gives an intermediate value between $M_{\text{CO}}^{01}$ and the total actual mass of the star.

4.2.1 Hertzsprung–Russell (HR) diagram

The models calculated in the present work follow the same tracks as the models from Meynet & Maeder (2003). This is expected since the only difference between the two sets of models is the inclusion of dynamical shear in the present models. Here we concentrate on the 20 $M_\odot$ models. For that purpose, we also calculated 20 $M_\odot$ models with initial rotation velocities of 100 and 200 km s$^{-1}$ (Hirschi et al. 2003b). The tracks of the 20 $M_\odot$ models are presented in Figs. 4.1, 4.2 and 4.3. Figure 4.1 shows the evolutionary tracks of the four different 20 $M_\odot$ stars in the HR-diagram. The non-rotating model ends up as a red supergiant (RSG) like the model of other groups (see Heger & Langer 2000; Limongi et al. 2000). However, the rotating models show very interesting features. Although the 100 km s$^{-1}$ model remains a RSG, the 200 km s$^{-1}$ model undergoes a blue loop to finish as a yellow–red supergiant whereas
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Figure 4.1: HR-diagram for 20 $M_\odot$ models: solid, dashed, dotted-dashed and dotted lines correspond respectively to $v_{\text{ini}}=0$, 100, 200 and 300 km s$^{-1}$. We also indicate the position of the progenitor of SN1993J.

Figure 4.2: 3D HR diagram with central helium mass fraction as the third dimension for non-rotating and rotating 20 $M_\odot$ models.
4.2. Hertzsprung–Russell (HR) diagram and lifetimes

![HR diagram](image)

Figure 4.3: $T_{\text{eff}}$ vs central helium mass fraction for 20 $M_{\odot}$ models: solid, dashed, dotted-dashed and dotted lines correspond respectively to $v_{\text{ini}} = 0$, 100, 200 and 300 km s$^{-1}$.

The 300 km s$^{-1}$ model ends up as a blue supergiant (BSG). Thus rotation may have a strong impact on the nature of the supernova progenitor (red, blue supergiant or even Wolf–Rayet star) and thus on some observed characteristics of the supernova explosion. For instance the shock wave travel time through the envelope is proportional to the radius of the star. Since RSG radii are about hundred times BSG ones, this travel time may differ by two orders of magnitude depending on the initial rotational velocity. Note that rotation also changes the surface abundances in the mass range $\sim 20–30$ $M_{\odot}$. Indeed non–rotating models have unchanged surface abundances (initial ones) while rotating models show H–burning products at their surface (see Figs. 4.15, 5.4 and 5.5).

When does the star evolve back to the blue after a RSG phase and why? Figure 4.2 is a 3D plot of the HR–diagram (in the plane) and the extra dimension represents the central helium mass fraction, $X_c(^4\text{He})$. The extra dimension allows us to follow H, He and post He–burnings within a same diagram. Indeed, $X_c(^4\text{He})$ increases during the main sequence, then decreases during He–burning and finally is equal to zero during the post He–burning evolution. We can see that:

- For the non–rotating model, He–burning starts when the star crosses the HR–diagram ($\log T_{\text{eff}} \sim 4$) and the star only reaches the RSG stage halfway through He–burning. Finally, the star luminosity rises during shell He–burning.
• For the $v_{\text{ini}} = 300 \text{ km s}^{-1}$ model, the star is more luminous and becomes a RSG before He–burning ignition. These two factors favour higher mass loss rates and the star loses most of its hydrogen envelope before He–burning is finished. Thus the star evolves towards the zone of the HR diagram where homogeneous helium stars are found, i.e. in the blue part of the HR diagram. We can see that the star track still evolves during shell He–burning.

Figure 4.3 is a projection of Fig. 4.2 in the Log $T_{\text{eff}}$ versus $X_\odot(^4\text{He})$ plane. Although less intuitive than the 3D plot, it is more quantitative and still allows us to follow the various burning stages described above. Figure 4.3 shows that all the rotating models become RSG before the beginning of the He–burning phase. The 100 km s$^{-1}$ model luminosity is lower than for the 300 km s$^{-1}$ model and therefore less mass is lost during He–burning and the burning ends before the hydrogen envelope is removed. The star therefore remains a RSG. The 200 km s$^{-1}$ model evolution is similar to the 300 km s$^{-1}$ model but the extent of its blue loop is smaller. At the end of He–burning for the 200 km s$^{-1}$ model, Log $T_{\text{eff}} = 4.28$ and the star becomes redder before C–burning starts.

Although the models discussed here are for solar metallicity, one can note that the behaviours of the models with $v_{\text{ini}}$ between 200 and 300 km s$^{-1}$ are reminiscent of the evolution of the progenitor of SN1987A. Let us recall that this supernova had a blue progenitor which evolved from a RSG stage (see e.g. the review by Arnett et al. 1989). In Fig. 4.1, we also indicate the position of the progenitor of SN 1993J. SN 1993J probably belongs to a binary system (Podsiadlowski et al. 1993). Nevertheless it has common points with our $v_{\text{ini}} = 200 \text{ km s}^{-1}$ 20 $M_\odot$ model: the star model and the progenitor of SN 1993J have approximately the same metallicity, they have a similar position in the HR–diagram taking into account the uncertainties and they both have a small hydrogen rich envelope, making possible a change from type II to type Ib some time after the explosion.

### 4.2.2 Lifetimes

The lifetimes are presented in Tables 4.1 and 4.2 and plotted in Fig. 4.4. We focus here our discussion on the effects of rotation on the lifetimes of the advanced burning phases. A discussion of the earlier stellar evolutionary phases can be found in previous papers (Meynet & Maeder 2003; Heger et al. 2000). For C–burning onwards, we have two patterns:

$M \lesssim 30M_\odot$: Since the He–burning temperature is higher in rotating stars, the $\frac{C}{O}$ ratio is smaller at the end of He–burning and therefore the C–burning lifetimes are shorter. If C–burning is less important, less neon is produced and neon burning is also shorter. The trends for O– and Si–burnings are similar.

$M \gtrsim 30M_\odot$: The rotating stars become more rapidly WR stars and are more eroded by winds. The central temperatures for rotating models are therefore equal or even smaller than for non-rotating models. This leads to higher $\frac{C}{O}$ ratios, longer C– and Ne–burnings phases.
4.2. *Hertzsprung–Russell (HR) diagram and lifetimes*

Figure 4.4: Burning lifetimes as a function of the initial mass and velocity. Solid and dotted lines correspond respectively to rotating and non-rotating models. Long-dashed and dotted-dashed lines are used for rotating and non-rotating Ne-burning lifetimes to point out that they are to be considered as estimates (see text).
Figure 4.5: \( \Omega \) variations as a function of the radius inside 15 \( M_\odot \) models: the dashed line is a profile from a model without dynamical shear and the solid line from a model with dynamical shear and \( Ri_c = 1 \) during core O-burning. The long and short arrows indicate the zones where \( Ri < 1 \) in the model without and with dynamical shear respectively. Note that the profiles do not differ significantly.

These two groups correspond to mass ranges where rotational mixing (\( M \lesssim 30 M_\odot \)) or mass loss (\( M \gtrsim 30 M_\odot \)) dominates the other process.

### 4.3 Rotation evolution

#### 4.3.1 Dynamical shear

As said in Sect. 2.2.1, we calculated three \( v_{\text{ini}} = 300 \text{ km s}^{-1} \) 15 \( M_\odot \) models to see the impact of dynamical shear and the importance of the value of \( Ri_c \): one model without dynamical shear, one with \( Ri_c = 1/4 \) and the last one with \( Ri_c = 1 \). In Fig. 4.5, the variation of the angular velocity, \( \Omega \), as a function of the radius is shown inside 15 \( M_\odot \) stellar models in the core O-burning phase. Arrows indicate the zones which are unstable against dynamical shear instability. These zones remain unstable during the whole post core He-burning phase. Our simulations show that the characteristic time scale of the dynamical shear (\( \propto R^2/D \)) is always very short when using Eq. (2.8) for the dynamical shear diffusion coefficient. Indeed, we obtain diffusion coefficients between \( 10^{12} \) and \( 10^{14} \text{ cm}^2 \text{ sec}^{-1} \). This is in general one or two orders of magnitude larger than
4.3. Rotation evolution

![Angular velocity graph]

Figure 4.6: Angular velocity as a function of the lagrangian mass coordinate, \( m_r \) inside the 25 \( M_\odot \) model (\( v_{\text{ini}} = 300 \text{ km s}^{-1} \)) at various evolutionary stages.

using the expressions given by Brüggen & Hillebrandt (2001) or Heger et al. (2000). However, the extent of the unstable zones is very small, a few thousandths of \( M_\odot \). Therefore the shear mainly smoothens the sharp \( \Omega \)-gradients as can be seen in Fig. 4.5 but does not transport angular momentum or chemical species over long distances. The general structure and the convective zones are similar between the model without dynamical shear and the one with dynamical shear.

Concerning the Richardson criterion, there is no significant difference between the models using \( Ri_c = 1/4 \) and \( Ri_c = 1 \). Except for the 15 \( M_\odot \) model discussed in this subsection, all the other models were computed with \( Ri_c = 1/4 \).

4.3.2 Angular velocity, \( \Omega \), and momentum evolution

Figure 4.6 shows the evolution of \( \Omega \) inside the 25 \( M_\odot \) model from the ZAMS until the end of the core Si–burning phase. The evolution of \( \Omega \) results from many different processes: convection enforces solid body rotation, contraction and expansion respectively increases and decreases \( \Omega \) in order to conserve angular momentum, shear (dynamical and secular) erodes \( \Omega \)-gradients while meridional circulation may erode or build them up and finally mass loss may remove angular momentum from the surface. If during the core H–burning phase, all these processes may be important, from the end of the MS phase onwards, the evolution of \( \Omega \) is mainly determined by convection, the local
Figure 4.7: Local specific angular momentum profiles for the 25 $M_{\odot}$ model ($v_{\text{ini}} = 300$ km s$^{-1}$) at different evolutionary stages.

conservation of the angular momentum and, for the most massive stars only during the core He–burning phase, by mass loss.

During the MS phase, $\Omega$ decreases in the whole star. When the star becomes a red supergiant (RSG), $\Omega$ at the surface decreases significantly due to the expansion of the outer layers. Note that the envelope is gradually lost by winds in the 25 $M_{\odot}$ model. In the centre, $\Omega$ significantly increases when the core contracts and then the $\Omega$ profile flattens due to convection. $\Omega$ reaches values of the order of 1 s$^{-1}$ at the end of Si–burning. It never reaches the local break-up angular velocity limit, $\Omega_c$, although, when local conservation holds, $\Omega_r/\Omega_c \propto r^{-1/2}$.

Figure 4.7 shows the evolution of the specific angular momentum, $j_r = 2/3 \Omega_r r^2$, in the central region of a 25 $M_{\odot}$ stellar model. The specific angular momentum remains constant under the effect of pure contraction or expansion, but varies when transport mechanisms are active. One sees that the transport processes remove angular momentum from the central regions. Most of the removal occurs during the core H–burning phase. Still some decrease occurs during the core He–burning phase, then the evolution is mostly governed by convection, which transports the angular momentum from the inner part of a convective zone to the outer part of the same convective zone. This produces the teeth seen in Fig. 4.7. The angular momentum of the star at the end of Si–burning is essentially the same as at the end of He–burning (by end of He–burning, we mean the time when the central helium mass fraction becomes less than $10^{-5}$). This
result is very similar to the conclusions of Heger et al. (2000) on this issue. They find that the angular momentum profile does not vary substantially after C–burning ignition (see Sect. 4.6.5 for a comparison). It means that we can estimate the pre-supernova angular momentum by looking at its value at the end of He–burning. We calculated, for the 25 $M_\odot$ model, the angular momentum of its remnant (fixing the remnant mass to 3 $M_\odot$). We obtained $L_{\text{rem}} = 2.15 \times 10^{50}$ g cm$^2$ s$^{-1}$ at the end of He–burning and $L_{\text{rem}} = 1.63 \times 10^{50}$ g cm$^2$ s$^{-1}$ at the end of Si–burning. This corresponds to a loss of only 24%. In comparison, the angular momentum is decreased by a factor $\sim$5 between the ZAMS and the end of He–burning. This shows the importance of correctly treating the transport of angular momentum during the Main Sequence phase.

4.4 Internal structure evolution

4.4.1 Central evolution

Figure 4.8 (left) shows the tracks of the 15 and 60 $M_\odot$ models throughout their evolution in the central temperature versus central density plane ($\log T_c$–$\log \rho_c$ diagram). Figure 4.8 (right) zooms in the advanced stages of the 12, 20 and 40 $M_\odot$ models. It is also very instructive to look at Kippenhahn diagrams (Figs. 4.11 and 4.12) in order to follow the evolution of the structure. Figure 4.9 helps understand the cause of the movements in the $\log T_c$–$\log \rho_c$ diagram. We clearly identify two categories of stellar models: those whose evolution is mainly affected by mass loss (with an inferior mass limit of about 30 $M_\odot$), and those whose evolution is mainly affected by rotational mixing (see also Sect. 4.2.2). We can see that for the 12, 15, 20 $M_\odot$ models, the rotating tracks have a higher temperature and lower density due to bigger cores. The bigger cores are due to the effect of mixing, which largely dominates the structural effects of the centrifugal force. On the other hand, for the 40 and 60 $M_\odot$ models, mass loss dominates mixing effects and the rotating model tracks in the $\log T_c$–$\log \rho_c$ plane are at the same level or below the non-rotating ones.

In order to understand the evolutionary tracks in the $\log T_c$–$\log \rho_c$ plane, we need to look at the different sources of energy at play. These are the nuclear energy, the neutrino and photon energy losses and the gravitational energy (linked to contraction and expansion). The different energy production rates at the star center are plotted in Fig. 4.9 as a function of the time left until core collapse. Going from the left to the right of Fig. 4.9, the evolution starts with H–burning where $\varepsilon_H$ dominates. In response, a small expansion occurs ($\varepsilon_q$ negative and very small movement to lower densities in the 15 $M_\odot$ model during H–burning in Fig. 4.8). At the end of H–burning, the star contracts non-adiabatically ($T \sim \rho^{1/3}$, every further contraction is also non-adiabatic). The contraction increases the central temperature. This happens very quickly and is seen in the sharp peak of $\varepsilon_q$ between H– and He–burnings. When the temperature is high enough, He–burning starts, $\varepsilon_{He}$ dominates and contraction is stopped. Note that during the H– and He–burning phases, most of the energy is transferred by radiation on thermal time scale. After He–burning, neutrino losses ($\varepsilon_\nu < 0$) overtake photon losses.
Figure 4.8: Log $T_e$ vs Log $\rho_e$ diagrams: Left: evolutionary tracks for the 15 and 60 $M_\odot$ models. Right: evolutionary tracks zoomed in the advanced stages for the 12, 20 and 40 $M_\odot$ models. Solid lines are rotating models and dashed lines are non-rotating models. The ignition points of every burning stage are connected with dotted lines. The additional long dashed line corresponds to the limit between non-degenerate and degenerate electron gas ($P_{\text{perfect gas}} = P_{\text{degenerate gas}}$).
4.4. Internal structure evolution

Figure 4.9: Log of the energy production rate per unit mass at the star center as a function of the time left until core collapse for the non-rotating (left) and rotating (right) 20 $M_\odot$ models. Nuclear energy production rates during H- and He-burnings are shown in dotted ($\varepsilon_H$) and dashed ($\varepsilon_{He}$) lines respectively. The solid line corresponds to the nuclear energy production rate in absolute value during the advanced stages ($\varepsilon_C-\varepsilon_{Si}$). Black crosses are drawn on top of the line whenever the energy production rate is negative. The thick long dashed line is the energy loss rates due to neutrinos multiplied by -1 ($-\varepsilon_\nu$). Finally the gravitational energy production rate in absolute value is plotted in the dotted-dashed line ($\varepsilon_g$). Blue squares are plotted on top when this energy is negative. Note that negative gravitational energy production corresponds to an expansion.
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This accelerates the evolution because neutrinos escape freely. During burning stages, the nuclear energy production stops the contraction if $\varepsilon_{\text{nucl}} \sim -\varepsilon_\nu$ (see C-burning for the rotating model) or even provoke an expansion when $\varepsilon_{\text{nucl}} > -\varepsilon_\nu$ (most spectacular during Si-burning). Central density decreases when the central regions expand (see Fig. 4.8). Once the iron core is formed, there is no more nuclear energy available while neutrino losses are still present and the core collapses.

Figure 4.10 shows the variation of the energy production as a function of the mass fraction inside a 20 $M_\odot$ stellar model at a stage during the shell C-burning phase. At the different burning shells, expansion occurs due to positive nuclear energy production. In the outer part, contraction and expansion are controlled by the photon luminosity and therefore by the opacities. In the inner regions, the energy produced either by the nuclear reactions or by contraction is evacuated by the neutrinos. In the non-rotating star, partial ionisation of helium I in the outermost layers produces a peak in the opacity ($\kappa / \tau$). This induces an expansion of the star ($\varepsilon_g < 0$). The situation is different for the rotating model because it has lost most of its envelope and temperatures are higher than the ionisation transition zone.

Numerically, it is important to note that the largest value for energy production rates corresponds to the nuclear one. Its maximum value is therefore used in order to determine the evolutionary time steps in our code.

The fate of the 12 $M_\odot$ models

By looking at the track of the 12 $M_\odot$ models in Fig. 4.8, we can see that rotation has a noticeable effect on the post C-burning phases. Indeed, the non-rotating model starts Ne-burning off-centre and the burning never reaches the centre. The unburnt Ne–O core, $M_{\text{Ne–O}}$ is equal to 0.096 (see Table 4.2). On the other hand, the rotating model starts Ne and O–burnings in the centre. This can be seen in Fig. 4.11 (top). The computation of the 12 $M_\odot$ models were stopped during the Ne/O-burning phase. To explore their further evolution, one can use the mass limits for the Ne–cores, $M_{\text{Ne}}$, given by Nomoto (1984):

- $M_{\text{Ne}} = 1.46 M_\odot$ is the lower limit for neon ignition in the centre.
- $M_{\text{Ne}} = 1.42 M_\odot$ is the lower limit for off-centre neon ignition where the subsequent neon burning front reaches the centre.
- $M_{\text{Ne}} = 1.37$ is the lower limit for off-centre neon ignition. In the mass range between 1.37 and 1.42 $M_\odot$, neon burning never reaches the centre.

Our rotating 12 $M_\odot$ model has $M_{\text{Ne}} \gtrsim 1.6$ well above the lower mass limit for neon ignition in the centre. In the non-rotating model the Ne–core mass is around 1.4 $M_\odot$ (more quantitatively carbon mass fraction decreases from 0.01 to 0.001 between 1.45 and 1.34 $M_\odot$). As expected from the mass limits above, in this model, neon burning, which starts off-centre, will probably reach the centre. Then (see Nomoto & Hashimoto 1988, Sect. 3.2: fate of stars with $10 M_\odot < M_{\text{ins}} < 13 M_\odot$ and references therein),
4.4. Internal structure evolution

Figure 4.10: Log of the energy production rate per unit mass as a function of $m_r/M_{\text{tot}}$ during shell C–burning for the non–rotating (left) and rotating (right) 20 $M_\odot$ models. The solid line corresponds to the nuclear energy in absolute value during C–burning ($\varepsilon_C$). Nuclear energy during H– and He–burnings are shown in dashed ($\varepsilon_{\text{He}}$) and dotted ($\varepsilon_{\text{He}}$) lines respectively. The long dashed line is the energy loss rates due to neutrinos multiplied by -1 ($-\varepsilon_\nu$). Finally the gravitational energy production rate in absolute value is plotted in the dotted–dashed line ($\varepsilon_\varphi$). Blue squares are plotted on top when this energy is negative. Note that negative gravitational energy corresponds to an expansion.
electron capture will help the star to collapse making the neon/oxygen burning explosive and possibly ejecting the H and He–rich layers. Note that in our models we only follow multiple–α elements. We did not follow the evolution of the electron mole number, $Y_e$, or of neutron excess, $\eta$, neither include Coulomb corrections. Let us recall that the electron mole number, $Y_e = \sum_i Z_i Y_i$, and the neutron excess, $\eta = \sum_i (N_i - Z_i) Y_i$, are linked by the following relation: $Y_e = (1 - \eta)/2$ ($N_i$, $Z_i$ and $Y_i$ are respectively the number of neutron $s$, protons and the number abundance of element $i$; $Y_i = X_i/A_i$, where $X_i$ and $A_i$ are the mass fraction and the mass number of element $i$). Therefore the electron mole number, $Y_e$, is always equal to 0.5\(^1\). Lower values of $Y_e$ (due to electron captures) and the inclusion of Coulomb corrections in the equation of state have an impact in this context. Electron captures remove electrons. This decreases the electron pressure and facilitates the collapse. Coulomb corrections generally act to decrease the iron core mass by about 0.1 $M_\odot$ (Woosley et al. 2002, and references therein). These omissions can be the cause of the failure of our models to follow the evolution of the 12 $M_\odot$ models further. These two effects however do not affect significantly the evolution of more massive stars before the shell Si–burning phase.

4.4.2 Kippenhahn diagrams

Figures 4.11–4.12 show the Kippenhahn diagrams for the different models. The $y$–axis represents the mass coordinate and the $x$–axis the time left until core collapse. The black zones represent convective zones. Since our calculations have not reached core collapse yet, we estimate that there is 10\(^{-5}\) yr between the last model and the collapse. This value has no significant influence since it is only a small additive constant. The graph is built by drawing vertical lines at each time step where the star is convective. this discrete construction shows its weakness at the right edge of each diagram and during shell He–burning where time steps are too distant from each other to cover the surface properly. The abbreviations of the various burning stages are written below the graph at the time corresponding to the central burning stages.

We can see the effect of the blue loops (Meynet & Maeder 2003) in the 12 $M_\odot$ models on the external convective zone during the core He–burning phase. The blueward motion reduces the external convective zone or even suppresses it. We also note the complex succession of the different convective zones between central O and Si–burnings (for instance in the non–rotating 15 $M_\odot$ model). The difference between non–rotating and rotating models is striking in the 20 and 25 $M_\odot$ models. We can see that small convective zones above the central H–burning core disappear in rotating models. Also visible is the loss of the hydrogen rich envelope in the rotating models. On the other hand non–rotating and rotating 40 and 60 $M_\odot$ models all have very similar convective zones history after He–burning. Nevertheless, the structure of the very massive stars (40, 60 $M_\odot$) is different from the models around 20 $M_\odot$. Indeed, the size of the carbon shell burning is much larger in the higher mass range. This leads to pre–SN structures composed mostly of oxygen (see Fig. 5.5). The size of the oxygen and silicon burnings

\(^1\)The mass limits given by Nomoto (1984) were also obtained from calculations with $Y_e = 0.5$. 

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4.4. Internal structure evolution

Figure 4.11: Kippenhahn diagrams for the non-rotating (left) and $v_{\text{ini}} = 300 \text{ km s}^{-1}$ (right) 12 (top), 15 (middle) and 20 (bottom) $M_\odot$ models. The black zones correspond to convective regions (see text).
Figure 4.12: Kippenhahn diagram for the non-rotating (left) and \( v_{\text{ini}} = 300 \ \text{km s}^{-1} \) (right) 25 (top), 40 (middle) and 60 (bottom) \( M_\odot \) models. The black zones correspond to convective regions (see text).
convective cores are only slightly bigger than for the models around 20 $M_\odot$.

**Convection during core C–burning?**

Recent calculations (Heger et al. 2000) show that non–rotating stars with masses less than about 22 $M_\odot$ have a convective central C–burning core while heavier stars have a radiative one. Our non–rotating models agree with this. What about models of rotating stars? Figure 4.11 (bottom) shows the Kippenhahn diagrams for the non–rotating and rotating 20 $M_\odot$ models. We can see that the rotating model has a radiative core during central C–burning. It is due to the fact that the nuclear energy production rate $\varepsilon_C$ does not overtake $-\varepsilon_\nu$ (see Fig. 4.9 right) and therefore the central entropy does not increase enough to create a convective zone. This behaviour results from the bigger He–cores formed in rotating models. Bigger cores imply higher central temperatures during the core He–burning phase and higher central temperatures imply lower carbon content at the end of the He–burning phase. Thus less fuel is available for the core C–burning phase which does not succeed to develop a convective core. The same explanation works for more massive (rotating or non–rotating) stars. Thus the upper mass limit for a convective core during the C–burning phase is lowered by rotation, passing from about 22 $M_\odot$ to a value inferior to 20 $M_\odot$ when the initial velocity increases from 0 to 300 km s$^{-1}$.

**4.4.3 Abundances evolution**

Figures 4.14 and 4.15 (where the abundance of $^{44}$Ti, dotted–long dashed line, is enhanced by a factor 1000 for display purposes) show the evolution of the abundances inside the non–rotating (left) and rotating (right) 20 $M_\odot$ models at the end of each central burning episode. At the end of H–burning, we notice the smoother profiles in the rotating model, consequence of the rotational mixing. At the end of He–burning, we can already see the difference in core sizes and total mass. We also notice the lower C/O ratio for rotating models. At the end of O–burning, we can see that the rotating model produces much more oxygen compared to the non–rotating model (about a factor two). At the end of Si–burning, the iron and Si–cores are slightly bigger in the rotating model (see also Table 4.1). The yields of oxygen are therefore expected to increase significantly with rotation. This will be discussed in an forthcoming article.

As discussed in the previous section, the rotating and non–rotating models for masses around 50 $M_\odot$ do not differ significantly after He–burning. Oxygen is the most abundant element as can be seen in Fig. 5.5 and a large fraction of helium is burnt.

**4.5 Pre–supernova models**

**4.5.1 Core masses**

Figure 4.16 shows the core masses (Tables 4.1 and 4.2) as a function of initial mass for non–rotating (dotted lines) and rotating (solid lines) models. Since rotation increases
Figure 4.13: Variations of the abundance (in mass fraction) as a function of the lagrangian mass coordinate, $m_r$, at the end of central Si-burning for the rotating $60 \, M_\odot$. Note that the $^{44}$Ti abundance (dotted–long dashed line) is enhanced by a factor 1000 for display purposes.
4.5. Pre-supernova models

Figure 4.14: Variation of the abundances in mass fraction as a function of the lagrangian mass at the end of central hydrogen (top), helium (middle) and carbon (bottom) burnings for the non-rotating (left) and rotating (right) 20$M_\odot$ models.
Figure 4.15: Variation of the abundances in mass fraction as a function of the lagrangian mass at the end of central neon (top), oxygen (middle) and silicon (bottom) burnings for the non–rotating (left) and rotating (right) 20M\(_\odot\) models.
4.5. Pre–supernova models

![Graph: Core masses as a function of the initial mass and velocity at the end of core Si–burning.](image)

Figure 4.16: Core masses as a function of the initial mass and velocity at the end of core Si–burning.

mass loss, the final mass, $M_{\text{final}}$, of rotating models is always smaller than that of non–rotating ones. Note that for very massive stars ($M \gtrsim 60M_{\odot}$) mass loss during the WR phase is proportional to the actual mass of the star. This produces a convergence of the final masses (see for instance Meynet & Maeder 2004). We can also see a general difference between the effects of rotation below and above $30M_{\odot}$. For $M \lesssim 30M_{\odot}$, rotation significantly increases the core masses due to mixing. For $M \gtrsim 30M_{\odot}$, rotation makes the star enter at an earlier stage into the WR phase. The rotating star spends therefore a longer time in this phase characterised by heavy mass loss rates. This results in smaller cores at the pre–supernova stage. We can see on Fig. 4.16 that the difference between rotating and non–rotating models is the largest between 15 and 25 $M_{\odot}$.

Concerning the initial mass dependence, one can make the following remarks:

$M_{\text{final}}$: There is no simple relation between the final mass and the initial one. The important point is that a final mass between 10 and 15 $M_{\odot}$ can correspond to any star with an initial mass between 15 and 60 $M_{\odot}$.

$M_{\alpha}$ and $M_{\text{CO}}$: The core masses increase significantly with the initial mass. For very massive stars, these core masses are limited by the very important mass loss rates undergone by these stars: typically $M_{\alpha}$ is equal to the final mass for $M \gtrsim 20M_{\odot}$ for rotating models and for $M \gtrsim 40M_{\odot}$ for the non–rotating ones. The mass of the carbon–oxygen core is also limited by the mass loss rates for $M \gtrsim 40M_{\odot}$ for
both rotating and non-rotating models.

\( M_{\text{Si}} \) **(at the end of central Si–burning):** For rotating models, \( M_{\text{Si}} \) oscillates between 2 and 2.5 \( M_{\odot} \). For non-rotating models, the mass increases regularly between 15 \( M_{\odot} \) (\( M_{\text{Si}} \simeq 1.56 M_{\odot} \)) and 40 \( M_{\odot} \) (\( M_{\text{Si}} \simeq 2.6 M_{\odot} \)) and stays constant for higher masses (due to mass loss).

\( M_{\text{Fe}} \) **(at the end of central Si–burning):** Follows the same trend as \( M_{\text{Si}} \).

**Final iron core masses**

For non-rotating models, the masses of the iron core \( M_{\text{Fe}} \) in the last computed model (end of shell Si–burning) are very close (within 8%) to the silicon core masses, \( M_{\text{Si}} \), at the end of central Si–burning. This occurs because the extent of shell Si–burning is limited by the entropy increase produced by the second episode of shell O–burning. Therefore even though our rotating models have not reached core collapse, we can have an estimate of the final iron core mass by taking the value of \( M_{\text{Si}} \) at the end of central Si–burning. In this way, we obtain iron core masses for rotating models between 2 and 2.5 \( M_{\odot} \). For non-rotating models, the mass is between 1.56 \( M_{\odot} \) and 2.6 \( M_{\odot} \). Rotating models have therefore more massive iron cores and we expect the lower mass limit for black hole (BH) formation to decrease with rotation.

As said in Sect. 4.4.1 about the fate of the 12 \( M_{\odot} \), we did not follow the evolution of the electron mole number, \( Y_e \), neither include Coulomb corrections. Coulomb corrections generally act to decrease the iron core mass by about 0.1 \( M_{\odot} \) (Woosley et al. 2002, and references therein). Electron captures during Si–burning increases neutron excess and also reduces the electron pressure and this (with photodisintegration) will allow the core to collapse (Woosley et al. 2002). It is therefore possible that some of our models should collapse before shell Si–burning occurs. Taking this argument into account and the fact that we used Schwarzschild criterion for convection, we have to consider the value of \( M_{\text{Si}} \) at the end of core Si–burning as an upper limit for the final iron core mass.

### 4.5.2 Internal structure

As well as the chemical composition (abundance profiles and core masses) of the pre–supernova star, other parameters, like the density profile, the neutron excess (not followed in our calculations), the entropy and the total radius of the star, play an important role in the supernova explosion. Figure 4.17 shows the density, temperature, radius and pressure variations as a function of the lagrangian mass coordinate at the end of the core Si–burning phase. Since the rotating star has lost its envelope, this truly affects the parameters towards the surface of the star. The radius of the star (BSG) is about one percent that of the non–rotating star (RSG). As said above this modifies strongly the supernova explosion. We also see that temperature, density and pressure profiles are flatter in the interior of rotating models due to the bigger core sizes.

Concerning the dependence of these profile on the initial mass of the star, two main comments can be made: a) the larger the initial mass, the flatter the profile due to
4.6. Comparison with the literature

Figure 4.17: Profiles of the radius, $r$, density, $\rho$, temperature, $T$ and pressure $P$ at the end of core Si-burning for the non–rotating (left) and rotating (right) 20 $M_\odot$ models. The pressure has been divided by $10^{10}$ to fit it in the diagram.

...the larger cores. b) the models which retain the hydrogen rich envelope have profiles near the surface of the star like the ones of the non–rotating model on Fig. 4.17, while the stars who eject the H–rich envelope have profile resembling the ones of the rotating model on Fig. 4.17.

4.6 Comparison with the literature

In this section, we compare our results (HMM hereinafter) with four other recent papers: Limongi et al. (2000) (LSC hereinafter), Woosley et al. (2002) (WHW), Rauscher et al. (2002) (RHW) and Heger et al. (2000) (HLW). Before we start the comparison, we need to mention which physical ingredients (treatment of convection, $^{12}$C($\alpha,\gamma$)$^{16}$O reaction rate, ...) they use:

- LSC use Schwarzschild criterion for convection without overshooting (except for core He–burning for which semiconvection and an induced overshooting are taken into account). For $^{12}$C($\alpha,\gamma$)$^{16}$O, they use the rate of Caughlan et al. (1985) (CF85). Mass loss is not included.

- WHW use Ledoux criterion for convection with semiconvection. They use a relatively large diffusion coefficient for modeling semiconvection. Moreover non–convective zones immediately adjacent to convective regions are slowly mixed on the order of a radiation diffusion time scale to approximately allow for the effects
of convective overshoot. For $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, they use the rate of Caughlan & Fowler (1988) (CF88) multiplied by 1.7.

- RHW use Ledoux criterion for convection with semiconvection. They use the same method as WHW for semiconvection. For $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, they use the rate of Buchmann (1996) (BU96) multiplied by 1.2.

- HLW use Ledoux criterion for convection with semiconvection using a small diffusion coefficient (about one percent of WHW’s coefficient) and without overshooting. For $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, they use a rate close to Caughlan et al. (1985) (CF85). They present models with and without rotation.

- In this paper (HMM), we used Schwarzschild criterion for convection with overshooting for core H and He–burnings. For $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, we used the rate of Angulo et al. (1999) (NACRE).

4.6.1 HR diagram

We remark that the present evolutionary tracks (as well as those from LSC) do not decrease in luminosity when they cross the Hertzsprung gap. This is in contrast with the tracks from HLW which present a significant decrease in luminosity when they evolve from the MS phase to the RSG phase. Models computed with the present code but using the Ledoux criterion for convection (without semiconvection) present a very similar behaviour to those of HLW. Thus the difference between the two sets of models mainly results from the different criterion used for convection.

4.6.2 Lifetimes

We can compare the lifetimes of the non–rotating 15, 20, 25 $M_\odot$ models with recent calculations presented in WHW and LSC. The comparison is shown in Table 4.3. As said earlier, LSC use Schwarzschild criterion with overshooting only for He–burning. WHW use Ledoux criterion with a very efficient semiconvection and allow for some overshoot. Despite important differences in the treatment of convection, all the models give very similar H–burning lifetimes which differ by less than 10%. For the He–burning lifetimes, during which the convective core grows in mass, one can expect that the results will be significantly different depending on which convection criterion is used. This is indeed the case. Inspecting Table 3, one sees that our results are shorter by 30–50% with respect to those of WHW. In contrast when the Schwarzschild criterion is used with some overshooting as in LSC, the results are very similar (differences inferior to six percents). In the advanced stages one sees that the lifetimes obtained by the different groups are of the same order of magnitude. Let us note that the definition of the duration of the nuclear burning stages may differ between the various authors and this tends to enhance the scatter of the results. Keeping in mind this source of difference
4.6. *Comparison with the literature*

Table 4.3: Lifetimes of central burning stages of solar metallicity models. Lifetimes are in years with exponent in brackets (2.14 (-2)=2.14 \( \times \) 10\(^{-2} \)).

<table>
<thead>
<tr>
<th>( M_{ZAMS} )</th>
<th>15 (HMM)</th>
<th>15 (WHW)</th>
<th>15 (LSC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{H} )</td>
<td>1.13 (7)</td>
<td>1.11 (7)</td>
<td>1.07 (7)</td>
</tr>
<tr>
<td>( \tau_{He} )</td>
<td>1.34 (6)</td>
<td>1.97 (6)</td>
<td>1.4 (6)</td>
</tr>
<tr>
<td>( \tau_{C} )</td>
<td>3.92 (3)</td>
<td>2.03 (3)</td>
<td>2.6 (3)</td>
</tr>
<tr>
<td>( \tau_{Ne} )</td>
<td>3.08</td>
<td>0.732</td>
<td>2.00</td>
</tr>
<tr>
<td>( \tau_{O} )</td>
<td>2.43</td>
<td>2.58</td>
<td>2.43</td>
</tr>
<tr>
<td>( \tau_{Si} )</td>
<td>2.14 (-2)</td>
<td>5.01 (-2)</td>
<td>2.14 (-2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( M_{ZAMS} )</th>
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<th>20 (WHW)</th>
<th>20 (LSC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{H} )</td>
<td>7.95 (6)</td>
<td>8.13 (6)</td>
<td>7.48 (6)</td>
</tr>
<tr>
<td>( \tau_{He} )</td>
<td>8.75 (5)</td>
<td>1.17 (6)</td>
<td>9.3 (5)</td>
</tr>
<tr>
<td>( \tau_{C} )</td>
<td>9.56 (2)</td>
<td>9.76 (2)</td>
<td>1.45 (3)</td>
</tr>
<tr>
<td>( \tau_{Ne} )</td>
<td>0.193</td>
<td>0.599</td>
<td>1.46</td>
</tr>
<tr>
<td>( \tau_{O} )</td>
<td>0.476</td>
<td>1.25</td>
<td>0.72</td>
</tr>
<tr>
<td>( \tau_{Si} )</td>
<td>9.52 (-3)</td>
<td>3.15 (-2)</td>
<td>3.50 (-3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( M_{ZAMS} )</th>
<th>25 (HMM)</th>
<th>25 (WHW)</th>
<th>25 (LSC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{H} )</td>
<td>6.55 (6)</td>
<td>6.70 (6)</td>
<td>5.93 (6)</td>
</tr>
<tr>
<td>( \tau_{He} )</td>
<td>6.85 (5)</td>
<td>8.39 (5)</td>
<td>6.8 (5)</td>
</tr>
<tr>
<td>( \tau_{C} )</td>
<td>3.17 (2)</td>
<td>5.22 (2)</td>
<td>9.7 (2)</td>
</tr>
<tr>
<td>( \tau_{Ne} )</td>
<td>0.882</td>
<td>0.891</td>
<td>0.77</td>
</tr>
<tr>
<td>( \tau_{O} )</td>
<td>0.318</td>
<td>0.402</td>
<td>0.33</td>
</tr>
<tr>
<td>( \tau_{Si} )</td>
<td>3.34 (-3)</td>
<td>2.01 (-3)</td>
<td>3.41 (-3)</td>
</tr>
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</table>
Table 4.4: Final core masses at the pre-supernova stage for different models of non-rotating stars at solar metallicity.

<table>
<thead>
<tr>
<th>$M_{ZAMS}$</th>
<th>15 (HMM)</th>
<th>15 (RHW)</th>
<th>15 (HLW)</th>
<th>15 (LSC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{total}$</td>
<td>13.232</td>
<td>12.612</td>
<td>13.55</td>
<td>15</td>
</tr>
<tr>
<td>$M_{\alpha}$</td>
<td>4.168</td>
<td>4.163</td>
<td>3.82</td>
<td>4.10</td>
</tr>
<tr>
<td>$M_{CO}$</td>
<td>2.302</td>
<td>2.819</td>
<td>1.77</td>
<td>2.39</td>
</tr>
<tr>
<td>$M_{Si}^{50}$</td>
<td>1.842</td>
<td>1.808</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$M_{Fe}^{50}$</td>
<td>1.514</td>
<td>1.452</td>
<td>1.33</td>
<td>1.429</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M_{ZAMS}$</th>
<th>20 (HMM)</th>
<th>20 (RHW)</th>
<th>20 (HLW)</th>
<th>20 (LSC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{total}$</td>
<td>15.694</td>
<td>14.740</td>
<td>16.31</td>
<td>20</td>
</tr>
<tr>
<td>$M_{\alpha}$</td>
<td>6.208</td>
<td>6.131</td>
<td>5.68</td>
<td>5.94</td>
</tr>
<tr>
<td>$M_{CO}$</td>
<td>3.840</td>
<td>4.508</td>
<td>2.31</td>
<td>3.44</td>
</tr>
<tr>
<td>$M_{Si}^{50}$</td>
<td>2.002</td>
<td>1.601</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$M_{Fe}^{50}$</td>
<td>1.752</td>
<td>1.461</td>
<td>1.64</td>
<td>1.552</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M_{ZAMS}$</th>
<th>25 (HMM)</th>
<th>25 (RHW)</th>
<th>25 (HLW)</th>
<th>25 (LSC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{total}$</td>
<td>16.002</td>
<td>13.079</td>
<td>18.72</td>
<td>25</td>
</tr>
<tr>
<td>$M_{\alpha}$</td>
<td>8.434</td>
<td>8.317</td>
<td>7.86</td>
<td>8.01</td>
</tr>
<tr>
<td>$M_{CO}$</td>
<td>5.834</td>
<td>6.498</td>
<td>3.11</td>
<td>4.90</td>
</tr>
<tr>
<td>$M_{Si}^{50}$</td>
<td>2.577</td>
<td>2.121</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$M_{Fe}$</td>
<td>1.985</td>
<td>1.619</td>
<td>1.36</td>
<td>1.527</td>
</tr>
</tbody>
</table>

and the fact that the lifetimes vary by eight or nine orders of magnitude between the H-burning and the Si-burning phases, the agreement between the various authors appears remarkable.

### 4.6.3 Kippenhahn diagrams and convection during central C-burning

Our Kippenhahn diagrams for the non-rotating models are in good agreement with those of Rauscher et al. (2002) except that in our model the carbon and oxygen shells do not merge for the 20 $M_\odot$ model. The only noticeable difference between the structures in the advanced phase obtained in the present work and those obtained by LSC is that their 20 $M_\odot$ model does not have a central convective core during C-burning. This can be explained by the fact that they use the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate from Caughlan et al. (1985). This rate is larger than the NACRE rate we use in our models (see Fig. 4.21) and therefore more $^{12}\text{C}$ is burnt during He-burning.

### 4.6.4 Core masses

#### Non-rotating models

In Table 4.4 the final masses and the core masses at the pre-supernova stage are given for different 15, 20 and 25 $M_\odot$ stellar models. The second column corresponds to the present non-rotating models, the third shows the results of Rauscher et al. (2002),
Figure 4.18: Variation of the He core masses, $M_\alpha$ (light lines), and of the CO core masses, $M_{CO}$ (heavy lines), at the pre-supernova stage in different initial mass models. Only non-rotating models are shown. The different types of line correspond to results obtained by different groups: HMM labels our results, W86 those of Woosley & Weaver (1986), RHW those of Rauscher et al. (2002), LSC those of Limongi et al. (2000) and HLW those of Heger et al. (2000).
CHAPTER 4. MODELS AT SOLAR METALLICITY

Figure 4.19: $M_{\text{iron}}$ (and $M_{\text{Si}}$) as a function of $M_\alpha$ for non-rotating models from different authors (see Table 4.4). The labels are the same as in Fig. 4.18. The light lines show the variation of $M_{\text{Si}}$, the heavy lines those of $M_{\text{iron}}$.

Figure 4.20: $M_{\text{iron}}$ (and $M_{\text{Si}}$) as a function of $M_{\text{CO}}$ for non-rotating models from different authors (see Table 4.4). The labels are the same as in Fig. 4.18. The light lines show the variation of $M_{\text{Si}}$, the heavy lines those of $M_{\text{iron}}$.  

79
the fourth those of Heger et al. (2000) and the fifth those of Limongi et al. (2000). In Fig. 4.18, we see that for $M_\alpha$, the results are very similar (within 5%) between our models and those of LSC and RHW. This can be understood by the similar outcome of the convection treatment. HLW use a small diffusion coefficient for semiconvection and logically obtain slightly smaller helium cores.

The differences between the mass of the CO cores are much greater. Let us recall here that the size of this core depends a lot on the convective criterion and also on the rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. This reaction becomes one of the main source of energy at the end of the core He-burning phase. A faster rate implies smaller central temperatures and thus increases the He-burning lifetime. This in turn will produce larger CO cores (Langer 1991) with a smaller fraction of $^{12}\text{C}$. Figure 4.21 shows the rates used by various authors divided by the NACRE rate for the temperature range of interest.

Since HLW and LSC use the same rate for this reaction, most of the difference between the mass of the CO cores must have its origin in the different treatment of convection. One notes also that LSC still have slightly smaller cores than us even though they added some semiconvection and use the CF85 rate for $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ which is greater than the one adopted in our models. RHW, although they had slightly smaller $M_\alpha$, have larger $M_{\text{CO}}$. This can be explained in part by the use of the rate BU96x1.2 for $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ which is larger than the NACRE rate at the end of He-burning (see Fig. 4.21).
In Figs 4.19 and 4.20 the Si and iron core masses obtained at the end of shell Si-burning are plotted as a function of \( M_\alpha \) and \( M_{\text{CO}} \) respectively. The present results are well in the range of values obtained by different authors for \( M_\alpha \lesssim 6 \, M_\odot \) (\( M_{\text{ini}} \lesssim 20 \, M_\odot \)). Above this mass range, our results are in agreement with those of Woosley & Weaver (1986) and significantly above the results obtained more recently by the other groups. As discussed in Sect. 4.5.1, we did not follow the evolution of the neutron excess or include Coulomb corrections. This does not affect our results until the end of core Si-burning but may affect the results plotted in Figs. 4.19 and 4.20 obtained at the end of the shell Si-burning. In this last case, the present results have to be considered as upper limits. This might be part of the explanation why our iron core masses appear to be systematically greater than those obtained in recent calculations. However one notes that Woosley et al. (2002) give a Chandrasekhar mass (lower mass limit for collapse) of 1.79 \( M_\odot \) for the 25 \( M_\odot \) model which is large compared to the iron core we obtain at the end of core Si-burning, implying that our 25 \( M_\odot \) \( (M_\alpha = 8.4 \, M_\odot) \) model may experience shell Si-burning before collapsing. Thus if we cannot discard that the final iron core masses are overestimated due to the above reason, they may also be greater than the masses obtained by other groups for other reasons. In this context it is interesting to compare the masses of the Si-burning core. The Si-cores are created by O-burning before Si-burning (except possibly a small fraction due to an additional shell O-burning during Si-burning). Their sizes are thus not dependent on the neutron excess or the Chandrasekhar mass. Looking at Fig. 4.19 and 4.20 where our Si-core masses are compared to those obtained by RHW, we see that our core masses are systematically larger. In that case the difference cannot be attributed to the neglect in our models of the electron capture reactions and of the Coulomb corrections. Our bigger cores result from the different prescription we used for convection in our models. Thus it is possible that the bigger iron cores we have obtained are due, at least in part, to the prescriptions we used for convection.

Rotating models

We can also compare core masses of the rotating 15, 20, 25 \( M_\odot \) models with recent calculations by Heger et al. (2000) (HLW). For \( M_{\text{Fe}} \), we use \( M_{\text{Si}}^{50} \). As discussed in section 4.5.1, this assumes that shell Si-burning occurs before the collapse and our value has to be considered as an upper limit.

The comparison is shown in Table 4.5. “F..B” models are the models with the same initial rotational velocity and inclusion of the \( \mu \)-gradients inhibiting effects on rotational mixing. These are the models which should give approximately the same results as us if uncertainties concerning the treatment of convection and particular reaction rates were small. We also show the “E..” models with a lower initial rotational velocity but without the \( \mu \)-gradients inhibiting effects.

One can see by comparing the results from the two models of HLW, the great dependence of the core masses on the treatment of the \( \mu \)-gradient inhibiting effect. The more efficient the rotational mixing (or less strong are the inhibiting effects of the
4.6. Comparison with the literature

Table 4.5: Final core masses at the pre–supernova stage for different models of rotating stars at solar metallicity. Note that we use $M_{\text{Si}}$ at the end of central Si–burning for the value of $M_{\text{Fe}}$ as discussed in the text.

<table>
<thead>
<tr>
<th>$M_{\text{ZAMS}}$</th>
<th>$M_{\text{total}}$</th>
<th>$M_{\text{Fe}}$</th>
<th>$M_{\text{Si}}$</th>
<th>$M_{\text{CO}}$</th>
<th>$M_{\text{total}}$</th>
<th>$M_{\text{Fe}}$</th>
<th>$M_{\text{Si}}$</th>
<th>$M_{\text{CO}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15</td>
<td>20</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F15B$ (HLW)</td>
<td>10.316</td>
<td>5.604</td>
<td>3.325</td>
<td>1.936</td>
<td>11.763</td>
<td>5.999</td>
<td>7.864</td>
<td>2.75</td>
</tr>
<tr>
<td>$E15$ (HLW)</td>
<td>10.86</td>
<td>3.88</td>
<td>2.01</td>
<td>1.38</td>
<td>10.00</td>
<td>5.999</td>
<td>5.01</td>
<td>1.73</td>
</tr>
</tbody>
</table>

$\mu$–gradients), the greater the core masses. Compared to the results obtained by HLW, one sees that our core masses are significantly greater. This essentially results from two facts: first the effects of rotation are included in our models in a different way than in the models by HLW. In particular in our models, the treatment of rotational mixing includes the inhibiting effect of $\mu$–gradients without any ad hoc parameters, and the transport of the angular momentum by the meridional circulation is properly accounted for by an advective term (Maeder & Zahn 1998). Secondly, the present stellar models were computed with the Schwarzschild criterion with a moderate overshooting while the models of HLW were computed with the Ledoux criterion with semiconvection using a small diffusion coefficient and without overshooting.

4.6.5 Final angular momentum

Long soft gamma–ray bursts (GRBs) were recently connected with SNe (see Matheson 2003, for example). One scenario for GRB production is the collapsar mechanism devised by Woosley (1993). In this mechanism, a star collapses into a black hole and an accretion disk due to the high angular momentum of the core. Accretion from the disk onto the central black hole produces bi–polar jets. These jets can only reach the surface of the star (and be detected) if the star loses its hydrogen rich envelope before the collapse. WR stars are therefore good candidates for collapsar progenitors since they lose their hydrogen rich envelope during the pre–SN evolution. The question to answer is whether the core of WR stars contains enough angular momentum at the pre–SN stage (the specific angular momentum, $j$, of the material just outside the core must be
Figure 4.22: Comparison of the final local specific angular momentum profiles for different $20 \, M_\odot$ models. Models with different initial velocities, $v_{\text{ini}} = 100, 200$ and $300 \, \text{km s}^{-1}$ are drawn with dotted, solid and thick solid lines respectively. We can see the convergence of the final of the final angular momentum of the core above $v_{\text{ini}} = 200 \, \text{km s}^{-1}$. 
4.6. Comparison with the literature

Figure 4.23: Comparison of the final local specific angular momentum profiles for different 20 $M_\odot$ models all with the same initial rotational velocity, $v_{\text{ini}}=200$ km s$^{-1}$. The thick solid line (HMM) corresponds to our model. The models from Heger et al. (2000) are drawn with a dashed line for model E20 (no $\mu$-barrier) and with a dotted-dashed line for model E20B ($\mu$-barrier with $f_\mu = 0.05$). Finally, model m20b5 from Heger et al. (2003) including the effect of the magnetic fields according to Spruit (2002) is drawn with the dotted line.
larger than $10^{16} \text{ cm}^2 \text{s}^{-1}$). So far only Heger and co–workers have obtained values for the angular momentum of the cores of massive stars at the pre–SN stage (Heger et al, 2000, 2003). The physical ingredients of their model have been given in Sect. 4.6.4. The comparison between our models and theirs shows that the size of the various cores depends significantly on the treatment of both convection and rotation. The evolution of angular velocity and angular momentum in the models of Heger et al. (2000, HLW00 hereinafter; in Figs. 8 and 9). The evolution of angular velocity and momentum in our models is described in Meynet & Maeder (2000) and in Sect. 4.3.2.

HLW00 show with their Fig. 9 the convergence of the final angular momentum of the core for a wide range of initial angular momentum. The dependence of the final angular momentum on the initial one for our models is displayed in Fig. 4.22. Models with $v_{\text{ini}} = 100$ and 200 km s$^{-1}$ have been computed until the end of O–burning. This should not affect the comparison since the angular momentum profile does not change during Si–burning. One can see in our case that convergence only occurs above $v_{\text{ini}} = 200$ km s$^{-1}$. Indeed, the average specific angular momentum of the core (assuming a $1.7 \, M_\odot$ core) at the end of the calculation is $1.326 \times 10^{16}$, $1.801 \times 10^{16}$ and $2.106 \times 10^{16} \text{ cm}^2 \text{s}^{-1}$ for $v_{\text{ini}} = 100, 200$ and 300 km s$^{-1}$ respectively.

Concerning the evolution of the angular momentum, the general picture is the following. Mass loss removes angular momentum from the surface and transport processes (convection and rotational mixings) redistribute angular momentum inside the star (see Sect. 4.3.2 and the references given above for details). Here we are only concerned about the evolution of the angular momentum of the core of the star. During H–burning, both our models and models without the inhibiting effect of the $\mu$–gradient on mixing (models without “B”) from HLW00 show a large decrease of the angular momentum of the core. On the other hand, in HLW00 models including the inhibiting effect of the $\mu$–gradient (models with “B”), the core does not lose much angular momentum during H–burning. In our models, thermal turbulence is taken into account and is able to overcome the inhibiting effect of the $\mu$–gradient. HLW00 do not include the thermal effects and in their situation, the inhibiting effect of the $\mu$–gradient is almost complete even with a reduction parameter $f_\mu$ equal to 0.05. The different treatment of rotation (and especially the different way the inhibiting effect of the $\mu$–gradient is included) has therefore a strong impact on the evolution of the angular momentum of the core during the MS and explains the difference between the various models.

At the end of H–burning, the core contracts and the envelope expands. This restructuring phase is accompanied by a formation of a very deep external convective zone. At the same time, shell H–burning creates a short–lived intermediate convective zone. These changes may affect the angular momentum profile. The largest change in our models is the creation of a large drop of the angular momentum at the bottom of the external convective zone (see Fig. 4.7). This is due to the fact that convection enforces solid body rotation and therefore angular momentum is transported at the outer edge of the convective zone. No significant change is seen in the core.

During He–burning, the trend is the opposite from H–burning. In both our models
and models without the inhibiting effect of the $\mu$-gradient on mixing from HLW00, the angular momentum in the core decreases slightly. On the other hand, in HLW00 models with the inhibiting effect of the $\mu$-gradient, the core loses a significant amount of angular momentum after H-burning. The reason is the following. During H-burning, in HLW00 models with the $\mu$-gradient effects on mixing, even though the core does not lose much angular momentum, the layers just above it lose angular momentum (due to various transport processes). This creates a large angular velocity gradient at the edge of the core which increases rotational mixing during He-burning. Furthermore the successive convective and semiconvective zones (due to the restructuring phase and shell H-burning) mix as well the angular momentum of the outer parts of the core with layers above the core and a large amount of angular momentum is transferred out of the core at this time. In our models (as well as those from HLW00 without $\mu$-gradient), angular momentum is transferred to the layers above the core during H-burning. This creates a smaller gradient of angular velocity at the edge of the core at the end of H-burning and thus rotational mixing is weaker during He-burning. Therefore, the angular momentum of the core does not change as much during He-burning. Note also that He-burning is ten times shorter than H-burning and that there is less time to mix. As said in Sect. 4.3.2, during the advanced stages, the angular momentum profile does not change substantially. Only convective zones create spikes along the profile.

The comparison of the final angular momentum profile of the different models, all with the same initial mass and surface angular velocity, is shown in Fig. 4.23. The thick solid line (HMM) corresponds to our model. The models from Heger et al. (2000) are drawn with a dotted-dashed line for model E20 (no $\mu$-barrier) and with a dashed line for model E20B ($\mu$-barrier with $f_{\mu} = 0.05$). Finally, model m20b5 from Heger et al. (2003) including the effect of the magnetic fields according to Spruit (2002) is drawn with the dotted line. Even though the evolution of angular momentum differs between our model and model E20B (with the $\mu$-gradient effects on mixing) from Heger et al. (2000), the final value of the angular momentum of the core is very similar for these two models. This confirms the possibility of the formation of GRBs via collapsars from rotating massive stars (Woosley & Heger 2003; Heger et al. 2003) if the effects of magnetic field (not included in our work) are small. Indeed, for example, the 25 $M_\odot$ model ends up as a WR star with a core having enough angular momentum ($j \gtrsim 10^{45}$ cm$^2$ s$^{-1}$ for the material just outside the core, see Fig. 4.7) to create a collapsar. The difference between our model and model E20 (without the $\mu$-gradient effects on mixing) from Heger et al. (2000) is probably due to the combination of the non-inclusion of the $\mu$-gradient effects on mixing and of the different treatment of meridional circulation (see Sect. 4.6.4). From model m20b5 (see Fig. 4.23), one sees that the inclusion of the effects of magnetic fields according to Spruit (2002) decreases significantly the final angular momentum of the core. In this situation, the core rotates too slowly and cannot produce a collapsar. We can also compare our models with the observed rotation period of young pulsars. Rotating models without the effects of magnetic fields have about 100 times more angular momentum at the pre-SN stage than the observed young pulsars (Heger et al. 2000). Models including the effects of magnetic fields according to Spruit
(2002) have about 5–10 times more angular momentum at the pre–SN stage than the observed young pulsars (Heger et al. 2003). This means that in any case, additional slow down is necessary during the core collapse (Woosley & Heger 2003; Fryer & Warren 2004) in order to reproduce the observed rotation periods of young pulsars. The question that needs to be answered is when and how this slow down occurs. Further developments will therefore be of great importance for the formation of both NSs and GRBs. The topic of the final angular momentum of our models and its implications for further evolution will be developed in a future article.

4.6.6 Lower mass limit for models to reach iron core collapse

As said above, our 12 $M_\odot$ models have not been pursued beyond the O and Ne–burning phases for the rotating and non–rotating models respectively. Nevertheless, we think that the rotating model has the potential to reach an iron core while the non–rotating model does not. Recent calculations done by Heger et al. (2000) are similar to ours on that point: Their non–rotating models as well as the rotating models E12B, F12B and G12B neither reach core–collapse. Only the model E12 reaches core collapse but the physics used in that last model does not include $\mu$–gradient inhibiting effects on rotationally induced mixing. At the same time, recent non–rotating 13 $M_\odot$ models in Woosley et al. (2002) and Limongi et al. (2000) reach core–collapse. Therefore we expect the lower mass limit for non–rotating models to reach the standard iron core collapse to be around 12–13 $M_\odot$ in agreement with Nomoto & Hashimoto (1988). Our rotating models tend to show that this limit should be lower for rotating stars. A finer grid of models around 12 $M_\odot$ would help constraining this limit.
4.6. Comparison with the literature
Chapter 5

Yields of rotating stars at solar metallicity

This chapter presents the stellar yields calculated from the models presented in the previous chapter. First, the formulae used to calculate the yields are given. The separate contributions from the stellar winds and the SN explosion are then presented. They are added to compute the total stellar yields. These total stellar yields are then compared to the literature. Finally, uncertainties concerning the stellar yields are discussed.

5.1 Abstract

We present a new set of stellar yields obtained from rotating stellar models at solar metallicity covering the massive star range (9–120 $M_\odot$). The stellar models were calculated with the latest version of the Geneva evolution code described in Hirschi et al. (2004). Evolution and nucleosynthesis are in general followed up to Silicon burning. Our non-rotating models are consistent with other calculations and differences can be understood in the light of the treatment of convection and the rate used for $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. This verifies the accuracy of our calculations and gives a safe basis for studying the effects of rotation on the yields. The contributions from stellar winds and supernova explosions to the stellar yields are presented separately. For each, we compare our models with the calculations from Maeder (1992). We then add the two contributions to compute the total stellar yields. Below $\sim 30\ M_\odot$, rotation increases the yields for heavy elements and in particular of carbon and oxygen by a factor 1.5–2.5. For very massive stars ($\sim 60\ M_\odot$), rotation increases the yields of helium and other hydrogen burning
products but does not affect much the yields of elements produced in more advanced evolutionary stages.

5.2 Description of the stellar models

The computer model used to calculate the stellar models is described in details in Hirschi et al. (2004). Convective stability is determined by the Schwarzschild criterion. Convection is treated as a diffusive process from oxygen burning onwards. The overshooting parameter is 0.1 \( H_P \) for \( H \) and He-burning cores and 0 otherwise. On top of the meridional circulation and secular shear, an additional instability induced by rotation, dynamical shear, is introduced in the model. The reaction rates are taken from the NACRE (Angulo et al. 1999) compilation for the experimental rates and from the NACRE website (http://pntpm.ub.ac.be/nacre.htm) for the theoretical ones.

Since mass loss rates are a key ingredient for the yields of massive stars, we recall here the prescriptions used. The changes of the mass loss rates, \( M \), with rotation are taken into account as explained in Maeder & Meynet (2000a). As reference mass loss rates, we adopt the mass loss rates of Vink et al. (2000, 2001b) who account for the occurrence of bi-stability limits which change the wind properties and mass loss rates. For the domain not covered by these authors we use the empirical law devised by de Jager et al. (1988). Note that this empirical law, which presents a discontinuity in the mass flux near the Humphreys–Davidson limit, implicitly accounts for the mass loss rates of LBV stars. For the non-rotating models, since the empirical values for the mass loss rates are based on stars covering the whole range of rotational velocities, we must apply a reduction factor to the empirical rates to make them correspond to the non rotating case. The same reduction factor was used as in Maeder & Meynet (2001). During the Wolf-Rayet phase we use the mass loss rates by Nugis & Lamers (2000). These mass loss rates, which account for the clumping effects in the winds, are smaller by a factor 2–3 than the mass loss rates used in our previous non-rotating “enhanced mass loss rate” stellar grids (Meynet et al. 1994). Wind anisotropy (described in Maeder 1999) was only taken into account for \( M \geq 40 \, M_\odot \) since its effects is only important for very massive stars. Later on, we compare our results with the “high mass loss rate” (high \( M \)dot) and “low mass loss rate” (low \( M \)dot) models of Maeder (1992). For the “low mass loss rate” models, Maeder (1992) uses mass loss rates given by de Jager et al. (1988) for non-WR stars and by Langer (1989) for WR stars. For the “high mass loss rate” models, the mass loss rates from de Jager et al. (1988) is increased by a factor two in the post-MS stages.

The initial composition of our models is given in Table 3.1. For a given metallicity \( Z \) (in mass fraction), the initial helium mass fraction \( Y \) is given by the relation \( Y = Y_p + \Delta Y/\Delta Z \cdot Z \), where \( Y_p \) is the primordial helium abundance and \( \Delta Y/\Delta Z \) the slope of the helium-to-metal enrichment law. We used the same values as in Maeder & Meynet (2001) i.e. \( Y_p = 0.23 \) and \( \Delta Y/\Delta Z = 2.25 \). For the solar metallicity, \( Z = 0.02 \), we thus have \( X = 0.705 \) and \( Y = 0.275 \). For the mixture of the heavy elements, we adopted the same mixture as the one used to compute the opacity tables for solar composition. For
elements heavier than Mg, we used the values from Anders & Grevesse (1989).

We calculated stellar models with initial masses of 9, 12, 15, 20, 25, 40, 60, 85 and 120 $M_\odot$ at solar metallicity, with initial rotation velocities of 0 and 300 km s$^{-1}$. The value of the initial velocity corresponds to an average velocity of about 220 km s$^{-1}$ on the Main Sequence (MS) which is very close to the observed average value (see for instance Fukuda 1982). The calculations start at the ZAMS. The rotating 15, 20, 25, 40 and 60 $M_\odot$ models were computed until the end of core silicon (Si) burning and their non-rotating counterparts until the end of shell Si-burning. For the non-rotating 12 $M_\odot$ star, neon (Ne) burning starts at a fraction of solar mass away from the centre but does not reach the centre and the calculations stop there. For the rotating 12 $M_\odot$ star, the model ends after oxygen (O) burning. The evolution of the models with initial mass between 12 and 60 $M_\odot$ is described in Hirschi et al. (2004). The 9, 85 and 120 $M_\odot$ models are presented in (Meynet & Maeder 2003) and their evolution was followed until the end of the core He-burning. Note that shell He-burning was not followed. This means that the yields for hydrogen burning products ($^4$He and $^{14}$N) may be overestimated and that the yields for helium burning products ($^{12}$C and $^{16}$O) may be underestimated for the 9, 85 and 120 $M_\odot$ models. This is also the case for yields published in Chiappini et al. (2003). Concerning the yields published in Chiappini et al. (2003), one has to note that the physical ingredients (especially mass loss) also changed. In Maeder (1992), evolution was followed until the end of core C-burning and therefore include the contribution from shell He-burning.

### 5.3 Yield calculations

In this paper, we calculated separately the yield contributions from stellar winds and the SN explosion. The wind contribution from a star of initial mass, $m$, to the stellar yield of an element $i$ is:

$$m\rho_{\text{wind}}^{\text{wind}} = \int_0^{\tau(m)} \dot{M}(m, t)[X_i^S - X_i^0] \, dt$$

(5.1)

where $\tau(m)$ is the final age of the star, $\dot{M}(m, t)$ the mass loss rate, $X_i^S$ the surface abundance in mass fraction of element $i$ at time $t$ and $X_i^0$ its initial mass fraction (see Table 3.1). Mass loss occurs mainly during hydrogen (H) and helium (He) burnings. Indeed, the advanced stages of the hydrostatic evolution are so short in time that only a negligible amount of mass is lost during these phases. Since all our stellar models were evolved at least until the end of He-burning, we can calculate the wind contribution to the yields directly from our models using Eq. 5.1.

In order to calculate the SN explosion contribution to stellar yields of all the chemical elements, one needs to model the complete evolution of the star from the ZAMS until and including the SN explosion. However, elements lighter than silicon are marginally modified by explosive nucleosynthesis (Thielemann et al. 1996) and are mainly determined by the hydrostatic evolution. In this work, we calculate SN yields at the end
5.3. Yield calculations

of core Si–burning. We do the calculation at that point so that we can compare the results for our rotating and non–rotating models and we present yields for chemical elements which are not significantly affected by the subsequent evolution. Furthermore, we discuss yield uncertainties related to the core collapse and SN explosion in Sect. 5.7. The SN contribution from a star of initial mass, $m_i$, to the stellar yield of an element $i$ is:

$$m_{SN}^{i} = \int_{m(\text{rem})}^{m(\tau)} \left[ X_i(m_r) - X_i^0 \right] dm_r \quad (5.2)$$

where $m(\text{rem})$ is the remnant mass, $m(\tau)$ the final stellar mass, $X_i^0$ the initial abundance in mass fraction of element $i$ and $X_i(m_r)$ the final abundance in mass fraction at the lagrangian mass coordinate, $m_r$.

The remnant mass in Eq. 5.2 corresponds to the final baryonic remnant mass that includes fallback that may occur after the SN explosion. The exact determination of the remnant mass would again require the simulation of the core collapse and SN explosion, which is not within the scope of this paper. Even if we had done the simulation, the remnant mass would still be a free parameter because explosion models including the latest physics still cannot reproduce explosions (Janka et al. 2003, and references therein). When comparing models to observations, the remnant mass is usually chosen so that the amount of radioactive $^{56}$Ni ejected by the star corresponds to the value determined from the observed light curve. So far, mostly 1D models are used for explosive nucleosynthesis but a few groups have developed multi–D models (see Travaglio et al. 2004; Maeda & Nomoto 2003). Multi–D effects like mixing and asymmetry could play a role in the choice of the mass cut if some iron group elements are mixed with oxygen or silicon rich layers.

In this work, we used the relation between $M_{\text{CO}}$ and the remnant mass described in Maeder (1992). The resulting remnant mass as well as the major characteristics of the models are given in Table 5.1. The determination of $M_\odot$ and $M_{\text{CO}}$ is described in Hirschi et al. (2004). The pre–collapse iron core mass given in column six is in fact the silicon (Si) core mass at the end of core Si–burning. This approximation is done because our models of rotating stars do not follow shell Si–burning. Nevertheless, the approximation is justified because the evolution of our non–rotating models show that the size of the iron core after shell Si–burning is very similar to the silicon core at the end of core Si–burning. This occurs because the extent of shell Si–burning is limited by the entropy increase produced by the shell O–burning. Therefore, if the shell Si–burning occurs, the silicon core at the end of core Si–burning is a very good estimate of the pre–collapse iron core. This point is further discussed in Sect. 5.7.1 (see also Hirschi et al. 2004).

In Fig. 5.1, we present the pre–collapse iron core and the remnant masses (columns six and seven of Table 5.1 respectively) as a function of the initial stellar mass. At first sight, the remnant masses appear large. Indeed, except for stars with $M_{\text{ini}} \lesssim 18 M_\odot$, the remnant masses are larger than the pre–collapse iron core masses and $M_{\text{rem}}$ becomes much larger for stars with $M_{\text{ini}} \gtrsim 30 M_\odot$. Taken at face, these would imply that all star with an initial mass larger than about 18 $M_\odot$ would not eject iron. However, one must
Table 5.1: Initial mass (column 1) and velocity (2), final mass (3), masses of the helium (4), carbon-oxygen (5) and iron (6) cores and the remnant mass (7) of our solar metallicity models. All masses are in solar mass units. Velocities are in km s\(^{-1}\). "A" next to the 300 means that wind anisotropy was taken into account.

<table>
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<th>(M_{\text{ini}})</th>
<th>(v_{\text{ini}})</th>
<th>(M_{\text{final}})</th>
<th>(M_{\alpha})</th>
<th>(M_{\text{CO}})</th>
<th>(M_{\text{iron}})(^a)</th>
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<td>11.270</td>
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</table>

\(^a\) pre-collapse iron core assuming that shell Si-burning occurs before collapse.
5.3. Yield calculations

Figure 5.1: Remnant masses as a function of the initial mass and velocity. $M_{\text{PreSN}}$ is the pre-collapse iron core mass (Table 5.1). Asterisks are the pre-collapse value of $M_{\text{Si}}$ (the silicon core mass) of our non-rotating models.
note that for stars which eject iron group elements ($M_{\text{ini}} \lesssim 30 M_\odot$), these elements are produced during the SN explosion (by explosive burnings) in the oxygen or silicon rich layers. Furthermore, the remnant masses are consistent with the idea (Woosley et al. 2002) that the upper limit of the remnant mass is the position of the oxygen shell due to a jump in entropy and a drop in density. This can be seen in Fig. 5.1 where $M_{\text{SN PreSN}}^n$ (value after shell Si-burning) is larger than $M_{\text{rem}}$ (only shown for non-rotating models). For stars more massive than about 30 $M_\odot$, black hole formation is expected and they do not eject iron group elements except through jets. In Sect. 5.7.2, we discuss the dependence of the results on the choice of the remnant mass.

From C–burning onwards, we only follow the abundance evolution of multiple–$\alpha$ elements. We therefore do not follow $^{22}\text{Ne}$ after He–burning (see Fig. 5.13) and have to apply a special criterion to calculate its SN yield. During He–burning, $^{22}\text{Ne}$ is produced by $^{18}\text{O}(\alpha, \gamma)$ and destroyed by an $\alpha$–captures which create $^{25}\text{Mg}$ or $^{26}\text{Mg}$. $^{22}\text{Ne}$ is totally destroyed by C–burning. We therefore consider $^{22}\text{Ne}$ abundance to be null inside of the C–burning shell. Numerically, this is done when the mass fraction of $^4\text{He}$ is less than $10^{-4}$ and that of $^{12}\text{C}$ is less than 0.1. The 9, 85 and 120 $M_\odot$ models have only been computed until the end of He–burning. In order to obtain SN yields, we use the same approach as in Maeder (1992). We use the relation between $M_{\text{CO}}$ and the $^{16}\text{O}$ yields derived from the results of our rotating 12 to 60 $M_\odot$ models. We then interpolate this relation to obtain $^{16}\text{O}$ yields for the 85 and 120 $M_\odot$ models using their respective $M_{\text{CO}}$.

For the $^{12}\text{C}$ and $^{22}\text{Ne}$ yields, we consider their abundances to be null when the mass fraction of $^4\text{He}$ is less than $10^{-4}$. For the 9 $M_\odot$ models, $M_{\text{rem}}$ is (almost) equal to $M_{\text{CO}}$ and the standard method can be applied.

Once both the wind and SN contributions are calculated, the total stellar yield of an element $i$ from a star of initial mass, $m$, is:

$$m_{\text{p}_{\text{im}}} = m_{\text{p}_{\text{SN}_{\text{im}}}} + m_{\text{p}_{\text{wind}}_{\text{im}}}$$  \hspace{1cm} (5.3)

$m_{\text{p}_{\text{im}}}^\text{tot}$ corresponds to the amount of element $i$ newly synthesised and ejected by a star of initial mass $m$ (see Maeder 1992).

Other authors give their results in ejected masses, $EM$, (which some authors confusingly call yields, see Sect. 5.6.1):

$$EM_{\text{im}} = \int_0^{r(m)} \dot{M}(m, t) X_i^S dt + \int_{m(\text{rem})}^{m(r)} X_i(m_r) dm_r$$  \hspace{1cm} (5.4)

and production factors (PF) (see Woosley et al. 1995):

$$f_{\text{im}} = EM_{\text{im}}/[X_i^0(m - m(\text{rem}))]$$  \hspace{1cm} (5.5)

We also give our final results as ejected mass in order to compare our results with the recent literature.
# 5.4. Stellar wind contribution

Table 5.2: Initial mass and velocity, lifetime, final mass and stellar wind contribution to the yields (\(m_{\text{rm}}^{\text{wind}}\)). All masses and yields are in solar mass units, velocities are in \(\text{km s}^{-1}\) and lifetimes in million years. "A" means wind anisotropy has been included in the model. \(Z\) is the total metal content and is defined by: \(Z = 1 - X_{\text{H}} - X_{\text{He}} - X_{\text{He}}\).

<table>
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<th>(\tau_{\text{life}})</th>
<th>(M_{\text{final}})</th>
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Figure 5.2: Final masses as a function of the initial mass and velocity.

Table 5.3: Continuation of Table 5.2.

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5.4. Stellar wind contribution

Figure 5.3: Wind contribution to the yields of $^4$He (top left), $^{12}$C (top right), $^{14}$N (middle left), $^{22}$Ne (middle right), $^{16}$O (bottom left) and Z (bottom right), $m_{p_{\text{wind}}}$, as a function of the initial mass for the solar metallicity models.
5.4 Stellar wind contribution

Before we discuss the wind contribution to the stellar yields, it is instructive to look at the final masses shown in Fig. 5.2. We see that, below 25 $M_{\odot}$, all the non-rotating models have a similar final mass (for the same initial mass) while the rotating models lose significantly more mass. This is due to the fact that rotation enhances mass loss (see for example Maeder & Meynet 2000b). For WR stars ($M \gtrsim 30 M_{\odot}$), the new prescription by Nugis & Lamers (2000), including the effects of clumping in the winds, results in mass loss rates that are a factor two to three smaller than the rates from Langer (1989). The final masses of WR stars in Maeder (1992) are therefore noticeably smaller than in the present calculation.

The wind contribution to the stellar yields is presented in Tables 5.2 and 5.3 and Fig. 5.3. We can see that H-burning products (represented by $^4$He and $^{14}$N in Fig. 5.3) are ejected by stellar winds in the entire massive star range. On the other hand, He-burning products (represented by $^{12}$C and $^{22}$Ne in Fig. 5.3) are only ejected by winds in WR stars ($M \gtrsim 25 M_{\odot}$).

Concerning H-burning products, the wind contribution to the yields of our non-rotating models are comparable to those of the models of Maeder (1992) with lower mass loss rates for $M \lesssim 40 M_{\odot}$ as expected (similar final mass and core masses). For our rotating models with $M \lesssim 40 M_{\odot}$, these yields are significantly larger for two reasons. The first one is a slightly larger mass loss but the second reason, which is the most important, is the diffusion of H-burning products in the envelope of the star due to rotational mixing. For more massive stars ($M \gtrsim 40 M_{\odot}$), the mass loss effect is dominant and the yields from the different models are similar.

The He-burning products are produced deeper into the star. They are therefore ejected only by WR star winds. Since the new mass loss rates are reduced by a factor two to three (see Sect. 5.2), the yields from the winds in $^{12}$C and $^{22}$Ne are much smaller for the present WR stellar models (rotating and not). As is shown below, the SN contribution to the yields of $^{12}$C are larger in the present calculation and, as a matter of fact, the new $^{12}$C total yields are larger. Since $^{16}$O is produced even deeper in the star, the present contribution by winds to this yield are even smaller. $^{12}$C constituting the largest fraction of ejected metals, the conclusion for the wind contribution to the total metallic yield is the same as for $^{12}$C.

In general, the yields from rotating stars are larger than from non-rotating ones due to the extra mass loss and mixing. For very massive stars ($M \gtrsim 60 M_{\odot}$), the situation is reversed for He-burning products because of the different mass loss history. Indeed, rotating stars enter into the WR regime in the course of the main sequence (MS). The long time spent in particular in the WNL phase (see Meynet & Maeder 2003) results in the ejection of large amounts of H-burning products. Therefore, the star enters the WC phase with a smaller total mass and less He-burning products are ejected by winds (the mass loss being proportional to the actual mass of the star). We note that the comparison between the various non-rotating models (present calculation and Maeder 1992) shows the strong dependence on the mass loss prescription of the stellar wind
5.5 SN explosion contribution and abundance profiles

As said above, our SN yields, $m_{\text{SN}}$, have been calculated at the end of core Si-burning using the remnant mass, $M_{\text{rem}}$, given in Table 5.1. We calculated the SN yields at the end of core Si-burning instead of at the last time step of our calculation in order to compare our rotating and non-rotating models at the same evolutionary stage (the rotating model have not been followed through shell Si-burning). Furthermore, it is not the scope of this paper to include the contribution from explosive burnings. We therefore concentrate on yields of light elements which do not depend on the evolution beyond core Si-burning. The uncertainties due to the fact that the SN yields are calculated at the end of core Si-burning are discussed in Sect. 5.7.

5.5.1 Abundance profiles at the pre-supernova stage

Before discussing the SN yields, it is interesting to look at the abundance profiles of the models at the time at which the yields are calculated. Figures 5.4 and 5.5 show the abundance profiles for the main elements at the end of core Si-burning. They clearly show the enhancement of core sizes due to rotational mixing. They also allow us to see where elements are most abundant in the star. We can see in these graphs that shell He-burning burns more helium and therefore produces more carbon and oxygen when the initial mass (and to a lesser extent when the initial rotation) increases.

5.5.2 SN yields

The SN yields for $^4\text{He}$, $^{12}\text{C}$, $^{16}\text{O}$ and Z are shown in Fig. 5.6 and a more complete list is given in Tables 5.4, 5.5 and 5.6.

Negative SN yields of $^4\text{He}$ for WR stars

The carbon–oxygen (CO) core increases with the initial mass (see Table 5.1) until it occupies most of the star when mass loss is important ($M \gtrsim 30 M_\odot$, see Fig. 5.5). The CO core is free of helium while the initial $^4\text{He}$ abundance is 0.275. In the definition of $m_{\text{SN}}$, we subtract the initial composition to the final one. Therefore the CO core has a negative contribution to the $^4\text{He}$ yields. This is why $^4\text{He}$ SN yields become negative in WR stars. However, if mass loss is important before helium burning, the CO core will be rather small and the $^4\text{He}$ SN yields will be less decreased. This explains why the models with the high mass loss rates from Maeder (1992) (AM92 hereinafter), which gives the smallest final core masses, reach the least negative values. WR stars being produced from lower initial masses for rotating models, we see that the yields drop first for these models. We note that the total $^4\text{He}$ yields will stay positive due to the wind contribution.
Figure 5.4: Abundance profile at the end of core silicon burning for the non-rotating (left) and rotating (right) 12 (top), 15 (middle) and 20 (bottom) \( M_\odot \) models.
5.5. SN explosion contribution and abundance profiles

![Diagram showing abundance profiles for different models](image)

Figure 5.5: Abundance profile at the end of core silicon burning for the non-rotating (left) and rotating (right) 25 (top), 40 (middle) and 60 (bottom) $M_\odot$ models.
Carbon, oxygen and metallic yields

Let us first consider the total metallic yields. If mass loss is not dominant ($M \lesssim 30 \, M_\odot$), the larger the initial mass, the larger the metallic yields (because the various cores become larger). Rotation increases the core sizes by extra mixing and therefore the metallic yields are larger for rotating models. Overshooting also plays a role in the core sizes. The larger the overshooting parameter, the larger the cores. Since in AM92, the overshooting parameter is 0.2 $H_P$ and in the present calculation it is 0.1 $H_P$, we obtain smaller metallic yields for our non–rotating models compared to the yields of AM92, as expected. We see that our rotating models with a smaller overshooting parameter give higher yields than the AM92 ones showing the importance of rotational induced mixing in stars. For very massive stars ($M \gtrsim 30 \, M_\odot$), the higher the mass loss, the smaller the final mass and metallic yields.

The same explanations work well in general for carbon and oxygen. We therefore expected the yields in AM92 to be larger than the non–rotating star yields in this paper due to the larger overshooting parameter. However, we find that they are almost equal. This probably means that the carbon and oxygen SN yields have been slightly underestimated in AM92 by calculating these yields at the end of core C–burning.

If we compare our rotating and non–rotating models, we see that the SN metallic yields in general and $^{12}$C and $^{16}$O yields in particular are enhanced by rotation by a factor 1.5–2.5.

Table 5.4: SN explosion contribution to the yields ($m_{\text{SN}}$) for the 9, 85 and 120 $M_\odot$ models calculated from the structure at the end of He–burning.

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Table 5.5: Initial mass and velocity, final mass, remnant mass and SN explosion contribution to the yields \( m_{SN}^{SN} \) of solar metallicity models. All masses and yields are in solar mass units and velocities are in km s\(^{-1}\). "A" means wind anisotropy has been included in the model. \( Z \) is the total metal content and is defined by: \( Z = 1 - X_{H} - X_{He} - X_{He} \).

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5.6. Total stellar yields

Figure 5.6: SN contribution to the yields of $^4$He (top left), $^{16}$O (top right), $^{12}$C (bottom left) and Z (bottom right), $m_{\text{SN}}$, as a function of the initial mass for the solar metallicity models. The solid and dotted lines represent rotating and non–rotating models from the present calculations. The dotted–dashed and dashed lines represent respectively high and low mass loss rates models from Maeder (1992).

5.6 Total stellar yields

The total stellar yields, $m_{\text{tot}} = m_{\text{SN}} + m_{\text{wind}}$ (to be used for chemical evolution models using Eq. 2 from Maeder 1992), are presented in Tables 5.7, 5.8 and 5.9.
Table 5.7: Initial mass and velocity, final mass, remnant mass and total stellar yields \( (mp_{\text{SN}} + mp_{\text{wind}}) \) of solar metallicity models. All masses are in solar mass units and velocities in \( \text{km s}^{-1} \). "A" marks wind anisotropy has been included in the model. \( Z \) is the total metal content and is defined by: 

\[ Z = 1 - X_{\text{H}} - X_{\text{He}} - X_{\text{He}}^4 \]

These are the yields to be used in chemical evolution models.

|  \( M_{\text{ini}} \)  |  \( v_{\text{ini}} \)  |  \( M_{\text{final}} \)  |  \( M_{\text{rem}} \)  |  \( ^4\text{He} \)  |  \( ^{12}\text{C} \)  |  \( ^{14}\text{N} \)  |  \( ^{16}\text{O} \)  |  \( ^{20}\text{Ne} \)  |  \( ^{22}\text{Ne} \)  |  \( ^{24}\text{Mg} \)  |  \( Z \)  |
|----------------|-----------|----------------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------|
| 12             | 0         | 11.52          | 1.34           | 1.20E+0     | 8.69E-2     | 3.68E-2     | 2.07E-1     | 1.05E-1     | 6.80E-3     | 6.75E-3     | 4.57E-1     | 1.48E+1  |
| 12             | 300       | 10.20          | 1.46           | 1.60E+0     | 1.63E-1     | 4.04E-2     | 3.92E-1     | 1.58E-1     | 1.55E-2     | 1.36E-2     | 7.97E-1     | 1.76E+1  |
| 15             | 0         | 13.23          | 1.51           | 1.67E+0     | 1.53E-1     | 4.33E-2     | 4.34E-1     | 1.10E-1     | 1.60E-2     | 6.06E-2     | 9.19E-1     | 1.87E+1  |
| 15             | 300       | 10.32          | 1.85           | 1.57E+0     | 3.61E-1     | 3.52E-2     | 1.01E+0     | 2.26E-1     | 3.33E-2     | 4.27E-2     | 1.92E+0     | 2.45E+1  |
| 20             | 0         | 15.69          | 1.95           | 2.08E+0     | 2.16E-1     | 4.96E-2     | 1.20E-1     | 4.83E-1     | 3.64E-2     | 1.28E-1     | 2.17E+0     | 3.35E+1  |
| 20             | 300       | 8.76           | 2.57           | 1.62E+0     | 4.33E-1     | 4.33E-2     | 2.57E+0     | 6.80E-1     | 4.26E-2     | 1.19E-1     | 3.98E+0     | 4.47E+1  |
| 25             | 0         | 16.00          | 2.49           | 2.57E+0     | 3.68E-1     | 7.40E-2     | 2.17E+0     | 8.41E-1     | 5.22E-2     | 1.38E-1     | 3.74E+0     | 5.52E+1  |
| 25             | 300       | 10.04          | 3.06           | 2.14E+0     | 7.37E-1     | 7.38E-2     | 3.55E+0     | 1.08E+0     | 2.26E-2     | 1.48E+1     | 5.75E+0     | 6.63E+1  |
| 40             | 0         | 13.97          | 4.02           | 3.01E+0     | 6.49E-1     | 1.35E-1     | 6.12E+0     | 1.36E+0     | 5.58E-3     | 1.52E-1     | 8.65E+0     | 8.85E+1  |
| 40             | 300A      | 12.65          | 3.85           | 5.89E+0     | 3.11E+0     | 1.63E-1     | 5.36E+0     | 1.42E+0     | 8.05E-2     | 1.20E+1     | 1.05E+1     | 1.28E+1  |
| 60             | 0         | 14.52          | 4.30           | 6.96E+0     | 3.94E+0     | 2.26E-1     | 6.47E+0     | 1.81E+0     | 1.35E-1     | 1.73E-1     | 1.30E+1     | 1.85E+1  |
| 60             | 300A      | 14.57          | 4.32           | 1.28E+1     | 4.51E+0     | 2.98E-1     | 6.41E+0     | 1.75E+0     | 1.74E-1     | 1.44E-1     | 1.35E+1     | 1.85E+1  |
Table 5.8: Continuation of Table 5.7. These are the yields to be used in chemical evolution models.

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Table 5.9: Total stellar yields ($m_{\text{im}}^{\text{SN}} + m_{\text{im}}^{\text{wind}}$) for the 9, 85 and 120 $M_\odot$ models. These are the yields to be used in chemical evolution models.

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What is the relative importance of the wind and SN contributions? Figures 5.7 and 5.8 display the total stellar yields divided by the initial mass of the star, $p_{\text{im}}^{\text{tot}}$, as a function of its initial mass, $m$, for the non–rotating and rotating models respectively. The different shaded areas correspond from top to bottom to $p_{\text{im}}^{\text{tot}}$ for $^{4}\text{He}$, $^{12}\text{C}$, $^{16}\text{O}$ and the rest of the heavy elements. The fraction of the star locked in the remnant as well as the expected explosion type are shown at the bottom. The dotted areas show for $^{4}\text{He}$, $^{12}\text{C}$ and $^{16}\text{O}$ the wind contribution.

For $^{4}\text{He}$ (and other H–burning products like $^{14}\text{N}$), the wind contribution increases with mass and dominates for $M \gtrsim 22M_\odot$ for rotating stars and $M \gtrsim 35M_\odot$ for non–rotating stars. As said earlier, for very massive stars, the SN contribution is negative and this is why $p_{\text{Hem}}^{\text{tot}}$ is smaller than $p_{\text{Hem}}^{\text{wind}}$. For $^{12}\text{C}$, the wind contributions only start to be significant above the mass limits given above (22 and 35 $M_\odot$ for rotating and non–rotating models respectively). This is expected because a star must have ejected most of its helium before it can eject carbon. Above these mass limits, the contribution from the wind and the SN are of similar importance. For $^{16}\text{O}$, the wind contribution remains very small because in the present calculation, winds do not peel off the CO core significantly.

5.6.1 Comparison with the literature

We first compare our results with Maeder (1992) (AM92) because of the comparable mass range and physical ingredients of the model. This allows us to better quantify the impact of rotation. Afterwards, we compare the yields of the non–rotating models with other authors.

Comparison with Maeder (1992)

The comparison is shown in Fig. 5.9 where the solid and dotted lines represent rotating and non–rotating models from the present calculations. The dotted–dashed and dashed lines represent respectively high and low mass loss rates models from Maeder (1992). For H–burning products, the yields of the rotating models are usually higher than those of non–rotating models. This is due to larger cores and larger mass loss.
5.6. Total stellar yields

![Diagram](image)

\[ \nu_{\text{ini}} = 0 \text{ km s}^{-1} \]

Figure 5.7: Stellar yields divided by the initial mass, \( p^{\text{tot}}_{\text{ini}} \), as a function of the initial mass for the non-rotating models at solar metallicity. The different total yields (divided by \( m \)) are piled up. \(^4\text{He} \) yields are delimited by the top solid and long dashed lines (top shaded area), \(^{12}\text{C} \) yields by the long dashed and short–long dashed lines, \(^{16}\text{O} \) yields by the short–long dashed and dotted–dashed lines and the rest of metals by the dotted–dashed and bottom solid lines. The bottom solid line also represents the mass of the remnant \( (M^{\text{rem}}/m) \). The corresponding SN explosion type is also given. The wind contributions are superimposed to these total yields for the same elements between their bottom limit and the dotted line above it. Dotted areas help quantify the fraction of the total yields due to winds. Note that for \(^4\text{He} \), the total yields is smaller than the wind yields due to negative SN yields (see Section 5.5.2).
Figure 5.8: Stellar yields divided by the initial mass, $p_{im}^{\text{tot}}$, as a function of the initial mass for the rotating models at solar metallicity. The different total yields (divided by $m$) are piled up. $^4$He yields are delimited by the top solid and long dashed lines (top shaded area), $^{12}$C yields by the long dashed and short-long dashed lines, $^{16}$O yields by the short-long dashed and dotted-dashed lines and the rest of metals by the dotted-dashed and bottom solid lines. The bottom solid line also represents the mass of the remnant ($M_{\text{rem}}^{\text{int}}/m$). The corresponding SN explosion type is also given. The wind contributions are superimposed to these total yields for the same elements between their bottom limit and the dotted line above it. Dotted areas help quantify the fraction of the total yields due to winds. Note that for $^4$He, the total yields is smaller than the wind yields due to negative SN yields (see Section 5.5.2).
5.6. Total stellar yields

Figure 5.9: Total stellar yields of $^4$He (top left), $^{12}$C (top right), $^{14}$N (middle left), $^{16}$O (middle right) and Z (bottom left), $m_{\text{H}_2}\text{tot}$ and ratio $\Delta Y/\Delta Z$ (bottom right) as a function of the initial mass for the solar metallicity models.
CHAPTER 5. YIELDS OF ROTATING STARS AT SOLAR METALLICITY

\[ \nu_{\text{ini}} = 0 \text{ km s}^{-1} \]

\[ \nu_{\text{ini}} = 300 \text{ km s}^{-1} \]

Figure 5.10: Product of the stellar yields, \( m_{\text{tot}} \), by Salpeter’s IMF (multiplied by an arbitrary constant: \( 1000 \times M^{-2.35} \)), as a function of the initial mass for the non–rotating (left) and rotating (right) models at solar metallicity. The different shaded areas correspond from top to bottom to \( m_{\text{tot}} \times 1000 \times M^{-2.35} \) for \(^4\text{He}, ^{12}\text{C}, ^{16}\text{O}\) and the rest of the heavy elements. The dotted lines show for \(^4\text{He}, ^{12}\text{C}\) and \(^{16}\text{O}\) the wind contribution. Note that for \(^4\text{He}\), the total yields are smaller than the wind yields due to negative SN yields (see Section 5.5.2).

Nevertheless, between about 15 and 25 \( M_\odot \), the rotating yields are lower. This is due to the fact that the winds do not expel much H–burning products yet and more of these products are burnt later in the pre–supernova evolution (giving negative SN yields). The same dependence of the initial mass of the yields of non–rotating models as for the yields of rotating models is seen but shifted to higher masses (plateau phase around 20 \( M_\odot \) for rotating models and around 30 \( M_\odot \) for non–rotating models). We also note that our rotating model yields are similar or larger than the high mass loss rates models from AM92.

Concerning He–burning products, below 40 \( M_\odot \), most of the \(^{12}\text{C}\) comes for the SN contribution. In this mass range, rotating models having larger cores also have larger yields (factor 1.5–2.5). We notice once again that the same dependence on the initial mass of the yields of non–rotating models as for the yields of rotating models is seen but shifted to higher masses. Above 40 \( M_\odot \), where mass loss dominates, the yields from the rotating models are closer to those of the non–rotating models and are smaller than those of AM92. Since very massive stars are much less numerous, we expect the overall \(^{12}\text{C}\) yield of rotating models to be larger than those of AM92. This can be seen in Fig. 5.10 where the yields are multiplied by Salpeter’s IMF. The situation for \(^{16}\text{O}\) and metallic yields is similar to carbon. Therefore \(^{16}\text{O}\) and metallic yields are larger for

113
our rotating models compared to our non–rotating ones by a factor 1.5–2.5. Metallic yields from AM92 are found in between our rotating and non–rotating models because of the higher value of overshooting used in AM92 (See Section 5.5.2).

An important result is the ratio between $^4\text{He}$ and metallic yields, $\Delta Y/\Delta Z$. Observed values of $\Delta Y/\Delta Z$ are around 2.8 (2.8 ± 0.5 in Izotov & Thuan 2004, and references therein). In order to obtain a theoretical value of $\Delta Y/\Delta Z$, we need to include the contribution of the low and intermediate mass stars ($M < 9 M_\odot$), which we did not consider in this work. We nevertheless calculated an estimate of $\Delta Y/\Delta Z$ using the following method. For stars less massive than 9 $M_\odot$, the metallic yields are very small and we neglected them. For $^4\text{He}$ yields, we used the results of AM92 multiplied by a factor (1.38 and 1.22 for the rotating and non–rotating models respectively) so that the yields match for the 9 $M_\odot$ models. We then calculated $\Delta Y/\Delta Z$ using the following formula:

$$\Delta Y/\Delta Z = \frac{\int_{M_l}^{M_u} \frac{\dot{M}_{\text{mer}}}{\dot{M}_{\text{BH}}} \varphi(m) \, dm + \int_{M_l}^{M_u} \frac{\dot{M}_{\text{ind}}}{\dot{M}_{\text{BH}}} \varphi(m) \, dm}{\int_{M_l}^{M_u} \frac{\dot{M}_{\text{mer}}}{\dot{M}_{\text{BH}}} \varphi(m) \, dm}$$

where $m$ is the initial total mass, $M_l = 1$ and $M_u = 120$ are respectively the lower and upper mass limits of the integration, $M_{\text{BH}}$ is the lower mass limit for black hole (BH) formation and $\varphi(m)$ the initial mass function (IMF). We used Salpeter’s IMF:

![Figure 5.11: Ratio between $^4\text{He}$ and metallic yields, $\Delta Y/\Delta Z$, as a function of the minimum mass for BH formation, $M_{\text{BH}}$. The solid, dotted, dotted–dashed and dashed lines correspond respectively to the rotating models, non–rotating models, AM92 solar metallicity models (all using Salpeter’s IMF) and the observed value from Izotov & Thuan (2004) with its uncertainty.](image)
\[ \varphi(m) = \frac{dN}{dM} = m^{-2.35} \]

For stars with an initial mass higher than \( M_{\text{BH}} \), we only considered the wind contributions (assuming that the whole pre–SN star will be swallowed by the central black hole).

Due to extreme mass loss, the very massive stars in AM92 have a very small final mass and do not produce BH. However, in this work, the final core masses of very massive stars (\( \sim 100 \, M_\odot \)) are larger than in AM92 by a factor about four. We therefore expect the same outcome (BH formation without ejection) for the very massive stars as for the stars with masses around \( 50 \, M_\odot \) at solar metallicity. The results are presented in Fig. 5.11. A ratio \( \Delta Y/\Delta Z \) of 2.8 corresponds respectively to \( M_{\text{BH}} \) values of 30 and 37.5 \( M_\odot \) for rotating and non–rotating models respectively. The value of 37.5 for non–rotating stars is close to the lower limit for direct BH formation of 40 \( M_\odot \) derived by Fryer (1999). This being said, we want to precise that these values are only estimates and are to be used with caution. The trends in Fig. 5.11 are however clear. We see that \( M_{\text{BH}} \) must be lower for rotating models if \( M_{\text{BH}} \) is the dominant parameter for \( \Delta Y/\Delta Z \).

We also see that both ratios in this work are larger than the value from AM92. This is due to larger \(^4\text{He} \) yields in both rotating and non–rotating models and to smaller metallic yields for the non–rotating models in the mass range 10–30 \( M_\odot \) for the present calculation. For \( \Delta Y/\Delta Z \) from individual masses (Fig. 5.9 bottom right), as in AM92, we note an increase at the very massive star end due to extreme mass loss.

**Comparison with other authors**

In order to compare our results with other authors, the ejected masses, \( EM \), defined by Eq. 5.4 in Sect. 5.3, are presented in Tables 5.10 and 5.11.
Table 5.10: Initial mass and velocity, remnant mass and total ejected masses (EM) of solar metallicity models. All masses are in solar mass units and velocities are in km s\(^{-1}\). "A" means wind anisotropy has been included in the model. Note that this table is given for comparison with other recent publications and does not correspond to our definition of yields (see Sect. 5.3)!

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5.6. Total stellar yields

Figure 5.12: Total ejected masses (EM) of $^1$H, $^4$He, $^{12}$C and $^{16}$O (left) and $^{17}$O, $^{18}$O, $^{20}$Ne and $^{22}$Ne (right) for different non-rotating models at solar metallicity. Solid lines (HMM04) represent our results, dotted lines (LC03) show the results from Limongi & Chieffi (2003), long–short dashed lines (TNH96) show the results from Thielemann et al. (1996), dashed lines (RHW02) represent the results from Rauscher et al. (2002) and dotted–dashed (WW95) lines show the results from Woosley & Weaver (1995).

Figure 5.12 shows the comparison with four other calculations: Limongi & Chieffi (2003) (LC03), Thielemann et al. (1996) (TNH96), Rauscher et al. (2002) (RHW02) and Woosley & Weaver (1995) (WW95). For LC03, we chose the remnant masses that are closest to ours (models 15D, 20B, 25A). The uncertainties related to convection and the $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction are dominant. Therefore, before we compare our results with other models, we briefly mention here which treatment of convection and $^{12}$C($\alpha$, $\gamma$)$^{16}$O rate other authors use:

- Limongi & Chieffi (2003) use Schwarzschild criterion (except for the H convective shell that forms at the end of core H–burning) for convection without overshooting. For $^{12}$C($\alpha$, $\gamma$)$^{16}$O, they use the rate of Kunz et al. (2002) (K02).

- Thielemann et al. (1996) use Schwarzschild criterion for convection without overshooting. For $^{12}$C($\alpha$, $\gamma$)$^{16}$O, they use the rate of Caughlan et al. (1985) (CF85).

- Woosley & Weaver (1995) use Ledoux criterion for convection with semiconvection. They use a relatively large diffusion coefficient for modeling semiconvection. Moreover non–convective zones immediately adjacent to convective regions are slowly mixed on the order of a radiation diffusion time scale to approximately allow for the effects of convective overshoot. For $^{12}$C($\alpha$, $\gamma$)$^{16}$O, they use the rate

- Rauscher et al. (2002) use Ledoux criterion for convection with semiconvection. They use the same method as WW95 for semiconvection. For $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, they use the rate of Buchmann (1996) (BU96) multiplied by 1.2.

- In this paper (HMM04), we use Schwarzschild criterion for convection with overshooting. For $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, we use the rate of Angulo et al. (1999) (NACRE).

A comparison of the different reaction rates and treatment of convection is presented in Hirschi et al. (2004). The comparison of the ejected masses is shown in Fig. 5.12. The $^{4}\text{He}$ and $^{16}\text{O}$ yields are larger when respectively the helium and carbon–oxygen cores are larger. This can be seen by comparing our models with those of RHW02 and LC03 (Fig. 5.12 and respective tables of core masses).

For $^{12}\text{C}$ yields, the situation is more complex because the larger the cores, the larger the central temperature and the more efficient the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. If we only consider the effect of this reaction we have that the larger the rate, the smaller the $^{12}\text{C}$ abundance at the end of He–burning and the smaller the corresponding yield (and the larger the $^{16}\text{O}$ yield). This can be seen in Fig. 5.12 by comparing our $^{12}\text{C}$ and $^{16}\text{O}$ yields with those of LC03 (We both use Schwarzschild criterion). Indeed the NACRE rate is larger than the K02 one so our $^{12}\text{C}$ yield is smaller. THN96 (who also use Schwarzschild criterion) using the rate of Caughlan et al. (1985) which is even larger, obtain an even smaller $^{12}\text{C}$ yield. When both the convection treatment and the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate are different, the comparison becomes more complicated. Nevertheless, within the model uncertainties, the yields of various models agree.

As will be discussed in the next section, our $^{20}\text{Ne}$ yields are upper limits due to the possible destruction of this element by shell and explosive burnings. Figure 5.12 shows that our results lies above the other group results but the difference is small. For $^{17}\text{O}$ yields, all recent calculations agree rather well and differ from the WW95 results because of the change in the reaction rates (especially $^{17}\text{O}(p, \alpha)^{14}\text{N}$, see Aubert et al. 1996). $^{18}\text{O}$ and $^{22}\text{Ne}$ are produced by $\alpha$–captures on $^{14}\text{N}$ (see Fig. 5.13). As said in Sect. 5.3, $^{22}\text{Ne}$ is not followed during the advanced stages and we had to use a special calculation for its yield. We see nevertheless that the result is very close to other calculations.

### 5.7 Uncertainties of pre–supernova structure and SN yields

We chose a value of $M_{\text{rem}}$ based on the size of the CO core and we calculated the SN yields at the end of core Si–burning. In this section, we discuss the possible uncertainties of our SN yields. We divide this discussion into three parts. Firstly, we answer the question: What happens between core Si–burning and the collapse of the star? Secondly, we use another value of $M_{\text{rem}}$ to see the impact the choice of the remnant mass has on the yields. Finally we look at the explosive burning contribution.
5.7. Uncertainties of pre-supernova structure and SN yields

Figure 5.13: Abundance profile of light elements at the end of central silicon burning for the non–rotating (left) and rotating (right) 20 $M_{\odot}$ models.

5.7.1 What happens after core Si–burning?

Before we answer this question, we need to introduce neutron excess ($\eta(t) = \sum_i (N_i - Z_i)Y_i$) and electron abundance or mole number ($Y_e = (1 - \eta)/2 = (\sum_i Z_i Y_i)$). We do not follow neutron excess (or $Y_e$) in our calculations due to the small nuclear reaction network used in the advanced stages. Neutron excess is important for the nucleosynthesis of neutron rich elements (s and r–processes), the star collapse. Indeed neutron excess is necessary in order to produce neutron rich elements. Electron captures during Si–burning increases neutron excess and also reduces the electron pressure and this (with photo–disintegration) will allow the core to collapse (Woosley et al. 2002). Neutron excess is however not important for the structure evolution and the energy production before core Si–burning (Thielemann & Arnett 1985). Even during core Si–burning, its main effect on structure is a mean molecular weight ($\mu = (Y_e + \sum_i Y_i)^{-1}$) gradient which prevents the growth of the core if one uses Ledoux criterion for convection. We use Schwarzschild criterion for convection and therefore a change in $Y_e$ would not alter the size of the Si–burning core. Our results until the end of core Si–burning are therefore not affected by the fact that we did not follow $Y_e$. One could still argue that the use of Schwarzschild criterion is less adapted in this situation than the Ledoux criterion but different authors still use different prescriptions (Limongi & Chieffi 2003; Rauscher et al. 2002). So what happens between core Si–burning and the core collapse? Either the core is heavy enough (more massive than its Oppenheimer-Volkov mass) and collapses or a shell episode of Si–burning (or possibly two) occurs before the collapse. For a reasonable range in the explosion parameters (like the neutron excess and the entropy) which
vary with the initial mass, the Oppenheimer-Volkov mass can vary between 1.15 and 1.79 $M_\odot$ (Woosley et al. 2002). Our non-rotating models were followed until first shell Si-burning but we did not follow $Y_e$. It is therefore possible that some of our models should collapse before shell Si-burning occurs. Note also that using Ledoux criterion instead of Schwarzschild criterion would probably produce smaller iron cores.

How different are the yields calculated after core Si-burning and first shell Si-burning? Even though our value of $M_{\text{rem}}$ is large, we can already see a large difference between the $^{28}\text{Si}$ yields calculated after shell (Table 5.12) and core (Table 5.5) Si-burning (See also Fig. 5.14). We also see in Fig. 5.14 that explosive nucleosynthesis will change significantly these yields. Figure 5.14 shows as well that $^{20}\text{Ne}$ has also burnt in a shell to produce $^{16}\text{O}$ and $^{24}\text{Mg}$. The reality of this feature is not certain. Indeed, in other calculations like those of Nomoto et al. (1997), $^{20}\text{Ne}$ does not show such a feature. Nevertheless, particular processes can happen in shells like the merging of C and O-burning shells in the 20 $M_\odot$ model in Rauscher et al. (2002). If $^{20}\text{Ne}$ burns, we can see that the $^{20}\text{Ne}$ and $^{24}\text{Mg}$ yields are strongly affected (Tables 5.12 and 5.5) while $^{16}\text{O}$ yield is only slightly modified (shell O-burning destroys some $^{16}\text{O}$ while shell Ne-burning produces it). We also see that the products of H and He-burnings are the same at the end of core and shell Si-burning.
Table 5.12: Initial and final masses, remnant mass and SN yields ($m_p^{SN}$) of non-rotating models after shell Si burning. All masses are in solar mass units.

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Figure 5.14: Abundance profile at the end of core silicon burning (left) and at last model (right) for the non-rotating 25 $M_\odot$ models. Shaded areas represent the mass range where explosive C, Ne, O, Si–burnings (from right to left) would take place. The shaded area at the centre of the star corresponds to the zone where abundances are determined by nuclear statistical equilibrium and most abundant elements belong to the iron group (+ $^4$He). See Sect. 5.7.3 for explanations.

5.7.2 SN yields dependence on $M_{\text{rem}}$

In order to see the influence of $M_{\text{rem}}$ on the yields, we present here yields with an unrealistic $M_{\text{rem}} = 0$. Comparing the SN yields obtained with $M_{\text{rem}} = 0$ (Tables 5.13 and 5.14) with the "normal" yields (Tables 5.5 and 5.6), we see the following differences: a) For H and He-burning products, the difference is usually very small and it is due to the fact that we deduct the initial abundance to the final one. This is best seen for $^4$He (for which the initial abundance is large) where the smaller the remnant mass, the more initial $^4$He deducted and the lower the SN yield, we can see that the $^{12}$C SN yields are unaffected by the change in $M_{\text{rem}}$ (as expected since these elements are produced in the outer shells). b) For heavier multiple–$\alpha$ elements, the situation is different: the lower the remnant mass, the more elements are ejected and the higher the SN yields. Once again this affects most significantly $^{28}$Si (and heavier elements) yields. The increase in $^{16}$O yield is important but is less correct because explosive burning will destroy part of $^{16}$O as we shall see now.
Table 5.13: Initial mass and velocity, final mass, remnant mass and SN yields ($m_{SN}^{\text{final}}$) using $M_{\text{rem}} = 0$ of solar metallicity models. All masses are in solar mass units and velocities are in $\text{km s}^{-1}$. "A" means wind anisotropy has been included in the model.

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<td>-4.25E-4</td>
<td>-6.00E-4</td>
<td>-5.54E-5</td>
<td>-5.68E-5</td>
<td>-3.22E-4</td>
<td>6.97E-3</td>
<td>2.72E-1</td>
<td>3.60E-2</td>
<td>1.30E-1</td>
<td>5.55E-6</td>
</tr>
</tbody>
</table>
Figure 5.15: Abundance profile at the end of central silicon burning for the non–rotating (left) and rotating (right) 20 $M_\odot$ models. Shaded areas represent the mass range where explosive C, Ne, O, Si–burnings (from right to left) would take place. The shaded area at the centre of the star corresponds to the zone where abundances are determined by nuclear statistical equilibrium and most abundant elements belong to the iron group (+ $^4$He).

5.7.3 Explosive nucleosynthesis

We did not simulate the explosion or the explosive nucleosynthesis. Nevertheless, in order to have a rough estimate of where explosive burning might occur in our models, we calculated the maximum shock temperature as a function of radius using Eq. (37) in Woosley et al. (2002) and fixing the kinetic energy, $KE$, to $2 \times 10^{51}$ erg. This equation assumes that the matter inside the shock is isothermal and that its heat capacity is in the radiation field. Woosley et al. (2002) also gives the temperature range (in 10$^9$ K units) of each explosive burning stage: 4–5 for silicon, 3–4 for oxygen, 2.5–3 for neon and 1.8–2.5 for carbon. Using this information, we superimposed these limits on our pre–supernova structures (Figs. 5.14 and 5.15). However, in case of a delayed explosion, matter has time to fall towards the central object and the radius at the time of the shock passage is smaller than the initial one. Indeed, using $KE = 10^{51}$ erg, matter that sits at a radius of 4412 km at the onset of collapse will be found at 3700km (limit for complete Si exhaustion in explosive Si–burning) at the time of the shock passage for a delay time of 0.5s (see Thielemann et al. 1996, for details). If we use $KE = 2 \times 10^{51}$ erg, the limit for complete Si exhaustion in explosive Si–burning is at 4400 km (equal to the limit using $KE = 10^{51}$ erg in an explosion with a 0.5s delay). Therefore the complete Si exhaustion limit ($T = 5 \times 10^9$ K) in Fig. 5.15 (and Fig. 5.14) is realistic if the star does not contract.
significantly before the explosion and if a) We have a prompt explosion and $KE = 2 \times 10^{51}$ erg or b) The delay time is 0.5 s and $KE = 10^{51}$ erg. Figure 5.14 seems to indicate that central parts around the iron core contract during shell Si-burning. In Figs. 5.14 and 5.15, starting from the cooler parts, we see that explosive carbon burning does not reach carbon rich layers and therefore carbon abundance as well as the abundance of the other H and He-burnings products will not be affected. However, for heavier elements (starting with $^{20}$Ne), explosive burning plays an important role. For $^{20}$Ne, our yields are upper limits since it cannot be created by explosive carbon burning and a fraction of it is destroyed by explosive neon burning. $^{24}$Mg can be created by explosive neon burning and destroyed by explosive oxygen burning and its abundance can still be changed significantly. $^{16}$O, like $^{24}$Mg, can be created by explosive neon burning and destroyed by explosive oxygen burning. However oxygen is the most abundant element between the CO and silicon cores. This means that even if a fraction of $^{16}$O is destroyed or produced by explosive burnings, the total SN yields will not change significantly (less than ten percents with our choice of $M_{\text{rem}}$). O and Si-burnings products ($^{28}$Si and heavier elements) are strongly affected by explosive burning. Comparison with observation suggest that the quantity of these elements ejected by the SN are mostly created in explosive burning (Woosley et al. 2002). Therefore our yields for elements heavier than $^{28}$Si (included) should be taken as rough estimates at best.

At the end of this section about uncertainties of SN yields, we can draw the following conclusions:

- Products of H and He-burnings are independent of the SN explosion and explosive nucleosynthesis and their yields are correctly calculated. Our results for the non-rotating models correspond therefore well with other calculations and differences can be understood in the light of the treatment of convection and the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction rate used.

- Even if $^{16}$O yields are affected slightly by explosive nucleosynthesis, the final total results is not modified significantly with our choice of $M_{\text{rem}}$. The comparison of our non-rotating models with other calculations is in good agreement, especially with LC03 who also use Schwarzschild criterion.

- The $^{20}$Ne yields can be changed due to shell burnings and explosive nucleosynthesis. They are to be considered as upper limits because $^{20}$Ne is destroyed by both processes. Nevertheless, by comparing its yield with other groups’ results, we find a good agreement.

- Even though $^{22}$Ne is not followed during the advanced stages and we had to use a special calculation for its yield, we see that the result is very close to other calculations.

- $^{28}$Si and heavier elements are mainly determined by explosive nucleosynthesis and the yields we provide are to be taken at best as rough estimates.
5.7. Uncertainties of pre-supernova structure and SN yields
Chapter 6

Conclusion and perspectives

This chapter summarises the work done, the results obtained and future prospects.

6.1 Geneva stellar evolution code

The Geneva evolution code was improved in order to model the pre-supernova evolution of rotating massive stars. A new adaptative reaction network was implemented in the Geneva code for the advanced burning stages. At the present time, the network used is a multiple–α elements chain between carbon and nickel (and α–particles). This small network is sufficient to follow the energy production and the evolution of the abundance of the main elements. It is also very fast. In the future, it is planned to increase the network size in order to follow the evolution of the neutron excess and of minor elements. The discretisation of the structure equations was modified in order to damp instabilities occurring during the advanced stages. For this purpose, Sugimoto’s prescription was used (Sugimoto 1970). Dynamical shear was introduced and added to the other mixing processes (horizontal turbulence, secular shear and meridional circulation). Other smaller modifications, like the treatment of convection as a diffusive process from O–burning onwards, were implemented.

6.2 Pre-supernova evolution at solar metallicity

Using the updated model, a set of stellar models at solar metallicity with and without rotation and with masses equal to 12, 15, 20, 25, 40 and 60 $M_\odot$ was calculated.

Concerning the evolution of rotation during the advanced stages, the angular velocity increases regularly with the successive contraction of the core while the angular momen-
6.3. Stellar yields

Tum does not change significantly (only convection creates spikes along its profile). This
means that one can have a good estimate of the pre-collapse angular momentum at the
end of He-burning. Comparing the pre-SN models with the criteria for collapsar pro-
genitors (Woosley & Heger 2003), one can see that WR stars are possible progenitors
of collapsars. However, in this work the effects of magnetic fields were neglected. Fur-
ther developments will be very interesting for the formation of both GRBs and neutron
stars. Dynamical shear, although very efficient, only smoothen sharp angular velocity
gradients but does not transport angular momentum over great distances.

Rotation significantly affects the pre-supernova models by the impact it has during
H and He-burnings. Two mass groups are distinguishable, where either rotationally
induced mixing dominates for $M < 30 M_\odot$ or rotationally increased mass loss dominates
for $M > 30 M_\odot$ as already discussed in Meynet & Maeder (2003).

Rotation affects the lower mass limits for the presence of convection during central
carbon burning, for iron core collapse supernovae and for black hole formation. The
effects of rotation on pre-supernova models are most spectacular for stars between 15
and 25 $M_\odot$. Indeed, rotation changes the supernova type (IIb or Ib instead of II),
the total size of progenitors (Blue instead of Red SuperGiant) and the helium and
CO core sizes by a factor $\sim 1.5$ (bigger in rotating models). For Wolf-Rayet stars
($M > 30 M_\odot$), even if the pre-supernova models are similar between rotating and non-
rotating models, their previous evolution is different (Meynet & Maeder 2003). The
models were compared with the literature. The biggest differences are the final mass
and the various core masses. Bigger cores are obtained in the present calculation.
This is due in part to the treatment of convection (use of Schwarzschild criterion and
overshooting) but mainly to rotation.

6.3 Stellar yields

A new set of stellar yields of rotating stars at solar metallicity was calculated, cov-
ering the entire massive star range (9–120 $M_\odot$). The evolution and nucleosynthesis
were followed in general until core silicon burning. Since the SN explosion and explosive
nucleosynthesis were not simulated, the yields of elements heavier than and including
silicon are to be taken at best as rough estimates. Anyway, these yields should be
determined by explosive nucleosynthesis calculations. The results for the non-rotating
models are consistent with other calculations and differences can be understood in the
light of the treatment of convection and the rate used for $^{12}$C($\alpha, \gamma$)$^{16}$O. This assesses
the accuracy of the calculations and gives a safe basis for the yields of the rotating stars.

The contributions to stellar yields by winds and Supernova explosion were calculated
separately. For each, the results were compared with the calculations from Maeder
(1992) (AM92). For the wind contribution, the rotating models have larger yields than
the non-rotating ones because of the extra mass loss and mixing due to rotation. Since
mass loss was decreased compared to AM92, heavy elements yields are lower in the
present calculations. For the SN yields and for masses below $\sim 30 M_\odot$, rotating models
have the highest yields followed by AM92 and the present non-rotating models (due to
a reduced overshooting parameter). The $^{12}$C and $^{16}$O yields are increased by a factor 1.5–2.5 by rotation in the present calculation. When mass loss is dominant (above $\sim 30 \, M_\odot$), the rotating and non-rotating models give similar yields which are larger than in AM92 due to the different mass loss prescription.

When the two contributions are added, the rotating model yields of all the heavy elements are the largest below $\sim 30 \, M_\odot$. Rotation increases the yields of heavy elements by a factor 1.5–2.5. For very massive stars, the situation can vary from one mass to the next due to the extreme mass loss. The ratio between $^4$He and metallic yields, $\Delta Y/\Delta Z$ is comparable with recent observed values (Izotov & Thuan 2004) if the minimum mass for black hole formation, $M_{BH}$, is around 30 and 37.5 $M_\odot$ for rotating and non-rotating stars respectively. Even though this comparison has to be considered as an estimate, it confirms that $M_{BH}$ is lower for rotating stars than for non-rotating stars.

6.4 Perspectives

6.4.1 Computer models and simulations

There are several modifications to the code planned in the near future. The first one is the extension of the nuclear reaction network. Then, there is the update of the neutrino energy loss rates and the modification of the equation of state. These modifications should allow the models to follow the evolution until the collapse of the core. It is planned to calculate grids of stellar models at different metallicities with the updated code. A denser grid than the set of models calculated here will allow to put tighter constraints on the mass limits for WD, NS and BH formation and for convection core C–burning at different metallicities.

Comparisons of these models with observations will be done. Rotating models have already passed many tests: a) Nitrogen surface enrichment during the MS (Maeder & Meynet 2000b; Heger & Langer 2000). b) the ratio of blue to red supergiants at different metallicities which is very sensitive to different parameters of the models like the treatment of convection, overshooting and mass loss (Maeder & Meynet 2001). c) The WR star populations at different metallicities (Meynet & Maeder 2003, 2004). Linked to the WR star populations are the number ratios between the different types of SNe. Prantzos & Boissier (2003) show that rotating model fit the observations for the ratio between SNIb and SNII.

Observations of the interaction between the SN and the circumstellar medium can give information about the density, structure and chemical composition of the matter lost in stellar winds (Chevalier & Fransson 2003). Comparisons with the models could further constrain the models and the mass loss. Observations of the angular momentum contained in NSs as well as WDs will constrain models on the transport of angular momentum.

The Geneva model can be developed in order to model the evolution of interacting binaries. The evolution of binary stars is important because they are thought to produce type Ia SNe and because binary systems are frequent among massive stars (works
6.4. Perspectives

by Langer, Vanbeveren and Podsiadlowski). On the observational side, it would be interesting to have better constraints on the fraction of interacting binaries at different metallicities.

6.4.2 Explosive nucleosynthesis

Calculation of the explosive nucleosynthesis as well as post-processing will allow to obtain the evolution of every element and the full set of stellar yields. These yields will be compared with observations of SN explosions and give constraints on the various explosion parameters (explosion energy, neutron excess, fallback, mixing). The yields of low metallicity massive stars can be compared with the abundance observed in extremely metal poor stars (Umeda & Nomoto 2003; Bessell et al. 2004).

The new set of yields will be used in models of the chemical evolution of galaxies and be compared to observations. It will be interesting to see the effect of rotation on the carbon and magnesium evolution. The yields of $^{56}\text{Ni}$, $^{56}\text{Co}$, $^{57}\text{Co}$, $^{44}\text{Ti}$ $^{26}\text{Al}$ and $^{60}\text{Fe}$ will also be compared with the INTEGRAL satellite observations (Prantzos 2004). The satellite will constrain the production site of $^{26}\text{Al}$ and $^{60}\text{Fe}$ (massive stars: wind or SNe, other sources like Novae). INTEGRAL will also give very interesting information about young SN remnants with $^{44}\text{Ti}$. We can hope that a close by SN will explode during the lifetime of INTEGRAL, in which case, we could have unique gamma–ray observations of a SN (Ni and Co).

Note that the simultaneous treatment of mixing and burning is an improvement that is planned for nucleosynthesis calculations.

6.4.3 Multi–dimensional simulations

The world is 3D, so the obvious goal is to built 3D simulations in the long run. 3D simulations are a real challenge for both numerical methods and computers. It will take decades before 3D simulations are common practice. Some problems might not even be treated in 3D. A example is the whole pre–SN evolution described in this work. The reason is simple: the time steps needed to follow convective motions are much smaller than the MS lifetime (leading to the assumption of instantaneous mixing in the early stages) and the calculations would go on forever. This pessimistic comment being said, every other problem will one day be approached with 3D simulations. Convection is clearly a 3D problem. Although maybe only snapshots of the evolution of the star might be followed, 3D simulations are crucial to improve our understanding of convection in cores and envelopes of stars as well as for constraining the extent of overshooting. Rotation is at least a 2D problem. Pulsations and magnetic fields are also multi–D. There is the study of circumstellar matter, mass loss, tidal interactions, disks and mass transfer between two interacting binaries. Mergers of stars also need to be simulated in multi–D. Finally, Novae and Supernova explosions are multi–D phenomena.

Multi–D simulations have already been developed in several areas. The largest efforts have been invested in the simulation of SNe. 3D models are now available for type Ia (Reinecke et al. 2002), type II (Janka et al. 2003; Livne et al. 2004; Blondin et al. 2003).
Models of disks around stars have also been developed (Townsend et al. 2004). In 2002, a conference has been dedicated to “3D stellar evolution”. The proceedings have been published (Turcotte et al. 2003). They show that the development of methods in multi-D is well underway concerning stellar evolution. Codes are already developed to study meridional circulation in 3D (Talon et al), convection in 3D (Rogers et al, Woodward et al, Asida for 2D). Radiative transfer models in 3D have also been developed and give great results (Asplund et al).

Concerning pre-SN evolution, multi-D works have been started (see for example Bazan & Arnett 1998). Two large projects are being developed for stellar evolution in 3D: Djehuty (Bazan, Eggleton et al) and FLASH (Plewa). All these works, and many others not cited here, show the way to follow. One plan is to develop a code to follow Si-burning and the early collapse of the iron core in 3D. Indeed, the time steps are small enough for reasonable computation times. Furthermore, since stars are rotating, the polar regions will probably collapse first. This situation cannot be simulated properly in 1D.
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Publications Related to this Thesis

Referred Articles


Publications in Conference Proceedings


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Stellar yields in CNO from rotating stellar models

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**Abstract.** For $^{12}$C and $^{16}$O, rotating models predict in general enhanced yields. At high metallicity, the carbon and oxygen yields from the very high mass stars, which go through a WR phase, are little affected by rotation. For $^{14}$N, rotation allows the production of important amounts of primary nitrogen in *intermediate mass stars* at very low metallicity. The process invoked for this production is different, from the classically accepted scenario i.e. the Hot Bottom Burning (HBB) in Asymptotic Giant Branch (AGB) stars. Rotating models also predict important productions of primary $^{13}$C, $^{17}$O and $^{22}$Ne at very low metallicity.

Rotation affects all the outputs of the stellar models (Heger & Langer 2000; Maeder & Meynet 2000a) and in particular the stellar yields. We shall not detail here the physical mechanisms driven by rotation. These are described in many other papers to which the reader may refer (Zahn 1992; Maeder & Zahn 1998; Heger & Langer 2000; Maeder & Meynet 2000a). We want to illustrate here, by a few numerical examples, how the effects of rotation on mixing and mass loss may affect the quantities of new CNO elements synthesized and ejected into the interstellar medium. For this we shall use recent grids of stellar models computed for three different metallicities: $Z = 0.00001$ (Meynet & Maeder 2002b), $Z = 0.004$ (Maeder & Meynet 2001) and $Z = 0.020$ (Meynet & Maeder in preparation). All these models were computed with the nuclear reaction rates of the NACRE compilation (Angulo et al. 1999; see also the review by M. Arnould in this volume).

1. **Rotation and the CO–core masses**

In Fig. 1, the variations of the CO–core masses as a function of the initial mass are indicated for various initial metallicities and for rotating and non–rotating stellar models. In stars with initial masses below 30–40 $M_\odot$, both at low and high metallicity, the CO cores are enhanced by rotation. As an example, at solar metallicity a rotating 20 $M_\odot$ stellar model with $v_{ini} = 300$ km s$^{-1}$ has a CO–core mass equal to that of a non–rotating 26 $M_\odot$ stellar model. Thus one expects that rotating models will inject more carbon and oxygen than their non–rotating counterparts.

The increase of the CO–cores due to rotation is less marked at low metallicity. Paradoxically this is a consequence of the more efficient rotational mixing at low $Z$ (Meynet & Maeder 2002b). Indeed the more efficient mixing continuously replenishes in hydrogen the H–burning shell. As a consequence the H–burning
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Figure 1. *Left panel:* variation as a function of the initial mass of the mass of the carbon–oxygen (CO) cores obtained at the end of the C–burning phase. The CO–core is defined as the region interior to the shell where the mass fraction of carbon plus oxygen is superior to 0.75. The models are those of Meynet and Maeder (2002b). *Right panel:* Same as the left panel for solar metallicity models (models from Meynet & Maeder, in preparation).

shell progresses less rapidly outwards and thus prevents the He–core mass and therefore the CO–core mass to increase as much as at higher metallicity.

For stars above 30–40 M\(_\odot\), at solar metallicity, the CO–cores in rotating models are smaller than in non–rotating ones. Due to rotational mixing, these models enter into the Wolf–Rayet phase at an earlier stage of their evolution. During the Wolf–Rayet phase, which lasts longer in rotating models, the stars loose mass at a higher rate, producing smaller final masses. As we shall see below, the CNO yields for these high mass stars are little affected by rotation.

2. Rotation and mass loss by stellar winds

In the framework of the radiative driven wind theory, it is possible to obtain an expression for the ratio of the mass loss rates from a rotating and a non–rotating star lying at the same position in the HR diagram (Maeder & Meynet 2000b). Typically for a 20 M\(_\odot\) star model at the end of the MS phase, the enhancement factor due to rotation is at most 1.67 at break–up while for a 60 M\(_\odot\) stellar model, in the same conditions, the factor amounts to 3.78. Rotation does not only increase the quantities of mass lost by stellar winds but also induces anisotropies in the winds. Typically, polar winds are expected for fast rotating hot stars, which, as a consequence, will loose small amounts of angular momentum and thus reach more easily the break–up limit (Maeder 2002). Other effects induced by rotation may change the quantity of mass lost by stellar winds: the increase of the MS lifetimes, the bluer evolutionary tracks, the easier Wolf–Rayet star formation.

Fig. 2 shows the final masses obtained for different initial masses, velocities and metallicities. At low metallicity, for stars with initial masses below 40 M\(_\odot\)
and with \( v_{\text{ini}} = 300 \text{ km s}^{-1} \), rotating and non-rotating stellar models end with final masses close to their initial values. Only in the case of the 60 M\(_{\odot}\) there is a significant difference. One expects that the difference will increase when the mass increases and/or when the initial velocity increases. Concerning this last point it is worthwhile to recall here that there are some indications that the relative number of fast rotators increases when the metallicity decreases (Maeder et al. 1999). This fact, together with the effect of rotation on both mixing and mass loss, may have interesting consequences for the nucleosynthesis expected from the first stellar generations in the Universe.

At solar metallicity, mass loss has an important impact on the values of the final masses. All stars with initial masses above 40 M\(_{\odot}\) end with final masses between 10–15 M\(_{\odot}\). In general, as expected, rotating models produce smaller final masses. One notes also that the new mass loss rates used in the recent computations are smaller than those used in the stellar grids of Schaller et al. (1992). This has some impact on the carbon yields from the most massive stars as we shall see in Sect. 4.

### 3. Effects of rotation on the carbon and oxygen yields

Stellar yields at the metallicities \( Z = 0.00001 \) and 0.004, for \(^4\text{He}, \ ^{12}\text{C}, \ ^{14}\text{N} \) and \(^{16}\text{O} \) from rotating and non-rotating models, in the mass range between 2 and 60 M\(_{\odot}\), are discussed in Meynet & Maeder (2002b). Let us recall here the main results. Figure 3 shows the yields in \(^{12}\text{C} \) for non-rotating and rotating stellar models \( (v_{\text{ini}} = 300 \text{ km s}^{-1}) \) for two metallicities and for mass ranges between 2 and 120 M\(_{\odot}\). For the metallicities considered here, the effects of
the stellar winds are small. We see that the yields in carbon are generally
enhanced in the rotating models. The enhancement factors are equal to 1.3 and
2.4 for the 20 $M_\odot$ models at $Z = 10^{-5}$ and 0.004 respectively. Larger carbon
yields in rotating models are a consequence of the greater CO–core masses in
rotating models (see Sect. 1 above). Interestingly, the yields from the rotating
models are not very different from those computed by Maeder (1992) for $Z =
0.001$ computed without rotation but with a moderate overshooting, while the
rotating models were computed without overshooting. Rotation, by enlarging
the CO core masses, acts in this respect as an overshoot.

Figure 4 shows the yields in $^{16}$O. As for carbon, the yields in oxygen are
enhanced by rotation. For the 20 $M_\odot$, the enhancement factor is 1.8 at $Z =
10^{-5}$ and 1.6 at $Z = 0.004$. Again the yields from rotating models (without
overshooting) are very similar to those obtained by Maeder (1992).

At solar metallicity, the main difference with respect to the situation found
at low metallicity essentially concerns the high mass star range (stars with initial
masses above about 40 $M_\odot$). For this mass range, mass loss by stellar winds
becomes the dominant effect. As a consequence the yields in carbon and oxygen
in these stars are much less affected by rotation. As a numerical example, the
yields in carbon and oxygen of a 60 $M_\odot$ star model differ by less than 11% between
the rotating and the non–rotating cases.

It was proposed by Maeder (1992) that massive stars (going through a WC
phase) may be important contributors of carbon at high metallicity, an idea
further sustained by various chemical evolution models (Prantzos et al. 1994;
Carigi 2000; Gustafsson et al. 1999). Since the mass loss rates have been reduced
since the work of Maeder (1992), the present yields of carbon from WC stars
are smaller, and those of oxygen greater. However these stars remain important
sources of carbon at solar and higher metallicities.
4. Effects of rotation on the nitrogen yields

Nitrogen is produced by transformation of carbon and oxygen through CNO cycle in H-burning zones. Nitrogen is said to have a primary origin when the carbon (oxygen) used for its synthesis is produced by the star itself. The secondary channel corresponds to the case when the carbon transformed into nitrogen is the one initially present in the star. When nitrogen is primary, its abundance in the interstellar medium evolves in lockstep with other primary elements like, for instance, oxygen. In that case, the N/O ratio remains constant when the metallicity increases. When nitrogen is mainly produced by the secondary channel, the N/O ratio is expected to increase steeply with the increasing metallicity.

If, at low metallicity, observations clearly require some source of primary nitrogen, the identification of these sources (massive or intermediate mass stars) remains controversial. Interestingly, there are some observational features which may give some hints: let us suppose that we observe starbursts in galaxies. If primary nitrogen is produced by massive stars, one expects to observe a very small scatter of the N/O ratios in starbursts of different ages, since both nitrogen and oxygen are released at the same time \(^1\). In case primary nitrogen is released by intermediate mass stars, there will be some time delay between the oxygen and the nitrogen release. Thus there is some chance to observe systems which have already been enriched in oxygen by the massive stars but which are in

\(^1\) However the observation of a small scatter of the N/O ratios at low metallicity does not necessarily imply that primary nitrogen is produced by massive stars. Indeed the systems may be sufficiently old for having allowed intermediate mass stars to have released their primary nitrogen. In that case, low star formation must be invoked in order to explain the observed low metallicity.
Figure 5. **Left panel:** Chemical composition inside a rotating 3 M\(\odot\) star model at the beginning of the TP-AGB phase. **Right panel:** Yields of \(^{14}\)N normalized to the initial metal content for different initial mass stellar models at various metallicities. Black circles are for models with rotation, black squares are for non-rotating models, the continuous lines show the models at \(Z = 10^{-5}\), the dotted lines, those at \(Z = 0.004\) and the dashed lines, those at \(Z = 0.020\).

Figure 6. **Left panel:** Simplified model for the galactic evolution of the N/O ratio as a function of the O/H ratio (in number). The dashed and continuous lines show the results deduced from the non-rotating and the rotating models respectively. The range of the initial masses used for computing the integrated yields are indicated. The empty symbols show the results when only stars more massive than 8 M\(\odot\) are considered. The initial velocity is indicated. The shaded area shows the region where most of the observations of extragalactic HII regions and stars are located (see e.g. Gustafsson et al 1999; Henry et al. 2000). **Right panel:** Same as left panel for the C/O ratio. The lifetimes corresponding to these injections are shown in Fig. 7.
CNO stellar yields from rotating stars

the process of being enriched in primary nitrogen by intermediate mass stars. One expects in that case some scatter in the observed values of the N/O ratios. Recently, numerous Damped Lyman Alpha (DLA) systems have been observed with values of the N/O ratios well below the plateau level. This strongly suggests that intermediate mass stars are responsible for the primary nitrogen production.

What are the predictions of the theoretical stellar models? In the classical scenario primary nitrogen is produced during the thermal pulse phase of AGB stars undergoing HBB (see van den Hoek and Groenewegen 1997; Marigo 2001). The quantities of primary nitrogen produced and expelled depends on various parameters as the mass loss along the AGB and the strength of the third dredge-up. Recently, we have proposed a new scenario involving rotational diffusion: carbon and oxygen produced in the He-core migrate by rotational diffusion in the H-burning shell where they are transformed into primary nitrogen. In contrast to the classical scenario, this process occurs in the whole mass range and is thus not restricted to the intermediate mass range.

The left panel of Fig. 5 shows the chemical abundances as a function of the lagrangian mass in a rotating 3 M\(_\odot\) star model at Z = 0.00001 at the beginning of the Thermal Pulse–AGB phase. The most striking feature is the very high \(^{14}\text{N}\) abundance obtained at the border of the He-core and in the outer convective zone. Its abundance is more than 70 times the initial metal content of the star! The right panel shows the \(^{14}\text{N}\) stellar yields normalised to the initial metal content obtained for different initial mass stars at various metallicities. At very low metallicity, all rotating models produce quantities of new \(^{14}\text{N}\) which are multiples of the initial metal content. However, for the initial velocities considered here (\(v_{\text{ini}} = 300\) km s\(^{-1}\)), the intermediate mass stars are the main producers. For higher metallicities, the secondary channel for nitrogen production is the dominant one.

Why the process of primary nitrogen production only works at low metallicity? At low metallicity rotational mixing of chemical species is more efficient due to the steeper gradients of angular velocity inside the stars. These steeper gradients arise as a result of less efficient transport by meridional circulation of the angular momentum in the outer shells (Meynet & Maeder 2002ab). In metal poor stars, the H-burning shell is also nearer from the He-burning core. This shortens the timescale for diffusion between these two zones.

Left panel of Fig. 6 shows the evolution of the N/O ratio as a function of the O/H ratio in the framework of a very simple model for the chemical evolution of galaxies (closed box model with instantaneous recycling). The hatched zones in Fig. 6 represent regions where many observed points are found (see caption). At low metallicity, only the rotating models involving the contribution of the intermediate mass stars can reproduce the plateau level (at about Log (N/O) = -1.7). Massive stars alone may contribute significantly only if they rotate much faster than the typical rotational velocities found at solar metallicity. While the evolution in the N/O versus O/H plane mainly depends on rotation, the evolution in the C/O versus O/H plane is mainly sensitive to the mass range of the stars contributing to the enrichments of the ISM. Thus, the combination of the diagrams C/O vs O/H, more sensitive to the mass interval, and of the diagram N/O vs. O/H, more sensitive to rotation, may be particularly powerful to disentangle the two effects of rotation and mass interval, and to specify the
Figure 7. Quantities of newly synthesized nitrogen (in solar masses) injected into the interstellar medium by 100,000 stars born at time $t = 0$ with initial masses between 0.1 and 60 $M_\odot$. The stellar models have an initial mass fraction of heavy elements equal to 0.00001 (1/2000 the solar metallicity). The left panel shows the situation for non-rotating stellar models, while the right panel shows the case for rotating stellar models. Note that the vertical axis have different scales in the left and right panel. No hot bottom burning process is accounted for. The labels along the curves indicate when a star of a given initial mass contributes.

properties of the star populations responsible for the early chemical evolution of galaxies.

The DLAs present also new and interesting constraints on the source of primary nitrogen at low metallicity. Recent discussions of this point may be found in the paper of Pettini et al. (2002) as well as in the contributions by Henry, Molaro in the present volume. As explained above, the numerous DLAs observed with N/O ratios below the plateau level favour a primary nitrogen production by intermediate mass stars. Secondly the time delay required to explain the frequency of the systems with low N/O ratios points towards a relative long time delay between the release of oxygen and that of nitrogen (of about 700 Myr according to Pettini et al. 2002). This is well in agreement with the predictions of the rotating models (see Fig. 7).

Finally some authors suggest that the N/O values of DLAs are distributed mainly along two plateaux. One is at $\log(N/O) \sim -1.7$, as shown on the left panel of Fig. 6. A second plateau seems to be present at about 0.7 dex below the first one (see the papers by Henry and Molaro in this volume). A possible interpretation of the lower plateau is that it is produced by stars with initial masses above 8 $M_\odot$ (see Molaro in this volume). Interestingly, the present massive fast rotating stellar models would be in agreement with this viewpoint. As shown on the left panel of Fig. 6, the contribution of rotating stars with masses superior to 8 $M_\odot$ and $v_{\text{ini}} = 400 \text{ km s}^{-1}$ predict a N/O value at $Z = 10^{-5}$ about 0.7 dex below the high plateau value.
5. Conclusion

We focused our discussion here on the main isotopes of carbon, oxygen and nitrogen. However rotation may also affect the other less abundant CNO isotopes. In particular, $^{13}\text{C}$ and to a lesser extent $^{17}\text{O}$ may also be abundantly produced in rotating models and have a primary production channel at low metallicity. As an example, at $Z = 0.00001$, the amounts of $^{13}\text{C}$ synthesized in rotating intermediate mass star models may be nearly 10 000 times the amount produced in non-rotating models. Interestingly some primary $^{13}\text{C}$ production seems to be required by the observations (Prantzos et al. 1996). Important productions of primary $^{22}\text{Ne}$ are also predicted by the rotating models.

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Dynamical shear instability

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Abstract. The dynamical shear instability is an important mixing process in the advanced stages of the evolution of massive stars. We calculated different models of $15\, M_\odot$ with an initial rotational velocity, $v_{\text{ini}} = 300$ km/s to investigate its efficiency. We found that the dynamical shear instability has a timescale shorter than Oxygen burning timescale and that it slightly enlarges the convective zones and smoothens the omega gradients throughout the evolution. However, its effect is too localized to slow down the core of the star.

1. Theory & computer model

The stability criterion for dynamical shear instability is the Richardson criterion: $Ri = \frac{N^2}{(\partial U/\partial z)^2} > \frac{1}{4} = Ri_c$, where $U$ is the horizontal velocity, $z$ the vertical coordinate and $N^2$ the Brunt-Väisälä frequency. Using this criterion, the timescale of the instability is the dynamical timescale and the instability is called dynamical shear instability (Endal & Sofia 1978). If heat losses are accounted for in $Ri$ (Maeder 1997), the timescale is the thermal timescale and in that case the instability is called secular shear instability. The critical value, $Ri_c = 1/4$, is used by most authors as the limit for the occurrence of the dynamical shear. However, recent studies (Canuto 2002; Brüggen & Hillebrandt 2001) show that turbulence may occur as long as $Ri < \sim 1$.

Different formulae for the corresponding diffusion coefficient, $D$, are used at the present time (Zahn 1992; Maeder 1997; Heger et al 2000; Brüggen & Hillebrandt 2001). The following dynamical shear diffusion coefficient suggested by J.-P. Zahn is used in this study:

$$D = \frac{1}{3} vl = \frac{1}{3} \frac{v}{l} l^2 = \frac{1}{3} r \frac{d\Omega}{dr} \Delta r^2 = \frac{1}{3} r \Delta \Omega \Delta r$$

(1)

where $r$ is the mean radius of the zone where the instability occurs, $\Delta \Omega$ is the variation of $\Omega$ over the zone and $\Delta r$ is the extent of the zone. The zone is the reunion of consecutive shells where $Ri < Ri_c$. This is valid if the Peclet number, $P_e > 1$.

The computer model used is the Geneva evolutionary code. Modifications (to be described in a future paper) have been made to study the advanced stages
of the evolution of massive stars. The models are calculated for 3 stars of 15 \( M_\odot \) at \( Z_\odot \) with \( v_{\text{ini}} = 300 \text{ km/s} \). Convective stability is determined by the Schwarzschild criterion. The overshooting parameter, \( d_{\text{over}}/H_P = 0.1 \) for H– and He–burning and 0 afterwards. In the first model, the dynamical shear was not included. Then one model was calculated with \( Ri_c = 1/4 \) and one with \( Ri_c = 1 \). The calculations reached shell O–burning. Note that the computer model includes secular shear and meridional circulation for any calculation.

2. Results & discussion

The characteristic timescale of the dynamical shear is very short (a fraction of a year) throughout evolution when using equation (1). We obtain diffusion coefficients between \( 10^{12} \) and \( 10^{14} \text{ cm}^2/\text{s} \). This is usually 1 or 2 orders of magnitude larger than the dynamical shear diffusion coefficients of Brüggen & Hillebrandt (2001) or Heger et al (2000).

However, the extent of the unstable zone is very small, a few \( 10^{-3} M_\odot \) so the shear mainly smoothens the sharp \( \Omega \)-gradients but does not transport angular momentum over long distances. The total angular momentum of our models, at the beginning and at the end of the calculations are of the same order of magnitude as the values obtained by Heger et al (2000) in their models E15 and E15B.

Although dynamical shear strengthens convection, especially the He–burning convective shell, the structure and the convective zones are similar between the model without dynamical shear and the one with dynamical shear and \( Ri_c = 1/4 \). Concerning the Richardson criterion, we see that there is a difference during Ne–burning between the model using \( Ri_c = 1/4 \) and \( Ri_c = 1 \). Indeed, using the latter, there is another large C–burning convective shell while the central convective core is smaller. One could argue that our diffusion coefficient is too high but we obtained a similar trend using the diffusion coefficient of Brüggen & Hillebrandt (2001) with \( Ri_c = 1 \).

The well known discrepancy between the observed and predicted pulsar rotation periods might be reduced with the interaction between rotation and magnetic field (see contributions by Heger and Spruit in this volume) or with a formula linking dynamical and secular shear in the advanced stages of the evolution of massive stars.

Poster copy @ http://obswww.unige.ch/~hirschi/work/cancun02.ps

References

Pre-supernova evolution of rotating massive stars

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Summary. The Geneva evolutionary code has been modified to study the advanced stages (Ne, O, Si burnings) of rotating massive stars. Here we present the results of four 20 M$_\odot$ stars at solar metallicity with initial rotational velocities, $v_{\text{ini}}$, of 0, 100, 200 and 300 km/s in order to show the crucial role of rotation in stellar evolution. As already known, rotation increases mass loss and core masses [4]. A fast rotating 20 M$_\odot$ star has the same central evolution as a non-rotating 26 M$_\odot$. Rotation also increases strongly net total metal yields. Furthermore, rotation changes the SN type so that more SNIb are predicted (see [5] and [6]). Finally, SN1987A–like supernovae progenitor colour can be explained in a single rotating star scenario.

1 Computer model

The computer model used is the Geneva evolutionary code (see [5]). Convective stability is determined by the Schwarzschild criterion. The overshooting parameter, $\alpha_{\text{over}} = d_{\text{over}}/H_P$ is equal to 0.1 for Hydrogen– and Helium–burning cores and equal to 0 afterwards. Modifications have been made to study the advanced stages of the evolution of rotating massive stars. Dynamical shear has been included using $Rc_c = 1/4$ [2]. Note that the computer model still includes secular shear and meridional circulation. The structure equations have been stabilised using Sugimoto’s prescription [7]. Furthermore, convection is treated as diffusion from the Oxygen (O) burning stage because convection is no longer instantaneous. The algorithm developed for rotational mixing is used for this purpose. Finally, the nuclear reaction network has been extended and contains all the multiple-$\alpha$ elements up to $^{56}\text{Ni}$ except $^8\text{Be}$. The reaction rates are taken from the NACRE compilation or Hauser-Feschbach code calculations (ULB, Belgium).
2 Evolution

The early evolutionary stages are presented in [5]. Here we concentrate on solar metallicity 20 M\(_{\odot}\) stars and study the effect of rotation by examining four models with initial rotational velocities of 0, 100, 200 and 300 km/s. Calculations have been followed until end of central O–burning for the \(v_{\text{ini}} = 100\) and 200 km/s models, end of central Si–burning for the \(v_{\text{ini}} = 300\) km/s model and end of first shell Si–burning for the non-rotating model.

2.1 Hertzsprung–Russell (HR) diagram

![HR diagram for the solar metallicity 20 M\(_{\odot}\) stars with initial rotational velocities of 0, 100, 200 and 300 km/s.](image)

Fig. 1. HR diagram for the solar metallicity 20 M\(_{\odot}\) stars with initial rotational velocities of 0, 100, 200 and 300 km/s.

Figure 1 shows the evolutionary tracks of the four different 20 M\(_{\odot}\) stars in the HR diagram. The non-rotating model ends up as a red supergiant (RSG) like other group models (see e.g. [1] or [3]). However, the rotating models show very interesting features. Although the 100 km/s model remains a RSG, the 200 km/s model undergoes a blue loop to finish as a yellow–red supergiant whereas the 300 km/s model ends up as a blue supergiant (BSG). Thus rotation may strongly affect the shock wave travel time through the envelope when the star explodes in a supernova event, since this time is proportional to the radius of the star (RSG radii are about hundred times BSG ones). Moreover, the behaviour of the models with \(v_{\text{ini}}\) between 200 and 300 km/s is reminiscent of the evolution of the progenitor of 1987A indicating that rotation may play a role in similar cases.
2.2 Central evolution

The central evolution is best seen in the central temperature, $T_c$, versus central density, $\rho_c$, diagram (Fig. 2). We can see that rotation makes the cores slightly less degenerate (higher $T_c$ and smaller $\rho_c$). This is explained by the bigger core masses. We also see that the “C–bump” due to the convective central C–burning fades away when rotation increases (see also Fig. 3). This is again a consequence of more massive cores in rotating models which implies higher neutrino loss rates and smaller central carbon abundance at the end of He–burning phase.

![Fig. 2. $T_c$ versus $\rho_c$ diagram.](image)

3 “Pre-SN” models

3.1 Mass loss and core masses

We can see in Fig. 4 that both mass loss and Helium (He) core masses, $M_\alpha$, increase with rotation as already known. There is a saturation effect at high rotation when the star is left with hardly any Hydrogen (H) envelope. As can be seen in [5], rotation noticeably increases the number of Wolf-Rayet stars (WR). Here we see that there is a smooth transition between SN type from IIP $\rightarrow$ IIL $\rightarrow$ IIb ($\rightarrow$ Ib). We also note that the $v_{\text{ini}} = 300$ km/s 20 $M_\odot$ model has a bigger He core than the non-rotating 25 $M_\odot$ model (it would correspond to the core of a non-rotating 26 $M_\odot$ model). The Carbon–Oxygen core mass, $M_{\text{CO}}$, increases with rotation in a similar way as $M_\alpha$. The Silicium (Si) core mass at the end of central O–burning only slightly increases with rotation.
3.2 Abundances profile and net yields

In Fig. 5 we notice the smoother profiles due to rotational mixing and also the very small quantity of remaining H. The “pre-SN” net yields calculated at this stage show that rotation increases the total metal yield and $^{16}$O yield. Typically, the total metal yield of the $v_{\text{ini}} = 300 \text{ km/s}$ model is twice the one of the non-rotating model. On the other hand, rotation decreases H–burning products yields (notably $^4$He) as can be seen in Fig. 6.
Fig. 5. Abundances profile for main elements at the end of central Si–burning. Left non-rotating 20\,M\odot model. Right $v_{\text{ini}} = 300\,\text{km/s}$ 20\,M\odot model.

Fig. 6. Net yields of the sum of all metals, $Y_{\Sigma}$, and individual elements as a function of $v_{\text{ini}}$.

References

Abstract. The inclusion of rotation in massive star models improves
the agreement between theory and observations on at least three im-
portant points: 1) Rotational mixing allows to produce variations of
the surface abundances already during the Main-Sequence phase as
is observed. The changes of the surface abundances are more impor-
tant when, for a given initial velocity, the initial masses are larger,
and/or the metallicities are lower; 2) The observed number of red super-
giants at the metallicity of the Small Magellanic Cloud (SMC) can
be accounted for; 3) The observed variation of the number ratio of
Wolf-Rayet to O-type stars as a function of the metallicity can be
reproduced. For all these comparisons non-rotating models give un-
satisfactory fits. Rotating models results also give interesting insights
on questions such as the origin of Be stars, the mechanisms responsible
for the huge mass loss rates undergone by the Luminous Blue Vari-
ables, the rotation rates of pulsars, the progenitors of collapsars and
the sources of primary nitrogen at low metallicity.

1 Introduction

Stellar rotation is an old subject. Indeed Galileo first observed that the Sun had
an axial rotation some 400 years ago; an important instability induced by rotation
in star, namely the meridional circulation, was described already three quarters of
a century ago (Vogt 1925; Eddington 1925). On the other hand, rotation is also
quite a topical subject: for instance, measurements of the oblateness of stars due
to fast rotation have recently been performed (van Belle et al. 2001; Domiciano
de Souza 2003), also rotation plays a key role in the collapsar model proposed by
Woosley (1993) for explaining the (long) Gamma Ray Bursts (GRB). Let us recall
that this model has received some support from the recent detection of a clear
spectroscopic supernova signature in the afterglow of the GRB 030329 (see e.g.
Hjorth et al. 2003).
The computation of new grids of massive star models accounting for the effects of rotation (see e.g. Heger & Langer 2000; Meynet & Maeder 2000) was stimulated by the numerous observations indicating surface enrichments not predicted by standard models (here standard models are models in which chemical mixing is only allowed in convective regions):

1) Howarth & Smith (2001) found that three of the most rapidly rotating late–O near–main–sequence stars known show substantially enhanced surface–helium abundances. Comparing the distribution of projected rotational velocities for ON (N–rich) and normal dwarf O stars, they also demonstrate that the ON stars have on the average larger rotation velocities.

2) Villamariz et al. (2002) derived the CNO abundances in four Galactic O9 stars. One of the star, showing a high value of $v \sin i$ (450 km s$^{-1}$), is N–rich. The surface N/C and N/O ratios amount to 6.3 and 4.5 times the initial ones.

3) Heap and Lanz (2004) observed O–type stars in the Small Magellanic Cloud. Nine of the 17 program stars present very important N–overabundances (by a factor 30 !). At this low metallicity, during the MS phase, the mass loss rates are too low for uncovering the inner layers. Thus the most probable explanation is that some extra mixing process is at work.

4) Venn et al. (2002) determined the boron abundance in Galactic B–type stars. Boron is a fragile element, which is easily destroyed by hot proton captures. In case of not too deep mixing, boron will be destroyed while other more resistant elements like carbon won’t. This is likely what happened in a few of the stars studied by Venn et al. (2002) which show boron depletion with no N–enrichment (which would result from the destruction of carbon). Interestingly, such a shallow mixing allows to discard binarity as one of the cause for this particular surface abundance pattern. Indeed, according to models, any mass transfer in a binary system will produce a boron depletion with a nitrogen enhancement.

5) Daflon et al. (2001) observed the N/O abundances of 17 young B–type main–sequence stars in Cep OB2. Two stars, the most massive, the most evolved and the most rapidly rotating ones, show significant N/O overabundances (about a factor of two above the initial value).

6) Smartt et al. (2002) measured the N/O ratios in four Galactic B–type supergiants. These stars have surface N/O ratios in the range 0.1–0.5 (by number), $i.e.$ higher than the initial value (around 0.1), but below the expected values if these stars would be on a blue loop after a red supergiant dredge–up episode. Indeed in that last case the N/O ratios are believed to be in the range 1–20. These modest N–enrichments give support to the idea that these stars underwent some mixing during the MS phase.

7) Venn & Przybilla (2004) measured N–abundances at the surface of A–type supergiants in the Galaxy and the Magellanic Clouds. In the SMC, the greatest N/H ratios are about 40 times the initial ones, while in the Galaxy the maximum enrichment factor is of the order of 8.

These inadequacies between standard model predictions and observations (for additional references see also the reviews by Maeder & Meynet 2000a; Smartt 2003)
clearly show that some additional physical mechanism(s) must be included in realistic stellar models. Some of the enrichments appear to be associated with fast rotation giving some support to the hypothesis that rotation could be the driving mechanism. The observation by Venn & Przybilla (2004), Heap and Lanz (2004) seem also to indicate that the mixing is stronger at low metallicity. In the following, after a brief recall of the different physical effects induced by rotation (Sect. 2), we discuss how rotation modifies the evolution of massive stars at different metallicities (Sect. 3). The consequences for the massive star populations and nucleosynthesis are presented in Sect. 4. Finally, Sect. 5 presents some future promising lines of research.

2 Physical effects induced by rotation

One can distinguish three classes of effects

1) The hydrostatic effects. They are the consequences of the centrifugal acceleration term in the equation of the hydrostatic equilibrium. The star is deformed and the stellar structure equations have to be modified in consequence. In case the centrifugal acceleration term can be deduced from a potential (conservative case), the stellar structure equations can be written as devised by Kippenhan & Thomas (1970). In the case of a “shellular” rotation law (see below), the equations can be kept the same as in the conservative case but with a slightly different interpretation of the variables (Meynet & Maeder 1997). On their own these hydrostatic effects have small impacts on the evolution of stars. However they imply that the local radiative flux and therefore the effective temperature of a rotating star depend on the latitude (consequence of the Von Zeipel theorem 1924). Thus the luminosity and the effective temperature of a star seen by an observer, depends on the angle of view. This may affect the position of a star in the HR diagram by a few tenths of a magnitude in luminosity and a few hundredths in log $T_{\text{eff}}$ (Maeder & Peytreman 1970).

2) The instabilities driven by rotation. Recent discussions of the various instabilities induced by rotation may be found in Maeder & Meynet (2000a), Heger et al. (2000), Talon (2004). Among the most important instabilities are the secular shear instability and the meridional circulation. These instabilities drive the transport of the chemical species and of the angular momentum (see below).

3) The effects on the stellar winds. Rotation induces anisotropies of the stellar winds. It also enhances the mass loss rate expected from a star lying at a given position in the HR diagram (see below).

2.1 The transport mechanisms

In a rotating star local radiative equilibrium cannot be achieved. As a result some parts of the star will be heated while other will cool. The buoyancy forces then drive a large scale motion. Complete expressions for the meridional velocities,
accounting, among other processes, for the effects of the molecular weight gradients (\(\mu\)-gradients) are given in Maeder & Zahn (1998). The radial component \(U(\mathbf{r})\) is given by

\[
U(\mathbf{r}) = \frac{P}{\rho g C_T (\nabla_{ad} - \nabla + (\varphi/\delta) \nabla_{\mu})} \times \left( \frac{L}{M_*} [E_\Omega + E_\mu] + \left( \frac{C_p}{\delta} \frac{\partial \Theta}{\partial t} \right) \right), \tag{2.1}
\]

where \(M_* = M \left( 1 - \frac{\Omega^2}{2\pi G \rho_\infty} \right)\) is the reduced mass and \(\Omega\) the mean angular velocity at level \(\mathbf{r}\). The other symbols are defined in the quoted paper. The driving term in the square brackets in the right hand side member is \(E_\Omega\). It behaves mainly like,

\[
E_\Omega \simeq \frac{8}{3} \left[ 1 - \frac{\Omega^2}{2\pi G \rho} \right] \left( \frac{\Omega^2 \rho^3}{GM} \right), \tag{2.2}
\]

The bar over some quantities means the average quantity on the considered isobar. The term with the minus sign in the square bracket is the Gratton–Opik term, which becomes important in the outer layers due to the decrease of the local density. It can produce negative values of \(U(\mathbf{r})\). A negative \(U(\mathbf{r})\) means a circulation going down along the polar axis and up in the equatorial plane. This makes an outwards transport of angular momentum, while a positive \(U(\mathbf{r})\) gives an inward transport of angular momentum.

Meridional circulation, contraction/expansion of the stellar layers, and/or convection, create gradients of the angular velocity inside the star. These gradients produce instabilities known as shear instabilities. The physical reason for this instability lies in the fact that the minimum energy state of a differentially rotating fluid is solid body rotation. The star will tend to approach this state by homogenizing the angular velocity by turbulent mixing.

In a radiative zone, the vertical stable density stratification counteracts the shear instability. In that respect the \(\mu\)-gradients play a key role as a stabilizing agent. These gradients may even, depending on the physics involved in the model, completely inhibit the mixing (see e.g. Meynet & Maeder 1997). There are different methods in the literature for accounting for the effects of the \(\mu\)-gradients on the mixing. Some authors choose a parametric approach consisting in multiplying the \(\mu\)-gradient by a free parameter, \(f_\mu\), smaller than one in order to weaken the stabilizing effect of the \(\mu\)-gradient. The value of \(f_\mu\) is chosen in order to enable the stellar models to reproduce the observed surface enrichments (Heger et al. 2000). The very small values chosen (of the order of \(f_\mu = 0.05\)) illustrates well the strong inhibiting effects of the \(\mu\)-gradients. Other methods devised by Maeder (1997), Talon & Zahn (1997) account for the fact that the medium in a rotating star is turbulent even before any vertical shear mixing occurs (see below). In that case, the energy available in the shear can always be used for performing some mixing. Expressions for the shear diffusion coefficients can then be deduced without the need of artificially reducing the inhibiting effect of the \(\mu\)-gradients.

Why can we consider that, even before any vertical shear instability sets in, the medium in a rotating star is already turbulent? As recalled above various processes will build vertical and horizontal gradients of \(\Omega\). In the horizontal direction,
In contrast to what happens in the vertical direction, the shear instability is not inhibited by a stable density stratification and the turbulence can develop without difficulty. Thus, as long as horizontal gradients are continuously built up by e.g. meridional circulation, a strong horizontal turbulence develops in the star (Zahn 1992). This strong horizontal turbulence will also erode very efficiently any horizontal gradient of $\Omega$. Thus the star can always be considered to be in a state of shellular rotation, characterized by constant values of $\Omega$ on isobars. Starting from this a priori, but reasonable hypothesis, Zahn (1992) proposed a consistent theory of the interaction between the shear instability and the meridional circulation. The theory can be kept one-dimensional thanks to the hypothesis of strong horizontal turbulence.

For shellular rotation, the equation of transport of angular momentum in the vertical direction is in Lagrangian coordinates (Zahn 1992),

$$\frac{d}{dt} \left( \rho^2 \Omega \right)_{M_r} = \frac{1}{5r^2} \frac{\partial}{\partial r} \left( \rho u^4 \Omega U(r) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho D_{\text{shear}} r^4 \frac{\partial \Omega}{\partial r} \right), \quad (2.3)$$

where $D_{\text{shear}}$ is the shear diffusion coefficient. From the two terms in the right member, we see that advection and diffusion are not the same. Thus, the transport of angular momentum by circulation cannot be treated as a diffusion.

For the changes of the chemical elements due to transport, we may use a diffusion equation with a diffusion coefficient which is the sum of the shear diffusion coefficient $D_{\text{shear}}$, and of $D_{\text{eff}} = \frac{\nu(r)^2}{\omega m_h n}$. $D_{\text{eff}}$ expresses the resulting effect of meridional circulation and of the large horizontal turbulence (Chaboyer & Zahn 1992). This expression of $D_{\text{eff}}$ tells us that the vertical advection of chemical elements is inhibited by the strong horizontal turbulence characterized by $D_h$. The usual estimate of $D_h = \frac{1}{c_h} |2V(r) - \alpha U(r)|$ was given by Zahn (1992). A recent study suggests that this coefficient is at least an order of magnitude larger (Maeder 2003).

### 2.2 Rotation and mass loss by stellar winds

Since, at the surface of a rotating star, the effective gravity is higher at the pole than at the equator, the Von Zeipel theorem (Von Zeipel 1924; see also Maeder 1999) implies that the radiative flux is more important at the pole than at the equator. In the radiative wind theory the mass loss rates are proportional to the radiative flux, which is the driving force, and to the opacity, which, in this context, measures the efficiency of the matter to be pushed out by radiation. Both factors, radiative flux and opacity, vary with the latitude at the surface of a rotating star and thus the mass loss rates are different at the pole and at the equator (Owocki et al. 1996; Maeder 1999). The winds are anisotropic. In the case of a hot star, the main source of opacity is the electron scattering opacity which only depends on the mass fraction of hydrogen. In that case, the dependance of the mass loss with the latitude will follow that of the effective gravity (“$g_{\text{eff}}$-effect”) and one expects to have higher rates at the pole than at the equator.
Interestingly, direct measurement of the size and shape of the present-day stellar wind of η Carinae by van Boekel et al. (2003) show that the wind nebula is elongated with a major axis aligned with that of the large bi-polar nebula that was ejected in the 19th century. This observation supports polar enhanced mass loss as predicted by Maeder & Desjacques (2001), Dwarkadas & Owocki (2002). Van Boekel et al. (2003), using the above models, estimate that η Carinae should rotate at about 90% of its critical velocity. Let us note also that recent HST spectroscopy (Smith et al 2003) of starlight reflected by dust in the Homonculus indicates a latitude-dependent wind velocity, with the highest velocities near the pole; again this is expected for a wind which is stronger at the poles.

For cooler stars, other sources of opacity intervene in addition to the electron scattering opacity. A bistability limit (i.e. a steep increase of the opacity, cf. Lamers et al. 1995) may occur somewhere between the pole and the equator, due to the decrease of $T_{\text{eff}}$ from pole to equator. This “opacity-effect” produces an equatorial enhancement of the mass loss. The anisotropies of mass loss influence the loss of angular momentum, in particular polar mass loss removes mass but relatively little angular momentum. This may strongly influence the evolution (Maeder 2002).

The Von Zeipel theorem also changes the expressions for the break-up velocities, for the maximum luminosity and for the global mass loss rates (Maeder & Meynet 2000b). Concerning this last point, the mass loss rates of a rotating star compared to that of a non-rotating star at the same location in the HR diagram is given by

$$\frac{\dot{M}(\Omega)}{\dot{M}(0)} \approx \frac{(1 - \Gamma)^{\frac{1}{2}}}{\left(1 - \frac{2}{\alpha}(\frac{v}{v_{\text{crit}}})^2 - \Gamma\right)^{\frac{1}{2}}},$$

(2.4)

where $\Gamma$ is the Eddington ratio corresponding to electron scattering opacity for a non-rotating star with the same mass and luminosity, $v_{\text{crit}} = (2GM/3R_{\text{pl}})^{1/2}$ and $\alpha$ is a force multiplier (Lamers et al. 1995). Values of $\alpha$ are given in Table 1, which shows, for stars of different masses and $\Gamma$, the values of the ratio of the mass loss rates for a star at critical rotation to that of a non-rotating star of the same $M$, $L$ and $T_{\text{eff}}$. The values $\infty$, when $\Gamma$ approaches 1, indicate that the critical velocity is met and quite large increase of the mass loss rates are expected (ΩΓ-limit). Interestingly, the huge mass loss rates are predicted for the region of the HR diagram where LBV stars are found, indicating that these stars may indeed be near the ΩΓ-limit.

### 3 Effects of rotation on the stellar models

In Fig. 1, different tracks for 20 $M_\odot$ stellar models are plotted for various initial velocities. Rotation extends the MS tracks towards higher values of the luminosities and lower values of the effective temperature. Rotation has thus a similar effect as core overshoot. An interesting question for future studies would be to disentangle the extension of the core due to rotation to that due to the overshooting mecha-
Table 1. Values of $\frac{M(0)}{M(0)}$ for different stellar masses. The four force multipliers $\alpha$ apply respectively to stars with: $4.70 \geq \log T_{\text{eff}} \geq 4.35$ (type B1.5 or earlier), $\log T_{\text{eff}} = 4.30$ (type B2.5), 4.00 (B9.5), 3.90 (A7) respectively.

<table>
<thead>
<tr>
<th>$M_{\text{ini}}$</th>
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<td>2.14</td>
<td>13.6</td>
<td>55.3</td>
<td>106.0</td>
</tr>
<tr>
<td>25</td>
<td>0.214</td>
<td>1.76</td>
<td>7.02</td>
<td>20.1</td>
<td>32.6</td>
</tr>
<tr>
<td>20</td>
<td>0.156</td>
<td>1.67</td>
<td>5.87</td>
<td>15.2</td>
<td>23.6</td>
</tr>
<tr>
<td>15</td>
<td>0.097</td>
<td>1.60</td>
<td>5.94</td>
<td>12.1</td>
<td>18.1</td>
</tr>
<tr>
<td>12</td>
<td>0.063</td>
<td>1.57</td>
<td>4.68</td>
<td>10.8</td>
<td>15.8</td>
</tr>
<tr>
<td>9</td>
<td>0.034</td>
<td>1.54</td>
<td>4.41</td>
<td>9.8</td>
<td>14.2</td>
</tr>
</tbody>
</table>

Fig. 1. Evolutionary tracks for 20 $M_\odot$ models at solar metallicity with different initial velocities. Evolution was computed from the ZAMS until the Si-core burning phase (Hirschi et al. 2004).

nism. The main sequence width in the mass range between 1.1 and 1.5 $M_\odot$ where the observed rotation velocities are much lower than in the higher mass range, can give indications on the extension due to mainly overshooting. From Fig. 1, one can note also that, depending on the initial velocity, the star may evolve to become a blue supergiant after the core He-burning phase. This illustrates the fact that the color of the supernova progenitor is quite sensitive to the chemical profiles inside the star (cf. Langer et al. 1989; Langer & Maeder 1995).
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Fig. 2. Evolution of the fraction $\frac{\Omega}{\Omega_c}$ of the angular velocity to the critical angular velocity at the surface of star models of different initial masses between 9 and 120 M$_\odot$ with account of anisotropic mass loss during the MS phase. On the ZAMS, the equatorial velocity of all the models is equal to 300 km s$^{-1}$.

The MS lifetime is in general increased by rotation. For solar metallicity models and $v_{\text{ini}} = 300$ km s$^{-1}$, the increase amounts to 25%. For the metallicity of the Small Magellanic Cloud, the increase is around 10%. For still lower metallicity models ($Z = 10^{-5}$), the MS lifetime of massive stars is even reduced by rotation. The reason is the following: on one hand rotation enlarges the convective core, increasing thus the reservoir of fuel and thus the MS lifetime. On the other hand the diffusion of helium in the radiative zone decreases the opacity. This increases the luminosity and thus decreases the MS lifetime. When the metallicity decreases the mixing of the chemical elements is more efficient (see below). Thus the factor decreasing the MS lifetime, namely the helium diffusion in the radiative zone, becomes more and more efficient, while that increasing the MS lifetime, the diffusion of H into the convective core, remains more or less at a constant level. Indeed hydrogen has just to cross the boundary between the radiative envelope and the convective core to increase the quantity of fuel available, while helium has to be diffused in the whole radiative envelope to affect significantly the opacity.

In Fig. 2, evolutions of the fraction $\frac{\Omega}{\Omega_c}$ of the angular velocity to the critical angular velocity at the surface of star models of different initial masses between 9 and 120 M$_\odot$ are shown. The effects of the wind anisotropies have been accounted for. The most massive stars slow down more rapidly than the less massive ones. This is an effect of mass loss. Interestingly this may be related to two observational facts: firstly the observed mean velocity on the MS of the O–type stars is inferior
to that of the B–type stars (Fukuda 1982), secondly, Be stars, i.e. B–type stars rotating near break–up, are observed among B–stars and not O–type stars (see e.g. Fabregat and Torrejón 2000).

In Fig. 3 the evolution as a function of time of $\Omega/\Omega_c$ is shown for metallicities between 0.004 and 0.040. When the metallicity decreases, the angular velocity, normalized to break up velocity increases during the MS phase, while it decreases at higher metallicity. This results from the weaker mass loss rates in metal poor regions and from the internal transport processes. Thus even if the initial distribution of the velocities on the ZAMS do not depend on the metallicity, one expects that the relative number of stars near break-up will increase for lower metallicities. Very interestingly the number fraction of Be stars with respect to the sum of B and Be stars in clusters of similar ages, increases when the metallicity decreases (Maeder et al. 1999, see also the contribution by Maeder in this volume). This may result from the evolutionary effect discussed above, but also from the fact that the initial distribution of the rotational velocities might contain more fast rotators at low metallicity. This is presently not known. Instrument like HARPS on the NTT or GIRAFFE on the VLT offer great opportunities for addressing such questions.

One can wonder if the evolution of fast rotating pop III stars would be strongly affected by the intense mass ejections expected at break–up. Marigo et al. (2003), who considered the evolution of rotating pop III stars assuming rigid body rotation, showed that the critical conditions of intense mass loss rates can only be maintained for short times. Indeed due to the efficient removal of angular momentum by intense mass loss, the star rapidly spins down. However it remains to be checked how the inclusion in the models of the different mechanisms for the transport of the angular momentum and of the chemical species, as well as the account for the mass loss anisotropies modify this result.

In Fig. 4, evolutions of the N/C ratios normalized to their initial values at the surface of rotating and non–rotating stars are shown. When the star is non–rotating there is no change of the surface abundance during the whole MS phase until it becomes a red supergiant (RSG). Changes occur only after the first dredge–up in the RSG phase. When rotation is accounted for, the N/C ratios increase already during the MS phase. The predicted enhancements for models with average rotational velocities during the MS phase in the range of the observed values ($\sim 200 – 250 \text{ km s}^{-1}$) are in good agreement with the observations.

For a given value of the initial velocity and of the initial mass, the surface enrichment increases when the metallicity decreases (see Fig. 4). This is due to the following reasons: when the metallicity decreases the stars are more compact. The higher values of the density in the outer layers compared to more metal rich stars, lead to a decrease of the Gratton–Opick circulation (cf. Eq. 2.2). Less angular momentum is transported from the core to the outer layers, making the gradients of $\Omega$ steeper in lower metallicity stars. Steeper $\Omega$–gradients imply stronger shear mixing. Other effects also contribute: at lower metallicity, the mass loss rates are weaker and thus less angular momentum is lost by stellar winds. Also, stars being more compact, the diffusion timescale which varies as the square of the radius is
Fig. 3. Evolution of the fraction $\frac{\Omega}{\Omega_c}$ of the angular velocity to the critical angular velocity at the surface of 40 M$_\odot$ star models of different initial metallicities. On the ZAMS, the equatorial velocity of all the models is equal to 300 km s$^{-1}$.

shortened.

Stronger enrichments at lower metallicities appear to be in good agreement with the trend shown by the data of Venn and Przybilla (2004) reported in the introduction. These authors have determined the N/H ratio at the surface of A-type supergiants both in the MW and in the SMC. They obtain that the range of values observed in the SMC are much greater than at solar metallicity. In addition to the evolutionary effect just mentioned, part of the effect might also be due to the fact that stars, in metal poor regions, could be born with higher rotational velocities.

4 Impact on the massive star populations and nucleosynthesis

The observation of the number ratio of blue to red supergiants in the SMC cluster NGC 330 indicates value between 0.5 and 0.8 (see e.g. Eggenberger et al. 2002). This cluster has a mass at the turn–off around 15 M$_\odot$. Classical non–rotating models predict that at such a low metallicity nearly the whole He–burning phase occurs in the blue part of the HR diagram. Classical models predict thus ratios of the number of blue to red supergiants of the order of 50, i.e. two orders of magnitude greater than observed. It has been shown by Maeder & Meynet (2001) that rotation favours the redwards evolution. In rotating stars, more helium is present in the zone of the H–burning shell, making the H–shell less active and the associated convective zone (if any) less extended. The ratio of the time spent in
Fig. 4. Evolution of the N/C ratios (normalized to their initial value) at the surface of various 9M☉ stellar models for different initial velocities and for various initial metallicities. The long-dashed line corresponds to a non-rotating 9 M☉ model.

the blue to that spent in the red agrees with the observed number ratio of blue to RSG for stars with rotational velocities during the MS phase between 200 and 300 km s⁻¹, i.e. for values of the rotational velocities well in the observed range for these stars. Thus rotation offers an interesting solution to this long-standing problem.

Rotation has also an important impact on the population of Wolf-Rayet stars. In non-rotating models the typical surface abundance pattern of WR stars are realized during the He-burning phase when, as a result of mass loss, layers processed by the CNO cycle appear at the surface. Rotating models show that the star may enter the WR phase already during the H-burning phase, while still an important H-rich envelope surrounds the core. This may occur because of rotational diffusion which brings CNO processed material to the surface (Fliegner & Langer 1995). One can immediately deduce two consequences: firstly, for a given initial mass, rotation increases the WR lifetime. Secondly, the part of the WR lifetime during which the wind is CNO enriched, namely the WN phase, is considerably increased by rotation. These two factors enable to reproduce the observed properties of the Wolf-Rayet populations at solar metallicities, properties which cannot be accounted for by non-rotating models (Meynet & Maeder 2003). Interestingly, Prantzos & Boissier (2003) show that models with rotation much better account for the observed ratio of type Ib/Ic (believed to originate from the explosion of WR stars) to type II supernovae, at least for progenitor stars with solar metallicity. Work in progress shows that the rotating models can also account for the
properties of the WR populations at different from solar metallicities (Meynet & Maeder, in preparation).

4.1 Effects of rotation on the yields

In Table 2, stellar yields from rotating and non-rotating stellar models are compared. For the 60 M\(_\odot\) model, which goes through a WR phase, rotation increases substantially the yields of the elements which are produced by the H-burning and are preferentially ejected by the stellar winds (helium and nitrogen). As indicated above, rotation allows the star to enter into the WR phase with still an important envelope, rich in H-burning products. Thus large amounts of these elements are ejected during the WN phase. This also explains why rotating models allows greater amounts of \(^{26}\)Al to be ejected by the winds of the WR stars (Vuissoz et al. 2004). This may have some interesting consequences for the interpretation of the 1.8 MeV diffuse emission observed in the direction of young star forming regions as for instance the Cygnus region (Knödlseder et al. 2002).

The helium-burning products (carbon and oxygen) are much less affected. A look at Fig. 5, which shows the evolution of the surface abundances for a rotating and a non-rotating 60 M\(_\odot\), shows that the entrance into the WC phase, characterized by strong and rapid enhancements of carbon and oxygen, occurs at the same value of the actual mass (20 M\(_\odot\)) in both models. This explains in that particular case that the carbon and oxygen yields are little affected. One notes however that the transition from the WN to the WC phase is smoother in the rotating model, allowing the simultaneous presence at the surface of both H- (\(^{14}\)N) and He-burning products (\(^{12}\)C, \(^{16}\)O, \(^{18}\)O, \(^{19}\)F, \(^{22}\)Ne). Such stars are indeed observed (e.g. Crowther et al. 1995) and their observed frequency is well accounted for by the rotating models (Meynet & Maeder 2003). From the point of view of the yields, this transition phase is too short for making a significant difference between the yields given in Table 2 for the rotating and the non-rotating model.

As proposed by Maeder (1992) the metallicity dependance of the mass loss rates may induce a metallicity dependance of the yields of some elements like carbon. When the metallicity increases, the stronger mass loss rates allow the star to enter into the WC phase at an earlier stage of the core He-burning phase, when still a lot of helium and carbon are present in the core. These elements can then escape further destruction by being removed from the core by stellar winds. In such a scenario high metallicity massive stars may be important sources of carbon. Many authors have given support to this scenario (Prantzos et al. 1994; Gustafsson et al. 1999; Carigi 2000; Henry et al. 2000; Akerman et al. 2004).

Does rotation change this point of view? On Fig. 6 the final masses obtained from rotating models at different metallicities are plotted as a function of the initial masses. As was already the case for the non-rotating models, smaller final masses are obtained at higher metallicities where the mass loss rates are greater. Thus qualitatively, one expects that rotating models would also predict an increase of the carbon yields from massive stars when the metallicity increases. On Fig. 7, the carbon yields of two grids of rotating models at solar metallicity are compared.
Fig. 5. Evolution of the abundances (logarithm of the mass fraction) at the surface of a rotating (left panels, \( v_{\text{ini}} = 300 \, \text{km} \, \text{s}^{-1} \)) and a non-rotating (right panels) 60 M\(_{\odot}\) as a function of the actual mass of the star (which decreases as a function of time). The initial metallicity is solar.

to the carbon yields of Maeder (1992). The two rotating grids differ mainly by the mass loss rates used (in paper V we used the recipe devised by Lamers & Cassinelli 1996, while in paper X the formula proposed by Vink et al. 2000 was adopted). We see that the mass loss rates from Vink et al. (2000) lead to much higher carbon yields than the prescription by Lamers & Cassinelli (1996). The chemical evolution models of Prantzos (2003), Chiappini et al. (2003) show that using the yields of paper V would significantly decrease the importance of the high metallicity massive stars as sources of carbon. Instead the yields from the models of paper X would favour the scenario suggested by Maeder (1992). One sees from these numerical examples, that the carbon yields from massive stars are
more sensitive to the mass loss rate prescriptions than to rotation.

If, at solar metallicity, for stars with initial masses superior to about 40 $M_\odot$, the mass loss by stellar winds appears as the main factor affecting the stellar yields, for lower initial mass stars, rotation appears as the most important factor. Typically, rotational mixing increases the CO core masses by 50% for masses between 9 and 25 $M_\odot$. From Table 2, one sees that the yields in carbon and
Table 2. Stellar yields in solar masses for two models at solar metallicity. The quantities in parenthesis indicate the fraction of the yields ejected by the stellar winds.

<table>
<thead>
<tr>
<th>$M_{\text{ini}}$</th>
<th>$v_{\text{ini}}$</th>
<th>Y($^4\text{He}$)</th>
<th>Y($^{12}\text{C}$)</th>
<th>Y($^{14}\text{N}$)</th>
<th>Y($^{16}\text{O}$)</th>
<th>Y(Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0</td>
<td>7.08</td>
<td>5.14</td>
<td>0.23</td>
<td>5.76</td>
<td>12.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(100%)</td>
<td>(49%)</td>
<td>(100%)</td>
<td>(8%)</td>
<td>(26%)</td>
</tr>
<tr>
<td>300</td>
<td>12.94</td>
<td>5.69</td>
<td>0.30</td>
<td>5.73</td>
<td>13.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(100%)</td>
<td>(53%)</td>
<td>(100%)</td>
<td>(7%)</td>
<td>(29%)</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>1.70</td>
<td>0.18</td>
<td>0.05</td>
<td>0.42</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0%)</td>
<td>(0%)</td>
<td>(3%)</td>
<td>(0%)</td>
<td>(0%)</td>
</tr>
<tr>
<td>300</td>
<td>1.55</td>
<td>0.32</td>
<td>0.03</td>
<td>1.08</td>
<td>1.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(19%)</td>
<td>(45%)</td>
<td>(0%)</td>
<td>(0%)</td>
<td>(0%)</td>
</tr>
</tbody>
</table>

Oxygen are enhanced by about a factor 2. Similar enhancements are obtained at lower metallicities (Meynet & Maeder 2002).

4.2 Rotation and primary nitrogen production

The observations of the N/O ratio at the surface of metal poor stars in the solar neighborhood, in HII regions and in DLAs, show that at low metallicity, a primary source of N is required (e.g. Henry et al. 2000). Classically, it is believed that primary N is produced by intermediate mass stars during the thermal pulse AGB phase. Rotation opens a new way for primary nitrogen production at low metallicity. Carbon produced in the He–core diffuses by rotational mixing in the H–burning shell where it is transformed into primary nitrogen. Nitrogen is primary because it is formed from carbon produced by the star itself. Both massive stars and intermediate mass stars may produce primary N through this process. However the production factors of intermediate mass stars is clearly much more important than the one of massive stars (Meynet & Maeder 2002). Let us note that this scenario only works at very low metallicity, where rotational mixing is expected to be more efficient (see above) and where the distance between the H–shell and the He–core is smaller.

What is the importance of this mechanism? Prantzos (2003), Chiappini et al.(2003), Carigi & Pettini (2004) have shown on the basis of chemical evolution models for galaxies that the yields from rotating models give results which can well reproduce the observed variation of the N/O ratio as a function of the metallicity in various environments. The results are similar to those obtained with yields (van den Hoek & Groenewegen 1997) from non–rotating models accounting for the effects of the third dredge-up and the HBB in a parametric way. More quantitative assessment on the relative importance of the two mechanisms for primary N production waits for further stellar computations including both processes simultaneously.

There is however one point which is already acquired at this stage: rotation significantly increases the time delay between the release of oxygen and that of...
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nitrogen (Meynet & Pettini 2004). This time delay passes from 400 Myr when non-rotating models are used, to 700 Myr when rotation is accounted for. Can we have access to that duration from the observations? Pettini et al. (2002) have suggested a way to do it following an argument proposed by Edmunds & Pagel (1978). Let us suppose that the DLA’s have all the same star formation history, that they form continuously, then the fraction of the DLA’s showing a N/O ratio below the plateau level observed at low metallicity becomes greater when the time delay increases. From this observed fraction, Pettini et al. conclude that the duration of the time delay should amount to about 700 Myr in very good agreement with the value just obtained from the rotating models. Of course, in view of the roughness of the hypotheses such a conclusion needs to be further examined. However the line of reasoning is worthwhile to be mentioned.

Interestingly, the rotating models at $Z = 10^{-5}$ predict that when the star is on the Asymptotic Giant Branch, the abundances of carbon, nitrogen and oxygen are all very much enhanced with respect to their initial values. Typically, at the beginning of the AGB, rotating models with initial masses between 3 and 7 $M_\odot$ present surface abundances of CNO elements which are enhanced by factors between 20–2000 for C, 3000–4000 for N, and 30–100 for O. Non-rotating models at the same evolutionary stage are C and O depleted, the surface abundance being reduced by about a factor 3 for C and 1.3 for oxygen; N is enhanced by a factor $\sim 10$. Of course such models are difficult to be constrained by observations. May be a possibility would be through observations of stars similar to CS29497-030. According to Sivarani et al.(2004), the known spectroscopic binary status of this star, together with the observed s-process abundance pattern, suggest that it had accreted matter from a companion, which formerly was an AGB star. This star exhibits large overabundances of carbon ([C/Fe]$=+2.38$), nitrogen ([N/Fe]$=+1.88$), and oxygen ([O/Fe]$=+1.67$) as well as large enhancements of neutron-capture elements. Sivarani et al.(2004) notes that if the mass transfer model might well account for the large enhancement of C, N and s-process elements, the large O abundance is not easily explained (nor is the overabundance of Na). It would of course be interesting to investigate the possible role of rotation in this context since, as illustrated by the numerical example above, rotation may deeply change the abundances in the AGB phase and may also affect the s-process elements synthesis (Herwig et al. 2003).

5 Future perspectives

Rotation has a great impact on the way massive stars evolve. In that respect, the initial rotational velocity becomes, in addition to the initial mass and the initial metallicity, an important factor governing the evolution of stars. Still many questions remain to be answered:

1) Does the initial distribution of rotation velocities depends on the initial metallicity? How does the surface velocity evolve as a function of time along the MS band for different initial metallicities? Answer to this last question will con-
strain the internal transport mechanisms of angular momentum provided that the observed stars do not undergo too heavy mass loss by stellar winds (otherwise mass loss is the key factor determining the surface velocity and the effects of the transport mechanisms will be blurred). Large and extended surveys are being now performed (North et al. 2004; Royer et al. 2004). In a near future, such observational programs will be addressed with new multi-object spectrographs as GIRAFFE/FLAMES. These spectrometers on the VLT will allow simultaneous spectroscopy of 130 stars, at resolutions up to 17 km s\(^{-1}\) (http://www.eso.org/instruments/flames).

2) Recently tight constraints on the values for the mass, luminosity and effective temperatures of bump Cepheids in the LMC have been obtained (Keller & Wood 2002). Their sample stars appear to be significantly more luminous than predicted by classical stellar models that do not incorporate extension of the convective core. As seen above, rotation can produce an extension of the core as would do an overshoot and thus would bring the theoretical results in better agreement with the observations. Rotation influences also the chemical profiles inside the stars and the surface compositions. Moreover when the star evolves from the red to the blue, its surface velocity increases. All these factors may have an impact on the pulsation properties of these stars.

3) Can the rotating models account for the observed rotation rates of young pulsars? It is not easy to determine the birth spin of pulsars. Values as different as 2 ms and more than 100 ms are found in the literature (Middleditch et al. 2000; Romani & Ng 2003; Marshall et al. 1998). A typical value of 20 ms is sometimes quoted (see e.g. Lorimer 2003). The angular momentum contained in the core at the end of the evolution in the models by Meynet & Maeder (2003) is sufficient for producing young neutron star near the break-up limit (with periods of the order of the millisecond). Similar result are obtained by Heger et al. (2004) in models without magnetic fields. This may indicate that the loss of angular momentum from the central stellar region is higher than predicted by the current models and/or that instabilities occurring at the time of the neutron star formation remove angular momentum. Models accounting for the presence of a magnetic field generated by differential rotation (see below) are able to extract great quantities of angular momentum from the core. According to Heger et al. (2004), this magnetic braking can slow down the core by about an order of magnitude, predicting periods of the order of 10 ms at pulsar birth. On the other hand, models of three dimensional collapse of rotating stars by Fryer & Warren (2004) indicate that during the collapse the inner core continues to lose angular momentum, weakening thus the necessity to lose the excess of angular momentum in the previous evolutionary phases.

4) What are the evolutionary scenarios leading to the collapsar model for the GRB? For the collapsar model to work, it is necessary that the core contains a great amount of angular momentum (in contrast to the problem of reproducing the observed rotation rate of young pulsars seen above). In fact the rotating models without magnetic braking would provide favourable conditions for the collapsar model. However these models are supposed to represent the behavior of an average
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star and not of the small fraction of stars which end their life as a GRB. Typically, it is believed that about 1% of all core collapse supernovae would occur as a GRB (Woosley & Heger 2004). Thus the conditions for a GRB to be produced remain quite particular. As before, a particular evolution can have occurred during the pre-supernova stage and/or at the time of the collapse (see the interesting discussion in the above quoted paper).

5) According to Spruit (2002), differential rotation could generate a magnetic field, whose consequences in stellar interior are now being explored (Heger et al. 2004; Maeder & Meynet 2003ab; see also the contribution by Maeder in the present volume). Let us just mention that the first definite detection of a magnetic field in an O star has recently been obtained by Donati et al. (2002). They deduced a dipole field with an intensity of $1.1 \pm 0.1$ kG.

There are many other problems such as the effects of rotation during the pre-MS phase (very interesting observational constraints are provided in Wolff et al 2004), in close binary evolution (Langer et al. 2004) or on the way massive stars explode (Fryer & Warren 2004). The subject is rich of further developments which will improve our knowledge in many fields of astrophysics.

We express our thanks to Marcel Arnould for the many fruitful collaboration in the past years and to the organizers of this very interesting conference held in the honor of Marcel’s 60th birthday.

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Rotating massive stars: Pre–SN models and stellar yields at solar metallicity

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We present a new set of stellar yields obtained from rotating stellar models at solar metallicity covering the massive star range (9–120 $M_\odot$). The stellar models were calculated with the latest version of the Geneva stellar evolution code described in [1]. Evolution and nucleosynthesis are in general followed up to Silicon burning. The contributions from stellar winds and from supernova explosions to the stellar yields were calculated separately. The two contributions were then added to compute the total stellar yields [2].

The effects of rotation on pre–supernova models are significant between 15 and 30 $M_\odot$. Above 20 $M_\odot$, rotation may change the radius or colour of the supernova progenitors (blue instead of red supergiant) and the supernova type (Ibc instead of II). Rotation increases the core sizes by a factor $\sim 1.5$. Thus, rotation increases the yields for heavy elements and in particular for carbon and oxygen by a factor 1.5–2.5. Rotating models produce larger yields for $^{12}$C and $^{16}$O in the mass range between 9 and about 35 $M_\odot$ compared to the 1992 calculations [3].

For Wolf-Rayet stars ($M \gtrsim 30M_\odot$), the pre–supernova structures are mostly affected by the intensities of the stellar winds and less by rotation [4]. In this mass range, rotation increases the yields of helium and other hydrogen burning products but does not affect much the yields of elements produced in more advanced evolutionary stages. Note that the final mass of the most massive stellar models ($\sim 120 M_\odot$) are similar to the final mass of less massive stars ($\sim 40 M_\odot$) due to the use of the revised mass loss rates from Nugis and Lamers 2000 [5]. The most massive stars are therefore also expected to form black holes.

1. Introduction

Over the last ten years, the development of the Geneva stellar evolution code has allowed the study of the evolution of rotating stars until carbon burning. The models reproduce very well many observational features at various metallicities, like surface enrichments [9], ratios between red and blue supergiants [10] and the population of Wolf–Rayet (WR hereinafter) stars [4]. In [1], we describe the recent modifications brought to the Geneva code and the evolution of our rotating models until silicon burning. In this contribution, we briefly present the stellar yields for a large initial mass range (9–120 $M_\odot$) for rotating stars at solar metallicity.
2. Computer model

The computer model used to calculate the stellar models is described in details in [1]. Convective stability is determined by the Schwarzschild criterion. Convection is treated as a diffusive process from oxygen burning onwards. The overshooting parameter is 0.1 $H_P$ for $H$ and He–burning cores and 0 otherwise. On top of the meridional circulation and secular shear, an additional instability induced by rotation, dynamical shear, was introduced in the model. The reaction rates are taken from the NACRE [11] compilation for the experimental rates and from the NACRE website (http://pntpm.ulb.ac.be/nacre.htm) for the theoretical ones. The mass loss rates used are described in [4]. In particular, during the Wolf–Rayet phase, we use the mass loss rates by Nugis and Lamers 2000 [5]. These mass loss rates, which account for the clumping effects in the winds, are smaller by a factor 2–3 than the mass loss rates used in our previous non–rotating “enhanced mass loss rate” stellar grids [12].

We calculated stellar models with initial masses of 9, 12, 15, 20, 25, 40, 60, 85 and $120 \, M_\odot$ at solar metallicity, with initial rotation velocities of 0 and 300 km s$^{-1}$. The value of 300 km s$^{-1}$ corresponds to an average velocity of about 220 km s$^{-1}$ on the Main Sequence (MS) which is very close to the observed average value [13]. The calculations start at the ZAMS. The rotating 15, 20, 25, 40 and $60 \, M_\odot$ models were computed until the end of core silicon (Si) burning and their non–rotating counterparts until the end of shell Si–burning. For the rotating 12 $M_\odot$ star, the model ends after oxygen (O) burning. For the non–rotating 12 $M_\odot$ star, neon (Ne) burning starts at a fraction of solar mass away from the centre but does not reach the centre and the calculations stop there. The evolution of the models with initial masses between 12 and 60 $M_\odot$ is described in [1]. The 9, 85 and $120 \, M_\odot$ models are presented in [4] and their evolution was followed until the end of the core He–burning.

3. Results

3.1. Contributions to yields from stellar winds and SN explosions

Before we discuss the stellar yields, it is useful to recall the influence of rotation on the final mass of the different models (presented in [4,2]). Below $30 \, M_\odot$, rotating models lose significantly more mass than non–rotating models [14]. For WR stars ($M \gtrsim 30 \, M_\odot$), the new mass loss prescription [5], including the effects of clumping in the winds, results in mass loss rates that are a factor two to three smaller than the rates from [15]. As a result, the final mass of WR stars in the present calculation are noticeably larger than in 1992 [3]. There is no clear difference between the final mass of rotating and non–rotating models. For a model with an initial mass larger than $30 \, M_\odot$, the final mass is always between 11 and $17 \, M_\odot$. A similar outcome, black hole formation, is therefore expected for all the very massive stars at solar metallicity.

What is the relative importance of the wind and SN contributions? Figure 1 displays the total stellar yields divided by the initial mass of the star as a function of its initial mass, $m$, for the non–rotating (left) and rotating (right) models. The different total yields (divided by $m$) are piled up. $^4$He yields are delimited by the top solid and long dashed lines (top shaded area), $^{12}$C yields by the long dashed and short–long dashed lines, $^{16}$O yields by the short–long dashed and dotted–dashed lines and the rest of metals by the
Figure 1. Stellar yields divided by the initial mass as a function of the initial mass for the non–rotating (left) and rotating (right) models at solar metallicity.

dotted–dashed and bottom solid lines. The bottom solid line also represents the fraction of the star locked in the remnant \(M_{\text{rem}}/m\). The corresponding SN explosion type is also given. The wind contributions are superimposed to the total yields for the same elements between their bottom limit and the dotted line above it. Dotted areas help quantify the fraction of the yields due to winds. Note that for \(^4\text{He}\), the total yields are smaller than the wind yields due to negative SN yields for \(^4\text{He}\).

For \(^4\text{He}\) (and other H–burning products like \(^{14}\text{N}\)), the wind contribution increases with mass and dominates for \(M \gtrsim 22M_\odot\) for rotating stars and for \(M \gtrsim 35M_\odot\) for non–rotating stars. These mass limits correspond to the lower mass limits for WR star formation. For very massive stars, the SN contribution for \(^4\text{He}\) is negative (this is possible because, in the yield calculation, the initial composition is deducted from the final one) and this is why the wind contribution is higher than the total one. For \(^{12}\text{C}\), the wind contributions only start to be significant above the mass limits for WR star formation (22 and 35 \(M_\odot\) for rotating and non–rotating models respectively). This is expected because a star must have ejected most of its helium before it can eject carbon. Above these mass limits, the contribution from the wind and the SN are of similar importance. For \(^{16}\text{O}\), the wind contribution remains very small because with the new mass loss prescription, the oxygen rich layers are not uncovered.

3.2. Total stellar yields

Our non–rotating models were compared to the literature \([6–8]\) and are consistent with other calculations. Differences can be understood in the light of the treatment of convection and the rate used for \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) \([2]\). This verifies the accuracy of our calculations.
and gives a safe basis for studying the effects of rotation on the yields.

For H–burning products, the yields of the rotating models are usually higher than those of non–rotating models. This is due to larger cores and larger mass loss. However, between about 15 and 25 $M_\odot$, the rotating yields are lower. This is due to the fact that the winds do not expel much H–burning products yet and more of these products are burnt later in the pre–supernova evolution (giving negative SN yields). For very massive stars ($M \gtrsim 60 M_\odot$), rotating stars enter into the WR regime in the course of the MS. The long time spent in particular in the WNL phase [4] results in the ejection of large amounts of H–burning products. Rotation therefore increases the H–burning products yields in this mass range. Note that our rotating model yields for H–burning products are similar or larger than the high mass loss rates models of 1992 [3]. Concerning He–burning products, below 40 $M_\odot$, most of the $^{12}$C comes for the SN contribution. In this mass range, rotating models, having larger cores, also have larger yields (factor 1.5–2.5). For very massive stars ($M \gtrsim 60 M_\odot$), the situation is reversed for He–burning products because of the different mass loss history. As said above, rotating stars enter into the WR regime in the course of the MS. The long time spent in the WNL phase [4] results in a large mass loss. Therefore, very massive rotating stars have a small total mass early in their evolution and end up with smaller cores. Compared to 1992 [3], the $^{12}$C yields are larger in the present rotating models for masses lower than 30 $M_\odot$ and lower for masses higher than 30 $M_\odot$. Since very massive stars are much less numerous, we expect the overall $^{12}$C yield of rotating models to be larger than those of 1992 [3]. The situation for $^{16}$O and the total metallic yields is similar to carbon. Therefore $^{16}$O and metallic yields are usually larger for our rotating models than for our non–rotating ones by a factor 1.5–2.5.

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Massive rotating stars: pre–SN evolution at solar metallicity

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1. Introduction

Over the last years, the development of the Geneva evolutionary code has allowed the study of rotating star evolution from the ZAMS until the end of the core carbon burning phase. Various checks of the validity of the rotating stellar models have been made. In particular, it has been shown that rotating models well reproduce the observed surface enrichments (Heger & Langer 2000; Meynet & Maeder 2000), the ratios of blue to red supergiants in the Small Magellanic Cloud (Maeder & Meynet 2001), and the variations of the Wolf–Rayet (WR hereinafter) star populations as a function of the metallicity (Meynet & Maeder 2003, 2004).

Recently, the Geneva stellar evolution code was developed in order to model the pre–supernova evolution of rotating massive stars. Rotating and non–rotating stellar models at solar metallicity with masses equal to 12, 15, 20, 25, 40 and 60 $M_\odot$ were computed from the ZAMS until the end of the core silicon burning phase (Hirschi et al. 2004a). The corresponding stellar yields have also been calculated (Hirschi et al. 2004b).

2. Evolution of rotation and GRBs

The specific angular momentum of the core changes (decreases) most during H–burning. This is due to the long duration of H–burning, time during which the different transport mechanisms (convection, rotationally induced mixings) remove angular momentum from the core. Still some decrease occurs during the core He–burning phase, then the evolution is mostly governed by convection, which transports the angular momentum from the inner part of a convective zone to the outer part of the same convective zone. The angular momentum of the star at the end of Si–burning is essentially the same as at the end of He–burning. It means that we can estimate the pre–supernova angular momentum by looking at its value at the end of He–burning. We calculated, for the 25 $M_\odot$ model, the angular momentum of its remnant (fixing the remnant mass to 3 $M_\odot$). We obtain $L_{\text{rem}} = 2.15 \times 10^{50}$ g cm$^2$ s$^{-1}$ at the end of He–burning and $L_{\text{rem}} = 1.63 \times 10^{50}$ g cm$^2$ s$^{-1}$ at the end of Si–burning. This corresponds to a loss of only 24%. In comparison, the angular momentum is decreased by a factor ~5 between the ZAMS and the end of He–burning. This shows the importance of correctly treating the transport of angular momentum during the Main Sequence phase.
Long soft gamma–ray bursts (GRBs) have recently been connected with SNe (see Matheson 2003, for example). One scenario for GRB production is the collapsar mechanism devised by Woosley (1993). In this mechanism, a star collapses into a black hole and an accretion disk due to the high angular momentum of the core. Accretion from the disk onto the central black hole produces bi–polar jets. These jets can only reach the surface of the star (and be detected) if the star loses its hydrogen rich envelope before the collapse. WR stars are therefore good candidates for collapsar progenitors since they lose their hydrogen rich envelope during the pre–SN evolution. The question to answer is whether the core of WR stars contains enough angular momentum at the pre–SN stage (the specific angular momentum, \( j_r \), of the material just outside the core must be larger than \( 10^{16} \text{ cm}^2 \text{ s}^{-1} \)). For example, our 25 \( M_\odot \) model has enough angular momentum in its core to produce a collapsar. It also loses its hydrogen rich envelope and therefore is a possible candidate for GRB production. This confirms the results from Heger et al. (2003) if the effects of magnetic fields are small. If the effects of magnetic fields are large, the core can lose a large amount of angular momentum and cannot produce a GRB. In any case, the pre–SN angular momentum contained in the core is larger than the one observed in young pulsars and additional breaking is necessary during the collapse. The question is how and when does the breaking occur (see Heger et al. 2003).

3. Stellar yields

The effects of rotation on pre–supernova models are significant between 15 and 30 \( M_\odot \). Indeed, rotation increases the helium and C–O core sizes by a factor 1.5–2. Above 20 \( M_\odot \), rotation may change the radius or colour of the supernova progenitors (blue instead of red supergiant) and the supernova type (Ib instead of II). For very massive stars \( (M \geq 40 M_\odot) \), the pre–supernova structures are mostly affected by the intensities of the stellar winds and less by rotational mixing. The stellar yields are presented in Hirschi et al. (2004b). The wind contribution is significant for WR stars for \( ^4\text{He} \) and \( ^{12}\text{C} \). Otherwise, the SN contribution dominates. Rotation increases by a factor 1.5–2 the yields in \( ^4\text{He} \) and metals around 20 \( M_\odot \). In fact the yields of a rotating 20 \( M_\odot \) model are equivalent to those of a non–rotating 26–30 \( M_\odot \) model. For very massive stars \( (M \geq 40 M_\odot) \), rotation only increases \( ^4\text{He} \) yields.

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